Generation of the D^* meson in the N/D formalism

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(Received 20 August 1980)

The generation of the D^* meson due to the exchanges of ρ and D^* in the crossed channels is studied in an N/D formalism; the channels considered are πD, ηD, and $\eta' D$. The D^* parameters calculated in this way are in reasonable agreement with theoretical expectations.

I. INTRODUCTION

The bootstrap mechanism came up in the sixties, which helped a great deal in ascertaining the dynamical properties of strongly interacting particles, the basic concept[1] being that all such particles are bound states or resonances of one another and are held by forces generated by the cross channels. First calculations were done[2] in this spirit, for the $\pi\pi \to \pi\pi$ interaction, from the requirement that if a ρ particle actually existed, it would generate itself by a bootstrap mechanism, by producing the necessary force between the pions. Attempts to calculate the K^* parameters in a similar way were done by a number of authors.[3,4] In a determinantal approach, Diu *et al.*[3] estimated the K^* mass and width from one-particle-exchange forces and obtained results which were in reasonable agreement with experiment. For heavier mesons, not many calculations exist; however, some time ago, Campbell[5] considered an SU(4) bootstrap model to study the mass spectrum of heavy particles in a single-baryon-exchange program. The results obtained at this level were quite encouraging.

Recently, we have considered[6] the generation of D^*, the charmed $J=1$, $I=\frac{1}{2}$ vector meson, in N/D formalism, by the exchange of a ρ particle in the cross channel. It was found that such a generation of D^* depended on the relative sign of $\rho\pi\pi$ and $\rho D\bar{D}$ coupling as was in the K^* case. The width of D^* turned out to be ~ 56 keV, which is well within the present experimental upper limit, while the magnitude of $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ came out to be smaller than that of $g_{\rho\pi\pi}g_{\rho K\bar{K}}/4\pi$ by about a factor of 2.

In this paper, we take up a more generalized treatment for the generation of the D^*, by considering other appropriate channels, and observe how their effects modify our earlier results. Accordingly, we consider the following channels: πD, ηD, and $\eta' D$, which we label as (1), (2), and (3), respectively. In Sec. II, we give the formalism and write the relevant N/D equations. Section III deals with the domain of cuts, and numerical results are presented in Sec. IV. A brief discussion of our work is given in the concluding Sec. V.

II. FORMALISM

Defining the partial-wave scattering amplitude t by

$$A_{ij}=\delta_{ij}+2i(p_i p_j)^{3/2}\frac{1}{\sqrt{s}}t_{ij}\,, \tag{1}$$

where s is the total energy squared and p_k the momentum in the center-of-mass system of channel k, we express the amplitude, following Zachariasen and Zemach's N/D determinantal bootstrap approach[7] as

$$t=ND^{-1}\,, \tag{2}$$

$$N=t_L\,, \tag{3}$$

$$D_{ij}=\delta_{ij}-\frac{s-s_0}{\pi}\int_{s_i}^{\infty}\frac{p_i^3(s')}{\sqrt{s'}}\frac{[t_L(s')]_{ij}ds'}{(s'-s)(s'-s_0)} \tag{4}$$

where s_0 is the subtraction point, and s_i are the threshold points given by

$$s_1=(m_\pi+m_D)^2\,,$$
$$s_2=(m_D+m_\eta)^2\,,$$
$$s_3=(m_D+m_{\eta'})^2\,.$$

As shown by Bjorken,[8] we generalize the N/D method to matrix amplitudes and thus treat several channels at one time. Projecting the results on $J=1$, $I=\frac{1}{2}$ channel, and noting that in $\pi D\to\pi D$, crossing leads to $\pi D\to\pi D$ as well as $\pi\pi\to D\bar{D}$, so that one can have exchange of the D^* and the ρ, respectively, in these channels, while in $\pi D\to\eta D$, $\pi D\to\eta' D$, $\eta D\to\eta D$ and $\eta' D\to\eta' D$ processes, only one-particle intermediate state, viz., the D^* is allowed, we write the following matrix:

$$(t_L)_{11}=\frac{1}{p_1^2}\left[\frac{g_{D^*D\pi}^2}{4\pi}F(\beta_{11}^{D^*},\alpha_{11}^{D^*})+\frac{g_{\rho\pi\pi}g_{\rho D\bar{D}}}{4\pi}F(\beta_{11}^{\rho},\alpha_{11}^{\rho})\right],$$

$$(t_L)_{22}=-\frac{1}{p_2^2}\frac{g_{D^*D\eta}^2}{4\pi}F(\beta_{22},\alpha_{22})\frac{1}{\cos^2\theta},$$

