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Spin-dependent forces in quantum chromodynamics

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In a manifestly gauge-independent formalism, all relativistic corrections to the fermion propagation function are determined and the general form of the spin-dependent forces in quantum chromodynamics for heavy-quark–antiquark ($q\bar{q}$) systems is derived. For example, the classical spin-orbit and Thomas-precession terms are found to be simple derivatives of the static potential. In addition to expressing the spin-dependent forces in terms of the minimal number of independent potentials, two new applications of this formulation are presented: (1) The effect of pseudoparticle solutions on the spin-dependent forces is analyzed, and (2) an electric-confinement assumption produces a zero-parameter spin-dependent potential. This potential determines the fine structure in heavy $q\bar{q}$ systems. Spin splittings in the Υ system are predicted and the J/ψ system splittings are compared with the experimentally observed values.

I. INTRODUCTION

One of the most important aspects of the J/ψ and Υ systems is that they are nonrelativistic systems possessing several states below the threshold for the production of mesons with the quantum numbers of their constituents. Therefore, extensive experimental investigation of the excitation spectrum of these systems has been possible. The observed masses can be used to build phenomenological models for the nonrelativistic potential acting between a heavy-quark–antiquark[1] pair and to obtain information about the spin-dependent corrections[2] to the nonrelativistic interaction.

There are a number of successful potential models of these states.[3] In particular the Cornell model[4] (linear plus Coulomb potential) describes well the spectra of states for both the J/ψ and Υ system.[5] Even though the static energy has not yet been calculated directly from the dynamics of the strong interactions,[6] we can analyze here the spin-dependent relativistic corrections to the static energy in quantum chromodynamics (QCD) and are able to determine their general structure and many phenomenological consequences independent of the actual form of the static energy. This work is an extended analysis based on a previously reported work by the present authors.[7]

The relativistic structures which occur in QCD have similarities to as well as important differences from relativistic corrections in QED. Therefore, in order to develop an intuitive understanding of the QCD relativistic effects, it is worthwhile to compare them to the analogous QED effects[8] in, for example, a lepton-antilepton system $(l_1, \bar{l}_2)$ where l_1 and l_2 are leptons with masses m_1 and m_2, respectively (if $l_1 = l_2$ pair creation and annihilation are ignored). For electrodynamics the static potential is known exactly, being given by the single Coulomb exchange, whereas in QCD the static potential even in perturbation theory is an infinite set of graphs, each fermion emitting any number of Coulomb excitations (in radiation gauge) which then interact through the full Yang-Mills couplings.[9] Nevertheless, in both theories there are two distinct types of relativistic spin-dependent corrections to the static potential. The first type consists solely of corrections to the fermion propagator due to its motion in the static potential. The numerator of the free fermion propagator may be written as

$$u(p)\bar{u}(p) = \frac{\not{p} + m}{2m} = \frac{1+\gamma^0}{2} - \frac{\vec{p}\cdot\vec{\gamma}}{2m} + O\left(\frac{1}{m^2}\right). \tag{1.1}$$

The leading term is simply the nonrelativistic projection operator $(1+\gamma^0)/2$, while the $\vec{p}\cdot\vec{\gamma}/2m$ term represents the first relativistic correction. Because of the projection operators a single $\vec{p}\cdot\vec{\gamma}/2m$ insertion on a fermion (or antifermion) line will vanish. However, two such momentum insertions with the static potential interacting between them give a nonzero contribution, the familiar classical spin-orbit interaction and Thomas precession [see Fig. 1(a)]. This contribution to the potential is given by

$$\delta_1 V(R) = \frac{1}{2m_1^2}\vec{L}\cdot\vec{S}_1 \frac{1}{R}\frac{dV_C(R)}{dR} + (1 \to 2), \tag{1.2}$$

where $V_C(R)$ is the static Coulomb potential and $\vec{S}_i$ is the spin of the ith lepton. The second type of correction is due to the exchange of transverse vector excitations, which in QED are the physical modes of the electromagnetic field [see Fig. 1(b)]. A simple computation of the lowest-order contribution of these terms yields the elementary result

$$\delta_2 V(R)=\frac{1}{m_1 m_2}\vec{L}\cdot(\vec{S}_1+\vec{S}_2)\frac{1}{R}\frac{dV_T(R)}{dR}+\frac{2}{3m_1m_2}\vec{S}_1\cdot\vec{S}_2\nabla^2 V_T(R)+\frac{1}{3m_1m_2}[3(\vec{S}_1\cdot\vec{R})(\vec{S}_2\cdot\vec{R})-(\vec{S}_1\cdot\vec{S}_2)]\left(\frac{1}{R}\frac{dV_T}{dR}-\frac{d^2V_T}{dR^2}\right), \tag{1.3}$$

where $V_T(R)$ is the potential associated with the exchange of transverse vector fields. Of course, $V_T(R)=V_C(R)$ in QED since $V_C(R)$ is the single Coulomb exchange which arises from the same interaction $J^\mu A_\mu$ in the Lagrangian as the transverse exchange. For the non-Abelian case, although there are still the same two types of interactions, the analog of $V_C(R)$ is not a single exchange and therefore is not necessarily equal to the analog of $V_T(R)$.

In Sec. II, as the first step toward determining the form of the spin-dependent forces in QCD, all relativistic corrections to the nonrelativistic fermion propagation function are determined in a gauge-independent formulation of QCD. In order to transform these corrections into perturbations of the static energy, the Wilson-loop formalism is introduced and reviewed in Sec. III. Then, the results of Secs. II and III are used in Sec. IV to produce the general form of the spin-dependent forces in QCD through order $1/m^2$. The general approach is similar to that of other authors[10-12] who make use of a Foldy-Wouthuysen transformation on the interacting Hamiltonian. The various terms involve expectation values of the non-Abelian electric and magnetic fields as well as the covariant kinetic energy all of which are evaluated in the presence of the interactions which produce the static potential. The formal expressions can be greatly simplified by use of the Jacobi identities for the gauge field and a non-Abelian generalization of Stokes' theorem applied to path-ordered integrals. Type-one contributions, the classical spin-orbit and Thomas-precession terms, are simply related to the static energy whereas type-two corrections cannot be reduced in this way.

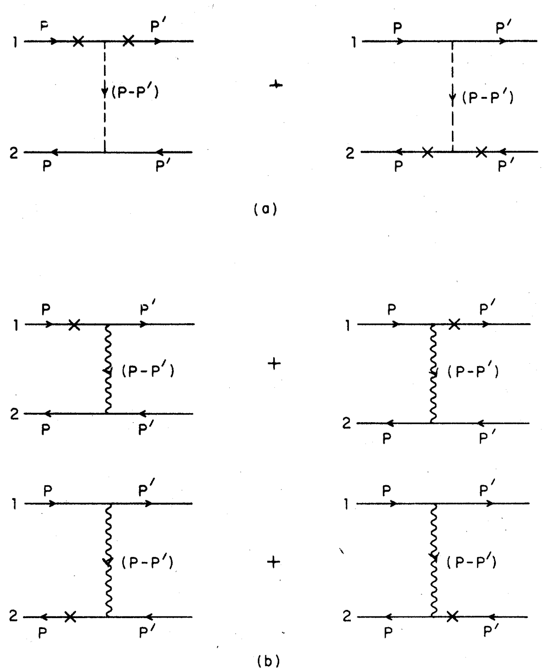

FIG. 1. The spin-dependent corrections in QED. A cross represents an insertion of $\vec{p}\cdot\vec{\gamma}/2m$, a dotted line represents a Coulomb exchange, and a wavy line a transverse photon exchange. Figure (a) displays those graphs that depend on $V_c(R)$, while the graphs in Fig. (b) depend on transverse photon excitations.

The applications of this general result are given in the remaining sections. In Sec. V the inclusion of pseudoparticle effects, that is, solutions of self-dual (or anti-self-dual) equations are analyzed. Using only the duality properties of these solutions without the usual dilute-gas approximation, we show that the spin-spin and tensor forces are simply determined by the pseudoparticle contribution to the static potential. The spin-orbit terms are more complicated but these can also be determined by the same static potential. This result is in disagreement with the conclusions of de Carvalho[13] and also of Callan *et al.*[11] It is explicitly shown by considering these instanton contributions that the number of independent spin-dependent potentials in QCD appearing in the general form derived in Sec. IV cannot be further reduced without additional assumptions about the nature of the spin-dependent forces.

The final sections consider the phenomenological applications of the formalism developed in Sec. IV. In Sec. VI a plausible argument is given for extrapolating the structure of the spin-dependent forces to mesons with one light and one heavy quark. The mass relations obtained are in good general agreement with data and lead to many testable predictions for the $(b\bar{u})$, $(b\bar{d})$, and $(b\bar{s})$ meson excitation spectra. Comparison with the spin-dependent interaction of De Rújula, Georgi, and Glashow[14] is made. To make a quantitative determination of the fine structure in heavy-quark–antiquark systems we assume that the large-distance behavior of the static potential is determined uniquely by the longitudinal component of the non-Abelian electric field. (The longitudinal modes of the Yang-Mills electric field may be isolated by a gauge-independent procedure.[15]) In Sec. VII it is shown that this assumption yields a zero-parameter determination of the spin-dependent forces in terms of the spin-independent static energy. Then the Cornell-model parameters for a linear plus Coulomb static potential are used to determine the fine structure in $(c\bar{c})$ and $(b\bar{b})$ systems. The comparison with experiment is discussed for the $(c\bar{c})$ system. In the last section our results are summarized and discussed.

II. RELATIVISTIC FERMION-PROPAGATOR CORRECTIONS

The physical system we investigate is that of a quark and antiquark bound together nonrelativistically in a color-singlet state by QCD. The standard Lagrangian which describes this interaction is

$$L(t)=\int d^3x[-\tfrac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})+\bar{\Psi}(i\not\partial+g\not A-m)\Psi], \tag{2.1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu]$, $A_\mu\equiv A^a_\mu t^a$. $\{f^{abc}\}$ are the structure constants of the gauge group (SU_3) and $\{t^a\}$ are the representation matrices for the fermions in the fundamental representation of the group. The fermion propagation function is denoted by $S(x,y,A)$ and satisfies the equation

$$(\not D-m)S(x,y;A)=\delta^4(x-y), \tag{2.2}$$

where $D_\mu=i\partial_\mu+gA_\mu$. The nonrelativistic propagation function S_0 is obtained from Eq. (2.2) by ignoring the spatial motion of the quark:

$$(D_0\gamma^0-m)S_0(x,y;A)=\delta^4(x-y). \tag{2.3}$$

Formally, this equation can be solved exactly with boundary conditions such that the fermion propagates forward in time and the antifermion backwards:

$$S_0(x,y;A^0)=-i\theta(x^0-y^0)e^{-im(x^0-y^0)}\frac{1+\gamma^0}{2}P\begin{pmatrix}x^0\\y^0\end{pmatrix}\delta(\vec{x}-\vec{y})-i\theta(y^0-x^0)e^{-im(y^0-x^0)}\frac{1-\gamma^0}{2}P\begin{pmatrix}x^0\\y^0\end{pmatrix}\delta(\vec{x}-\vec{y}), \tag{2.4}$$

in which

$$P\begin{pmatrix}x\\y\end{pmatrix}\equiv P\exp\left[ig\int_y^x dz_\mu A^\mu(z)\right]. \tag{2.5}$$

In Eq. (2.4) the path-ordered exponentials depend only on A^0. In terms of S_0 an integral equation for S is given by

$$S(x,y;A)=S_0(x,y;A)+\int d^4z\,S_0(x,z;A)\vec{\gamma}\cdot\vec{\mathrm{D}}S(z,y;A). \tag{2.6}$$

The operator $\vec{\gamma}\cdot\vec{\mathrm{D}}$, the kernel of the integral equation, gives rise to spatial motion and Eq. (2.6) provides a formal solution to Eq. (2.2). To extract the relativistic corrections most efficiently, projection operators onto the nonrelativistic Dirac eigenstates are introduced:

$$\begin{aligned}S^{++}&\equiv\frac{1+\gamma^0}{2}S(x,y;A)\frac{1+\gamma^0}{2},\\ S^{-+}&\equiv\frac{1-\gamma^0}{2}S(x,y;A)\frac{1+\gamma^{0'}}{2},\\ S^{+-}&\equiv\frac{1+\gamma^0}{2}S(x,y;A)\frac{1-\gamma^0}{2},\\ S^{--}&=\frac{1-\gamma^0}{2}S(x,y;A)\frac{1-\gamma^0}{2}.\end{aligned} \tag{2.7}$$

Clearly $S_0^{+-}=S_0^{-+}=0$. In terms of these projection operators Eq. (2.6) becomes

$$S^{++}(x,y)=S_0^{++}(x,y)+\int d^4z\,S_0^{++}(x,z)\vec{\gamma}\cdot\vec{\mathrm{D}}S^{-+}(z,y), \tag{2.8a}$$

$$S^{-+}(x,y)=+\int d^4z\,S_0^{--}(x,z)\vec{\gamma}\cdot\vec{\mathrm{D}}S^{++}(z,y), \tag{2.8b}$$

with two other equations obtained by exchanging + and −. Combining Eqs. (2.8a) and (2.8b) produces a closed equation for S^{++} (an equation for S^{--} is obtained similarly):

$$S^{++}(x,y;A)=S_0^{++}(x,y;A)+\int d^4z\,d^4w\,S_0^{++}(x,z;A)\vec{\gamma}\cdot\vec{\mathrm{D}}(z)\times S_0^{--}(z,w;A)\vec{\gamma}\cdot\vec{\mathrm{D}}(w)S^{++}(w,y;A). \tag{2.9}$$

Graphically, this equation is represented in Fig. 2. The quark propagates from x to z interacting with the gauge field nonrelativistically (the open circles indicate fully interacting nonrelativistic propagators). At z it suffers a "hard" interaction $\vec{\gamma}\cdot\vec{\mathrm{D}}$ after which the quark propagates backwards in time to w, where it suffers another "hard" interaction, and then propagates from w to y interacting fully relativistically with the gauge fields. To obtain the relativistic expansion for $S(x,y;A)$ it is first necessary to evaluate more explicitly the kernel in Eq. (2.9) by inserting into it the explicit form of S_0 from Eq. (2.4). The spatial integrations are then trivial because of the

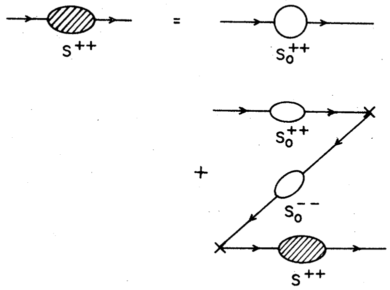

FIG. 2. The integral equation for the ++ component of the full propagator S. S_0 is the propagator in the nonrelativistic limit and a cross represents the insertion of a $\vec{\gamma}\cdot\vec{p}$ operator.

