O(5) × U(1) electroweak theory

C. Mukku and W. A. Sayed
International Centre for Theoretical Physics, Trieste, Italy
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An anomaly-free O(5) × U(1) theory of electroweak interactions is described which provides a unified description of electroweak phenomena for two families of standard leptons and quarks. No "new" nonsequential-type fermions are introduced, unlike the case for all past studies based on this group. The present scheme requires the introduction of two further charged and three more neutral gauge fields over and above those of SU(2) × U(1), giving rise to new neutral and charged currents.

The most economical spontaneously broken unified gauge theory of the strong and electroweak interactions,[1] based on the group SU(5), predicts what has come to be known as a desert region extending over 13 orders of magnitude in energy, where no new physics is to be expected. A prediction of this nature is to be expected if a minimal grand unified theory is constructed by straightforwardly interpreting the successes of the SU(2) × U(1) scheme as an indication of gauge unification of all forces, employing spontaneous symmetry breaking via the Higgs-Kibble mechanism. It seems more reasonable, however, to try to extend the SU(2) × U(1) theory[2] so as to understand what new features are possible in a spontaneously broken gauge theory when an extrapolation of around two to three orders of magnitude in energy is made. This leads one to investigate extended theories of electroweak phenomena and it is clear from past studies[3] along this line that one can expect several new features to arise which are not present in the SU(2) × U(1) theory and are also not indicated by the simplest grand unified theory of Georgi and Glashow. In any case, recent work[4] in the field of grand unification does seem to suggest that the desert region predicted by SU(5) may not in fact be so devoid of physics—several new interactions may be expected to make an appearance as one goes up in energy. Indeed, the earliest grand unified theory of Pati and Salam[5] incorporating ideas of lepton-quark unification also suggests the appearance of new interactions with increasing energies. In view of all the above, the Salam-Weinberg scheme may reasonably be expected to require enlargement. The success[6] of their theory in explaining electroweak phenomena simply indicating that at present energies all extended electroweak models must reproduce the results of $SU(2)_L \times U(1)$.

In the present note we shall consider an O(5) × U(1) extended electroweak model. All left-handed fermions shall be assigned to the four-dimensional spinorial representation of O(5), while all right-handed fermions will be required to be singlets under O(5). In a sense, the model we construct is intermediate between the O(4) × U(1) model of Matsuki and Okada[3] and the SU(4) × U(1) model of Deshpande, Hwa, and Mannheim[3] and differs from earlier models of Ovrut[3] and Munczek[3] based on the same group in that no new fermions are introduced in the present model. We shall break the O(5) × U(1) theory down to the standard SU(2) × U(1) model of Salam and Weinberg so that seven of the new vector bosons introduced in the extension SU(2) → O(5) acquire large masses $\sim 100 M_W$. It is one of these heavy vector bosons which allows the rare kaon decay $K_L \to \bar{\mu} e$ which allows us to determine the symmetry-breaking scale of the primary descent. Gauging the group O(5) × U(1) requires the introduction of ten gauge fields for the O(5) part and a single gauge boson for the U(1) part. As the observed leptons and quarks are to be assigned to the four-dimensional spinor representation of O(5), it will be useful to construct the corresponding 4 × 4 matrix representation for the O(5) generators. This is carried out by using a set of five anticommuting Hermitian matrices Γ_i satisifying

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \ .$$

The generators of O(5) are then given by

$$G_{ij} = \frac{1}{2i}\Gamma_i\Gamma_j, \quad i < j = 1, \ldots, 5 \ ,$$

while the Γ_i are constructed out of two independent sets of Pauli matrices σ, τ as follows:

$$\Gamma_1 = \sigma_1 \times \tau_1, \quad \Gamma_2 = \sigma_1 \times \tau_2, \quad \Gamma_3 = \sigma_3 \times 1_2 \ ,$$

$$\Gamma_4 = \sigma_1 \times \tau_3, \quad \Gamma_5 = \sigma_2 \times 1_2$$

($1_n \equiv n \times n$ identity matrix). There are just two diagonal generators, G_{12} and G_{45}. Taking the lepton and quark multiplets to be

$$l_L=\begin{pmatrix}\nu_e\\ e\\ \mu\\ \nu_\mu\end{pmatrix}_L\ ,\quad e_R,\mu_R$$

and

$$q_L=\begin{pmatrix}u\\ d_\theta\\ s_\theta\\ c\end{pmatrix}_L\ ,\quad u_R,s_R,d_R,c_R$$