$$(t_L)_{33}=-\frac{1}{p_3^2}\frac{g_{D^*D\eta'}^2}{4\pi}F(\beta_{33},\alpha_{33})\frac{1}{\sin^2\theta},\tag{5}$$

$$(t_L)_{12}=(t_L)_{21}=-\frac{\sqrt{3}}{p_1p_2}\frac{g_{D^*D\pi}g_{D^*D\eta}}{4\pi}F(\beta_{12},\alpha_{12})\frac{1}{\cos\theta},$$

$$(t_L)_{13}=(t_L)_{31}=-\frac{\sqrt{3}}{p_1p_3}\frac{g_{D^*D\pi}g_{D^*D\eta'}}{4\pi}F(\beta_{13},\alpha_{13})\frac{1}{\sin\theta},$$

$$(t_L)_{23}=(t_L)_{32}=-\frac{\sqrt{3}}{p_2p_3}\frac{g_{D^*D\eta}g_{D^*D\eta'}}{4\pi}F(\beta_{23},\alpha_{23})\frac{1}{\sin\theta\cos\theta},$$

where the g's are the strong VPP coupling constants, defined by the Hamiltonian $\mathcal{H}$ as[9,10]

$$\mathcal{H}=\left[g_{\rho\pi\pi}\vec{\pi}\times\partial_\mu\vec{\pi}+ig_{\rho D\bar{D}}\left(\partial_\mu D\frac{\vec{\tau}}{2}D-D\frac{\vec{\tau}}{2}\partial_\mu D\right)\right]\cdot\vec{\rho}_\mu$$
$$+g_{D^*D\pi}D_\mu^*\vec{\tau}\cdot(D\partial_\mu\vec{\pi}-\vec{\pi}\partial_\mu D)$$
$$+g_{D^*D\eta}D_\mu^*(D\partial_\mu\eta-\eta\partial_\mu D)$$
$$+g_{D^*D\eta'}D_\mu^*(D\partial_\mu\eta'-\eta'\partial_\mu D)+\text{H.c.}\tag{6}$$

Here $g_{\rho\pi\pi}$ can be extracted from the experimental[11] ρ width as

$$\frac{g_{\rho\pi\pi}^2}{4\pi}=\frac{12\Gamma_\rho}{(1-4m_\pi^2/m_\rho^2)^{3/2}m_\rho}\approx 2.9,\tag{7}$$

with p_1, p_2, p_3, the center-of-mass momenta given by

$$p_1^2=\frac{1}{4s}[s-(m_D+m_\pi)^2][s-(m_D-m_\pi)^2],$$
$$p_2^2=\frac{1}{4s}[s-(m_D+m_\eta)^2][s-(m_D-m_\eta)^2],\tag{8}$$
$$p_3^2=\frac{1}{4s}[s-(m_D+m_{\eta'})^2][s-(m_D-m_{\eta'})^2],$$

θ, the η,η' mixing angle[10] defined as

$$|\eta\rangle=\cos\theta\,|8\rangle-\sin\theta\,|0\rangle,$$
$$|\eta'\rangle=\sin\theta\,|8\rangle+\cos\theta\,|0\rangle,\tag{9}$$

$$F(\beta,\alpha)=\tfrac{1}{4}(\alpha+\beta)\left(2-\beta\,\mathrm{Ln}\frac{\beta+1}{\beta-1}\right)\tag{10}$$

and the α's and β's are

$$\alpha_{11}^{\rho}=1+\frac{2\omega_{D1}\omega_\pi}{p_1^2},\quad \beta_{11}^{\rho}=1+\frac{m_\rho^2}{2p_1^2},\quad \alpha_{11}^{D^*}=1+\frac{s}{2p_1^2}-\frac{(m_D^2-m_\pi^2)^2}{2p_1^2m_{D^*}^2},\quad \beta_{11}^{D^*}=1+\frac{m_{D^*}^2}{2p_1^2}-\frac{(\omega_{D1}-\omega_\pi)^2}{2p_1^2},$$

$$\alpha_{22}=1+\frac{1}{2p_2^2}\left[s-\frac{(m_D^2-m_\eta^2)^2}{m_{D^*}^2}\right],\quad \beta_{22}=1+\frac{1}{2p_2^2}[m_{D^*}^2-(\omega_\eta-\omega_{D2})^2],$$