δ functions. Moreover, it is expected that in the nonrelativistic limit the time interval during which the quark propagates backwards will be suppressed so that the large-mass limit should correspond to the time interval $w^0 - z^0$ being short. To expose this behavior analytically one performs the time integrations by repeated integrations by parts on the term $e^{-im(w^0-z^0)}$ in Eq. (2.9) up to order $1/m^2$. The $1/m$ term is the integrated kernel evaluated at $w^0=z^0$ so that to this order the kernel is effectively $(\vec{\gamma}\cdot\vec{D})^2=-\vec{D}^2+g\vec{\sigma}\cdot\vec{B}$. The $1/m^2$ terms require one time derivative acting on the kernel, a derivative which appears covariantly. Explicitly, this term is

$$[D^0,\vec{\gamma}\cdot\vec{D}]\vec{\gamma}\cdot\vec{D}=-ig(\delta^{ij}-i\epsilon^{ijk}\sigma_k)E_iD_j\,. \qquad (2.10)$$

In addition, in order $1/m^2$ there is a local term which arises from the nonrelativistic suppression of the region $w^0<x^0$. (This term does not contribute to spin-dependent effects in order $1/m^2$ but is included for completeness.) The details of the derivation of the $1/m$ expansion of Eq. (2.9) to arbitrary order are given in the Appendix. The final form of this equation, correct to order $1/m^2$, is

$$\left[1+\frac{1}{4m^2}(\vec{D}^2-g\vec{\sigma}\cdot\vec{B})\right]S^{++}(x,y;A)=S_0^{++}(x,y;A)-\int d^4w\,S_0^{++}(x,w;A)\left[\frac{1}{2m}(\vec{D}^2-g\vec{\sigma}\cdot\vec{B})+\frac{ig}{4m^2}(\delta_{ij}-i\epsilon_{ijk}\sigma^k)E^iD^j\right]$$
$$\times S^{++}(w,y;A)+O\left(\frac{1}{m^3}\right). \qquad (2.11)$$

Equation (2.11) describes explicitly all relativistic fermion-propagator corrections through order $(1/m)^2$. [Higher-order terms are given in Eq. (A17).] This equation, the basis of the derivation of all spin-dependent forces, is independent of gauge and formally the same as the corresponding expression in QED. The essential difference between QED and QCD is in the implicit non-Abelian fermion representation matrices in Eq. (2.11).

III. THE STATIC ENERGY IN POSITION SPACE

In order to incorporate the relativistic propagation-function corrections in Eq. (2.11) as correction terms to the static energy (potential) it is advantageous to develop a formulation of the nonrelativistic limit in such a way that relativistic effects may be easily included. The Wilson-loop formalism,[16] which is reviewed in this section, provides such a basis. A quark-antiquark bound-state wave function can be written gauge invariantly as

$$\mathfrak{M}(x-y)e^{ik(x+y)/2}=\left\langle 0\left|T^*\bar{\Psi}(x)\Gamma P\begin{bmatrix}x\\y\end{bmatrix}\Psi(y)\right|M\right\rangle, \qquad (3.1)$$

in which k^2 equals the mass squared of the bound state, T^* indicates time ordering, and Γ has the appropriate Dirac and flavor structure to ensure a nonzero coupling of the operator to the bound state denoted by $|M\rangle$. The path-ordered exponential is included to maintain gauge invariance. To derive the Wilson-loop form of the potential, however, it is convenient to introduce the four-point function

$$I=\left\langle 0\left|T^*\left[\bar{\Psi}(y_2)\bar{\Gamma}_B P\begin{bmatrix}y_2\\y_1\end{bmatrix}\Psi(y_1)\right]\right.\right.$$
$$\left.\left.\times\left[\bar{\Psi}(x_1)\Gamma_A P\begin{bmatrix}x_1\\x_2\end{bmatrix}\Psi(x_2)\right]\right|0\right\rangle, \qquad (3.2)$$

$\bar{\Gamma}_B=\gamma^0\Gamma_B^\dagger\gamma^0$. The kinematic region in which the bound state M contributes significantly to I is given by the time interval

$$T\equiv(y_1^0+y_2^0)/2-(x_1^0+x_2^0)/2\to\infty$$

with $x_2^0-x_1^0$ and $y_2^0-y_1^0$ fixed. Then, in Euclidean space, the connection between I and $\mathfrak{M}$ is easily established by inserting a complete set of physical states between $\Psi(y_1)$ and $\bar{\Psi}(x_1)$ in Eq. (3.2). Care must be taken in the order of limits since both time (T) and quark masses (m) are becoming large. For $T\to\infty$ first the lowest-mass state (presumably $|M\rangle$ with mass M_A) exponentially dominates the sum over states and

$$I\to\delta_{AB}\bar{\mathfrak{M}}_B(y_2-y_1)\mathfrak{M}_A(x_1-x_2)\exp(-TM_A)\,. \qquad (3.3a)$$

However, for deriving the static energy the appropriate order of limits is $m\to\infty$ then $T\to\infty$. In this limit the motion of the quark and antiquark can be neglected (actually treated perturbatively); hence the quark-antiquark separation commutes with the Hamiltonian. The eigenstates are therefore labeled by $\vec{R}_x=\vec{x}_1-\vec{x}_2$ ($\vec{R}_y=\vec{y}_1-\vec{y}_2$). Inserting a complete set of states and removing the trivial dependence of the energy on the quark masses results in the expression

$$I\to\delta_{AB}\,\delta(\vec{R}_x-\vec{R}_y)\bar{M}_{BR_y}M_{AR_x}e^{-2mT}\exp[-T\epsilon(R)]\,, \tag{3.3b}$$

as $T\to\infty$. $\epsilon(R)$ is called the static energy (or potential) between the quark and antiquark separated by a spatial distance $R=R_x=R_y$.

To actually compute $\epsilon(R)$, the nonrelativistic limit of I is obtained by using the Schwinger functional formalism to reexpress Eq. (3.2) in terms of fermion propagators in external fields. These external fields are $A_\mu(x)=\langle 0|A(x)|0\rangle_J=i(\delta/\delta J)\langle 0|0\rangle_J$. [$\langle 0|0\rangle_J$ is the vacuum expectation value of $A_\mu(x)$ in the presence of an external source $J^\mu(x)=J_a^\mu(x)t^a$.] In terms of the fermion-propagation function $S(x,y;A)$, I becomes

$$I=\left\{\mathrm{Tr}\left[S(x_2,y_2;\ -i\delta/\delta J)P\begin{pmatrix}y_2\\y_1\end{pmatrix}\bar{\Gamma}_B S(y_1,x_1;\ -i\delta/\delta J)P\begin{pmatrix}x_1\\x_2\end{pmatrix}\Gamma_A\right]\right.$$
$$\left.-\mathrm{Tr}\left[S(y_1,y_2;\ -i\delta/\delta J)P\begin{pmatrix}y_2\\y_1\end{pmatrix}\bar{\Gamma}_B\right]\mathrm{Tr}\left[S(x_2,x_1;\ -i\delta/\delta J)P\begin{pmatrix}x_1\\x_2\end{pmatrix}\Gamma_A\right]\right\}Z(J)\Bigg|_{J=0}\,, \tag{3.4}$$

where $Z(J)=W(J)/W(0)$ and

$$W(J)=\mathrm{Det}[S(-i\delta/\delta J)]\int[dA^\mu]\exp\left\{i\int d^4x[-\tfrac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})+J_\mu^a A_a]\right\}.$$

Now consider the various terms in Eq. (3.4) for quarks with very large masses. The fermion determinant factor, Det $[S(-i\delta/\delta J)]$, which produces quark loops is spin independent in order $1/m^2$ and therefore we may safely ignore heavy-quark loops. (Light quarks are not being considered here.[17]) Also, the double trace term, the annihilation contribution, will be ignored. Since this term is a short-distance effect ($R\sim 1/m$) the effective coupling constant $\alpha_s(R)$ will be small; and furthermore, in perturbation theory, this term does not contribute (to any spin-dependent effect) in lowest order. (For $C=-1$ states it contributes in order α^3 and for $C=+1$ states in order α^2.) Therefore (in this approximation), the $q\bar{q}$ sector of the Fock space is completely separate from all other sectors which have different quark content. The nonrelativistic form for I becomes

$$I^{\mathrm{NR}}=\mathrm{Tr}\left[S(x_2,y_2;\ -i\delta/\delta J)P\begin{pmatrix}y_2\\y_1\end{pmatrix}\bar{\Gamma}_B\right.$$
$$\left.\times S(y_1,x_1;\ -i\delta/\delta J)P\begin{pmatrix}x_1\\x_2\end{pmatrix}\Gamma_A\right]Z'(J)\Bigg|_{J=0}\,, \tag{3.5}$$

where $Z'(J)=W'(J)/W'(0)$ and

$$W'(J)=\int[dA_\mu]\exp\left\{i\int d^4x[-\tfrac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})+J_\mu^a A_a^\mu]\right\}.$$

Physically, Eq. (3.5) corresponds to a fermion and antifermion propagating in a potential $(-i\delta/\delta J^\mu)$ whose form is determined by a functional integral over a pure Yang-Mills theory with an external source $J^\mu(x)$. In bound-state systems such as the J/ψ and the Υ families, each quark moves in the presence of the Yang-Mills fields generated by itself and its partner.

It is now straightforward to obtain the Wilson loop from Eq. (3.5) by using for the fermion propagators the nonrelativistic form, given in Eq. (2.4), to produce

$$I^{\mathrm{NR}}=e^{-2imT}\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\mathrm{Tr}\left(\frac{1+\gamma^0}{2}\bar{\Gamma}_B\frac{1-\gamma^0}{2}\Gamma_A\right)$$
$$\times\mathrm{Tr}\left\{P\exp ig\oint_{C(R,T)}dz_\mu\left[-i\frac{\delta}{\delta J_\mu(z)}\right]\right\}Z'(J)\Bigg|_{J=0}\,, \tag{3.6}$$

where $T=|x^0-y^0|$, $R=|\vec{x}_2-\vec{x}_1|$, and $C(R,T)$ is the curve shown in Fig. 3. Also, the trace over Dirac matrices has been separated from the trace over fermion representation matrices. The closed path in Eq. (3.6), the Wilson loop, occurs because the temporal path-ordered exponentials of the nonrelativistic fermion propagators combine with the spatial path-ordered exponentials needed for gauge invariance to produce the closed path $C(R,T)$. This integral may be rewritten as

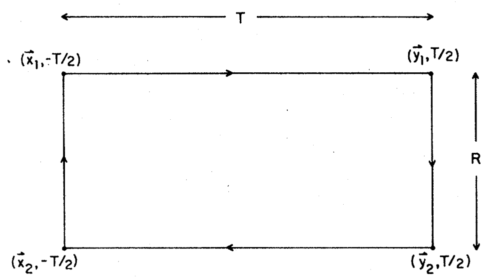

FIG. 3. The Wilson loop $C(R,T)$. The rectangular closed curve $C(R,T)$ has a spatial length R and a temporal length T.

$$\mathcal{C} \equiv \mathrm{Tr}\left\{P \exp ig \oint_{C(R,T)} dz_\mu \left[-i\frac{\delta}{\delta J_\mu(z)}\right]\right\} Z'(J)\Bigg|_{J=0}$$
$$= \int [dA_\mu] \,\mathrm{Tr}\left(P \exp ig \oint_{C(R,T)} dz^\mu A_\mu(z)\right) e^{iS_{\mathrm{YM}}(A)}, \tag{3.7}$$

where $S_{\mathrm{YM}}^{(A)}$ is the Yang-Mills action. Therefore, the Wilson-loop expression has been related to the large-mass limit of the fermion-propagation function. Also, $\mathcal{C}$ is related to the static energy $\epsilon(R)$. In Euclidean space for large T, $\mathcal{C}_E$ becomes

$$\mathcal{C}_E = A(R,T)e^{-\epsilon(R)T}. \tag{3.8}$$

Comparing Eq. (3.3b) and Eq. (3.7) it is clear that $A(R,T)$ behaves no worse than a power of T for T large. Thus, more precisely,

$$\lim_{T\to\infty} \frac{\ln \mathcal{C}_E}{T} = -\epsilon(R). \tag{3.9}$$

$\epsilon(R)$ is the static energy, that is, the potential in the nonrelativistic limit. Therefore,

$$\lim_{T\to\infty} -\frac{1}{T}\ln\left\{\mathrm{Tr}\, P\left[\exp ig \oint_{C(R,T)} \times dz_\mu \left(-i\frac{\delta}{\delta J^\mu(z)}\right)\right] Z'(J)\right\}^{\mathrm{Euclidean}}_{J=0} = \epsilon(R) \tag{3.10}$$

is a gauge-invariant expression for the nonrelativistic potential between a very heavy quark and antiquark in a color-singlet state. In QED since the action is quadratic in the field strengths, Z' may be explicitly evaluated (when a gauge-fixing term is included in the usual way) and the left-hand side of Eq. (3.10) immediately gives the Coulomb potential (independent of the gauge choice used to evaluate the functional integral). However, in QCD, the left-hand side cannot be calculated exactly and must be treated numerically[6] or by some other approximation scheme. Here, the static energy is not determined, but as will be shown in Sec. IV some of the spin-dependent forces can be related to it.

