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Z^0 decay into three gluons

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After pointing out that the two-gluon decay mode of the Z^0 vanishes, we calculate the differential and the total decay rates for $Z^0 \to ggg$. Using the standard Weinberg-Salam model and quantum chromodynamics, we find a branching ratio of 1.8×10^{-5}. We also discuss $Z^0 \to gg\gamma$ and $Z^0 \to \gamma\gamma\gamma$.

The next generation of electron-positron colliders is expected to achieve c.m. energies comparable to the mass of the weak intermediate neutral vector boson Z^0. In the standard Weinberg-Salam model, this mass is around 90 GeV/c^2 and the SLAC single-pass collider, the Cornell e^+e^- ring, and LEP are all projected to reach or exceed this energy. The purpose is to take advantage of the very large resonant cross section at $\sqrt{S} = M_Z$ and study rare decays of the Z^0.

Calculations[1-4] have been reported on several decay modes: $l\bar{l}$, $l\bar{l}\gamma$, $q\bar{q}$, $q\bar{q}\gamma$, $q\bar{q}g$, $H\gamma$, etc. In this paper we report on a new decay channel, namely, $Z^0 \to ggg$, and also discuss $Z^0 \to gg\gamma$ and $Z^0 \to \gamma\gamma\gamma$. One of the reasons why we study these processes is that the corresponding two-body decay modes $Z^0 \to \gamma\gamma$, $g\gamma$, and gg vanish: the first by Yang's theorem,[5] the second by color conservation, and the third because the two gluons have to carry the same color $[\mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}]$, and, therefore, Yang's theorem again applies.

Our calculations are based on the standard Weinberg-Salam model and quantum chromodynamics. Furthermore, to simplify our results, we will consider only the limit of small quark masses, i.e., $m_q/M_Z \to 0$. The Feynman diagrams can be divided into two sets: box diagrams [Fig. 1(a)] and triangle diagrams [Fig. 1(b)]. One must, of course, sum over color as well as flavor in the quark loops of Fig. 1. Then we find that the triangle diagrams sum up to zero, because each diagram is proportional to the axial-vector coupling b^i of the Z^0 to $q_i\bar{q}_i$, and, with $b^i = I_3^i$ in the standard model, the sum within each SU(2) doublet vanishes ($b^u = -b^d = b^c = -b^s = b^t = -b^d = \frac{1}{2}$). Clearly, this argument holds even for massive quarks as long as the members of each doublet have the same mass.

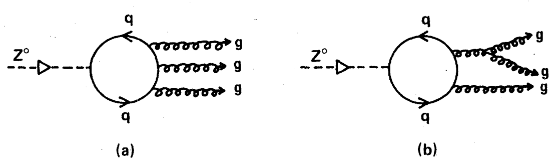

FIG. 1. Feynman diagrams for the decay $Z^0 \to ggg$.

We are left with the box diagrams which contain both vector and axial-vector couplings. The $VVVV$ ($AVVV$) diagrams involve the symmetric (antisymmetric) part d_{abc} (f_{abc}) of the trace over the color matrices: $\mathrm{Tr}(T_a T_b T_c) = \frac{1}{4}(d_{abc} + if_{abc})$. Again the sum over quark flavors eliminates the $AVVV$ box diagrams by the above argument, and, therefore, in the limit of equal masses within each doublet the decay $Z^0 \to ggg$ is proportional only to the vector couplings a^i, where $a^u = a^c = a^t = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$ and $a^d = a^s = a^b = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$ in the standard model.

Since we need the box diagram with only one massive external leg, we start with the expressions given by Costantini, De Tollis, and Pistoni[6] for photon splitting and take the limit of vanishing fermion mass. Though separate parts of the amplitude contain divergences, the final answer is free of any mass singularity, and can be written as a function of the dimensionless variables $x = 2E_a/M_Z$, $y = 2E_b/M_Z$, and $z = 2E_c/M_Z$, where the E's refer to the gluon energies in the Z^0 rest frame. Only two are independent, since $x + y + z = 2$. We also found, to our surprise, that the imaginary parts add up to zero: There are 24 helicity amplitudes and they are all real. Later we will comment on a possible connection between the absence of mass singularities and the absence of the imaginary part in each amplitude.