$$\alpha_{33}=1+\frac{1}{2p_3^2}\left[s-\frac{(m_D^2-m_{\eta'}^2)^2}{m_{D^*}^2}\right],\quad \beta_{33}=1+\frac{1}{2p_3^2}[m_{D^*}^2-(\omega_{\eta'}-\omega_{D3})^2],$$

$$\alpha_{12}=\frac{1}{2p_1p_2}\left[p_1^2+p_2^2+(\omega_{D2}+\omega_\pi)(\omega_{D1}+m_\eta)-\frac{(m_D^2-m_\pi^2)(m_D^2-m_\eta^2)}{m_{D^*}^2}\right],$$

$$\beta_{12}=\frac{1}{2p_1p_2}[p_1^2+p_2^2+m_{D^*}^2-(\omega_{D2}-\omega_\pi)^2],\tag{11}$$

$$\alpha_{13}=\frac{1}{2p_1p_3}\left[p_1^2+p_3^2+(\omega_{D3}+\omega_\pi)(\omega_{D1}+\omega_{\eta'})-\frac{(m_D^2-m_\pi^2)(m_D^2-m_{\eta'}^2)}{m_{D^*}^2}\right],$$

$$\beta_{13}=\frac{1}{2p_1p_3}[p_1^2+p_3^2+m_{D^*}^2-(\omega_{D3}-\omega_\pi)^2],$$

$$\alpha_{23}=\frac{1}{2p_2p_3}\left[p_2^2+p_3^2+(\omega_{D3}+\omega_\eta)(\omega_{D2}+\omega_{\eta'})-\frac{(m_D^2-m_\eta^2)(m_D^2-m_{\eta'}^2)}{m_{D^*}^2}\right],$$

$$\beta_{23}=\frac{1}{2p_2p_3}\left[p_2^2+p_3^2+m_{D^*}^2-(\omega_{D3}-\omega_\eta)^2\right],$$

with

$$\omega_{Di}=(p_i^2+m_D^2)^{1/2},\quad i=1,2,3,\quad \omega_\pi=(p_1^2+m_\pi^2)^{1/2},\quad \omega_\eta=(p_2^2+m_\eta^2)^{1/2},\ \text{and}\ \omega_{\eta'}=(p_3^2+m_{\eta'}^2)^{1/2}.\tag{12}$$

We now write the condition for the $(t_L)_{ij}$ matrix defined in Eq. (2), having a D^* pole:

$$[\mathrm{Re}\,(\det D)]_{s=m_{D^*}^2}=0\,. \tag{13}$$

The elements of (ReD) may be written, using Eq. (4), as

$$\mathrm{Re}D_{11}=1-\frac{g_{D^*D\pi}{}^2}{4\pi}I_{11}^{D^*}-\frac{g_{\rho\pi\pi}g_{\rho D\bar{D}}}{4\pi}I_{11}^{\rho},\quad \mathrm{Re}D_{22}=1+\frac{g_{D^*D\eta}{}^2}{4\pi}I_{22}\frac{1}{\cos^2\theta},\quad \mathrm{Re}D_{33}=1+\frac{g_{D^*D\eta'}}{4\pi}I_{33}\frac{1}{\sin^2\theta},$$

$$\mathrm{Re}D_{12}=\sqrt{3}\,\frac{g_{D^*D\pi}g_{D^*D\eta}}{4\pi}I_{12}\frac{1}{\cos\theta},\quad \mathrm{Re}D_{21}=\sqrt{3}\frac{g_{D^*D\pi}g_{D^*D\eta}}{4\pi}I_{21}\frac{1}{\cos\theta},\quad \mathrm{Re}D_{13}=\sqrt{3}\frac{g_{D^*D\pi}g_{D^*D\eta'}}{4\pi}I_{13}\frac{1}{\sin\theta}, \tag{14}$$

$$\mathrm{Re}D_{31}=\sqrt{3}\frac{g_{D^*D\pi}g_{D^*D\eta'}}{4\pi}I_{31}\frac{1}{\sin\theta},\quad \mathrm{Re}D_{32}=\sqrt{3}\,\frac{g_{D^*D\eta}g_{D^*D\eta'}}{4\pi}I_{31}\frac{1}{\sin\theta\,\cos\theta},\quad \mathrm{Re}D_{23}=\sqrt{3}\frac{g_{D^*D\eta}g_{D^*D\eta'}}{4\pi}I_{23}\frac{1}{\sin\theta\cos\theta},$$

where

$$I_{ij}=\frac{s-s_0}{\pi}\int_{s_i}^{\infty}\frac{p_1{}^2(s')}{p_j(s')\sqrt{s'}}\,\frac{F[(\beta_{ij}(s'),\alpha_{ij}(s'))]}{(s'-s)(s'-s_0)}ds'\,. \tag{15}$$