IV. THE SPIN-DEPENDENT FORCES

The results of the two previous sections can be used now to determine the form of the spin-dependent forces in QCD through order $(1/m^2)$. The static energy, Eq. (3.10), was derived from the Wilson-loop expression in Eq. (3.6) by substituting for the fermion-propagation function S in Eq. (3.5) its nonrelativistic limit S_0 given in Eq. (2.4). The relativistic corrections, however, are given by replacing S in Eq. (3.5) not by S_0 but by the propagation function with relativistic effects included as given in Eq. (2.11). For example, in order $(1/m)$, the propagation function $S^{++}(x,y)$ is replaced by

$$\int d^4w\, S_0^{++}(x,w)(1/2m)(\vec{\mathrm{D}}^2 - g\vec{\sigma}\cdot\vec{\mathrm{B}})S_0^{++}(w,y)$$

instead of by $S_0^{++}(x,y)$. The effect of this substitution is to modify the Wilson loop to include, along the time integrations, an insertion of the operator $-i(\vec{\mathrm{D}}^2 - \vec{g}\sigma\cdot\vec{\mathrm{B}})/2m$ some time t, $-T/2 \leq t < T/2$. Similarly, all other corrections from Eq. (2.11) are insertions into the Wilson loop of the appropriate operator. In order to avoid long and cumbersome formulas to describe these relativistic effects, the following expectation-value notation is introduced to describe the insertions into the Wilson loop:

$$\langle \mathcal{O}(x)\rangle \equiv \int [dA^\mu]\,\mathrm{Tr}\left\{P\left[\exp\left(ig\oint_{C(R,T)} dz_\mu A^\mu(z)\right)\mathcal{O}(x)\right]\right\}_{x\in C} e^{iS_{\mathrm{YM}}(A)}. \tag{4.1}$$

$\mathcal{O}$ is any operator. In particular

$$\langle 1\rangle = \mathcal{C} \tag{4.2}$$

and $\langle (1/2m)(\vec{\mathrm{D}}^2 - g\vec{\sigma}\cdot\vec{\mathrm{B}})\rangle$ is essentially the correction to $\mathcal{C}$ in order $1/m$. It is convenient to introduce a quantity $\tilde{I}$ which removes the unnecessary Dirac-matrix dependence in I. In order to consider the most general case, the quark mass (m_1) will not be assumed to be equal to the antiquark mass (m_2). $\tilde{I}$ is defined as follows:

$$I \equiv e^{-i(m_1+m_2)T}\,\mathrm{Tr}\left(\frac{1+\gamma^0}{2}\bar{\Gamma}\frac{1-\gamma^0}{2}\Gamma\tilde{I}\right). \tag{4.3}$$

$\tilde{I}$ must be inside the Dirac trace since it will have a nontrivial Dirac structure, unlike $\mathcal{C}$. In the nonrelativistic limit

$$\tilde{I}=\langle 1\rangle\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\,. \tag{4.4}$$

As discussed in Sec. III, $\tilde{I}$ is effectively the exponential of the static energy. Including all corrections to $\tilde{I}$ from Eq. (2.12) through order $(1/m)^2$ gives

$$\begin{aligned}\tilde{I}=&\langle 1\rangle\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)+i\left[\int_{-T/2}^{T/2}dz\,\frac{1}{2m_1}\langle\vec{D}^2(\vec{x}_1,z)-g\vec{\sigma}\cdot\vec{B}(\vec{x}_1,z)\rangle\right.\\&\left.+\int_{-T/2}^{T/2}dz\,\frac{1}{2m_2}\langle\vec{D}^2(\vec{x}_2,z)-g\vec{\sigma}_2\cdot\vec{B}(\vec{x}_2,z)\rangle\right]\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\\&+\left[\frac{1}{4m_1^2}(-g)\int_{-T/2}^{T/2}dz[(\delta_{ij}-i\epsilon_{ijk}\sigma_1^k)\langle E^i(\vec{x}_1,z)D^j(\vec{x}_1,z)\rangle+(1\leftrightarrow 2)]\right.\\&-\frac{1}{4m_1^2}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,\theta(z'-z)[\langle(D^2-g\vec{\sigma}_1\cdot\vec{B})(\vec{x}_1,z)(\vec{D}^2-g\vec{\sigma}_1\cdot\vec{B})(\vec{x}_1,z')\rangle+(1\leftrightarrow 2)]\\&\left.-\frac{1}{4m_1m_2}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\langle(D^2-g\vec{\sigma}_1\cdot\vec{B})(x_1,z)(\vec{D}^2-g\vec{\sigma}_2\cdot\vec{B})(\vec{x}_2,z')\rangle\right]\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\,.\end{aligned} \tag{4.5}$$

In order $1/m^2$, all terms except the first are iterations of the $1/m$ term. Equation (4.5) may be considerably simplified by using the following observations about the structure of the expectation values. First, note that only the spin-dependent part of $\tilde{I}$ is relevant so that all spin-independent terms, such as $\langle\vec{D}^2\rangle$, may be omitted. Also, parity invariance implies that some terms vanish. For example, $\langle\vec{\sigma}\cdot\vec{B}\rangle=\sigma^i\langle B^i\rangle$, but $\langle B^i\rangle$ must be proportional to R^i, the only three-vector available. Therefore, $\langle B^i\rangle=0$ since $\langle B^i\rangle$ and R^i have opposite parity. The form of $\tilde{I}$ now becomes

$$\begin{aligned}\tilde{I}=&\langle 1\rangle\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)+\left[\frac{+ig}{4m_1^2}\int_{-T/2}^{T/2}dz[\epsilon_{ijk}\sigma_1^k\langle E^i(\vec{x}_1,z)D^j(\vec{x}_1,z)\rangle+(1\leftrightarrow 2)]\right.\\&+\frac{g}{4m_1^2}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'[\sigma_1^i\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle+(1\leftrightarrow 2)]\\&+\frac{g}{4m_1m_2}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'[\sigma_1^i\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_2,z')\rangle+\sigma_2^i\langle\vec{D}^2(\vec{x}_1,z)B^i(\vec{x}_2,z')\rangle\\&\left.-g\sigma_1^i\sigma_2^j\langle B^i(\vec{x}_1,z)B^j(\vec{x}_2,z')\rangle]\right]\delta(\vec{x}_1-\vec{y}_1)(\vec{x}_2-\vec{y}_2)\,.\end{aligned} \tag{4.6}$$

Next, Eq. (4.6) can be reduced further by using the following identities involving the path-ordered exponentials:

$$P(x^0,y^0)P(y^0,z^0)=P(x^0,z^0)\,, \tag{4.7a}$$

$$D^i(x^0)P(x^0,y^0)-P(x^0,y^0)D^i(y^0)=\int_{y^0}^{x^0}dz\,P(x^0,z)gE^i(z)P(z,y^0)\,, \tag{4.7b}$$

$$P(\vec{y},t;\,\vec{x},t)D_i(\vec{x},t)P(\vec{x},t;\,\vec{y},t)=D_i(\vec{y},t)-\epsilon_{ijk}g\int_0^1 d\alpha\,(x-y)^j[P(\vec{y},t,\vec{z},t)B^k(\vec{z},t)P(\vec{z},t;\,\vec{y},t)]\,, \tag{4.7c}$$

where $\vec{z}\equiv\alpha\vec{y}+(1-\alpha)\vec{x}$ in Eq. (4.7c). The first identity is not trivial because of the path ordering necessitated by the noncommutativity of the vector potential. In Eqs. (4.7a) and (4.7b) all spatial points are taken to be equal. All three of these identities are established straightforwardly, for example, by expanding both sides of the equations in a power series in the coupling constant. The third identity, Eq. (4.7c), will be used for large times $t=\pm T/2$ where as $T\to\infty$ gauge magnetic fields are assumed to vanish.[18] In this limit the identity simplifies to

$$P(y,t;x,t)D_i(x,t)P(x,t;y,t)\xrightarrow[|t|\to\infty]{} i\partial_i^y\,. \tag{4.7d}$$

To illustrate the usefulness of these equations, consider the first nontrivial term in Eq. (4.6). Using Eqs. (4.7a) and (4.7b) one obtains

$$\int_{-T/2}^{T/2} dz\, \epsilon_{ijk}\langle gE^i(\vec{x}_1,z)D^j(\vec{x}_1,z)\rangle = \int_{-T/2}^{T/2} dz\, \epsilon_{ijk}\langle gE^i(x_1,z)D^j(x_1,T/2)\rangle \tag{4.8a}$$

since the term involving $\epsilon_{ijk}\langle E^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle$ vanishes by symmetry. Then using Eq. (4.7d)

$$\int_{-T/2}^{T/2} dz\, \epsilon_{ijk}\langle gE^i(\vec{x}_1,z)D^j(\vec{x}_1,T/2)\rangle = i\int_{-T/2}^{T/2} dz\, \epsilon_{ijk}\langle gE^i(\vec{x}_1,z)\rangle\partial_1^j \tag{4.8b}$$

and applying Eq. (4.7b) again to the right-hand side gives

$$\int_{-T/2}^{T/2} dz\, \epsilon_{ijk}\langle gE^i(\vec{x}_1,z)D^j(\vec{x}_1,z)\rangle = -\epsilon_{ijk}\partial_1^i\langle 1\rangle\partial_1^j\,. \tag{4.8c}$$

Similar manipulations lead to the following relationships:

$$\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle \simeq 2ig\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'(z'-z)\langle B^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle\partial_1^j\,, \tag{4.9a}$$

$$\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_2,z')\rangle \simeq 2ig\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'\langle B^i(\vec{x}_1,z)z'E^j(\vec{x}_2,z')\rangle\partial_2^j\,. \tag{4.9b}$$

Here $\simeq$ denotes equality for the spin-dependent terms only. The final form for $\tilde{I}$ now becomes, incorporating the results in Eqs. (4.8) and (4.9),

$$\begin{aligned}\tilde{I} = \langle 1\rangle\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)+\Bigg[&\frac{-i}{4m_1^2}\epsilon_{ijk}\sigma_{1k}\partial_1^i\langle 1\rangle\partial_1^j+\frac{g^2 i}{2m_1^2}\sigma_1^i\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'(z'-z)[\langle B^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle\partial_1^j+(1\leftrightarrow 2)]\\ &+\frac{g^2 i}{2m_1m_2}\sigma_1^i\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'[\langle B^i(\vec{x}_1,z)z'E^j(\vec{x}_2,z')\rangle\partial_2^j+(1\leftrightarrow 2)]\\ &-\frac{g^2}{4m_1m_2}\sigma_1^i\sigma_2^j\int_{-T/2}^{T/2} dz\int_{-T/2}^{T/2} dz'\langle B^i(\vec{x}_1,z)B^j(\vec{x}_2,z')\rangle\Bigg]\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\,.\end{aligned} \tag{4.10}$$

This equation determines all the spin-dependent forces in QCD to order $(1/m)^2$. However, only for the first correction term is the spin dependence given by the static limit. This term $\epsilon_{ijk}\partial_1^i\langle 1\rangle\partial_1^j$ includes the classical spin-orbit interaction and the Thomas precession. It is an effect which was called in Sec. I a type-1 correction to the potential; that is, a relativistic correction to the potential not involving the exchange of transverse gluons between the quark and the antiquark. All the other terms in Eq. (4.10) involve transverse gluon exchange and require new potentials.

