PHYSICAL REVIEW D VOLUME 23, NUMBER 11 1 JUNE 1981

Flavor unity in SU(7): Low-mass magnetic monopole, doubly charged lepton, and $Q=5/3,-4/3$ quarks

Jihn E. Kim
Department of Physics, Seoul National University, Seoul 151, Korea
(Received 24 November 1980)

A specific flavor unification is suggested in the SU(7) gauge group. This model can be trivially extended to O(14). A global symmetry Γ forbids mixings of the b ($Q = -1/3$) quark with the d and s quarks, and of the t ($Q = 2/3$) quark with the u and c quarks. Since the b and t quarks carry different Γ quantum numbers, they do not belong to the same $SU(2)_L$ doublet. A mechanism for the Γ-symmetry violation is suggested, which allows c-t mixing without b-quark mixing. There are unconventionally charged light (masses $\lesssim 300$ GeV) fermions: a doubly charged lepton T^{--}, a $Q = -4/3$ quark x, and a $Q = 5/3$ quark y. The bare value of the Weinberg angle $\sin^2\theta_W^0 = 3/20$ is renormalized to the low-energy value by introducing an intermediate mass scale M_1. A topologically stable magnetic monopole is light (mass $\approx M_1/\alpha$) and hence there does not exist a conflict arising from the grand unified theories and the hot-big-bang cosmology.

I. INTRODUCTION

The gauge principle[1] seems to be the most important one in understanding[2] superficially different interactions on the same footing. The first successful unification[3] (the electroweak theory) has been experimentally verified,[4] and the second[5] such unifications [grand unified theories (GUT's)] have emerged as candidates for strong, weak, and electromagnetic interactions. The single most important prediction of the electroweak theory is the existence of the weak neutral current. This weak neutral current has played an important role[6] at the time of nucleosynthesis (1 sec after the big bang) in the history of the Universe. The most important prediction of GUT's is that the baryon number is violated, or the proton can decay. This consequence comes from viewing quarks and leptons on the same footing. The baryon-number violation combined with the CP violation might have played[7] an important role in producing the baryon asymmetry of the Universe about 10^{-35} sec after the singularity.

The third effort along this line is a unification of fermion families. The flavor question[8, 9] "Why does Nature repeat herself?" is one of the unsatisfactory features of the standard electroweak theory[3] and of the standard SU(5) GUT model.[5] Recently, Georgi[10] proposed several laws toward a grand unified theory of flavor. The first law of grand unification requires that the representation of the left-handed (LH) fermions[11] must be real with respect to the color-SU(3) subgroup. A comprehensive study of this law extended to the reality under $SU(3)_c\times U(1)_{em}$ has been made.[12] The second law requires that the LH fermions must be complex with respect to the $SU(3)_c\times SU(2)\times U(1)$ subgroup. The third law of Georgi requires that no irreducible representation should appear more than once in the representation of the LH fermions. Several authors[13] argued that the third law need not be satisfied. Certainly, the third law seems to be an aesthetic requirement. Nevertheless, we satisfy the third law in this paper. This makes more sense in theories of dynamical symmetry breakdown such as in the schemes with heavy color,[14] hypercolor,[15] extra strong,[16] and sideway interactions.[17] In such theories the symmetry of the fermions is given from the gauge-invariant kinetic-energy terms. Then if one introduced n identical representations, he would have $U(n)$ symmetry[18] in addition to the gauge symmetry. We know that this $U(n)$ symmetry should be broken, since the fermion masses are not degenerate. Then, there should exist n^2-1 Goldstone bosons [U(1) can be left unbroken for fermion-number conservation]. This is a potential problem. In theories with fundamental scalar fields, however, this problem can be circumvented by complicated Yukawa and Higgs quartic couplings. Relaxing the third law of Georgi, we encounter too many possibilities.[19]

This leads us to consider theories with nonrepeating complex anomaly-free representations which are real under the $SU(3)_c\times U(1)_{em}$ subgroup. Complex representations occur in $SU(N)$, $O(4n+2)$, and E_6 groups. The simplest choice[20] appears in the spinor representation of SU(7). [The spinor representation of $SU(N)$ is defined as the representation of $SU(N)$ which results from the breakdown of the spinor representation of $O(2N)$.] This paper is a detailed presentation of the model presented in this spirit in Ref. 20. (Ma *et al.*[21] have also commented on the possibility of spinor representations discussed in this paper, but have not discussed the low-mass magnetic monopole, the b-quark decay mechanism, nor the renormalization of $\sin^2\theta_W$ to fit the data.)

The fermion representation in SU(7) is taken as the spinor

$$\psi_\alpha+\psi^{\alpha\beta}+\psi_{\alpha\beta\gamma}=(\bar{1})+(2)+(\bar{3}) \tag{1}$$

which is anomaly free.[10] The repeated indices mean antisymmetric combinations. This representation allows the charges for a fundamental representation[12]:

$$Q=\mathrm{diag}(a,a,a,b,b-1,1-3a-2b,0) \tag{2}$$

or

$$Q=\mathrm{diag}(a,a,a,1,0,c_1,-1-3a-c_1) \tag{3}$$

under the reality condition in $SU(3)_c\times U(1)_{em}$. We could include two generations in a single representation (1) for the case (3) if $a=-\frac{1}{3}$,

