Instability of colored baryons in quark potential models

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(Received 5 February 1981)

Evidence is presented, using reasonable potential models, that the three-quark system with color-octet symmetry has the same instability as the corresponding quark-antiquark system. The "falling-apart" nature of color-octet baryons occurs only when the basis set includes all allowed orbital symmetries.

Simple quark potential models based on quantum chromodynamics have been remarkably successful[1-5] in describing the internal properties of baryons and mesons. In general such potential models invoke a color-dependent interaction between quark pairs of the form

$$V_{ij}=\vec{F}_i\cdot\vec{F}_j v_{ij}\,, \qquad (1)$$

wherein $\vec{F}$ is an eight-component SU(3)-color vector and

$$v_{ij}\underset{r_{ij}\to\infty}{\sim}-|k|r_{ij}{}^{n} \qquad (2)$$

with $n>0$ providing a confinement mechanism for antisymmetric color pair states and an "anticonfinement" for all other color pair states.

At first sight the above color dependence of V_{ij} would suggest that only color-singlet states can be stable. For quark-antiquark ($q\bar{q}$) color-octet configurations this is clearly the case because $\vec{F}_i\cdot\vec{F}_j$ has a positive eigenvalue and the states "fall apart" due to the infinite "attraction" at infinite separations. For qqq baryons the situation is not as simple because the octet colored states (Young tableau [21]) involve a definite mixture of color-symmetric pairs and color-antisymmetric pairs. Recently it was stated[6] without proof that the color-octet baryon states do not exist as discrete eigenstates of the Hamiltonian. The purpose of this work is to provide detailed evidence that q^3 color-octet baryons fall apart in an analogous manner to $q\bar{q}$ color-octet mesons.

In this communication we show the results obtained using a quark potential model which gives quantitative agreement with both the meson[2] and baryon[3] spectra. The Hamiltonian for particles of mass m_i and momentum p_i is

$$H=\sum_i (p_i{}^2+m_i{}^2)^{1/2}+\sum_{i>j}\vec{F}_i\cdot\vec{F}_j v_{ij} \qquad (3)$$

with the constraint $\sum_i\vec{p}_i=0$. The interaction v_{ij} is discussed in detail in Ref. 2 and includes a linear confinement plus a short-range one-gluon-exchange potential with a strong but finite-range spin-spin term. The three-quark Hamiltonian is evaluated by expanding the orbital solutions in terms of a six-dimensional harmonic-oscillator basis, which is then used in conjunction with the SU(3)-color and SU(6)-flavor-spin subspaces in the correct combinations so that overall antisymmetry is achieved for each basis state. Eigenvalues are obtained by varying the oscillator frequency and increasing the total oscillator quanta in the basis set until the solution is obtained to a desired accuracy. Full details of the numerical method, which was also used in Ref. 3, are lengthy and will be presented in a later report.

Possible totally antisymmetric states which have color-singlet and color-octet nature are given in Table I. In the case of color-singlet baryons the only possible mixing between different orbital symmetries is through the spin-dependent parts of the interaction.[7] For color-octet baryons the situation is quite different. The nature of the states is such that terms in the force which do not mix flavor-spin possess matrix elements which mix different orbital symmetries. Thus a long-range potential will mix different orbital symmetries at large distances and lead to wave functions which differ greatly from those found in color-singlet baryons.

The quark-quark potential of Ref. 2 has been solved using wave functions with color-octet symmetry and the quantum numbers of N ($I=\frac{1}{2}$, $Y=1, J^\pi=\frac{1}{2}^+$), N^* ($I=\frac{1}{2}, Y=1, J^\pi=\frac{1}{2}^-$), and Δ ($I=\frac{3}{2}, Y=1, J^\pi, \frac{3}{2}^+$). Resulting masses as a function of the total basis oscillator quanta (N) are shown by the solid lines in Fig. 1. The corresponding color-singlet masses are shown by the dashed lines in Fig. 1. For all baryons shown the basis frequency was chosen as that for which the color-singlet masses minimized most rapidly. Clearly the color-octet solution behaves quite differently from the color-singlet solution. With a basis including up to $10\hbar$ quanta the color-singlet masses are relatively insensitive to variations in the oscillator frequency, although a definite minimum is obtained for an oscillator frequency $\nu=0.5$ GeV^2. Calculations of the color-singlet

TABLE I. Totally antisymmetric states of singlet- or octet-color symmetry possible in a three-quark system. Definite symmetries are denoted by [3] (symmetric), [21±] [mixed symmetry which is either symmetric (+) or antisymmetric (–) in particles one and two], and [1³] (antisymmetric). The subscripts c, f, and o refer to color, flavor-spin, and orbital spaces.

Color-singlet states	
(1)	$[1^3]_c \otimes [3]_f \otimes [3]_o$
(2)	$[1^3]_c \otimes [1^3]_f \otimes [1^3]_o$
(3)	$(\frac{1}{2})^{1/2}[1^3]_c \otimes \{[21+]_f \otimes [21+]_o + [21-]_f \otimes [21-]_o\}$
Color-octet states	
(1)	$(\frac{1}{2})^{1/2}\{[21+]_c \otimes [21-]_f \otimes [3]_o - [21-]_c \otimes [21+]_f \otimes [3]_o\}$
(2)	$(\frac{1}{2})^{1/2}\{[21+]_c \otimes [3]_f \otimes [21-]_o - [21-]_c \otimes [3]_f \otimes [21+]_o\}$
(3)	$(\frac{1}{2})^{1/2}\{[21+]_c \otimes [21+]_f \otimes [1^3]_o + [21-]_c \otimes [21-]_f \otimes [1^3]_o\}$
(4)	$(\frac{1}{2})^{1/2}\{[21+]_c \otimes [1^3]_f \otimes [21+]_o + [21-]_c \otimes [1^3]_f \otimes [21-]_o\}$
(5)	$\frac{1}{2}\{[21+]_c \otimes ([21-]_f \otimes [21+]_o + [21+]_f \otimes [21-]_o) - [21-]_c \otimes ([21-]_f \otimes [21-]_o - [21+]_f \otimes [21+]_o)\}$