Explicitly, the first correction term in Eq. (4.10) is

$$\begin{aligned}&\left(\frac{-i}{4m_1^2}\epsilon_{ijk}\sigma_{1k}\partial_1^i\langle 1\rangle\partial_1^j\right)\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)+(1\leftrightarrow 2)\\ &\qquad= -Te^{-\epsilon(R)T}\left[\frac{\vec{\sigma}_1\times\vec{R}}{4m_1^2}(i\vec{\nabla}_1)-\frac{\vec{\sigma}_2\times\vec{R}}{4m_1^2}(i\vec{\nabla}_2)\right]\frac{1}{R}\frac{d\epsilon(R)}{dR}\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2) \qquad (4.11a)\\ &\qquad= -Te^{-\epsilon(R)T}\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\frac{1}{R}\frac{d\epsilon(R)}{dR}\delta(\vec{x}_1-\vec{y}_1)\delta(\vec{x}_2-\vec{y}_2)\,, \qquad (4.11b)\end{aligned}$$

which when combined with the static energy term $\langle 1\rangle = e^{-\epsilon(R)T}$ results in the potential V' given by

$$V' = \lim_{T\to\infty}\left\{-\frac{1}{T}\ln e^{-\epsilon(R)T}\left[1-T\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\frac{1}{R}\frac{d\epsilon(R)}{dR}\right]\right\} \tag{4.12a}$$

$$= \epsilon(R)+\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\frac{1}{R}\frac{d\epsilon(R)}{dR}\,. \tag{4.12b}$$

Equation (4.12a) follows directly from Eq. (3.10), suitably generalized to include the spin-dependent effects in Eq. (2.11). To obtain Eq. (4.12b), the logarithm in Eq. (4.12a) has been expanded to lowest order in T/m^2 which is a small quantity since to obtain the nonrelativistic limit $m\to\infty$ first and only then does T become large.[19] The appearance of ϵ explicitly in the spin-orbit term in Eq. (4.12b) implies that not all spin-dependent forces are short range in QCD. This result follows only from QCD and the existence of the static limit, and is independent of any detailed mechanism for confinement or of perturbation theory.

The other terms in Eq. (4.10) may also be rewritten as potential terms leading to the total spin-dependent potential V_{SD} given by

$$V_{\mathrm{SD}}(R)=\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\left(\frac{1}{R}\frac{d\epsilon(R)}{dR}+\frac{2}{R}\frac{dV_1(R)}{dR}\right)+\left(\frac{\vec{\sigma}_2\cdot\vec{L}_1}{2m_1m_2}-\frac{\vec{\sigma}_1\cdot\vec{L}_2}{2m_1m_2}\right)\frac{1}{R}\frac{dV_2(R)}{dR}$$
$$+\frac{1}{12m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2V_3(R)+\frac{1}{12m_1m_2}(3\vec{\sigma}_1\cdot\hat{R}\vec{\sigma}_2\cdot\hat{R}-\vec{\sigma}_1\cdot\vec{\sigma}_2)V_4(R)\,, \tag{4.13}$$

with $\epsilon(R)$ the static potential and

$$R_k\frac{1}{R}\frac{dV_1}{dR}\equiv\lim_{T\to\infty}\epsilon_{ijk}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\left(\frac{z'-z}{T}\right)g^2/2\langle B^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle/\langle 1\rangle\,, \tag{4.14a}$$

$$R_k\frac{1}{R}\frac{dV_2}{dR}\equiv\lim_{T\to\infty}\epsilon_{ijk}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\left(\frac{z'}{T}\right)g^2/2\langle B^i(\vec{x}_2,z)E^j(\vec{x}_1,z')\rangle/\langle 1\rangle\,, \tag{4.14b}$$

$$[(\hat{R}^i\hat{R}^j-\tfrac{1}{3}\delta^{ij})V_3+\tfrac{1}{3}\delta^{ij}V_4]\equiv\lim_{T\to\infty}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz\,g^2\left(\frac{1}{T}\right)\frac{\langle B^i(\vec{x}_1,z)B^j(\vec{x}_2,z')\rangle}{\langle 1\rangle}\,. \tag{4.14c}$$

Finally, the potentials V_2 and V_4 are not independent. The expression for V_2 can be rewritten in terms of purely magnetic field correlations. For $V_2(R)$

$$\int_{-T/2}^{T/2}dz'\epsilon_{ijk}i\partial_2^k[\langle B^i(\vec{x}_1,z)E^j(\vec{x}_2,z')\rangle](z'-z)=\int_{-T/2}^{T/2}dz'\epsilon_{ijk}\langle B^i(\vec{x}_1,z)[D^k,E^j](\vec{x}_2,z')\rangle(z'-z)$$
$$+\text{ terms which vanish by symmetry.} \tag{4.15}$$

Now using the Bianchi identity

$$[D_0,B^i]+\epsilon^{ijk}[D_j,E_k]=0\,, \tag{4.16}$$

and then performing an integration by parts over z',

$$\int_{-T/2}^{T/2}dz'\epsilon_{ijk}(z'-z)\partial_2^k\langle B^i(\vec{x}_1,z)E^j(\vec{x}_2,z')\rangle=-\int_{-T/2}^{T/2}dz'\langle B^i(\vec{x}_1,z)B_i(\vec{x}_2,z')\rangle\,. \tag{4.17}$$

Thus the equation for V_2 may be reexpressed as

$$\vec{\nabla}^2V_2(R)=\lim_{T\to\infty}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\frac{-1}{2T}g^2\langle B^i(\vec{x}_1,z)B_i(\vec{x}_2,z')\rangle/\langle 1\rangle\,. \tag{4.18}$$

Comparing Eq. (4.18) with Eq. (4.14c) we have

$$\vec{\nabla}^2V_2(R)=\tfrac{1}{2}V_4(R)\,. \tag{4.19}$$

Thus $V_4(R)$ may be eliminated in Eq. (4.13) to obtain

$$V_{\mathrm{SD}}=\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\left[\frac{1}{R}\frac{d\epsilon(R)}{dR}+\frac{2}{R}\frac{dV_1(R)}{dR}\right]+\left(\frac{\vec{\sigma}_2\cdot\vec{L}_1}{2m_1m_2}-\frac{\vec{\sigma}_1\cdot\vec{L}_2}{2m_1m_2}\right)\frac{1}{R}\frac{dV_2(R)}{dR}$$
$$+\frac{1}{6m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\nabla}^2V_2(R)+\frac{1}{12m_1m_2}(3\vec{\sigma}_1\cdot\hat{R}\vec{\sigma}_2\cdot R-\vec{\sigma}_1\cdot\vec{\sigma}_2)V_3(R)\,. \tag{4.20}$$

Equation (4.20) is the complete spin-dependent potential in QCD through order $(1/m)^2$ with V_1, V_2, and V_3 given in Eq. (4.14), Eq. (4.17), and Eq. (4.14c), respectively, as expectation values of the appropriate electric and magnetic fields. In QED the V_i's may be evaluated explicitly to order α (in which case $V_1=0$) to reproduce the Breit equation. However, Eq. (4.20) is valid both in QED and QCD, independent of perturbation theory, and will serve as the basis of the examination of spin-dependent forces given in the remaining sections.

V. PSEUDOPARTICLE CONTRIBUTIONS TO SPIN-DEPENDENT FORCES

The general form for the spin-dependent forces in QCD as given in Eq. (4.13) can be further simplified when only instanton contributions are considered. The pseudoparticle solutions of the spin-dependent potential included here arise from all solutions of the Euclidean field equations which also satisfy the duality $\vec{E}=\vec{B}$ (or antiduality $\vec{E}=-\vec{B}$) condition and therefore the results are valid beyond the dilute-gas approximation.[20] They include arbitrary multi-instanton contributions provided only that the configuration is a local minimum of the Euclidean action (i.e., multi-instanton regions and multi-anti-instanton regions of space-time are well separated). Furthermore, the only property of the pseudoparticle solutions needed is their definition ($\vec{E}=\pm\vec{B}$); no particular construction of these configurations or gauge conditions need be specified. The potentials in this section are labeled by the subscript I to emphasize the fact that only the contribution from pseudoparticles is included.

To investigate the spin-dependent forces in the nonzero topological charge sector, we start with Eq. (4.20):

$$V_{\rm SD}^I(R)=\left(\frac{\vec{\sigma}_1\cdot\vec{L}_1}{4m_1^2}-\frac{\vec{\sigma}_2\cdot\vec{L}_2}{4m_2^2}\right)\left[\frac{1}{R}\frac{d\epsilon^I(R)}{dR}+\frac{2}{R}\frac{dV_1^I(R)}{dR}\right]+\left(\frac{\vec{\sigma}_2\cdot\vec{L}_1}{2m_1m_2}-\frac{\vec{\sigma}_1\cdot\vec{L}_2}{2m_1m_2}\right)\frac{1}{R}\frac{dV_2^I(R)}{dR}+\frac{1}{6m_1m_2}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\nabla}^2V_2^I(R)+\frac{1}{12m_1m_2}(3\vec{\sigma}_1\cdot\hat{R}\vec{\sigma}_2\cdot\hat{R}-\vec{\sigma}_1\cdot\vec{\sigma}_2)V_3^I(R)\,. \tag{5.1}$$

The potentials V_1^I, V_2^I, and V_3^I are given by the pseudoparticle contributions to the Euclidean space expectation values in Eqs. (4.14a) and (4.14c), using Eq. (4.19):

$$R_k\frac{d}{dR}V_1^I(R)=\lim_{T\to\infty}\epsilon_{ijk}\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,\frac{(z'-z)}{T}\frac{g^2}{2}\times\langle B^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle/\langle 1\rangle\,, \tag{5.2a}$$

$$[(\hat{R}^i\hat{R}^j-\tfrac{1}{3}\delta^{ij})V_3^I(R)+\tfrac{2}{3}\delta^{ij}\vec{\nabla}^2V_2^I(R)]=\lim_{T\to\infty}\left(\frac{1}{T}\right)\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'g^2\frac{\langle B^i(\vec{x}_1,z)B^j(\vec{x}_2,z')\rangle}{\langle 1\rangle}\,. \tag{5.2b}$$

It is easy to see that in the instanton sector the spin-spin and tensor force terms, $\vec{\nabla}^2V_2^I$ and V_3^I are simply related to the nonrelativistic potential ϵ^I since

$$g^2\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\langle B^i(\vec{x}_1,z)B^j(\vec{x}_2,z')\rangle=g^2\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\langle E^i(\vec{x}_1,z)E^j(\vec{x}_2,z')\rangle \tag{5.3a}$$

$$=-\partial_1^i\partial_2^j\langle 1\rangle\,, \tag{5.3b}$$

$$\lim_{T\to\infty}-\frac{1}{T}\frac{\partial_1^i\partial_2^j\langle 1\rangle}{\langle 1\rangle}=\frac{\partial}{\partial R^j}\frac{\partial}{\partial R^i}[\epsilon^I(R)]=\left[(\hat{R}^i\hat{R}^j-\tfrac{1}{3}\delta^{ij})\left(\frac{d^2}{dR^2}-\frac{1}{R}\frac{d}{dR}\right)\epsilon^I(R)+\tfrac{1}{3}\delta^{ij}\vec{\nabla}^2\epsilon^I(R)\right]. \tag{5.3c}$$

Using Eqs. (5.3c) and (5.3b) in Eq. (5.2b) we conclude

$$V_3^I(R)=\left(\frac{d^2}{dR^2}-\frac{1}{R}\frac{d}{dR}\right)\epsilon^I(R) \tag{5.4a}$$

and

$$V_2^I(R)=\tfrac{1}{2}\epsilon^I(R) \tag{5.4b}$$

in which Eq. (5.3b) is a consequence of Eq. (4.7).

There is an additional identity for pseudoparticle solutions which relates V_1^I and V_2^I. Using Eq. (4.7) and the dual property of pseudoparticle solutions ($\vec{E}=\pm\vec{B}$) it is straightforward to show that

$$\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,g\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle=\pm\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,g\langle E^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle=\pm\int_{-T/2}^{T/2}dz\,i\partial_1^i\langle\vec{D}^2(\vec{x}_1,z)\rangle\mp\int_{-T/2}^{T/2}dz\,\langle[D^i,\vec{D}^2](\vec{x}_1,z)\rangle \tag{5.5}$$

in which the commutator term may be expanded as follows:

$$[D^i,D^2]=ig\epsilon^{ijk}(B_kD_j+D_jB_k)=\pm 2ig\epsilon^{ijk}E_kD_j+\text{nonspin-dependent terms.} \tag{5.6}$$

Inserting Eq. (5.6) into Eq. (5.5) and observing that $\partial_1^i=-\partial_2^i$ produces

$$\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,g\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle\simeq\pm\int_{-T/2}^{T/2}dz\,i\partial_2^i\langle\vec{D}^2(\vec{x}_1,z)\rangle-2ig\epsilon^{ijk}\int_{-T/2}^{T/2}dz\,\langle E_kD_j(\vec{x}_1,z)\rangle \tag{5.7a}$$

$$\simeq-\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,g\langle B^i(\vec{x}_2,z)\vec{D}^2(\vec{x}_1,z')\rangle+2i\epsilon^{ijk}(\partial_1^k\langle 1\rangle)\,\partial_1^j\,. \tag{5.7b}$$