$$Q=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,c,-c). \tag{4}$$

The color and the electroweak indices are $\alpha=1,2,3$ and $\alpha=4,5$, respectively. We call $\alpha=6,7$ the flavor indices. One can see that the representation (1) is real under the subgroup $SU(3)_c\times SU(2)\times U(1)$ if $c=0$. This has been phrased in Ref. 10 as the representation (1) has no family. Therefore, we endow a nonzero value to c in order to allow families of fermions. Then, the fermion content of (1) under $SU(3)\times SU(2)\times U(1)$ is

$$\begin{aligned}\Psi_\alpha+\Psi^{\alpha\beta}+\Psi_{\alpha\beta\gamma}=&[(3^*,1,\tfrac{1}{3})+(1,2,-\tfrac{1}{2})_1+(1,1,-c)+(1,1,c)]\\&+[(3^*,1,-\tfrac{2}{3})+(3,2,\tfrac{1}{6})+(3,1,-\tfrac{1}{3}+c)+(3,1,-\tfrac{1}{3}-c)\\&+(1,1,1)+(1,2,\tfrac{1}{2}+c)_3+(1,2,\tfrac{1}{2}-c)_2+(1,1,0)]\\&+[(1,1,1)+(3,2,\tfrac{1}{6})+(3,1,\tfrac{2}{3}-c)+(3,1,\tfrac{2}{3}+c)+(3^*,1,-\tfrac{2}{3})+(3^*,2,-\tfrac{1}{6}-c)\\&+(3^*,2,-\tfrac{1}{6}+c)+(3^*,1,\tfrac{1}{3})+(1,1,-1-c)+(1,1,-1+c)+(1,2,-\tfrac{1}{2})_1].\end{aligned} \tag{5}$$

From this decomposition in the subgroup $SU(3)_c\times SU(2)\times U(1)$, we verify that the representation $\Psi_\alpha+\Psi^{\alpha\beta}+\Psi_{\alpha\beta\gamma}$ of SU(7) is complex under this subgroup. The weak-hypercharge operator is given by

$$Y=Q-I_3=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2},c,-c)\,. \tag{6}$$

Without loss of generality we fix $c=1$ which can include the τ-lepton doublet. The three orthogonal color-singlet hypercharges are

$$Y=\mathrm{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2},1,-1), \tag{7}$$

$$Y_a=\mathrm{diag}(\tfrac{1}{5},\tfrac{1}{5},\tfrac{1}{5},\tfrac{1}{5},\tfrac{1}{5},-\tfrac{1}{2},-\tfrac{1}{2}), \tag{8}$$

$$Y_b=\mathrm{diag}(\tfrac{2}{5},\tfrac{2}{5},\tfrac{2}{5},-\tfrac{3}{5},-\tfrac{3}{5},\tfrac{1}{2},-\tfrac{1}{2})\,. \tag{9}$$

The bare value of $\sin^2\theta_W^0$ at the grand unification mass scale is then $\frac{3}{20}$ which is smaller than the low-energy value 0.23. Hence, the symmetry-breaking pattern should not be $SU(7)\to SU(3)_c\times SU(2)\times U(1)$ at the grand-unification mass scale $\tilde{M}$. To increase the value of $\sin^2\theta_W$ from $\frac{3}{20}$, we have to introduce an intermediate mass scale M_1. This possibility is studied in detail in Sec. II. Two models are acceptable. Model 1 is discussed in the paper and model 2 will be presented elsewhere.[22] In Sec. III, we present the argument that there would not be a potential conflict between the present model and cosmology.[23] In Sec. IV, Higgs fields, Yukawa couplings, and a global symmetry are presented. The method we present to find a global symmetry may be useful in other gauge models. The phenomenology of neutral currents and the decay mechanism of the b quark are discussed in Sec. V. Since M_1 is around 1000 GeV, we present fermion contents in an extended electroweak theory $SU(3)_w\times U(1)$ in the Appendix. Section VI is the conclusion.

II. COUPLING-CONSTANT RENORMALIZATION

Since the SU(7) model has a small unrenormalized $\sin^2\theta_W^0=\frac{3}{20}$, there is a need to embed part of the weak hypercharge Y in a non-Abelian gauge group to increase the $\sin^2\theta_W$ from the bare value.[24] Therefore, there should exist at least one more mass scale M_1 between the grand-unification mass scale $\tilde{M}$ and the electroweak mass scale M_W. Namely, the symmetry-breaking pattern is

$$\begin{aligned}SU(7)\underset{\text{at }\tilde{M}}{\longrightarrow}&\;SU(n_c)_s\times SU(n_w)_w\times\tilde{U}(1)\\\underset{\text{at }M_1}{\longrightarrow}&\;SU(3)_c\times SU(2)\times U(1)\\\underset{\text{at }M_W}{\longrightarrow}&\;SU(3)_c\times U(1)_{em}\,,\end{aligned} \tag{10}$$

where $n_w+n_c\leq 7$. Quantum chromodynamics belongs to $SU(n_c)_s$ and the weak SU(2) belongs to $SU(n_w)_w$. The $\tilde{U}(1)$ which is separated at $\tilde{M}$ may or may not contain part of $U(1)_{em}$. If $n_c=3$, one cannot increase the Weinberg angle $\sin^2\theta_W$. This is because $U(1)_Y$ can only be embedded in $SU(n_w)_w$ and $\tilde{U}(1)$, leading to (g'/g) at $M_W < (g'/g)$ at $\tilde{M}$. Therefore, there are three possibilities for n_c indices: 1, 2, 3, 6 or 1, 2, 3, 7 or 1, 2, 3, 6, 7. Except for the last case, n_w can take two values $n_w=2$ or 3. Hence there are five possibilities. We discuss the renormalization of $\sin^2\theta_W$ in two allowable models.