We next compare[12] t_L with the scattering amplitude

$$T=\frac{1}{s-m_{D^*}{}^2-im_{D^*}\Gamma_{D^*}}\begin{bmatrix} -2\frac{g_{D^*D\pi}{}^2}{4\pi} & \frac{-2}{\sqrt{3}}\,\frac{g_{D^*D\pi}g_{D^*D\eta}}{4\pi} & \frac{-2}{\sqrt{3}}\,\frac{g_{D^*D\pi}g_{D^*D\eta'}}{4\pi} \\ -\frac{2}{\sqrt{3}}\,\frac{g_{D^*D\pi}g_{D^*D\eta}}{4\pi} & -\frac{2}{3}\,\frac{g_{D^*D\eta}{}^2}{4\pi} & -\frac{2}{\sqrt{3}}\,\frac{g_{D^*D\eta}g_{DD\eta'}}{4\pi} \\ -\frac{2}{\sqrt{3}}\,\frac{g_{D^*D\pi}g_{D^*D\eta'}}{4\pi} & \frac{-2}{\sqrt{3}}\,\frac{g_{D^*D\eta}g_{D^*D\eta'}}{4\pi} & -\frac{2}{3}\,\frac{g_{D^*D\eta'}}{4\pi}\end{bmatrix} \tag{16}$$

for the D^* pole, which appears in the direct channels. This leads to three more conditions when the diagonal terms are compared:

$$-2\frac{g_{D^*D\pi}{}^2}{4\pi}X=N_{11}(D_{22}D_{33}-D_{32}D_{23})+N_{12}(D_{32}D_{13}-D_{12}D_{33})+N_{13}(D_{12}D_{23}-D_{13}D_{22})\,,$$

$$-\frac{2}{3}\frac{g_{D^*D\eta}{}^2}{4\pi}X=N_{21}(D_{31}D_{23}-D_{21}D_{33})+N_{22}(D_{11}D_{33}-D_{13}D_{31})+N_{23}(D_{13}D_{21}-D_{11}D_{23})\,, \tag{17}$$

$$-\frac{2}{3}\frac{g_{D^*D\eta'}}{4\pi}X=N_{31}(D_{21}D_{32}-D_{31}D_{22})+N_{32}(D_{31}D_{12}-D_{11}D_{32})+N_{33}(D_{22}D_{11}-D_{21}D_{12})\,,$$

where $X=(\mathrm{Re}\det D)'$ and

$$\det D=\begin{vmatrix} D_{11} & D_{12} & D_{13}\\ D_{21} & D_{22} & D_{23}\\ D_{31} & D_{32} & D_{33}\end{vmatrix}\,. \tag{18}$$

In terms of the coupling constants $g_{D^*D\pi}/4\pi=a$, $g_{D^*D\eta}{}^2/4\pi=b$, $g_{D^*D\eta'}{}^2/4\pi=c$, and $g_{\rho D\bar{D}}g_{\rho\pi\pi}/4\pi=d$, the above equations become

$$\begin{aligned}1+[bI_{22}+cI_{33}-aI_{11}^{D^*}-dI_{11}^{\rho}]+[bcI_{22}I_{33}-3bcI_{32}I_{23}-ab(I_{11}^{D^*}I_{22}+3I_{21}I_{12})-dbI_{11}^{\rho}I_{22}-ac(I_{11}^{D^*}I_{33}+3I_{31}I_{13})-dcI_{11}^{\rho}I_{33}]\\ +[abc(3I_{32}I_{23}I_{11}^{D^*}-I_{22}I_{33}I_{11}^{D^*}+3\sqrt{3}I_{21}I_{13}I_{32}+3\sqrt{3}I_{13}I_{12}I_{23}-3I_{31}I_{13}I_{22}-3I_{12}I_{21}I_{33})\\ +bcdI_{11}^{\rho}(3I_{23}I_{32}-I_{22}I_{33})]=0,\end{aligned} \tag{19a}$$

$$\begin{aligned}-2aXp_1{}^2=a[F_{11}^{D^*}+dI_{11}^{\rho}][(1+bI_{22})(1+cI_{33})-3bcI_{23}I_{32}]-(p_1/p_2)3abF_{12}[\sqrt{3}cI_{23}I_{31}-(1+I_{33}c)I_{21}]\\ -(p_1/p_3)3acF_{13}[\sqrt{3}bI_{21}I_{32}-(1+bI_{22})I_{31}]\,,\end{aligned} \tag{19b}$$