masses with a basis including up to $16\hbar$ quanta changed the eigenvalues of low-lying states by only a few MeV which implies that the $10\hbar$ calculations are essentially converged to an accurate color-singlet state solution. On the other hand, the color-octet baryon masses show no indication of converging to a stable energy value. Our calculations for color-octet states were computationally limited to a maximum of $N = 10$; nevertheless, it is clear that the masses of color-octet states are approaching a linearly decreasing value. Variation of the basis frequency ν does *not* yield a minimum in the color-octet masses; instead we find they continuously decrease as the frequency ν is decreased.

We conclude that the color-octet spectrum as $N \to \infty$ eventually turns into a continuum with the lowest energy approaching $-\infty$. Such a continuum is analogous to the color-octet-meson spectrum wherein all eigenstates occupy an infinite spatial region. This falling apart of octet baryons occurs because the long-range part of v_{ij} can mix orbital symmetries at arbitrarily large distances, thereby allowing one quark to escape to infinity relative to the remaining quark pair which is bound in an antisymmetric color state $[1^2]$. The same color configurations occur in the octet meson states because the antiquark carries color $\bar{3}$ which is represented by the Young tableau $[1^2]$.

In order to clarify the role of orbital symmetry mixing as the origin of the instability of color-octet baryons we also performed calculations with a simpler Hamiltonian:

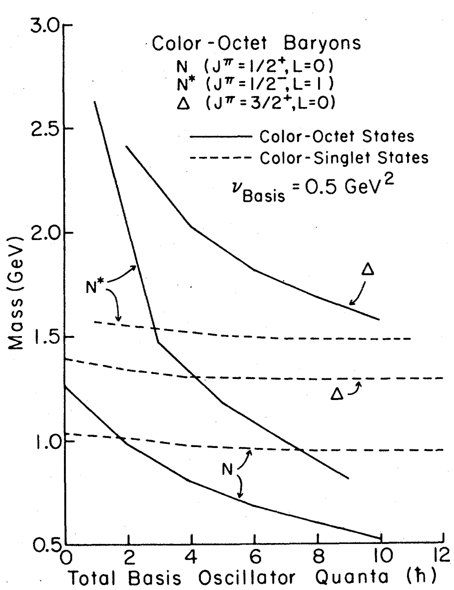

FIG. 1. Masses of color-singlet and color-octet baryons with the quantum numbers of N, N^*, and Δ, calculated as functions of the total basis oscillator quanta. The Hamiltonian used [Eq. (3)] was one found to reproduce the observed color-singlet meson (Ref. 2) and baryon (Ref. 3) spectra.

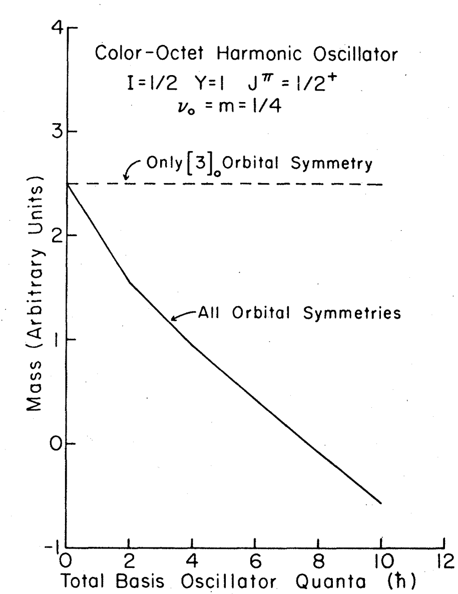

FIG. 2. Mass of a color-octet system with the quantum numbers of N as a function of the total basis oscillator quanta. The Hamiltonian used [Eq. (5)] assumes a harmonic confinement. The solid line was calculated with all possible orbital symmetries. The dashed line was calculated using only states which are completely symmetric in orbital space.

$$H_0 = \sum_{i=1}^{3} \frac{p_i^2}{2m} - \frac{\nu_0^2}{6m} \sum_{i>j} \vec{F}_i \cdot \vec{F}_j r^2_{ij} \quad (4)$$

corresponding to a harmonic confinement, which is often adopted by other workers for numerical convenience. For convenience we choose $\nu_0 = m = \frac{1}{4}$ in arbitrary units. Figure 2 shows the results of solving this Hamiltonian when all color-octet states are included and when only those states which have totally symmetric orbital states are included.

When the orbital symmetry is restricted to be fully symmetric the color-octet eigenvalues converge to finite values. The stability of the system when total orbital symmetry is artificially maintained is due to the fact that the three-quark system cannot isolate one quark from the other two if the spatial wave function is constrained to be totally symmetric. Removing this restriction again yields instability as indicated in Fig. 2 by the rapid linear fall of the eigenvalue as N gets larger.

Finally we note that calculations of quark dynamics with potential models require that care must be taken to include all symmetries in all subspaces which can contribute to a given eigenvector. Besides the example given in the above discussion for colored baryons we also noted that spin-dependent terms can mix different orbital symmetries for color-singlet states. This is expected to be important in the nucleon-nucleon potential generated from quark-quark interactions which describe the physical masses of mesons and baryons. Preliminary work by us on this latter mixing effect shows that there are indeed significant effects from this source.

This work was supported in part by the U. S. Department of Energy.

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