Model 1

This pattern of symmetry breaking has been discussed in Ref. 20. One interesting feature which is not shared by model 2 is that $\tilde{U}(1)$ which is separated at $\tilde{M}$ does not contribute to $U(1)_{em}$. Therefore, the mass of a magnetic monopole is not of order $\tilde{M}$, but of order M_1. It suggests that there would not exist the problem arising from the heavy mass of the SU(5) magnetic monopole.[23] The pattern of the symmetry breaking is

$$\begin{aligned} SU(7) &\to SU(4)_s\times SU(3)_w\times \tilde{U}(1) \text{ at } \tilde{M} \\ &\to SU(4)_s\times SU(3)_w \text{ at } M_t \\ &\to SU(3)_c\times SU(2)\times U(1) \text{ at } M_1 \\ &\to SU(3)_c\times U(1)_{em} \text{ at } M_W\,, \end{aligned} \tag{11}$$

where the $SU(4)_s$ contains 1, 2, 3, and 6 indices. We have indicated the possibility of one more mass scale M_t. The electroweak hypercharge

$$C^{-1}Y=\mathrm{diag}(-\tfrac{1}{3},\,-\tfrac{1}{3},\,-\tfrac{1}{3},\,\tfrac{1}{2},\,\tfrac{1}{2},\,1,\,-1)\frac{\sqrt{3}}{\sqrt{17}} \tag{12}$$

is the linear combination of two hypercharges

$$Y_4=\mathrm{diag}(\tfrac{1}{3},\,\tfrac{1}{3},\,\tfrac{1}{3},\,0,\,0,\,-1,\,0)\frac{\sqrt{3}}{2\sqrt{2}}\,, \tag{13}$$

$$Y_3=\mathrm{diag}(0,\,0,\,0,\,1,\,1,\,0,\,-2)\frac{1}{2\sqrt{3}}\,. \tag{14}$$

Namely,

$$C^{-1}Y=\cos\alpha\,Y_4+\sin\alpha\,Y_3=-\frac{2\sqrt{2}}{\sqrt{17}}\,Y_4+\frac{3}{\sqrt{17}}\,Y_3\,. \tag{15}$$

The $\tilde{U}(1)$ hypercharge

$$\tilde{Y}=(-\tfrac{3}{4},\,-\tfrac{3}{4},\,-\tfrac{3}{4},\,1,\,1,\,-\tfrac{3}{4},\,1)(\tfrac{2}{21})^{1/2} \tag{16}$$

does not contribute to the electroweak hypercharge Y.

First, let us look at the evolution of the strong-interaction coupling constant. At M_t,

$$\frac{1}{g_{4s}^2(M_t)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{44}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_t}\,, \tag{17}$$

where f is half of the number of quark flavors. Suppose a few quarks are removed at M_t whose number is denoted as Δ_t. Then at M_1,

$$\frac{1}{g_{4s}^2(M_1)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{44}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_1} -\frac{1}{8\pi^2}\frac{2}{3}\Delta_t\ln\frac{M_t}{M_1}\,. \tag{18}$$

If Δ_1 families are removed at M_1, the strong-interaction coupling constant at M_W is

$$\frac{1}{g_{3s}^2(M_W)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{44}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_W} +\frac{1}{8\pi^2}\,(\tfrac{11}{3}-\tfrac{4}{3}\Delta_1)\ln\frac{M_1}{M_W} -\frac{1}{8\pi^2}\frac{2}{3}\Delta_t\ln\frac{M_t}{M_1}\,. \tag{19}$$

Second, the weak coupling constant at M_1 is

$$\frac{1}{g_{3w}^2(M_1)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{33}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_1} \tag{20}$$

and at M_W it is

$$\frac{1}{g_{2w}^2(M_W)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{33}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_W} +\frac{1}{8\pi^2}\,(\tfrac{11}{3}-\tfrac{4}{3}\Delta_1)\ln\frac{M_1}{M_W}\,. \tag{21}$$

At M_1, the hypercharges from $SU(4)_s$ and $SU(3)_w$ groups continue to form the hypercharge Y of the electroweak theory. The corresponding coupling constants are related by

$$\begin{aligned}\frac{1}{g_1(M_1)^2}&=\frac{\cos^2\alpha}{g_4^2(M_1)}+\frac{\sin^2\alpha}{g_3^2(M_1)}\\ &=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\,(-\tfrac{44}{3}+\tfrac{4}{3}f)\ln\frac{\tilde{M}}{M_1}\\ &\quad+\frac{1}{8\pi^2}\left(\tfrac{11}{3}\sin^2\alpha\ln\frac{\tilde{M}}{M_1}-\tfrac{2}{3}\Delta_t\cos^2\alpha\ln\frac{M_t}{M_1}\right)\,.\end{aligned} \tag{22}$$