fixes the charge operator Q to be

$$Q=G_{45}+\tfrac{1}{2}G_B$$

with G_B being the U(1) generator, the eigenvalue of which for any given multiplet will be denoted by Y. In an obvious notation we have

$$Y_L^l=-1,\quad Y_R^l=-2,\quad Y_L^q=\tfrac{1}{3}\ ,$$

$$Y_R^{u,c}=\tfrac{4}{3},\quad Y_R^{d,s}=-\tfrac{2}{3}$$

exactly as in the $SU(2)\times U(1)$ scheme. In the quark multiplet given above, we have employed the Cabibbo-rotated quarks

$$d_\theta=d\cos\theta_C+s\sin\theta_C,\quad s_\theta=s\cos\theta_C-d\sin\theta_C$$

which we shall introduce in the present work in the conventional manner through mixings in the quark mass matrix.

A convenient basis for the generators and the corresponding charge eigenstates of the gauge fields is given by

$$U_\mu^\pm=\frac{1}{\sqrt{2}}(W_\mu^{15}\pm iW_\mu^{14}),\quad G_U^\pm=\frac{1}{\sqrt{2}}(G_{15}\mp iG_{14})\ ,$$

$$V_\mu^\pm=\frac{1}{\sqrt{2}}(W_\mu^{24}\mp iW_\mu^{25}),\quad G_V^\pm=\frac{1}{\sqrt{2}}(G_{24}\pm iG_{25})\ ,$$

$$W_\mu^\pm=\frac{1}{\sqrt{2}}(W_\mu^{35}\pm iW_\mu^{34}),\quad G_W^\pm=\frac{1}{\sqrt{2}}(G_{35}\mp iG_{34})\ ,$$

where the W_μ^{ij} refer to the 10 gauge fields of O(5). The neutral fields corresponding to the generators G_{12}, G_{13}, G_{23}, and G_{45} are labeled $C_\mu\equiv W_\mu^{12}$, $D_\mu\equiv W_\mu^{13}$, $E_\mu\equiv W_\mu^{23}$, and $F_\mu\equiv W_\mu^{45}$, while the gauge field belonging to the U(1) part is denoted by B_μ.

In order to carry out the first stage of symmetry breaking down to $SU(2)\times U(1)$, we introduce two Higgs-scalar multiplets ϕ, η which transform as vectors under O(5). Minimization of the potential[7] allows us to take the vacuum expectation values $\langle\phi_i\rangle=v_1\delta_{i1}$ and $\langle\eta_i\rangle=\delta_{i2}v_2$, which give large masses to the three neutral vector bosons C_μ, D_μ, and E_μ and also the charged vector bosons $U_\mu^\pm$ and $V_\mu^\pm$, while leaving us with the standard $SU(2)\times U(1)$ invariance generated by $G_{W^\pm}$, G_{45}, and G_B. It is the D_μ gauge field which gives rise to the $K_L\to\bar{\mu}e$ decay which we shall use below to get a lower limit on the scale of the symmetry breaking due to the vector Higgs. The primary breaking generates the following mass terms for the gauge fields:

$$\begin{aligned}\mathcal{L}_m=&\tfrac{1}{2}g^2(v_1{}^2+v_2{}^2)C_\mu{}^2+\tfrac{1}{2}g^2v_1{}^2D_\mu{}^2+\tfrac{1}{2}g^2v_2{}^2E_\mu{}^2\\&+\tfrac{1}{2}g^2v_1{}^2(U_\mu^+U^{-\mu}+U_\mu^-U^{+\mu})\\&+\tfrac{1}{2}g^2v_2{}^2(V_\mu^+V^{-\mu}+V_\mu^-V^{+\mu})\ .\end{aligned}$$

Notice that C_μ is the most massive boson and that the D_μ boson is degenerate with $U_\mu^\pm$ while E_μ is degenerate with $V_\mu^\pm$. The values of these masses will be discussed below.

For this first stage of symmetry breaking, it will be shown elsewhere that the above scheme satisfies the criterion for minimizing the most general—at most quartic, hence renormalizable—$O(5)\times U(1)$-invariant potential for the Higgs fields. The gauge fields corresponding to the $SU(2)\times U(1)$ symmetry we are left with are $\{W_\mu^\pm,F_\mu,B_\mu\}$. These gauge fields are still massless and will only acquire mass from the secondary stage of symmetry breaking which is carried out by introducing two spinorial Higgs fields χ_i^A, $A=1,2$, transforming under the four-dimensional spinor representation of O(5) with hypercharge $Y=+1$.