$$\begin{aligned}\tfrac{2}{3}\,bXp_2{}^2=bF_{22}[(1-aI_{11}^{D^*}-dI_{11}^{\rho})(1+cI_{33})-3acI_{13}I_{31}]+(p_2/p_1)3abF_{12}[\sqrt{3}cI_{31}I_{23}-(1+cI_{33})I_{21}]\\ +(p_2/p_3)bcF_{23}[\sqrt{3}aI_{13}I_{21}-(1-aI_{11}^{D^*}-dI_{11}^{\rho})I_{23}]\,,\end{aligned} \tag{19c}$$

$$\tfrac{2}{3}cXp_3^2 = cF_{33}[(1+bI_{22})(1-aI_{11}^{D^*}-dI_{11}^{\rho})-3abI_{12}I_{21}]+3ac(p_3/p_1)F_{13}[\sqrt{3}bI_{21}I_{32}-(1+bI_{22})I_{31}]$$
$$+3bc(p_3/p_2)F_{23}[\sqrt{3}\,aI_{31}I_{12}-(1-aI_{11}^{D^*}-dI_{11}^{\rho})I_{32}]\,. \tag{19d}$$

III. DOMAIN OF CUTS

In this section, we find out the various domains of cuts for $\pi D\to\pi D$, $\eta D\to\eta D$, and $\eta' D\to\eta' D$ processes.

A. $\pi D\to\pi D$ process

(i) ρ exchange. Here we need to solve Eq. (9) and

$$t+2p_1^2(1-\cos\theta)=0,\quad |\cos\theta|\leq 1\,, \tag{20}$$

which give the position of the singularities in the complex plane. Equations (9) and (20) then give

$$s^2+2s\left[\frac{t}{1-\cos\theta}-(m_D^2+m_\pi^2)\right]+(m_D^2-m_\pi^2)=0\,. \tag{21}$$

To get now the ρ-exchange region we set[3] $t=(2m_\rho)^2$ and solve for s from Eq. (21); we find

$$s=0.9;\quad 4.5 \tag{22}$$

indicating a cut from $-\infty$ to 0 and from 0.9 to 4.5.

(ii) D^* exchange. Here the singularities are determined from the following relation between s and u:

$$s+u=2(m_D^2+m_\pi^2)+2p_1^2(1-\cos\theta) \tag{23}$$

substituting p_1^2 from Eq. (9), we get

$$[(1+\cos\theta)s^2+2s[u-(m_D^2+m_\pi^2)(1+\cos\theta)]$$
$$-(1-\cos\theta)(m_D^2-m_\pi^2)^2=0\,, \tag{24}$$

which leads to the following solutions:

$$s=\frac{(m_D^2-m_\pi^2)^2}{u}>0 \text{ and } -\infty \text{ for } \cos\theta=-1\,,$$

and (25)

$$s=2(m_D^2+m_\pi^2)-u>0 \text{ and } 0 \text{ for } \cos\theta=+1\,.$$

B. $D\eta\to D\eta$ and $D\eta'\to D\eta'$ processes

In order to find the singularities for these processes, we replace m_π by m_η and $m_{\eta'}$ in Eq. (24), the solutions turn out to be

$$\frac{(m_D^2-m_p^2)^2}{u}>0 \text{ and } -\infty \text{ for } \cos\theta=-1\,,$$

and (26)

$$2(m_D^2+m_p^2)-u>0 \text{ and } 0 \text{ for } \cos\theta=+1$$

for $p=\eta$ or η'.

Thus, we find that in all the three processes described above, the left-hand cuts run from $-\infty$ to 0. However, the position of the right-hand cuts might vary between $(1/u)(m_D^2-m_p^2)^2$ and $2(m_D^2-m_p^2)-u$ accordingly as $p=\pi$, η, or η' for various values of u. For the problem at hand, the three threshold values are $u_1=(m_D+m_\pi)^2$, $u_2=(m_D+m_\eta)^2$, and $u_3=(m_D+m_{\eta'})^2$, so that our range of interest for the variation of $u=m_{D^*}^2$ would be between 4 and 7.9 GeV. The system of Eqs. (19) may now be solved by treating $m_{D^*}^2$ as a parameter and varying it in the region discussed above. Obviously for each value of $m_{D^*}^2$ one would get a prediction for the coupling constants $g_{D^*Dp}/4\pi (p=\pi,\ \eta$ or $\eta')$, $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ and the decay width $\Gamma(D^*\to D\pi)$. However, it may be noted that although the experimental mass of D^* is known, a precise value of Γ_{D^*} is still awaited. In view of this, we solve the system of equations taking $m_{D^*}^2$ as $(m_{D^*}^2)_{\text{expt}}=4.024$ GeV to predict Γ_{D^*}. Regarding the location of the subtraction point, we choose it at the start of the left-hand cut in keeping with the conventional practice.