Therefore, the U(1) coupling constant at M_W is

$$\frac{1}{g_1^2(M_W)}=\frac{1}{g_7^2}+\frac{1}{8\pi^2}\left(-\tfrac{44}{3}\right)\ln\frac{\tilde{M}}{M_1}+\frac{1}{8\pi^2}\left(\tfrac{11}{3}\sin^2\alpha\ln\frac{\tilde{M}}{M_1}-\tfrac{2}{3}\Delta_t\cos^2\alpha\ln\frac{M_t}{M_1}\right)+\frac{1}{8\pi^2}\,\frac{4}{3}\,f\ln\frac{\tilde{M}}{M_W}-\frac{1}{8\pi^2}\,\frac{4}{3}\,\Delta_1\ln\frac{M_1}{M_W}\,. \tag{23}$$

At M_W, g_2, e, and $\sin^2\theta_W$ are related in the usual way,

$$\frac{1}{g_{2w}{}^2(M_W)}=\frac{\sin^2\theta_W(M_W)}{e^2(M_W)}=\frac{\sin^2\theta_W}{4\pi\alpha_{\text{em}}}\,. \tag{24}$$

Hence $1/g_7{}^2(\tilde{M})$ is given by

$$\frac{1}{g_7{}^2}=\frac{\sin^2\theta_W}{4\pi\alpha_{\text{em}}}+\frac{1}{8\pi^2}\left(\tfrac{33}{3}-\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_W}-\frac{1}{8\pi^2}\left(\tfrac{11}{3}-\tfrac{4}{3}\Delta_1\right)\ln\frac{M_1}{M_W}\,. \tag{25}$$

Inserting (25) into (19), we obtain

$$\frac{1}{4\pi\alpha_c}=\frac{\sin^2\theta_W}{4\pi\alpha_{\text{em}}}-\frac{1}{8\pi^2}\,\frac{11}{3}\ln\frac{\tilde{M}}{M_W}-\frac{1}{8\pi^2}\,\frac{2}{3}\,\Delta_t\ln\frac{M_t}{M_1} \tag{26}$$

or

$$\ln\frac{\tilde{M}}{M_W}=\frac{6\pi}{11}\left(\frac{\sin^2\theta_W}{\alpha_{\text{em}}}-\frac{1}{\alpha_c}-\frac{1}{3\pi}\Delta_t\ln\frac{M_t}{M_1}\right). \tag{27}$$

The coupling constants on the right-hand side of Eq. (27) are at the mass scale M_W. Δ_t is the number of SU(2)-singlet quarks removed at M_t. If the t quark is the only quark removed at M_t, $\Delta_t=1$. If the t-quark mass is of order M_W, $\Delta_t=0$. From the relation

$$\sin^2\theta_W=\frac{1/g_{2w}{}^2}{1/g_{2w}{}^2+C^2/g_1{}^2}, \tag{28}$$

we obtain

$$\ln\frac{\tilde{M}}{M_1}=\frac{2\pi(1+C^2)\sin^2\theta_W-2\pi+\frac{22}{3}\alpha_{\text{em}}C^2\ln(\tilde{M}/M_W)+\frac{2}{3}\alpha_{\text{em}}\Delta_t\cos^2\alpha\ln(M_t/M_W)}{\alpha_{\text{em}}C^2(11-\frac{11}{3}\sin^2\alpha+\frac{2}{3}\Delta_t\cos^2\alpha)}\,. \tag{29}$$

For $\alpha_c=0.23$, $\sin^2\theta_W=0.20$, and $\Delta_t=0$, we have[25]

$$\tilde{M}=6.68\times10^{17}\ \text{GeV},\qquad M_1=483\ \text{GeV}. \tag{30}$$

For $\alpha_c=0.23$, $\sin^2\theta_W=0.20$, $\Delta_t=1$, and $M_t=10^7$ GeV,

$$\tilde{M}=1.3\times10^{17}\ \text{GeV},\qquad M_1=1030\ \text{GeV}. \tag{30$'$}$$

Model 2

This model unifies the interactions at relatively low energy $\approx 10^8$–10^9 GeV. The symmetry-breaking pattern is

$$\begin{aligned}\text{SU}(7)&\to\text{SU}(5)_s\times\text{SU}(2)_w\times\tilde{\text{U}}(1)\ \text{at}\ \tilde{M}\\&\to\text{SU}(4)_s\times\text{SU}(2)_w\times\text{U}(1)'\times\text{U}(1)''\ \text{at}\ M_2\\&\to\text{SU}(3)_c\times\text{SU}(2)_w\times\text{U}(1)\ \text{at}\ M_1\\&\to\text{SU}(3)_c\times\text{U}(1)_{\text{em}}\ \text{at}\ M_W,\end{aligned} \tag{31}$$

where the SU(5) carries 1, 2, 3, 6, and 7 and the $\text{SU}(4)_s$ carries 1, 2, 3, and 6. The possibility 1, 2, 3, and 7 for the $\text{SU}(4)_s$ gives smaller values of $\sin^2\theta_W$ and hence is not discussed. We have included one more mass scale M_2. The electroweak hypercharge Y is a linear combination of three hypercharges

$$Y_5=\text{diag}(1,1,1,0,0,1,-4)\,\frac{1}{2\sqrt{10}}\,, \tag{32}$$

$$Y_4=\text{diag}(1,1,1,0,0,-3,0)\,\frac{1}{2\sqrt{6}}\,, \tag{33}$$

$$\tilde{Y}=\text{diag}(1,1,1,-\tfrac{5}{2},-\tfrac{5}{2},1,1)\,\frac{1}{\sqrt{35}}\,, \tag{34}$$