Note that this result, Eq. (5.7b), is independent of whether the pseudoparticle configuration is self-dual or anti-self-dual.[21] Now using Eqs. (4.9a) and (4.9b), (4.13), and (4.14a) and (4.14b) we have

$$\lim_{T\to\infty}\frac{1}{T}g\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\frac{\langle B^i(\vec{x}_1,z)\vec{D}^2(\vec{x}_1,z')\rangle}{\langle 1\rangle}\simeq\lim_{T\to\infty}\frac{1}{T}2g^2 i\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,(z'-z)\langle B^i(\vec{x}_1,z)E^j(\vec{x}_1,z')\rangle\,\partial_1^j$$

$$\simeq 2L_1^i\frac{1}{R}\frac{dV_1^I(R)}{dR} \tag{5.8a}$$

and

$$\lim_{T\to\infty}-\frac{1}{T}g\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\frac{\langle B^i(\vec{x}_2,z)\vec{D}^2(\vec{x}_1,z')\rangle}{\langle 1\rangle}\simeq\lim_{T\to\infty}-\frac{1}{T}2g^2 i\int_{-T/2}^{T/2}dz\int_{-T/2}^{T/2}dz'\,z'\frac{\langle B^i(\vec{x}_2,z)E^j(\vec{x}_1,z')\rangle}{\langle 1\rangle}\,\partial_1^j$$

$$\simeq 2L_1^i\frac{1}{R}\frac{dV_2^I(R)}{dR}. \tag{5.8b}$$

Hence using Eqs. (5.8a), (5.8b), (4.11a), and (4.11b) in Eq. (5.7b) the desired relation between V_1^I and V_2^I results in the following:

$$V_1^I(R)=V_2^I(R)-\epsilon^I(R). \tag{5.9}$$

Therefore, all terms in the spin-dependent potential have been reexpressed in terms of the spin-independent potential for the pseudoparticle contributions. [See Eqs. (5.4a), (5.4b), and (5.9).] The final result of this analysis is given by

$$V_{SD}^I(R)=\frac{\vec{S}\cdot\vec{L}}{2m_1m_2}\frac{1}{R}\frac{d\epsilon^I(R)}{dR}+\frac{1}{3m_1m_2}\vec{S}_1\cdot\vec{S}_2\,\vec{\nabla}^2\epsilon^I(R)$$
$$+\frac{1}{3m_1m_2}[(\vec{S}_1\cdot\hat{R})(\vec{S}_2\cdot\hat{R})-\vec{S}_1\cdot\vec{S}_2]$$
$$\times\left(\frac{d^2}{dR^2}-\frac{1}{R}\frac{d}{dR}\right)\epsilon^I(R), \tag{5.10}$$

where $\vec{L}=\vec{L}_1=-\vec{L}_2$, $\vec{S}_1=\vec{\sigma}_1/2$, $\vec{S}_2=\vec{\sigma}_2/2$, and $\vec{S}=\vec{S}_1+\vec{S}_2$.

Unfortunately, although Eq. (5.10) expresses all the spin-dependent forces due to pseudoparticle solution in terms of their contribution to the static energy $\epsilon^I(R)$, it is not directly useful for the phenomenology of heavy-quark systems as the static energy $\epsilon^I(R)$ is difficult to calculate.[11] Even so, there is important information about the general form of spin-dependent forces in QCD contained in Eq. (5.10). We may conclude by comparing the contributions from instantons in Eq. (5.10) with the perturbative contribution to the spin-dependent force which in lowest order [see Eq. (7.1)] are essentially the same as in QED [Eq. (1.3)] that there can be no further relations between V_1, V_2, and V_3 without making specific dynamical assumptions in QCD (e.g., those to be discussed in Sec. VII). Comparing the two equations we find for perturbation theory [see Eq. (1.3)]

$$V_3=\left(\frac{1}{R}\frac{d}{dR}-\frac{d^2}{dR^2}\right)V_2$$

and $V_1(R)$ of order α_s^2; while for instantons [see Eqs. (5.4a), (5.4b), and (5.9)]

$$V_3=-\left(\frac{1}{R}\frac{d}{dR}-\frac{d^2}{dR^2}\right)V_2$$

and

$$V_1=V_2-\epsilon^I.$$

Thus no general relationship can exist between V_1, V_2, and V_3 in QCD.

VI. PHENOMENOLOGY: GENERAL CONSIDERATIONS

The general form for the spin-dependent forces in QCD through order $(1/m)^2$ is given by Eq. (4.20). This form may be directly applied to meson systems involving two sufficiently heavy quarks. We will discuss the quantitative applications to such systems in the next section. However, the success of nonrelativistic dynamics [such as SU(6) predictions for ordinary mesons and baryons] suggests that for dynamical reasons not yet completely understood the form of Eq. (4.20) may be more general than is apparent from its derivation. Our modest extension of it from the nonrelativistic limit is to meson systems involving one heavy and one light quark such as the charmed-meson systems $D^0(c\bar{u})$, $D^+(c\bar{d})$, and $F(c\bar{s})$, and the corresponding bottom-quark systems $B^-(b\bar{u})$, $B^0(b\bar{d})$, and $E^0(b\bar{s})$. If M is the heavy-quark mass and m is the mass of the lighter quark, then the dynamics is governed by the reduced mass $\mu=Mm/(M+m)\approx m$ for $M\gg m$. Even if m is not large, the Dirac structure obtained for heavy-quark systems may still be valid but now with the various $V_i(R)$ completely unknown functions of R and μ, and not given by the expectation values derived in Sec. IV. A simplification in the unequal mass case is that the annihilation term identically vanishes. The form of the spin-dependent potential for such heavy-light systems then becomes

$$V_{SD}=\vec{S}_l\cdot\vec{L}\,V_a(R;\mu)$$
$$+\frac{\mu}{M}(\vec{S}_l+\vec{S}_H)\cdot\vec{L}\,V_b(R;\mu)+\frac{\mu}{M}\vec{S}_l\cdot\vec{S}_H V_c(R;u)$$
$$+\frac{\mu}{M}(3\vec{S}_l\cdot\hat{R}\,\vec{S}_H\cdot\hat{R}-\vec{S}_l\cdot\vec{S}_H)V_d(R;\mu)+O\left(\frac{\mu^2}{M^2}\right), \tag{6.1}$$

where V_a, V_b, V_c, V_d are completely unknown functions of R and μ. $\vec{S}_H$ and $\vec{S}_l$ are the spins of the heavy and the light quark, respectively ($\mu/M \ll 1$). The dominant term is $\vec{S}_l \cdot \vec{L} \equiv \frac{1}{2}(\vec{J}_l^{\,2} - \vec{S}_l^{\,2} - \vec{L}_l^{\,2})$ where $\vec{J}_l = \vec{S}_l + \vec{L}$ so that the coupling scheme is jj and not ls in atomic physics terminology.

Equation (6.1) may be used to obtain scaling relations between mesons which differ only in the flavor of their heavy constituent. All the dependence of the energy on M is explicit in Eq. (6.1). The dynamics of the wave functions and spin-dependent functions $V_a, \ldots, V_d$ is determined by μ alone; thus the coupling between states $|nLJ\rangle$ and $|n'L'J\rangle$ due to the spin-dependent forces is given by

$$\epsilon_J(nL, n'L') = M + \epsilon_0(n,L)\delta_{n'n}\delta_{L'L} + \vec{S}_l \cdot \vec{L}\,\delta_{LL'}\epsilon_a(n,L;n') + \frac{\mu}{M}(\vec{S}_l + \vec{S}_H)\cdot \vec{L}\,\delta_{LL'}\epsilon_b(n,L;n')$$
$$+ \frac{\mu}{M}\vec{S}_H \cdot \vec{S}_l \delta_{LL'}\epsilon_c(n,L;n') + \frac{\mu}{M}\langle \vec{L}|3\vec{S}_l\cdot\hat{R}\,\vec{S}_H\cdot\hat{R} - \vec{S}_l\cdot\vec{S}_H|\vec{L}'\rangle\epsilon_d(n,L;n'L')\,. \tag{6.2}$$

Equation (6.2) is still a matrix equation in the spin degrees of freedom of the states. The quantities ϵ_0, ϵ_a, ϵ_b, ϵ_c, and ϵ_d are universal numbers for heavy-light systems. Once determined in some heavy-light system, the masses for any other such systems are obtained by simply scaling M appropriately. For example, the excitation spectrum of the charmed mesons will determine this quantities ϵ_0, ϵ_a, ϵ_b, ϵ_c, and ϵ_d and so the B-meson excitation spectrum can be predicted. For ground-state mesons ($L=0$), only ϵ_0 and ϵ_c have nonzero coefficients and the spin splitting will be proportional to the ratio of the masses of the heavy quarks:

$$m(B^*) - m(B) = \frac{m_c}{m_b}[m(D^*) - m(D)] \simeq 50 \text{ MeV}. \tag{6.3}$$

Using the experimental information on the D, D^*, F, and F^* masses,[22,23] and $m_c = 1.84$ GeV and $m_b = 5.17$ GeV the spectrum of ground states of the B meson can be determined. The results are shown in Fig. 4. Similarly, P-state splittings for the $c\bar{u}$ system determine those of $b\bar{u}$ via Eq. (6.1). Although those splittings have not yet been measured one could attempt to use the K ($s\bar{u}$) meson system to make these estimates.[24] Of course, treating the strange quark as heavy is a questionable approximation. In analogy to Eq. (6.2),

$$m(K^*) - m(K) = \frac{m_c}{m_s}[m(D^*) - m(D)] \approx 500 \text{ MeV}, \tag{6.4}$$

while actually the difference is ~400 MeV. The approximate validity of Eq. (6.4) supports the application of Eq. (6.1) even to the strange mesons.

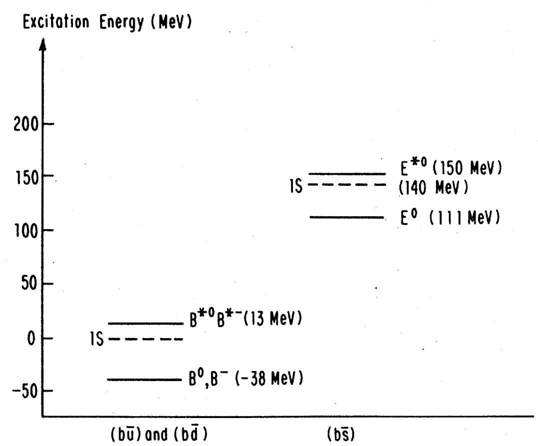

FIG. 4. The spin splittings for the ground-state (1S) $B^0(b\bar{u})$, $B^-(b\bar{d})$, and $E^0(b\bar{s})$ mesons. The small mass differences arising from electroweak interactions and the current-algebra u-d quark-mass difference have been ignored [e.g., $m(B^0) - m(B^-) \approx 4.4$ MeV]. The center of gravity (c.o.g.) of the 1S state is indicated by a dashed line.

Even more speculative is the assumption that Eq. (4.13) has the correct structure for the spin-dependent effects for light mesons. The form of the spin-dependent potential would be

$$V_{SD}(R) = \left(\frac{\vec{S}_1\cdot\vec{L}}{2m_1^{\,2}} + \frac{\vec{S}_2\cdot\vec{L}}{2m_2^{\,2}}\right)V_a'(R) + \frac{(\vec{S}_1+\vec{S}_2)\cdot\vec{L}}{2m_1m_2}V_b'(R) + \frac{\vec{S}_1\cdot\vec{S}_2}{m_1m_2}V_c'(R) + \frac{1}{m_1m_2}(3\vec{S}_1\cdot\hat{R}\,\vec{S}_2\cdot\hat{R} - \vec{S}_1\cdot\vec{S}_2)V_d'(R)\,, \tag{6.5}$$

with $V'_{a,b,c,d}$ completely unknown. Since the spin splittings in light mesons are not as large as the separations between the radial or orbital excitations, it might be possible to treat V_{SD} perturbatively. Furthermore, assuming SU(3) symmetry, the expectation values of $V_{a,b,c,d}$ are independent of the quark state (u, d, or s) so that many relationships between meson masses are obtained. In fact, the potential in Eq. (6.5) was used by De Rújula, Georgi, and Glashow[14] in the SU(3) limit. They obtained the above form by making the assumption of single-gluon exchange between the quarks which of course does dominate at very small quark-antiquark separation. The results of their analysis indicate that Eq. (6.5) is valid to within 30% [the order of SU(3) breaking]. Naively it would not be expected that Eq. (6.5), derived by extrapolating the nonrelativistic expression Eq. (4.20) into a highly relativistic region, would be so successful.

VII. PHENOMENOLOGY: HEAVY-QUARK-ANTIQUARK SYSTEMS

In this section the spin-dependent splittings are obtained for a bound state of a heavy quark and a heavy antiquark. Although Eq. (4.13) is directly applicable to these states, the potentials V_i ($i=1,\ldots,4$) are not presently calculable (except in perturbation theory) even though they are given explicitly in terms of various expectation values. Furthermore, a model of the static energy is needed to make explicit predictions.