The Yukawa terms for the quarks and leptons pose no problems and a suitable choice of these can be constructed to yield the desired Cabibbo mixing.

The final descent to U(1) through the spinorial Higgs yields the desired mixing between the gauge fields B_μ and F_μ which allows us to define the mass-eigenstate fields as

$$A_\mu=\frac{gB_\mu-g'F_\mu}{(g^2+g'^2)^{1/2}}\quad\text{and}\quad Z_\mu=\frac{gF_\mu+g'B_\mu}{(g^2+g'^2)^{1/2}}$$

and introduces the electroweak angle $\theta_W=\tan^{-1}(g'/g)$. These gauge fields along with the $W_\mu^\pm$ are the lightest gauge fields of our theory with the massless A_μ being identified with the photon and generate the standard theory as will be clear when we write out the fermion–gauge-field interaction a little later. The final breaking also gives the standard mass relation between the Z and $W^\pm$ gauge-boson masses $M_{W^\pm}=M_Z\cos\theta_W$. We are now able to write out the fermion–gauge-field interaction part of the Lagrangian:

$$\mathcal{L}_{\rm int}=\frac{gg'}{(g^2+g'^2)^{1/2}}A_\mu J^\mu_{\rm em}+\frac{Z_\mu}{(g^2+g'^2)^{1/2}}\left[\frac{(g^2-g'^2)}{2}(\bar e_L\gamma^\mu e_L+\bar\mu_L\gamma^\mu\mu_L)-g'^2(\bar e_R\gamma^\mu e_R+\bar\mu_R\gamma^\mu\mu_R)\right]$$

$$-\tfrac{1}{2}(g^2+g'^2)^{1/2}Z_\mu(\bar\nu_{eL}\gamma^\mu\nu_{eL}+\bar\nu_{\mu L}\gamma^\mu\nu_{\mu L}+\bar u_L\gamma^\mu u_L-\bar d_L\gamma^\mu d_L-\bar s_L\gamma^\mu s_L+\bar c_L\gamma^\mu c_L)$$

$$+\frac{g'^2}{(g^2+g'^2)^{1/2}}Z_\mu(\tfrac{2}{3}\bar u\gamma^\mu u-\tfrac{1}{3}\bar d\gamma^\mu d-\tfrac{1}{3}\bar s\gamma^\mu s+\tfrac{2}{3}\bar c\gamma^\mu c)$$

$$+\frac{g}{\sqrt2}W^+_\mu(\bar u_L\gamma^\mu d_{L\theta}+\bar c_L\gamma^\mu s_{L\theta}+\bar\nu_{eL}\gamma^\mu e_L+\bar\nu_{\mu L}\gamma^\mu\mu_L)+\text{H.c.}$$

$$+\frac{g}{2}C_\mu[\tilde c^\mu c-\tilde u^\mu u+\cos2\theta_C(\tilde s^\mu s-\tilde d^\mu d)-\sin2\theta_C(\tilde s^\mu d+\tilde d^\mu s)-\tilde\nu^\mu_e\nu_e-\tilde e^\mu e+\tilde\mu^\mu\mu+\tilde\nu^\mu_\mu\nu_\mu]$$

$$+\frac{ig}{2}D_\mu(\tilde d^\mu s-\tilde s^\mu d+\tilde c^\mu u-\tilde u^\mu c-\tilde\nu^\mu_e\nu_\mu+\tilde e^\mu\mu-\tilde\mu^\mu e+\nu^\mu_e\nu_e)$$

$$+\frac{g}{2}E_\mu[\cos2\theta_C(\tilde d^\mu s+\tilde s^\mu d)+\sin2\theta_C(\tilde s^\mu s-\tilde d^\mu d)-\tilde c^\mu u-\tilde u^\mu c-\tilde\nu^\mu_e\nu_\mu+\tilde e^\mu\mu+\tilde\mu^\mu e-\tilde\nu^\mu_\mu\nu_e]$$

$$+\frac{g}{\sqrt2}U^+_\mu(\tilde c^\mu d_\theta-\tilde u^\mu s_\theta+\tilde\nu^\mu_\mu e-\tilde\nu^\mu_e\mu)+\text{H.c.}-\frac{g}{\sqrt2}V^+_\mu(\tilde u^\mu s_\theta+\tilde c^\mu d_\theta+\tilde\nu^\mu_e\mu+\nu^\mu_\mu e)+\text{H.c.}\ ,$$

where we have used the notation $\hat f^\mu=\frac{1}{2}\bar f\gamma^\mu(1+\gamma_5)$.