where $\tilde{Y}$ is separated at $\tilde{M}$, Y_5 is separated at M_2, and Y_4 is separated at M_1:

$$\begin{aligned}C^{-1}Y&=\text{diag}(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3};\tfrac{1}{2},\tfrac{1}{2},1,-1)\,\frac{\sqrt{3}}{\sqrt{17}}\\&=\frac{2\sqrt{6}}{\sqrt{85}}\,Y_5-\frac{2\sqrt{2}}{\sqrt{17}}\,Y_4-\left(\frac{21}{85}\right)^{1/2}\tilde{Y}\\&=aY_5+bY_4+c\tilde{Y}\end{aligned} \tag{35}$$

with $a^2+b^2+c^2=1$.

We will assume for simplicity that no fermions are removed above the mass scale M_W. (Removal of a few fermions does not affect drastically the estimate of mass scales.)

At M_2, the coupling constants are

$$\frac{1}{g_{5s}{}^2(M_2)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\left(-\tfrac{55}{3}+\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_2}, \tag{36}$$

$$\frac{1}{\tilde{g}^2(M_2)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\frac{4}{3}f\ln\frac{\tilde{M}}{M_2}, \tag{37}$$

where f is the number of families. At M_2, a and c terms in (35) combine to form a $\mathrm{U}(1)_a$ hypercharge Y_a, and the corresponding coupling constant g_a is

$$\begin{aligned}\frac{1}{g_a{}^2(M_2)}&=\frac{\frac{8}{15}}{g_5{}^2(M_2)}+\frac{\frac{7}{15}}{\tilde{g}^2(M_2)}\\&=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\frac{4}{3}f\ln\frac{\tilde{M}}{M_2}-\frac{11}{9\pi^2}\ln\frac{\tilde{M}}{M_2}.\end{aligned} \tag{38}$$

At M_1, the coupling constants are

$$\frac{1}{g_{4s}{}^2(M_1)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\left(-\tfrac{44}{3}+\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_1}-\frac{1}{8\pi^2}\frac{11}{3}\ln\frac{\tilde{M}}{M_2}, \tag{39}$$

$$\frac{1}{g_a{}^2(M_1)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\frac{4}{3}f\ln\frac{\tilde{M}}{M_1}-\frac{11}{9\pi^2}\ln\frac{\tilde{M}}{M_2}. \tag{40}$$

At M_1, three terms in (35) combine to form the electroweak U(1). The corresponding coupling constant is

$$\begin{aligned}\frac{1}{g_1{}^2(M_1)}&=\frac{\frac{8}{17}}{g_{4s}{}^2(M_1)}+\frac{\frac{9}{17}}{g_a{}^2(M_1)}\\&=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\frac{4}{3}f\ln\frac{\tilde{M}}{M_1}\\&\quad-\frac{44}{51\pi^2}\left(\ln\frac{\tilde{M}}{M_1}+\ln\frac{\tilde{M}}{M_2}\right).\end{aligned} \tag{41}$$

At M_W, the coupling constants are

$$\frac{1}{g_c{}^2(M_W)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\left(-\tfrac{33}{3}+\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_W}-\frac{1}{8\pi^2}\frac{11}{3}\left(\ln\frac{\tilde{M}}{M_2}+\ln\frac{\tilde{M}}{M_1}\right), \tag{42}$$

$$\frac{1}{g_2{}^2(M_W)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\left(-\tfrac{22}{3}+\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_W}, \tag{43}$$

$$\frac{1}{g_1{}^2(M_W)}=\frac{1}{g_7{}^2}+\frac{1}{8\pi^2}\frac{4}{3}f\ln\frac{\tilde{M}}{M_W}-\frac{44}{51\pi^2}\left(\ln\frac{\tilde{M}}{M_1}+\ln\frac{\tilde{M}}{M_2}\right). \tag{44}$$

The relation (43) can be rewritten as

$$\frac{1}{g_7{}^2}=\frac{\sin^2\theta_W(M_W)}{4\pi\alpha_{\mathrm{em}}(M_W)}+\frac{1}{8\pi^2}\left(\tfrac{22}{3}-\tfrac{4}{3}f\right)\ln\frac{\tilde{M}}{M_W}. \tag{45}$$

Inserting this into (42) and (44) we obtain

$$\ln\frac{\tilde{M}}{M_W}+\ln\frac{\tilde{M}}{M_1}+\ln\frac{\tilde{M}}{M_2}=\frac{6\pi}{11}\left(\frac{\sin^2\theta_W}{\alpha_{\mathrm{em}}}-\frac{1}{\alpha_c}\right) \tag{46}$$

and

$$(1+C^2)\sin^2\theta_W-1=-\frac{11}{3\pi}\alpha_{\mathrm{em}}C^2\ln\frac{\tilde{M}}{M_W}+\frac{176}{51\pi}\alpha_{\mathrm{em}}C^2\left(\ln\frac{\tilde{M}}{M_1}+\ln\frac{\tilde{M}}{M_2}\right). \tag{47}$$