To proceed with a phenomenological analysis, the following basic assumption is made: The longitudinal color electric field alone is responsible for the long-range part of the static energy. Such an assumption, that the confinement mechanism is due to longitudinal electric fields, is a direct generalization of the confinement mechanism in the two-dimensional Schwinger model and the mechanism indicated by the lattice gauge theories (at least in the strong-coupling region) as well as string models. Furthermore, the longitudinal component of the electric field may be isolated by a gauge-invariant procedure[15] so that the results are not limited to a particular gauge, such as the radiation gauge, in which isolation of the longitudinal electric field is particularly easy. Magnetic field correlations are therefore short range and should be calculable in perturbation theory. In particular, since V_1, V_2, and V_3 as given in Eqs. (4.14a), (4.14c), (4.19), and (4.20) are determined by magnetic field correlations, they should be given reasonably accurately by their lowest-order perturbation-theory expressions. Therefore, one obtains a potential for spin-dependent forces in QCD without the introduction of any new parameters:

$$V_{SD}(R)=\left(\frac{1}{2m_1^2}\vec{L}\cdot\vec{S}_1+\frac{1}{2m_2^2}\vec{L}\cdot\vec{S}_2\right)\frac{1}{R}\frac{d\epsilon(R)}{dR}$$
$$+(\tfrac{4}{3}\alpha_s)\frac{1}{m_1m_2}\frac{1}{R^3}\vec{L}\cdot\vec{S}$$
$$+(\tfrac{4}{3}\alpha_s)\frac{2}{3m_1m_2}\vec{S}_1\cdot\vec{S}_2 4\pi\delta(\vec{R})$$
$$+(\tfrac{4}{3}\alpha_s)\frac{1}{m_1m_2}(3\vec{S}_1\cdot\hat{R}\,\vec{S}_2\cdot\hat{R}-\vec{S}_1\cdot\vec{S}_2)\frac{1}{R^3}, \tag{7.1}$$

where $\vec{S}=\vec{S}_1+\vec{S}_2$, α_s is the running coupling constant in QCD, and $\epsilon(R)$ is the static energy. The only corrections to Eq. (7.1) arise from higher orders in perturbation theory, possible inclusion of pseudoparticle contributions, and higher-order relativistic corrections.

Some general comments about the assumption of electric confinement and a short-range magnetic force are appropriate. For the spin-spin and tensor forces this assumption leads to the dominance of single-gluon exchange, in Eq. (7.1), for these forces (the result of lowest-order perturbation theory). This is the same assumption made previously by De Rújula, Georgi, and Glashow,[14] and by Jaffe[25] who investigated the force associated with the "magnetic" component of single-gluon exchange within the context of the MIT bag model. However, the treatment of the spin-orbit interaction arises here as the natural consequence of the underlying vector gauge character of elementary quark-gluon interactions and the assumption of a short-range magnetic force, and does not agree with these works. The assumption of only a single gluon exchange of course does not give any long-range component ot the spin-orbit force. Within the MIT bag model[26] there is a long-range component associated with the quark motion in the effective bag potential (a Thomas term). However, Eq. (7.1) contains a long-range term arising from the vector nature of the interaction in addition to a Thomas term.

Finally, a phenomenological model for $\epsilon(R)$ is needed if Eq. (7.1) is to be used to estimate spin splittings. For this purpose the linear plus Coulomb potential is used with parameters as determined by the Cornell model[4]:

$$\epsilon(R)=-\frac{K}{R}+\frac{R}{a^2}, \tag{7.2}$$

where $a=2.34\ \text{GeV}^{-1}$, and $K_\psi=0.52$ and $K_\Upsilon=0.48$. This potential gives a very satisfactory description of the nonrelativistic structure for both the charmonium ($c\bar{c}$) and the ($b\bar{b}$) systems with $m_c=1.84$ GeV and $m_b=5.17$ GeV, respectively. Substituting $\epsilon(R)$ given in Eq. (7.2) into Eq. (7.1) for the special case $m_1=m_2$ (appropriate to the J/ψ and Υ families of resonances) gives

$$V_{SD}(R)=\frac{1}{2m^2}\vec{L}\cdot\vec{S}\left(\frac{K}{R^3}+\frac{1}{a^2R}\right)+\frac{1}{m^2}\vec{L}\cdot\vec{S}\frac{4\alpha_s}{3}\frac{1}{R^3}$$
$$+\frac{2}{3m^2}\vec{S}_1\cdot\vec{S}_2\frac{4\alpha_s}{3}4\pi\delta(\vec{R})$$
$$+\frac{1}{m^2}(3\vec{S}_1\cdot\hat{R}\,\vec{S}_2\cdot\hat{R}-\vec{S}_1\cdot\vec{S}_2)\frac{4\alpha_s}{3}\frac{1}{R^3}, \tag{7.3}$$

where α_s is the running coupling evaluated at $q^2=4m^2$. It is interesting to compare this potential with a general phenomenological analysis based on the Breit equation in QED reviewed by Appelquist, Barnett, and Lane.[2] An instantaneous Bethe-Salpeter kernel consisting of vector and scalar interaction terms is typically assumed:

$$V_{\rm Coulomb}(k^2)\gamma_1^\mu\gamma_{2\mu}+V_V(k^2)\Gamma_1^\mu(k)\Gamma_{2\mu}(K)+V_S(k^2)1_1 1_2\,, \tag{7.4a}$$

where 1 is a unit matrix in Dirac space and

$$\Gamma_\mu(k)=\gamma_\mu-\frac{i\lambda}{2m}\sigma_{\mu\nu}k^\nu \tag{7.4b}$$

with λ being a color magnetic moment of the quark. Further, $V_{\rm Coulomb}=-K/R$, $V_V=\eta(R/a^2)$, and $V_S=(1-\eta)(R/a^2)$. λ and η are adjustable parameters $(0\leq\eta\leq 1)$ and $K\equiv\frac{4}{3}\alpha_s$. The resulting potential is

$$V(R,K,\lambda)=\frac{1}{2m^2}\left[\frac{4K}{R^3}+4(1+\lambda)\eta\frac{1}{Ra^2}-\frac{1}{R}\frac{d}{dR}\left(-\frac{K}{R}+\frac{R}{a^2}\right)\right]\vec{L}\cdot\vec{S}+\frac{2}{3m^2}\left[4\pi K\delta(\vec{R})+(1+\lambda)^2\eta\frac{1}{Ra^2}\right]\vec{S}_1\cdot\vec{S}_2$$
$$+\frac{1}{3m^2}\left[\frac{3K}{R^3}+(1+\lambda)^2\eta\frac{1}{Ra^2}\right](3\vec{S}_1\cdot\hat{R}\vec{S}_2\cdot\hat{R}-\vec{S}_1\cdot\vec{S}_2)\,. \tag{7.5}$$

The last contribution to the $\vec{L}\cdot\vec{S}$ interaction is the Thomas term. The scalar part of the potential $V_S=(1-\eta)R/a^2$ contributes only here. Considering only the $\vec{L}\cdot\vec{S}$ terms in Eq. (7.5) one would conclude $\eta=\frac{1}{2}$, $\lambda=0$ gives agreement with Eq. (7.3). That is, that the confining potential is a mixture of a vector and scalar exchange. However, in reality, the assumption that went into deriving Eq. (2.3) was the vector nature of the color magnetic interaction.

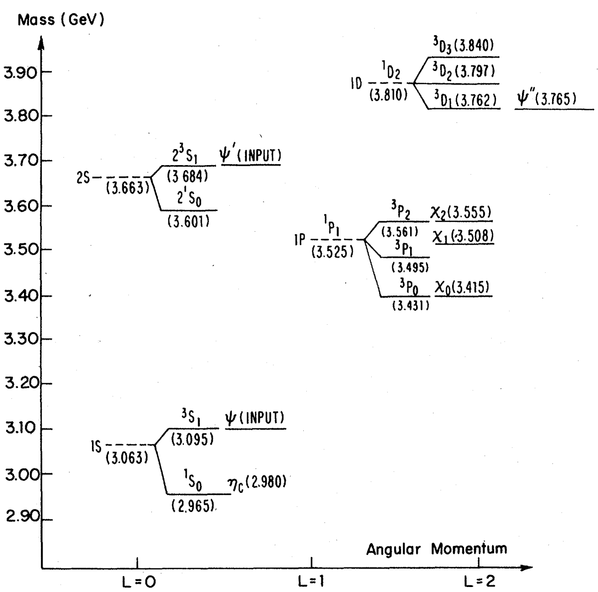

FIG. 5. The spectrum of low-lying ($c\bar{c}$) meson states. The dashed lines are the spectrum of states arising from the nonrelativistic potential alone. The solid lines, the results of including the spin-dependent corrections. The experimental masses of observed states are also included. The J/ψ and ψ' masses and the center of gravity of the $1\,^3P_J$ states was used as input to the nonrelativistic potential model employed.

Comparing the full form of Eqs. (7.5) and (7.3) it is clear that an analysis based on the Breit equation and the instantaneous Bethe-Salpeter kernel of Eqs. (7.4) is inadequate. Equation (7.3) cannot be obtained for any values of η and λ. The Bethe-Salpeter kernel can be further generalized by allowing for the exchange of all possible (parity-conserving) Dirac invariants, scalar V_S, pseudoscalar V_P, vector V_V, axial vector V_A, and tensor V_T. This more general form still does not allow agreement with Eq. (7.3).[27]

The spin splittings induced by the potential in Eq. (7.3) for the $1S$, $1P$, $2S$, $1D$ states of the $c\bar{c}$ system are given in Fig. 5. The running coupling constant $\alpha_s(q^2)$ is given by

$$\alpha_s(q^2)=\tfrac{12}{27}\pi[\ln(q^2/\Lambda_c^2)]^{-1}\,,$$

where Λ_c is taken to be 400 MeV. We choose $q^2=4m_c^2$ and hence $\alpha_s=\alpha_s(4m_c^2)=0.341$ as the expansion parameter. This choice of α_s would not be expected to lead to large logarithmic terms in the next order in α_s for the $c\bar{c}$ states. The expectation values of the powers of R needed were evaluated using the Cornell-model potential given in Eq. (7.2). For comparison, the experimental values[28] for the masses are also given in the figure. For the ^3P_J states the qualitative agreement with the experiment is good. The Cornell potential model was used to fit the center of gravity (c.o.g.) of the P states. Therefore, only the splittings directly test the spin-dependent potential of Eq. (7.3). One parameter which has been used to characterize the P-state splittings is[29]

$$r=\frac{m(^3P_2)-m(^3P_1)}{m(^3P_1)-m(^3P_0)}\,. \tag{7.6}$$

Experimentally $r_{\rm obs}=0.506\pm0.018$ while Eq. (7.3) gives $r_{th} = 1.02$. For comparison a pure $\vec{L}\cdot\vec{S}$ coupling gives $r = 2$ whereas a pure Coulomb potential gives $r = \frac{4}{5}$. The details of the contributions to the spin splitting from the perturbative term and confining potential terms in Eq. (7.5) are shown in Table I. The splitting, J/ψ-η_c has recently been measured experimentally[30]: $m(J/\psi) - m(\eta_c) = 116 \pm 9$ MeV. The result agrees well with the lowest-order splitting calculated above. However, the theoretical calculation has uncertainties that must be mentioned. The spin-spin force here is the lowest-order perturbative result first suggested by Appelquist and Politzer,[31] a result which depends linearly on α_s and the wave function at the origin squared. However, caution must be exercised in using relations depending on the wave function at the origin since even in the nonrelativistic limit they may have large perturbative corrections. For example, the leptonic width of the J/ψ is related to the wave function at the origin in lowest order in α_s by the Van-Royen–Weisskopf relation:

$$\Gamma(J/\psi \to l^+l^-) = 4(\tfrac{2}{3})\alpha^2 |\Psi(0)|^2 / M_{J/\psi}^2 . \tag{7.7a}$$

This relation does not agree with experiment for the value of $|\Psi(0)|^2$ calculated with the potential model of Eq. (7.2). However, first-order corrections in α_s have been calculated[32] and are indeed large. The corrected relation is

$$\Gamma(J/\psi \to l^+l^-) = 4(\tfrac{2}{3})\alpha^2 / M_{J/\psi}^2 |\Psi(0)|^2 [1 - \tfrac{16}{9}\alpha_s + O(\alpha_s^2)] . \tag{7.7b}$$

Equation (7.7b) to order α_s actually works well for the J/ψ resonance but, of course, there is no assurance that there are not also large corrections in order α_s^2. Similarly there might be large perturbative corrections to the spin-spin splittings, but these corrections are not identical to those for the leptonic width, and the agreement between the lowest-order result and experiment suggests that the total correction term is actually small. A one-loop α_s calculation[33] should resolve this question. However, the separation of the short- and long-distance parts of the correction for the spin-spin force is not as simple as it is for the leptonic width.

The perturbative corrections to the lowest-order relation between singlet-triplet splitting for S states and the wave function squared at the origin will cancel out in ratios of splitting for different radial quantum number. Thus the relation

$$\frac{m(\psi') - m(\eta_c')}{m(J/\psi) - m(\eta_c)} = \frac{|\Psi_{2s}(0)|^2}{|\Psi_{1s}(0)|^2} = \frac{\Gamma(\psi' \to e^+e^-) M_{\psi'}^2}{\Gamma(\psi \to e^+e^-) M_{J/\psi}^2} \tag{7.8a}$$

should hold independent of the size of the perturbative corrections. Thus the splitting between ψ' and η_c' is expected to be

$$m(\psi') - m(\eta_c') = 80 \pm 15 \text{ MeV} \tag{7.8b}$$

using Eq. (7.8a) and the experimental values of the leptonic width of the J/ψ and ψ' and the $m(\eta_c)$.