IV. NUMERICAL CALCULATIONS

With the location of the subtraction point fixed, it is now possible to solve the system of equations given by (19) for the coupling constants $g_{D^*D\pi}^2/4\pi$, $g_{D^*D\eta}^2/4\pi$, $g_{D^*D\eta'}^2/4\pi$, and $g_{\rho\pi\pi}g_{DD\bar{D}}/4\pi$. We begin by considering the one-channel process $\pi D\to\pi D$ where the D^* meson is generated in a direct channel through the exchange of ρ and D^* in the crossed channels. In our previous analysis,[6] we had neglected the effects of the D^* force; for the present work, we include its effects as well.

Setting $b=c=0$ in Eq. (19), we get

$$1-aI_{11}^{D^*}-DI_{11}^{\rho}=0\,,$$
$$-2aXp_1^2=aF_{11}^{D^*}+dF_{11}^{\rho}\,, \tag{27}$$

which when solved, yield

$$\frac{g_{D^*D\pi}^2}{4\pi}=1.5 \text{ and } \frac{g_{\rho\pi\pi}g_{D\bar{D}}}{4\pi}=-5.9\,.$$

Γ_{D^*} may now be calculated from

$$\Gamma_{D^*}=\frac{2g_{D^*D\pi}^2p_1^3}{4\pi m_{D^*}^2}\,. \tag{28}$$

For $g_{D^*D\pi}^2/4\pi=1.5$, Γ_{D^*} turns out to be ~ 50 keV, which is not much different from what was calculated with the ρ force alone. However, the product of the coupling constants remains insensitive to the addition of the D^* force, and indicates that $g_{\rho\pi\pi}{}^2/4\pi$ is of the order of 10 (using symmetry relation $\langle g_{\rho D\bar{D}}\rangle = \frac{1}{2}\langle g_{\rho\pi\pi}\rangle$) in contrast to its experimental value of ≈ 3, given in Eq. (7).

We next investigate the effects of the inclusion of other channels such as $D\eta$ or $D\eta'$ on these results and find out whether such channels reduce significantly the product of the coupling constants $g_{\rho\pi\pi}$ and $g_{\rho D\bar{D}}$ so that a better agreement of $g_{\rho\pi\pi}{}^2/4\pi$ with experimental value is obtained.

We consider first the effect of the inclusion of $D\eta$ channel. The governing equations may easily be read off from the relations given in (19), which are

$$1+bI_{22}-dI_{11}^{\rho}-ab(I_{11}^{D^*}I_{22}+3I_{21}I_{12})-dbI_{11}^{\rho}I_{22}=0\,,$$

$$\tfrac{2}{3}Xp_2{}^2=-3a(p_2/p_1)F_{12}I_{21}+F_{22}(1-aI_{11}^{D^*}-dI_{11}^{\rho})\,, \qquad (29)$$

$$-\tfrac{2}{3}Xp_1{}^2=(aF_{11}^{D^*}+dF_{11}^{\rho})(1+bI_{22})+(p_1/p_2)3abI_{21}F_{12}\,,$$

where

$$X=-aI_{11}'^{D^*}-dI_{11}'^{\rho}+bI_{22}' -ab(I_{11}'^{D^*}I_{22}+I_{22}'I_{11}^{D^{*5}}+3I_{12}'I_{21}+3I_{12}I_{21}') -db(I_{11}'^{\rho}I_{22}-I_{22}'I_{11}^{\rho})\,. \qquad (30)$$

Equations (29) and (30) are now solved for a, b, and d. It may be remarked that for points below s_2, the second threshold point p_2 is imaginary and F_{12} is taken as

$$F_{12}=-\frac{i}{2}|a+b|\left[1-|b|\tan^{-1}\frac{1}{|b|}\right] \qquad (31)$$

so that F_{12}/p_2 is always real. Using either the form (8) or (31) as the case may be, the I's may be calculated in a straightforward manner. We have evaluated the I's numerically up to 20 000 $m_\pi{}^2$ and the remainder using asymptotic expressions. The results obtained are

$$\frac{g_{D^*D\pi}{}^2}{4\pi}=a=0.06, \quad \text{implying } \Gamma(D^*\to D\pi)\approx 3\text{ keV}$$
$$\frac{g_{D^*D\eta}{}^2}{4\pi}=b=26.6, \quad \frac{g_{\rho\pi\pi}g_{\rho D\bar{D}}}{4\pi}=-1.7\,. \qquad (32)$$

The asymmetric parameter $P=|t_{12}/T_{12}|$ turns out to be ~1.1, thereby indicating that the t matrix is an approximately symmetric one.