From (46) and (47) we obtain

$$\ln\frac{\tilde{M}}{M_W}=\frac{17\pi}{121\alpha_{\mathrm{em}}}\left[\left(\frac{15}{17}-\frac{1}{C^2}\right)\sin^2\theta_W-\frac{32}{17}\frac{\alpha_{\mathrm{em}}}{\alpha_c}+\frac{1}{C^2}\right], \tag{48}$$

$$\ln\frac{\tilde{M}}{M_1}+\ln\frac{\tilde{M}}{M_2}=\frac{17\pi}{121\alpha_{\mathrm{em}}}\left[\left(3+\frac{1}{C^2}\right)\sin^2\theta_W-2\frac{\alpha_{\mathrm{em}}}{\alpha_c}-\frac{1}{C^2}\right]. \tag{49}$$

For $\alpha_c=0.23$, $\alpha_{\mathrm{em}}=1/128.5$, and $\sin^2\theta_W=0.20$, we obtain

$$\begin{aligned}&\tilde{M}=1.55\times10^8\ \mathrm{GeV},\\&(M_1M_2)^{1/2}=2360\ \mathrm{GeV}.\end{aligned} \tag{50}$$

Since $\tilde{M}$ is of order 10^8 GeV, this is a low-energy grand unification. This pattern of symmetry breaking can be successful if one guarantees the proton stability up to 10^{31} yr. This will be discussed elsewhere.[22] Then this model will not have the conflict of high-mass magnetic monopoles.[26]

III. LOW-MASS MAGNETIC MONOPOLES

In this section, we explain why model 1 of Sec. II does not have the magnetic-monopole conflict. Subsequent sections describe model 1 in detail.

The symmetry-breaking pattern (11) allows one to express the electroweak hypercharge in terms of unbroken generators above the mass scale M_1,

$$C^{-1}Y=-\frac{2\sqrt{2}}{\sqrt{17}}Y_4+\frac{3}{\sqrt{17}}Y_3. \tag{51}$$

The $\tilde{\mathrm{U}}(1)$ hypercharge $\tilde{Y}$ in (11) does not contribute to the electroweak hypercharge Y, and hence not to the electromagnetic charge Q. This can be intuitively understood by noting that the charge generator

$$Q=\mathrm{diag}\left(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,1,-1\right) \tag{52}$$

gives traceless conditions both for the $\mathrm{SU}(4)_s$ ($\alpha=1,2,3,6$) and $\mathrm{SU}(3)_w$ ($\alpha=4,5,7$) groups. Hence, we argue that the symmetry-breaking pattern (11) avoids the potential conflict arising in the SU(5) grand-unification scheme applied to the hot-big-bang cosmology.[23]

The magnetic-monopole conflict in the SU(5), O(10), and E_6 theories has been discussed[27] and an estimate of helium abundance at the time of nucleosynthesis shows that the magnetic monopoles in the universe are 10 orders of magnitude larger[23] than the value permitted by the cosmological observation. In grand unified theories in a simple group, a unification group G is ultimately broken down to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$. By a general topological argument, these theories then necessarily have the 't Hooft–Polyakov-type magnetic monopoles. The mass of a topologically stable magnetic monopole is determined at the stage of symmetry breaking where the first nontrivial, i.e., $\mathrm{U}(1)_{\mathrm{em}}$-dependent, U(1) factor group appears. In the prototype SU(5) theory, the first such U(1) factor appears at $M \approx 10^{14}$ GeV corresponding to a 10^{16}-GeV monopole mass.

Unless the grand unified theory has a trivial intersection between $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ subgroups, it is not known at present how these monopoles and antimonopoles annihilate sufficiently fast.[27] Also, there does not exist a grand unified model which has a trivial intersection of the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ subgroups. In our model (52), the conflict does not exist, not because the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ subgroup have a trivial intersection, but because the mass of the monopole is sufficiently small so that its contribution to the mass density of the universe is negligible. We follow Preskill[23] and present this possibility numerically.

The rate equation of the density, n, of magnetic monopoles is

$$\frac{dn}{dt} = -Dn^2 - 3\frac{\dot{R}}{R}n, \tag{53}$$

where R is the cosmic scale factor and D characterizes the annihilation rate which can be parametrized as a power form

$$D = \frac{A}{m}\left(\frac{m}{T}\right)^p . \tag{54}$$

The above equations allow the following solution:

$$r(T) = \left\{\frac{1}{r(T_i)} + \frac{A}{p-1}\frac{Cm_P}{m}\left[\left(\frac{m}{T}\right)^{p-1} - \left(\frac{m}{T_i}\right)^{p-1}\right]\right\}^{-1}, \tag{55}$$

where m_P is the Planck mass, $\approx 1.2\times 10^{19}$ GeV, and $C = 0.6 N^{-1/2}$ in terms of spin degrees of freedom N. As discussed in Ref. 23, the above solution is approximately

$$r(T_f) \approx \frac{1}{Bh^2}\left(\frac{4\pi}{h^2}\right)^2 \frac{m}{Cm_p}, \tag{56}$$

where $p=2$, and B and h are constants.