Now we analyze the $b\bar{b}$ system in which the heavy-quark expansion should be better. The parameters for the $b\bar{b}$ resonances are $\alpha_s(4m_b^2) = \alpha_s(4m_c^2)[1 + (25/12\pi)\ln(m_b^2/m_c^2)]^{-1} = 0.227$, $K_\Upsilon = 0.483$, $m_b = 5.17$ GeV, and $a = 2.34$ GeV^{-1}. The resulting spectrum[34] including the spin-dependent forces is shown in Fig. 6. The detailed contributions to the spin splittings from each of the terms

TABLE I. Details of spin splittings in the $(c\bar{c})$ system (in MeV).

State	Perturbative terms	Potential terms Linear	Coulomb	Total
$^3S_1$	+32.4	0	0	32.4
$^1S_0$	−97.4	0	0	−97.4
$^3P_2$	15.9	10.1	10.45	36.5
$^1P_1$	0	0	0	0.0
$^3P_1$	−8.8	−10.1	−10.45	−29.4
$^3P_0$	−53.1	−20.2	−20.9	−94.1
$^3S_1$	20.7	0	0	20.7
$^1S_0$	−62.2	0	0	−62.2
$^3D_3$	8.8	5.4	15.6	29.8
$^1D_2$	0	0	0	0
$^3D_2$	−2.4	−2.7	−7.8	−12.9
$^3D_1$	−16.5	−8.7	−23.3	−48.5

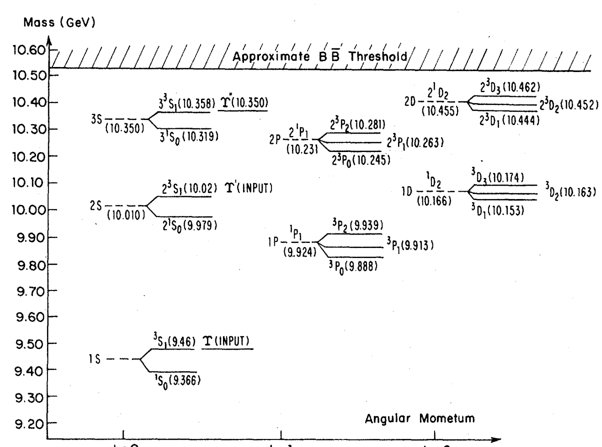

FIG. 6. The spectrum of low-lying $(b\bar{b})$ meson states. Notation as in Fig. 5. The Υ and Υ' masses were used as input to the nonrelativistic potential model. The threshold for Zweig-allowed decays is indicated.

TABLE II. Details of spin splittings in $b\bar{b}$ system (in MeV).

State	Perturbative term	Potential terms Linear	Coulomb	Total
$^3S_1$	+23.7	0	0	23.7
$^1S_0$	−71.2	0	0	−71.2
$^3P_2$	6.3	2.0	5.5	13.8
$^1P_1$	0	0	0	0.0
$^3P_1$	−3.5	−2.0	−5.5	−11.0
$^3P_0$	−20.9	−3.9	−11.0	−35.8
$2\,^3S_1$	+10.3	0	0	10.3
$2\,^1S_0$	−30.9	0	0	−30.9
$^3D_3$	2.6	3.0	2.2	7.8
$^1D_2$	0	0	0	0.0
$^3D_2$	−0.7	−1.5	−1.1	−3.3
$^3D_1$	−4.9	−4.4	−3.3	−12.6
$2\,^3P_2$	4.6	1.6	4.0	10.2
$2\,^1P_1$	0	0	0	0.0
$2\,^3P_1$	−2.5	−1.6	−4.0	−8.2
$2\,^3P_0$	−15.2	−3.2	−8.0	−26.4
$3\,^3S_1$	+7.8	0	0	7.8
$3\,^1S_0$	−23.6	0	0	−23.6
$2\,^3D_3$	2.4	2.2	2.1	6.7
$2\,^1D_2$	0	0	0	0.0
$2\,^3D_2$	−0.7	−1.1	−1.0	−2.8
$2\,^3D_1$	−4.6	−3.3	−3.1	−11.0

in Eq. (7.3) are given in Table II.

The predictions for the spin splittings in the $b\bar{b}$ as well as the $c\bar{c}$ systems are the result of three separate approximations within the electric-confinement hypothesis:

(1) The nonrelativistic assumption—the spin-dependent interaction is treated only to lowest order in v^2/c^2. The expected magnitudes of neglected terms are $\sim\langle v^2/c^2\rangle$ × typical splittings, and $\langle v^2/c^2\rangle_\psi = 0.2$ while $\langle v^2/c^2\rangle_\Upsilon = 0.1$.

(2) Short-range perturbative assumption—the short-range (magnetic) terms of the spin-dependent interaction are treated in lowest-order perturbation theory, i.e., $O(\alpha_s)$. For the $c\bar{c}$ system $\alpha_s = 0.34$ while for the $b\bar{b}$ system $\alpha_s = 0.23$. Therefore, the nominal order of the neglected terms is ~35% in the $c\bar{c}$ and ~25% in the $b\bar{b}$ system—of course, the actual correction will depend on the particular term considered.

(3) Neglected contributions are small—instantons, light-quark pairs, and annihilation terms.[35] The contribution from pseudoparticle solutions was given in Sec. V in terms of the pseudoparticle contributions to the static energy which, however, are difficult to estimate quantitively. Certainly the contributions to a low-lying $(b\bar{b})$ state will be smaller than the corresponding $(c\bar{c})$ state. The effects of light-quark pairs have been studied by the Cornell group[4] by including the effects of coupling to virtual-charmed-meson channels. The results indicate that the dominant effects of this coupling can be absorbed into a redefinition of the potential parameters K, a, and the mass m_c (or m_b). Finally, the annihilation terms begin only in order α_s^2 for $C = +1$ mesons and α_s^3 for $C = -1$ mesons. Therefore, they are higher order in α_s and fall under category (2) above.

In all cases we find (as expected) much more reliable calculations may be performed for the $b\bar{b}$ system than the $c\bar{c}$ system.

Finally, we turn to an unusual heavy-quark system—$(b\bar{c})$. This system is interesting because it is the lowest-mass system with both quarks heavy but unequal in mass. Since the threshold for Zweig-rule-allowed decays into $B + \bar{D}$ is above the $2S$ state the $1P$ states are narrow, spin splittings can be studied in detail. The excitation spectrum of low-lying $(b\bar{c})$ mesons including the spin-dependent forces is shown in Fig. 7. The general considerations of Sec. VI for systems with one quark much heavier than the other apply here and the notation intorduced there to classify the states is used in Fig. 7. The reduced mass of the $b\bar{c}$ system is 2.71 GeV. The other parameters used were $\alpha_s = 0.26$, $K = 0.51$, and $a = 2.34$ (GeV)$^{-1}$. The details of the spin splittings are given in Table III. In Table III we have included the mixing between states having the same total angular momentum $\vec{J}$ and differing $\vec{j}_1 = \vec{L} + \vec{s}_1$ through order $(1/m_c m_b)$ (i.e., we have not included

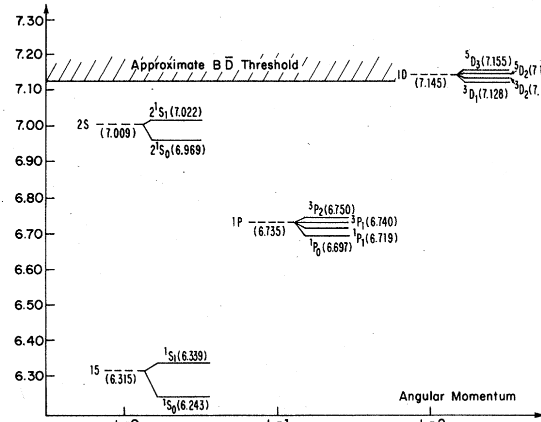

FIG. 7. The spectrum of low-lying $(b\bar{c})$ meson states. Notation as in Figs. 5 and 6.

TABLE III. Details of spin splittings in the $(b\bar{c})$ system (in MeV).

State $(n\ 2l+1L_J)$	Perturbative term		Potential terms Linear	Potential terms Coulomb	Total
$^1S_1$	+23.9		0	0	23.9
$^1S_0$	−71.8		0	0	−71.8
$2\,^1S_1$	13.4		0	0	13.4
$2\,^1S_0$	−40.4		0	0	−40.4
$^3P_2$	7.14		3.21	4.11	14.5
$^3P_1$	−2.64	−1.87	3.21	4.11	4.8
$^1P_1$	−1.87	−1.38	−6.42	−8.22	−16.2
$^1P_0$	−23.8		−6.42	−8.27	−38.4
$^5D_3$	3.75		4.52	2.09	10.4
$^5D_2$	−0.61	−0.50	4.52	2.09	6.2
$^3D_2$	−0.50	−0.40	−6.79	−3.14	−10.4
$^3D_1$	−7.06		−6.79	−3.14	−17.0

the $1/m_b^2$ terms). The full mixing matrix is given in the table where necessary. The column "total" gives the energy shift of the true eigenstate associated with diagonalizing this matrix in these cases.

It may be possible to study this interesting system since $(b\bar{c})$ mesons can be directly pair produced in e^+e^- annihilation. In particular for the 1P states: The $\bar{b}c(^1S_0)+\bar{b}c(^1P_1)$, $\bar{b}c(^1S_1)+b\bar{c}(^1P_0)$, and $bc(^1S_1)+b\bar{c}(^1P_1)$ final states can be produced in a relative S wave with thresholds at 12.96, 13.04, and 13.06 GeV, respectively, while the $\bar{b}c(^1S_0)+b\bar{c}(^3P_1)$, $\bar{b}c(^1S_0)+b\bar{c}(^3P_2)$, and $\bar{b}c(^1S_1)+b\bar{c}(^3P_1)$ final states can be produced in a relative D wave with thresholds at 12.98, 13.08, and 13.08 GeV, respectively. Unfortunately the cross section for production of these final states is expected to be quite small.

VIII. SUMMARY

The main results of this paper are the following: (1) the derivation of the general form of the spin-dependent forces between a heavy quark and antiquark within QCD through order $(1/m)^2$ as given in Eq. (4.20); (2) the reduction of the spin-dependent forces due to pseudoparticle configurations (which are minima of the Euclidean action) to their associated contribution to the spin-independent static energy, Eq. (5.10); (3) determination of a parameter-free spin-dependent potential, Eq. (7.1), based on QCD as expressed in Eq. (4.20), and the additional assumption of electric confinement, or equivalently, short-range gauge magnetic interactions (no *ad hoc* parameters or interactions have been included.); and (4) application of the resulting phenomenological potential to the $(c\bar{c})$, $(b\bar{b})$, and $(b\bar{c})$ systems, yielding good agreement with the observed spin splittings in the first system and numerous predictions for spin splitting in the others (see Sec. VII and Figs. 5–7).

The phenomenological spin-dependent potential presented here does not fit into the framework used in previous discussions of the spin-dependent interaction, because that framework presupposes that the potential arises from the instantaneous exchange of elementary excitations with definite Lorentz character (Lorentz scalar, four-vector, etc.). This simple classification is not useful in QCD. The mechanism of confinement is nonperturbative and the nonrelativistic limit need not resemble the nonrelativistic limit of a single-exchange potential as in QED. Even in perturbation theory the spin-dependent potential receives contributions from all possible Lorentz structures which survive in the nonrelativistic limit. In particular the spin-dependent potential we find cannot be reproduced by assuming the static potential arises from any combination of a Lorentz-scalar- and Lorentz-vector-exchange potential in a Breit equation.

There are several ways that the work presented here may be extended. First, it should be relatively easy to extend the general analysis presented in Secs. IV and VII to baryons and obtain the general form of the spin-dependent forces for baryons analogous to Eq. (4.20) for mesons.

It is also possible to attempt to evaluate the electric and magnetic correlation functions, Eqs. (4.14a) and (4.14c), explicitly in a lattice gauge theory and use Monte Carlo methods to attempt to extract the continuum limit. This would allow a direct evaluation of the spin-dependent terms without recourse to perturbation theory. Furthermore, the assumption of short-range magnetic correlations could be checked explicitly. The corrections to the phenomenological spin-dependent potential presented in Eq. (7.1) should also be investigated. In particular the higher-order perturbative corrections and the contribution of the annihilation terms should be estimated. It would also be useful to obtain an estimate of the contributions of pseudoparticle solutions to the spin-splitting in heavy-quark systems. It is possible that including these contributions will improve the comparison of $^3P_J(c\bar{c})$ spin splittings with experiment. A clearer picture of the validity of Eq. (7.1) will emerge when the spin splittings in the $b\bar{b}$ system have been measured and compared to the predictions presented here.