We thus find that the width of the D^* is considerably reduced from what was obtained in the one-channel case. It is not possible at present to say whether this reduction is in the right direction since only a upper limit on Γ_{expt} is available.[11]

$g_{D^*D\eta}{}^2/4\pi$ turns out to be rather large in comparison with $g_{D^*D\pi}{}^2/4\pi$. It indicates that a large amount of $\bar{c}$ leakage may be present in the η state.

The product of the coupling constants $g_{\rho\pi\pi}$ and $g_{\rho D\bar{D}}$ continues to remain negative. However, the magnitude has shrunk a good deal which is welcome. Using SU(4), this value now predicts $g_{\rho\pi\pi}{}^2/4\pi\approx 3.4$, which is in very good agreement with the experimental value of $g_{\rho\pi\pi}{}^2/4\pi\approx 3$.

However, the value of $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ as obtained above is very sensitive to the choice of any particular channel. This can be seen if we introduce the channel $D\eta'$ in place of $D\eta$ and solve the equations as before. The relevant equations in this case are

$$\tfrac{2}{3}p_3{}^2X=-3a\frac{p_3{}^2F_{13}}{p_1\;p_3}I_{31}+F_{33}(1-aI_{11}^{D^*}-dI_{11}^{\rho})\,, \qquad (33a)$$

$$-\tfrac{2}{3}Xp_1{}^2=(aF_{11}^{D^*}+dF_{11}^{\rho})(1+cI_{33})+p_1 3ac\,\frac{F_{13}}{p_3}I_{31}\,, \qquad (33b)$$

$$X=-aI_{11}'^{D^*}-dI_{11}'^{\rho}+cI_{33}' -ac(I_{11}'^{D^*}I_{33}+I_{33}'I_{11}^{D^*}+3I_{13}'I_{31}+3I_{13}I_{31}') -\left(\frac{I_{11}'^{\rho}}{I_{11}^{\rho}}+\frac{I_{33}'}{I_{33}}\right)[1+cI_{33}-dI_{11}^{\rho}-ac(I_{11}^{D^*}I_{33}+3I_{31}I_{13})]\,. \qquad (34)$$

It is interesting to note that for a fixed value of a, the right-hand side of Eq. (34) involves terms which are linear in c and d. Thus, when Eq. (34) is substituted into Eqs. (33) one gets a couple of linear equations which can be solved for c and d. The solutions turn out to be

$$\frac{g_{D^*D\eta'}{}^2}{4\pi}=c=1.26\times 10^2=4.7g_{D^*D\eta}{}^2/4\pi\,,$$
$$\frac{g_{\rho\pi\pi}g_{\rho D\bar{D}}}{4\pi}=d=-6.4 \qquad (35)$$

for $a=0.06$, obtained in the earlier case.

Obviously such a large value of $g_{D^*D\eta'}/4\pi$ implies that the $c\bar{c}$ leakage in the η' state is quite substantial. The other parameter, viz., $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$, also turns out to be large, much larger than what was obtained from $D\pi-D\eta$ channels. This indicates that the value of $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ does depend on the types of the channels used; a more complete set of forces may therefore be necessary to generate the D^*. However, this problem cannot be taken up right now owing to the nonavailability of sufficient experimental informations on the parameters of the D^*.

V. CONCLUSION AND DISCUSSIONS

To conclude, we have considered in this paper the generation of D^* by the exchange of ρ and D^* in the crossed channels in an N/D formalism.