On the other hand, the observed galactic mass density of the Universe ($\approx 10^{-30}$ g/cm^3) sets a limit on $r(2.7\,^\circ\mathrm{K})$,

$$r(2.7\,^\circ\mathrm{K}) \lesssim \frac{6.7}{m}\ \mathrm{eV} \tag{57}$$

and the mass density of the Universe at the time of helium synthesis ($T \approx 1$ MeV) in the history of the Universe sets a limit

$$r(1\ \mathrm{MeV}) \lesssim \frac{1}{m}\ \mathrm{MeV}. \tag{58}$$

The weaker limit (58) gives

$$r(1\ \mathrm{MeV}) \lesssim 10^{-19} \tag{59}$$

for 10^{16}-GeV magnetic monopoles. Roughly, the SU(5) theory produces[23] $r(T_i) \approx 10^{-6}$ which is not much reduced in the course of the Universe's evolution. However, monopoles of 10^5 GeV relevant for model 1 of Sec. II give a limit

$$r(1\ \mathrm{MeV}) \lesssim 10^{-8} \tag{60}$$

from Eq. (58). In the course of evolution of the Universe, $r(T_i) \approx 10^{-6}$ will be reduced to 10^{-13} $(T'/m)^{p-1} \lesssim 10^{-13}$ from Eq. (55). Hence, in model 1 of Sec. II, we do not have a conflict between the grand unified theory and the standard hot-big-bang cosmology.

Breaking of the quantum electrodynamics[28] is not required at higher temperature to resolve the magnetic-monopole conflict. Electromagnetism is a good gauge symmetry over the entire energy range.[29]

IV. YUKAWA COUPLINGS AND A GLOBAL SYMMETRY

To give masses to fermions, we introduce 7 (H^α), 35 ($H^{\alpha\beta\gamma}$), 140 ($H_\gamma^{\alpha\beta}$), and 588 ($H_{\mu\nu\rho}^{\alpha\beta}$) of Higgs fields.[30] We introduce 140 and 588 to give enough terms to remove degeneracy between fermion masses. These Higgs fields can be fundamental or composite. The most general Yukawa couplings are given by

$$\begin{aligned}-\mathcal{L}_Y = {} & f_1\tilde{\psi}^{\alpha\beta}C\psi_{\alpha\beta\gamma}H^\gamma + f_2\tilde{\psi}_\alpha C\psi^{\alpha\beta}H_\beta + f_1'\tilde{\psi}^{\alpha\beta}C\psi^{\gamma\delta}H^{\mu\nu\rho}\epsilon_{\alpha\beta\gamma\delta\mu\nu\rho} + f_2'\tilde{\psi}_{\alpha\beta\gamma}C\psi_\delta H_{\mu\nu\rho}\epsilon^{\alpha\beta\gamma\delta\mu\nu\rho} \\ & + g_1\tilde{\psi}^{\alpha\beta}C\psi_{\mu\nu\rho}H_{\alpha\beta}^{\mu\nu\rho} + g_2\tilde{\psi}_{\alpha\beta\gamma}C\psi_{\mu\nu\rho}H_{\sigma\delta\lambda}^{\alpha\beta}\epsilon^{\gamma\mu\nu\rho\sigma\delta\lambda} \\ & + h_1\tilde{\psi}_{\alpha\beta\gamma}C\psi_{\delta\mu\nu}H_{\rho\sigma}^\alpha\epsilon^{\beta\gamma\delta\mu\nu\rho\sigma} + h_2\tilde{\psi}^{\alpha\beta}C\psi_\gamma H_{\alpha\beta}^\gamma + h_3\tilde{\psi}^{\alpha\beta}C\psi_{\alpha\mu\nu}H_\beta^{\mu\nu} + \mathrm{H.c.},\end{aligned} \tag{61}$$

where the tilde indicates the transpose on the Dirac spinor and C is the charge-conjugation matrix. The 21-dimensional Higgs field $H^{\alpha\beta}$ does not give a renormalizable Yukawa term. A possible term

$$\tilde{\psi}_\alpha C\psi_\beta H^{\alpha\beta} = \tilde{\psi}_\beta C\psi_\alpha H^{\alpha\beta} = -\tilde{\psi}_\beta C\psi_\alpha H^{\beta\alpha} = 0 .$$

vanishes. The first equality follows from two minus signs of C and interchange of fermion fields, and the second equality follows from the antisymmetric property of $H^{\alpha\beta}$. Another possible term

$$\begin{aligned}\tilde{\psi}_{\mu\nu\rho} C\psi_{\alpha\beta\gamma} H_\delta \epsilon^{\mu\nu\rho\alpha\beta\gamma\delta} &= \tilde{\psi}_{\alpha\beta\gamma} C\psi_{\mu\nu\rho} H_\delta \epsilon^{\mu\nu\rho\alpha\beta\gamma\delta} \\ &= -\tilde{\psi}_{\alpha\beta\gamma} C\psi_{\mu\nu\rho} H_\delta \epsilon^{\alpha\beta\gamma\mu\nu\rho\delta} \\ &= 0\end{aligned}$$

vanishes. We have used the antisymmetric property of the ϵ symbol. Similar arguments apply to

$$\tilde{\psi}_{\alpha\beta\gamma} C\psi_{\mu\nu\rho} H^{\alpha\mu}_{\sigma\delta\lambda} \epsilon^{\beta\gamma\nu\rho\sigma\delta\lambda} .$$

Terms containing $\sum_\alpha H^{\alpha\beta}_{\alpha\mu\nu}$ are not allowed, because 588 is defined to be traceless. Also terms with $\sum_\alpha H^\alpha_{\alpha\beta}$ are not allowed.