It is clear from the above that we must identify $gg'/(g^2+g'^2)^{1/2}$ with e the electric charge and $G_F/\sqrt2$ with $g^2/8M_W{}^2$ in order to reproduce the $SU(2)\times U(1)$ theory. Furthermore, the interactions mediated by A_μ, $W^\pm_\mu$, and Z_μ are clearly those of the standard theory. However, at higher energies, we see that among the many new interactions that are possible we have that $K_L\to\bar\mu e$ through the direct exchange of a neutral D boson. The amplitude for this process is easily seen to be[8]

$$\text{Amp}(K_L\to\bar\mu e)=\frac{g^2}{8M_D{}^2}F_K K^\mu l_\mu\ ,$$

where F_K is the kaon decay constant and $l_\mu=\bar u_e\gamma_\mu(1-\gamma_5)v_\mu$. Now the amplitude for $K^+\to\mu^+\nu_\mu$ decay mediated by W^+_μ is known to be

$$\text{Amp}(K^+\to\mu^+\nu_\mu)=G_F\sin\theta_C F_K K^\mu l_\mu\ ,$$

where $G_F=g^2/4(2M_W{}^2)^{1/2}$. This allows us to write

$$\frac{\Gamma(K_L\to\bar\mu e)}{\Gamma(K^+\to\bar\mu\nu_\mu)}=\frac{1}{2\sin^2\theta_C}\left(\frac{M_W}{M_D}\right)^4\ .$$

Experimentally,[9] we have the upper limit that

$$\frac{\Gamma(K_L\to\bar\mu_e)}{\Gamma(K^+\to\bar\mu\nu_\mu)}<7.6\times10^{-10}\ .$$

Hence, putting in all the values, we find

$$M_D>300M_W\ .$$

We can also obtain a constraint on the mass difference of the vector bosons contributing to the K_L and K_S self-energies, namely, D_μ and E_μ. Using the interaction terms obtained after the d and s quarks are rotated through the Cabibbo angle θ_C, we find that

$$\frac{m_L-m_S}{M_K}=\frac{G_F}{\sqrt2}F_K{}^2\left(\frac{M_W{}^2}{M_E{}^2}\cos^2 2\theta_C-\frac{M_W{}^2}{M_D{}^2}\right)\ ,$$

where, experimentally, $\cos\theta_C\sim0.97$ implies that $\cos^2\theta_C\sim0.8$. This gives

$$\frac{m_L-m_S}{m_K}\simeq G_F F_K{}^2\left(0.8\frac{M_W{}^2}{M_E{}^2}-\frac{M_W{}^2}{M_D{}^2}\right)\ .$$

However, experimentally we have that

$$\frac{m_L-m_S}{m_K}=7.14\times10^{-15}\ .$$

This therefore yields the result that

$$|0.8M_E{}^{-2}-M_D{}^{-2}|\sim6\times10^{-8}M_W{}^{-2}$$

for the $(\text{mass})^{-2}$ difference between the D and E vector bosons. These two results above lead us to conclude that either the masses of the D and E bosons are almost equal and greater than $300M_W$, or that they are both extremely massive at $\sim10^4M_W$. These two results are sufficient to give limits on the masses of the gauge fields after the first stage of symmetry breaking. We have, assuming that $M_D=M_E>300M_W$,

$$M_C>600M_W,\quad M_U>300M_W,\quad\text{and}\quad M_V>300M_W\ .$$

We can also obtain other limits on the scale of the primary symmetry breaking through a study of separate-lepton-number-violating interactions such as

$$\frac{g}{\sqrt{2}} U_\mu^+ (\bar{\nu}_{\mu L} \gamma^\mu e - \bar{\nu}_{eL} \gamma^\mu \mu)$$

leading to $\mu^- \rightarrow e^- + \bar{\nu}_\mu + \nu_e$ decay. This gives us a mass limit through

$$\frac{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu)} = \left(\frac{M_V}{M_W}\right)^4 > (1.4)^4 \ .$$

or $M_U > 1.4 M_W$. Clearly this is a much less stringent limit than that obtained through $K_L \rightarrow \bar{\mu} e$ decay. It should be pointed out, of course, that we do not have total-lepton-number-violating interactions in the model.

A more complete presentation of the model and analysis of other interactions (such as $\mu \rightarrow e\gamma$ or $e^- e^+ e^-$) that can arise in this scheme together with their implications will be presented elsewhere.

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