After averaging/summing over the helicities, the totally differential decay rate can be written as

$$\frac{1}{\Gamma_0}\frac{d^2\Gamma(Z^0 \to ggg)}{dx\,dy} = \frac{1}{256}\left(\frac{\alpha_s}{\pi}\right)^3 C^{ggg} \frac{\left(\sum_i a_i\right)^2}{\sum_i (a_i^2 + b_i^2)} \frac{d^2F}{dx\,dy}, \tag{1}$$

where Γ_0 is the "total" hadronic decay width

$$\Gamma_0 = \sum_i \Gamma(Z^0 \to q_i\bar{q}_i) = \frac{\sqrt{2}M_Z^3 G_F}{4\pi} \sum_i (a_i^2 + b_i^2), \tag{2}$$

and $C^{ggg}=\frac{10}{3}$ is a group factor obtained by summing over the gluon colors. The function $d^2F/dx\,dy$ is given by

$$\frac{d^2F}{dx\,dy}=\frac{16}{3}\left\{\frac{y(1-y)}{x(1-x)}[E^{(1)}(x,y,z)]^2+\frac{z(1-z)}{x(1-x)}[E^{(1)}(x,z,y)]^2 -\frac{2(1-y)(1-z)}{x(1-x)}E^{(1)}(x,y,z)E^{(1)}(x,z,y)+[E^{(2)}(x,y,z)]^2\right\}+(x\leftrightarrow y)+(x\leftrightarrow z)+\tfrac{64}{3}\ , \quad (3)$$

where

$$E^{(1)}(x,y,z)=2\left(\frac{1-z}{y}\right)+\left[3-\frac{1}{y}+2\left(\frac{1-y}{1-x}\right)-2\left(\frac{1-x}{y^2}\right)\right]\ln(1-y) +\left[-1+\frac{1}{z}+2\left(\frac{1-z}{1-x}\right)\right]\ln(1-z)+\left(\frac{y-z}{1-x}+2\frac{(1-y)(1-z)}{(1-x)^2}\right)G(y,z) \quad (4a)$$

and

$$E^{(2)}(x,y,z)=\left[1-\frac{1}{y}+2\left(\frac{1-y}{1-x}\right)\right]\ln(1-y)+\left[1-\frac{1}{z}+2\left(\frac{1-z}{1-x}\right)\right]\ln(1-z)+\left(\frac{x}{1-x}+2\frac{(1-y)(1-z)}{(1-x)^2}\right)G(y,z)\ . \quad (4b)$$

The function G appearing in Eqs. (4a) and (4b) is

$$G(y,z)=\ln(1-y)\ln(1-z)+\mathrm{Li}_2(y)+\mathrm{Li}_2(z)-\pi^2/6\ , \quad (5)$$

where $\mathrm{Li}_2(x)=-\int_0^x(dt/t)\ln(1-t)$. An important property of G used to prove the infrared finiteness of the amplitude (see below) is $G(y,1-y)=0$. In our numerical work we found it necessary to calculate the dilogarithms very accurately[7] to obtain this cancellation.

The function $d^2F/dx\,dy$ is clearly symmetric under $x\leftrightarrow y$, $x\leftrightarrow z$, or $y\leftrightarrow z$. In Fig. 2 we show it as a function of x for several values of y. The range shown is $1.01\leq x+y\leq 1.98$, or, equivalently, $0.02\leq z\leq 0.99$. We characterize the divergence near $z\to 0$ as infrared and near $z\to 1$ as collinear. Both are logarithmic and integrable, i.e., $d^2F/dx\,dy\to\ln^2 z$ and $d^2F/dx\,dy\to\ln^2(1-z)$ as $z\to 0$ and $z\to 1$, respectively.

Leaving the details of our calculation to be reported elsewhere, we point out that there are several delicate cancellations which serve as a check of our calculation. The absence of an infrared $1/z$ divergence is expected because there is no bremsstrahlung from external legs. This is particularly welcome, and in fact necessary, since had there been an infrared divergence, then the usual mechanism to cancel it would not be available—the usual mechanism being the inclusion of virtual corrections to $Z^0\to gg$ which, as we stated earlier, vanishes.