In the one-channel $D\pi\to D\pi$ case, where the D^* may be generated in the direct channel by the exchanges of ρ as well as D^* in the crossed channels, we have found that the addition of the D^* force does not make any significant change in the value of Γ_{D^*} than when it is calculated using the ρ force alone; Γ_{D^*} turns out to be ~50 keV which is well within the present experimental upper limit[11] on Γ_{D^*}. However, the product of the coupling constants $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ comes out to be rather large in contrast to out usual expectations. It may also be noted that although the sign of $g_{\rho\pi\pi}g_{\rho DD}/4\pi$ comes out to be negative which is similar to what had been observed by Diu *et al.*[3] for $\rho\pi\pi$ and $\rho K\bar{K}$ couplings in the K^* case, the magnitude of $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ comes out to be smaller than that of $g_{\rho\pi\pi}g_{\rho K\bar{K}}/4\pi$ by more than a factor of 2. This is an encouraging result, since this in turn indicates that

$$\left|\frac{g_{\rho D\bar{D}}}{g_{\rho\pi\pi}}\right| \leq \left|\frac{g_{\rho K\bar{K}}}{g_{\rho\pi\pi}}\right| .$$

The inclusion of the $D\eta$ channel greatly reduces the width of the D^*, and the reduction factor turns out to be ~15. However, it is not possible to say at present whether this reduction factor is welcome until a precise experimental value on Γ_{D^*} becomes available. The product of the coupling constants $g_{\rho\pi\pi}g_{\rho DD}/4\pi$ is also considerably shrunk-corresponding to the value obtained, SU(4) symmetry gives $g_{\rho\pi\pi}^2/4\pi \approx 3.4$ which is quite close to the experimental value of ≈ 3. However, $g_{\rho\pi\pi}g_{\rho D\bar{D}}/4\pi$ is very sensitive to the types of channels used.

The $g_{D^*D\eta}^2/4\pi$ and $g_{D^*D\eta'}/4\pi$ couplings come out to be rather large in comparison with $g_{D^*D\pi}^2/4\pi$. This indicates that a large amount of $c\bar{c}$ component may be present in both η and η'—a theory advanced by a number of authors[13] to explain the nonzero decay rates of $\Psi \to \eta\gamma$ and $\Psi \to \eta'\gamma$ processes.

ACKNOWLEDGMENT

We thank Dr. B. Bagchi for many useful discussions and Dr. C. R. Mandal for helping us with the computer program.

APPENDIX

In this appendix, we shall determine the mixing angle θ appearing in Eq. (9) in the (u,d,c) quark sector. Following Gell-Mann, Oakes, and Renner[14] (GMOR), one can express the meson masses as

$$m_\pi^2 = \frac{\sqrt{2}+l}{3f^2}(\sqrt{2}\xi_0 + \xi_8)\,, \tag{A1}$$

$$m_D^2 = \frac{2\sqrt{2}-l}{3f^2}(2\sqrt{2}\xi_0 - \xi_8)\,, \tag{A2}$$

$$m_\eta^2 = -\frac{2}{3f^2}\Big[(\sin\theta - l\cos\theta)(\cos\theta\xi_8 - \sin\theta\xi_0) + \Big(\cos\theta - \frac{1}{\sqrt{2}}l\cos\theta - l\sin\theta\Big) \times\Big(\frac{1}{\sqrt{2}}\xi_8\cos\theta + \xi_8\sin\theta - \xi_0\cos\theta\Big)\Big]\,, \tag{A3}$$

$$m_{\eta'}^2 = -\frac{2}{3f^2}\Big[-(\cos\theta + l\sin\theta)(\sin\theta\xi_8 + \cos\theta\xi_0) + \Big(\sin\theta - \frac{1}{\sqrt{2}}l\sin\theta + l\cos\theta\Big) \times\Big(\frac{1}{\sqrt{2}}\xi_8\sin\theta - \xi_8\cos\theta - \xi_0\sin\theta\Big)\Big]\,, \tag{A4}$$

where $\xi_i = \langle 0|u_i|0\rangle$, f is a pseudoscalar decay constant, l is a parameter corresponding to the symmetry-breaking Hamiltonian density $H = -u_0 - lu_8$ and the u_i's are the scalar densities which belong to the $(3,\bar{3})+(3,\bar{3})$ representation of chiral SU(3) $\times$SU(3) in the (u,d,c) quark[15] sector. Equations (A1) and (A2) give the following estimate of l by making the GMOR assumption that the vacuum is SU(3) symmetric:

$$l = -2\sqrt{2}\,\frac{m_D^2 - m_\pi^2}{2m_D^2 - m_\pi^2} \approx -1.4\,. \tag{A5}$$

Eliminating f from Eqs. (A3) and (A4) which gives

$$\frac{m_\eta^2}{m_{\eta'}^2} = \frac{(\sqrt{2}-l) - 2\sqrt{2}\,l\tan\theta + \sqrt{2}\tan^2\theta}{(\sqrt{2}-l)\tan^2\theta + 2\sqrt{2}\,l\tan\theta + \sqrt{2}}\,. \tag{A6}$$

θ may be calculated using Eq. (A5); θ turns out to be $\approx -26°$.

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