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APPENDIX: $1/m$ EXPANSION FOR THE FERMION PROPAGATION FUNCTION

In this appendix the details of the derivation of Eq. (2.11) are presented. In fact, it is shown how to formally expand the fermion propagation function to any order in $(1/m)$. The explicit form of the nonrelativistic propagation function given in Eq. (2.4) is substituted into Eq. (2.9) (The dependence of S^{++} on A_μ is suppressed):

$$S^{++}(x,y)=S_0^{++}(x,y)+\left(\frac{1+\gamma^0}{2}\right)\int d^4z\, d^4w(-i)\theta(x^0-z^0)e^{-im(x^0-z^0)}P\begin{bmatrix}x^0\\ z^0\end{bmatrix}\delta(\vec{x}-\vec{z})$$
$$\times\vec{\gamma}\cdot\vec{D}(z)(-i)\theta(w^0-z^0)e^{-im(w^0-z^0)}P\begin{bmatrix}z^0\\ w^0\end{bmatrix}\delta(\vec{z}-\vec{w})\vec{\gamma}\cdot\vec{D}(w)S^{++}(w,y). \tag{A1}$$

It is convenient to remove the time dependence of the propagation function as follows by defining $\tilde{S}$:

$$\tilde{S}^{++}(w,y)\equiv e^{im(w^0-y^0)}S^{++}(w,y). \tag{A2}$$

Therefore,

$$\tilde{S}^{++}(x,y)=\tilde{S}_0^{++}(x,y)-\frac{1+\gamma^0}{2}e^{-im(x^0-y^0)}I, \tag{A3}$$

where

$$I\equiv\int dz^0dw^0\,\theta(x^0-z^0)\theta(w^0-z^0)e^{-2im(w^0-z^0)}f(z^0), \tag{A4}$$

$$f(\xi)\equiv P\begin{bmatrix}x^0\\ \xi\end{bmatrix}\vec{\gamma}\cdot\vec{D}(\xi)P\begin{bmatrix}\xi\\ w^0\end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y). \tag{A5}$$

(all spatial points equal)

I may be rewritten as

$$I=\int_{-\infty}^{\infty}dw^0\int_{-\infty}^{\infty}dz^0e^{-2im(w^0-z^0)}\theta(x^0-z^0)\theta(w^0-z^0)[\theta(x^0-w^0)+\theta(w^0-x^0)]f(z^0) \tag{A6a}$$

$$=\int_{-\infty}^{\infty}dw^0\left[\int_{-\infty}^{w^0}dz^0\theta(x^0-w^0)+\int_{-\infty}^{x^0}dz^0\theta(w^0-x^0)\right]e^{-2im(w^0-z^0)}f(z^0) \tag{A6b}$$

$$=\int_{-\infty}^{\infty}dw^0\left[\int_{-\infty}^{0}dz^0\theta(x^0-w^0)e^{2imz^0}f(z^0+w^0)+e^{-2im(w^0-x^0)}\int_{-\infty}^{0}dz^0e^{2imz^0}f(z^0+x^0)\theta(w^0-x^0)\right]. \tag{A6c}$$

The z^0 integrals are now evaluated by integrating by parts:

$$\int_{-\infty}^{0} dz^0 e^{2imz^0} f(z^0+w^0) = \sum_{n=0}^{\infty} \left(\frac{+i}{2m}\right)^{n+1} (-1) f^{(n)}(w^0)\,, \tag{A7a}$$

$$\int_{-\infty}^{0} dz^0 e^{2imz^0} f(z^0+x^0) = \sum_{n=0}^{\infty} \left(\frac{+i}{2m}\right)^{n+1} (-1) f^{(n)}(x^0)\,. \tag{A7b}$$

The derivatives on the right-hand side of Eq. (A7) are evaluated directly from the definition of f given in Eq. (A5). The first derivative is given by

$$f'(\xi) = P\begin{bmatrix} x^0 \\ \xi \end{bmatrix}[-igA^0(\xi)\vec{\gamma}\cdot\vec{D}(\xi) + \vec{\gamma}\cdot\vec{D}'(\xi) + \vec{\gamma}\cdot\vec{D}(\xi) igA^0(\xi)]\, P\begin{bmatrix} \xi \\ w^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y)\,. \tag{A8}$$

The A^0 terms come from time derivatives of the path-ordered exponentials. The term in brackets in Eq. (A8) may be rewritten as

$$-i[D^0, \vec{\gamma}\cdot\vec{D}] \tag{A9}$$

so that

$$f'(\xi) = (-i)P\begin{bmatrix} x^0 \\ \xi \end{bmatrix}[D^0, \vec{\gamma}\cdot\vec{D}](\xi)P\begin{bmatrix} \xi \\ w^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y)\,. \tag{A10}$$

Similarly, each higher derivative introduces one new commutator with D^0 with the result that

$$f^{(n)}(\xi) = (-i)^n P\begin{bmatrix} x^0 \\ \xi \end{bmatrix}[D_0, [D_0, \cdots [D^0, \vec{\gamma}\cdot\vec{D}]\cdots]](\xi)P\begin{bmatrix} \xi \\ w^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y)\,, \tag{A11}$$

where there are n commutators. Putting Eq. (A11) into Eq. (A7a) then produces

$$\int_{-\infty}^{0} dz^0 e^{2imz^0} f(z^0+w^0) = \sum_{n=0}^{\infty} (-i)\left(\frac{1}{2m}\right)^{n+1} P\begin{bmatrix} x^0 \\ w^0 \end{bmatrix}[D^0, \cdots [D^0, \vec{\gamma}\cdot\vec{D}]\cdots]\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y) \tag{A12a}$$

and plugging Eq. (A11) into Eq. (A7b) gives

$$\int_{-\infty}^{0} dz^0 e^{2imz^0} f(z^0+x^0) = T(x^0)P\begin{bmatrix} x^0 \\ w^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y)\,, \tag{A12b}$$

where

$$T(x^0) \equiv (-i)\sum_{n=0}^{\infty}\left(\frac{1}{2m}\right)^{n+1}[D^0, \cdots [D^0, \vec{\gamma}\cdot\vec{D}]\cdots]\vec{\gamma}\cdot\vec{D}(x^0)\,. \tag{A13}$$

Next, Eqs. (A12a) and (A12b) are inserted into Eq. (A6c) to produce the final form for I. Putting Eq. (A12b) into the second term of Eq. (A6c) results in

$$\int_{-x^0}^{\infty} dw^0 e^{-2im(w^0-x^0)} T(x^0)P\begin{bmatrix} x^0 \\ w^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w,y)$$

$$= \int_0^{\infty} dw^0 e^{-2imw^0} T(x^0)P\begin{bmatrix} x^0 \\ w^0+x^0 \end{bmatrix}\vec{\gamma}\cdot\vec{D}(w^0+x^0)\tilde{S}^{++}(w^0+x^0, y^0) \tag{A14a}$$

$$= \sum_{n=0}^{\infty} (+i)\left(\frac{-1}{2m}\right)^{n+1} T(x^0)(D^0)^n\vec{\gamma}\cdot\vec{D}(x^0)\tilde{S}^{++}(x^0,y^0)\,. \tag{A14b}$$

To obtain Eq. (A14b) from Eq. (A14a) integrations by parts were performed on e^{-2imw^0}. Finally, Eq. (A14b) and Eq. (A12a) are used in Eq. (A6c) to give

$$I = \int_{-\infty}^{\infty} dw^0\,\theta(x^0-w^0)P\begin{bmatrix} x^0 \\ w^0 \end{bmatrix}(-i)\sum_{n=0}^{\infty}\left(\frac{1}{2m}\right)^{n+1}[D^0, \cdots [D^0, \vec{\gamma}\cdot\vec{D}]\cdots]\vec{\gamma}\cdot\vec{D}(w)\tilde{S}^{++}(w)$$

$$+ i\sum_{n=0}^{\infty}\left(\frac{-1}{2m}\right)^{n+1} T(x^0)(D^0)^n\vec{\gamma}\cdot\vec{D}(x^0)\tilde{S}^{++}(x^0,y^0)\,. \tag{A15}$$

To reexpress Eq. (A15) in terms of S and S_0, Eqs. (A2) and (A3) are used as well as Eq. (2.4), the equation for the nonrelativistic propagation function S_0:

$$\tilde{S}^{++}(x,y)=\tilde{S}^{++}(x,y)+\int d^4w\left\{\tilde{S}_0^{++}(x,w)\sum_{n=0}^{\infty}\left(\frac{1}{2m}\right)^{n+1}[D^0,\cdots[D^0,\vec{\gamma}\cdot\vec{\mathrm{D}}]\cdots]\vec{\gamma}\cdot\vec{\mathrm{D}}(w)\tilde{S}^{++}(w,y)\right\}$$
$$+i\sum_{n=0}^{\infty}\left(\frac{-1}{2m}\right)^{n+1}T(x^0)(D^0)^n\vec{\gamma}\cdot\vec{\mathrm{D}}(x)\tilde{S}^{++}(x,y)\,, \tag{A16}$$

which may be rewritten as

$$\left[1+i\sum_{n=0}^{\infty}\left(\frac{-1}{2m}\right)^{n+1}T(x)(D^0)^n\vec{\gamma}\cdot\vec{\mathrm{D}}(x)\right]S^{++}(x,y)$$
$$=S_0^{++}(x,y)+\int d^4w\,S_0^{++}(x,w)\sum_{n=0}^{\infty}\left(\frac{1}{2m}\right)^{n+1}[D^0,\cdots[D^0,\vec{\gamma}\cdot\vec{\mathrm{D}}]\cdots]\vec{\gamma}\cdot\vec{\mathrm{D}}(w)S^{++}(w,y)\,. \tag{A17}$$

Equation (A17) describes all relativistic propagator corrections to the fermion propagation function. To obtain Eq. (2.11), Eq. (A17) is expanded to second order in $1/m$. On the left-hand side only $h=0$ contributes since $T(x)$, given in Eq. (A13), is at least of order $1/m$, whereas on the right-hand side both $n=0$ and $n=1$ contribute. Also, note that

$$-(\vec{\gamma}\cdot\vec{\mathrm{D}})^2=\vec{\mathrm{D}}^2-g\vec{\sigma}\cdot\vec{\mathrm{B}}\,. \tag{A18}$$

Therefore

$$\left[1+\frac{1}{4m^2}(\vec{\mathrm{D}}^2-g\vec{\sigma}\cdot\vec{\mathrm{B}})\right]S^{++}(x,y)=S_0^{++}(x,y)-\int d^4w S_0^{++}(x,w)\left[\frac{1}{2m}(\vec{\mathrm{D}}^2-g\vec{\sigma}\cdot\vec{\mathrm{B}})+\frac{ig}{4m^2}(\delta_{ij}-i\epsilon_{ijk}\sigma^k)E^iD^j\right]$$
$$\times S^{++}(w,y)+O\left(\frac{1}{m^3}\right), \tag{A19}$$

where $\gamma_i\gamma_j=-\delta_{ij}+i\epsilon_{ijk}\sigma^k$ has been used. Equation (A19) is identical to Eq. (2.11).

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[18]This assumption is actually unnecessarily strong. If the gauge magnetic fields did not vanish as $|t|\to\infty$, Eq. (4.7d) would contain such terms on the right-hand side. However, these terms would still not contribute to the spin-dependent potential. This is because the static energy and spin-dependent potentials are terms proportional to T in the ln of $\tilde{I}$ [see Eq. (4.6)]. This T factor arises from the invariance under time translation of the contribution being considered, while any

contribution including a term appearing on the right-hand side of Eq. (4.7d) is of order one in T and can be ignored if a finite correlation length between gauge magnetic fields is assumed. This is consistent with the assumption of electric confinement which will be made in Sec. VII.

[19]As pointed out by L. S. Brown and W. I. Weisberger [Phys. Rev. D 20, 3239 (1979)] the virial theorem relates the average potential and kinetic energy for the ground state (or any fixed excited state) of a heavy-quark–antiquark system. Thus the kinetic energy cannot be completely ignored for a fixed bound state even as $m_Q \to \infty$. Of course, the mean radius of the state $\langle R^2\rangle^{1/2}$ approaches zero as $m_Q \to \infty$ so that for sufficiently large m_Q the properties of the state may be calculated in perturbation theory. Formally this application of the virial theorem implies limit $T \to \infty$ and then $m_Q \to \infty$; while the static energy and spin-dependent potentials are obtained with the opposite order of limits. For large but finite m_Q the kinetic effects may be rather complicated.

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$$V = V_S(R) + V_p(R)\gamma^5 \otimes \gamma^5 - V_V(R)\gamma^\mu \otimes \gamma_\mu - \frac{V_A(R)}{4m_1m_2}\gamma^5\gamma^\mu \otimes \gamma^5\gamma_\mu + \frac{V_T(R)}{4m_1m_2}\sigma^{\mu\nu}\otimes\sigma_{\mu\nu}$$

has been analyzed by D. Gromes, Nucl. Phys. B131, 80 (1977). The spin-dependent forces in leading order of $1/m^2$ are dependent on V_T and V_A only through the combination $(V_T - V_A)$. The constraint of Eq. (4.19) requires $V_p + 3(V_T - V_A) = 0$. The nonrelativistic static energy determines $V_S + V_V$. Finally, requiring agreement with Eq. (7.3) imposes three conditions (on the remaining two invariants) which have no solutions.

[28]The most recent data from the Mark II detector at SPEAR on the $1P$ states is reported in T. M. Himel *et al.*, Phys. Rev. Lett. 44, 920 (1980) while the result of the Crystal Ball Detector are presented by E. Bloom, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 1979,* edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1979).

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[33]The logarithmic terms in one loop have been calculated by M. Dine, Phys. Lett. 81B, 339 (1979) and Yale thesis (unpublished).

[34]For the details of the Υ spectrum see E. Eichten, Phys. Rev. D 22, 1819 (1980) and the references therein.

[35]The annihilation terms are formally higher order in α_s. One method for estimating the magnitude of the coefficient is given in Ref. 14. Applying this method to the $c\bar{c}$ and $b\bar{b}$ systems, we find the contribution to ${}^3S_1 - {}^1S_0$ mass differences typically a few MeV. This justifies the neglect of these terms in our considerations.