Though not necessary, we introduce a cutoff parameter ϵ in the standard manner[8] of treating three-jet events in e^+e^- collisions. Experimental cuts will require that each gluon jet carry a minimum energy of $E\geq\epsilon M_Z$ for some $0<\epsilon<\frac{1}{3}$. Following Ref. 8 (see also Ref. 4) we introduce this cut symmetrically to calculate

$$\frac{dF}{dx}=\int_{1-x+\epsilon}^{1-\epsilon}dy\frac{d^2F}{dx\,dy}$$

and

$$F(\epsilon)=\int_{2\epsilon}^{1-\epsilon}dx\frac{dF}{dx}\ .$$

The upper limits guarantee that the opening angle θ

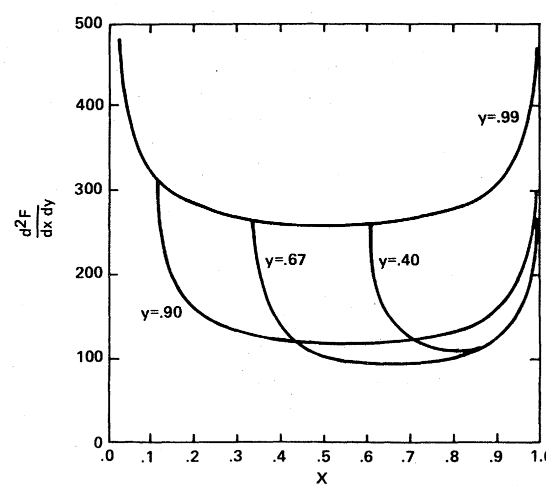

FIG. 2. The double-differential decay spectrum $d^2F/dx\,dy$ as a function of x for several values of y in the physical region $1.01\leq x+y\leq 1.98$.

between any two gluons satisfies $\sin(\theta/2) \geq 2\sqrt{\epsilon}/(1+\epsilon)$.

In Fig. 3 we show dF/dx for several values of ϵ, and in Fig. 4 we plot $F(\epsilon)$. Except at $x=1$, dF/dx and F are finite for $\epsilon=0$. One can extract the value of $F(0)$ by using $F(0)-F(\epsilon)=128[1+\zeta(2)-2\zeta(3)]\epsilon\ln^2\epsilon$ and by plotting $F(\epsilon)$ as a function of $\epsilon\ln^2\epsilon$; the intercept of the resulting straight line is $F(0)$. We find $F(0)\simeq 80$. The value of the constant multiplying $d^2F/dx\,dy$ in Eq. (2) is 2.3×10^{-7} in the standard model with three quark doublets,[9] and, therefore, the branching ratio $\Gamma(Z^0\to ggg)/\Gamma_0 = 1.8\times10^{-5}$.

The process $Z^0\to gg\gamma$. For this decay mode we need only change the value of the constant multiplying $d^2F/dx\,dy$ in Eq. (2). It becomes

$$\frac{1}{256}\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\alpha}{\pi}\right)C^{gg\gamma}\frac{\left(\sum_i a_i q_i\right)^2}{\sum_i (a_i^2+b_i^2)} \approx 6.1\times10^{-8} ,$$

where q_i is the electric charge of quark i and the color factor $C^{gg\gamma}=8$. We obtain a relatively large branching ratio $\Gamma(Z^0\to gg\gamma)/\Gamma_0 = 4.9\times10^{-6}$. This is an interesting process because all three interactions, weak, strong, and electromagnetic, are involved.

The process $Z^0\to\gamma\gamma\gamma$. This decay channel has a very small branching ratio as expected. The constant factor in Eq. (2) becomes

$$\frac{1}{256}\left(\frac{\alpha}{\pi}\right)^3\frac{4\left(3\sum_i a_i q_i^3 + \sum_{\text{leptons}} a_i q_i^3\right)^2}{\sum_i (a_i^2+b_i^2)} \simeq 5.8\times10^{-11} ,$$

where we have included both quark loops and lepton loops. In addition, a third class of diagrams must be added in this case, namely, W-loop diagrams which contribute coherently to the $Z^0\to\gamma\gamma\gamma$ amplitude. We have not calculated those diagrams, but we see no reason to suspect that W-loop contributions are much larger than fermion-loop contributions. Our estimate, based on fermion loops and including the statistical factor 1/3!, is $\Gamma(Z^0\to\gamma\gamma\gamma)/\Gamma_0 \simeq 7.7\times10^{-10}$

Remarks. (a) With the flourishing of jet physics, particularly in e^+e^- collisions, it is hoped that it will be possible to distinguish gluon jets from quark or antiquark jets.[10] Clearly such distinction will be very helpful in separating the ggg from the more common $q\bar{q}g$ final state, but the task remains difficult because of the small branching ratio. It is, however, quite feasible since the proposed Z^0 factories are expected to produce several Z^0 bosons every second. This gives, using our result for the branching ratio, a few $Z^0\to ggg$ events per day, which should be sufficient for experimental observation.

(b) We have no physical explanation for the vanishing of the imaginary part of the amplitude for $Z^0\to ggg$, but feel that it is connected with the ab-

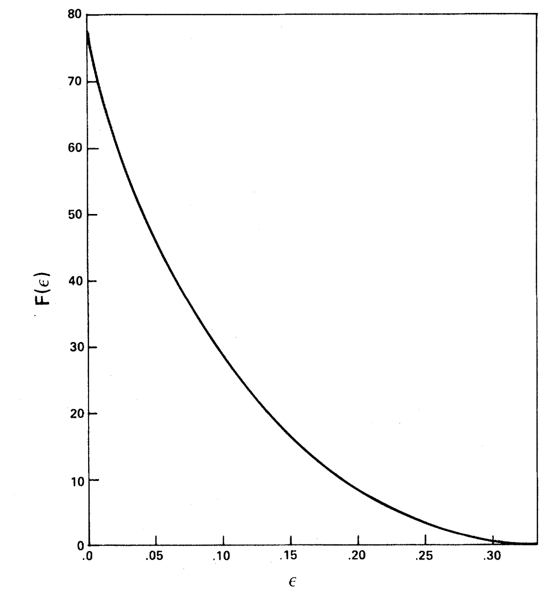

FIG. 4. The function $F(\epsilon)=\int_{2\epsilon}^{1-\epsilon}dx\int_{1-x+\epsilon}^{1-\epsilon}dy\, d^2F/dx\,dy$. $F(\frac{1}{3})=0$ and $F(0)\simeq 80$.

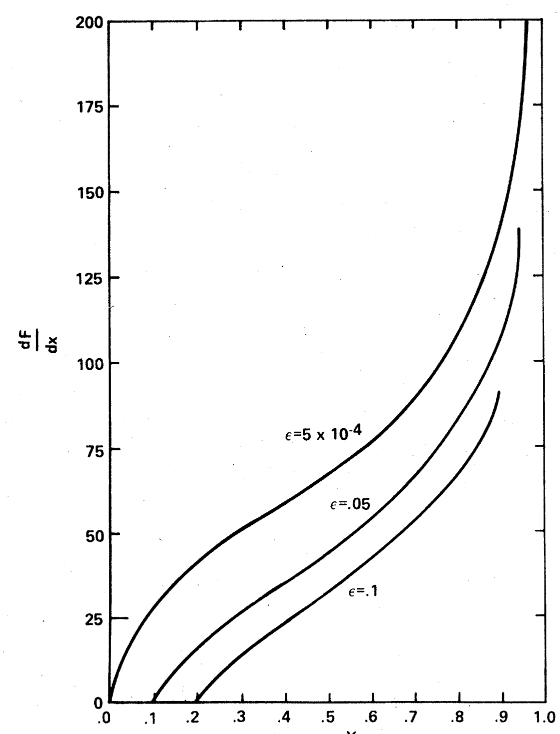

FIG. 3. The single-differential decay spectrum dF/dx as a function of x for three values of the cutoff parameter ϵ.

sence of mass singularities in each helicity amplitude. This conjecture is based on the relationship between mass divergences and imaginary parts of certain diagrams as pointed out by Fabricius and Schmitt.[11]

(c) Finally, we point out that the pure three-gluon state can be produced also in e^+e^- collisions through the decay of a virtual photon even before the Z^0 resonance is reached, and is, therefore, accessible in the energy region presently covered by PEP and PETRA. Calculations on $e^+e^- \rightarrow ggg$ are in progress and the results will be reported elsewhere.

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