The Lagrangian (61) has a global symmetry in addition to the local gauge symmetry SU(7). Let us call the quantum number of this global symmetry as X. The complex fields carry the following X quantum numbers:

$$X(H^\alpha) = -\tfrac{2}{7}, \quad X(H^{\alpha\beta\gamma}) = -\tfrac{6}{7}, \quad X(H^{\alpha\beta}_{\mu\nu\rho}) = \tfrac{2}{7}, \quad X(H^\alpha_{\beta\gamma}) = \tfrac{2}{7} ,$$
$$X(\psi_\alpha) = -\tfrac{5}{7}, \quad X(\psi^{\alpha\beta}) = \tfrac{3}{7}, \quad X(\psi_{\alpha\beta\gamma}) = -\tfrac{1}{7} . \tag{62}$$

Symmetry breaking of SU(7) at $\tilde{M}$ by an adjoint Higgs field Φ leaves the X quantum number unbroken, since Φ is a real field.

Therefore, there are five unbroken color-singlet generators above M_1 (let us neglect for a moment the mass scale M_t),

$$I_3,\ Y,\ Y_a,\ Y_b, \text{ and } X . \tag{63}$$

To break the corresponding symmetries at the scales M_1 and M_W, one generally endows vacuum expectation values to Higgs fields. One can easily recognize that one vacuum expectation value breaks one linear combination of (63). To break the four generators (except Q_{em}) completely, one needs four independent vacuum expectation values for the neutral components of Higgs fields.

Since the intermediate mass scale M_1 is small[31] ($\ll 10^5$–10^6 GeV), we have to introduce a mechanism to suppress flavor-changing neutral currents. For this purpose, we wish to point out that there are two Weinberg-Salam-Glashow-type fermion generations in the SU(7) model as discussed in Eq. (5). To naturally suppress flavor-changing neutral currents we satisfy the Glashow-Weinberg theorem.[32] This is achieved by introducing a conserved quantum number, say Γ. Endowing the same Γ number to the d and s quarks but a different Γ number to the b quark, we forbid mixings of the b quark with the d and s quarks,[33] but allow the Cabibbo mixing between d and s quarks. A natural choice for this Γ number is a linear combination of generators in Eq. (63):

$$\Gamma = xX + yY + y_a Y_a + y_b Y_b , \tag{64}$$

where we have neglected contribution of I_3 to give the same quantum number to u and d quarks. Three linearly independent hypercharge generators for a fundamental representation are

$$Y = \mathrm{diag}(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1, 0, 1, -1) , \tag{65}$$

$$Y_a = \mathrm{diag}(\tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, -\tfrac{1}{2}, -\tfrac{1}{2}) , \tag{66}$$

$$Y_b = \mathrm{diag}(\tfrac{2}{5}, \tfrac{2}{5}, \tfrac{2}{5}, -\tfrac{3}{5}, -\tfrac{3}{5}, \tfrac{1}{2}, -\tfrac{1}{2}) . \tag{67}$$

We present a scheme to determine the constants x, y, y_a, and y_b without mixings of the b quark with light quarks. For this purpose, we present X, Y, Y_a, and Y_b quantum numbers of neutral components of Higgs fields in Table I. From the Gell-Mann–Nishijima formula $I_3 = Q - Y$, the neutral Higgs fields with $Y = \pm 1$, i.e., $I_3 = \mp 1$, are not allowed to develop vacuum expectation values.

TABLE I. Quantum numbers of neutral Higgs fields.

Higgs	X	Y	Y_a	Y_b	Γ [Eq. (70)]
H^5	$-\frac{2}{7}$	$\frac{1}{2}$	$\frac{1}{5}$	$-\frac{3}{5}$	0
H^{567}	$-\frac{6}{7}$	$\frac{1}{2}$	$-\frac{4}{5}$	$-\frac{3}{5}$	0
H^{457}	$-\frac{6}{7}$	0	$-\frac{1}{10}$	$-\frac{17}{10}$	-2
H^{57}_{abc}	$\frac{2}{7}$	$\frac{1}{2}$	$-\frac{9}{10}$	$-\frac{23}{10}$	0
$H^{\alpha\beta}_{5\alpha\beta}$	$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{1}{5}$	$\frac{3}{5}$	0
H^{67}_{457}	$\frac{2}{7}$	0	$-\frac{9}{10}$	$\frac{17}{10}$	2
H^{6a}_{45a}	$\frac{2}{7}$	0	$-\frac{9}{10}$	$\frac{17}{10}$	2
H^{a5}_{a47}	$\frac{2}{7}$	1	$\frac{1}{2}$	$\frac{1}{2}$	2
H^{56}_{467}	$\frac{2}{7}$	1	$\frac{1}{2}$	$\frac{1}{2}$	2
H^{47}_{567}	$\frac{2}{7}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-2
H^{a4}_{a56}	$\frac{2}{7}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-2
H^{45}_{467}	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{3}{5}$	0
H^{a5}_{a67}	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{3}{5}$	0
$H^{\alpha}_{\alpha 5}$	$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{1}{5}$	$\frac{3}{5}$	0
H^{6}_{45}	$\frac{2}{7}$	0	$-\frac{9}{10}$	$\frac{17}{10}$	2
H^{5}_{47}	$\frac{2}{7}$	1	$\frac{1}{2}$	$\frac{1}{2}$	2
H^{4}_{56}	$\frac{2}{7}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-2
H^{5}_{67}	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{3}{5}$	0

Otherwise, the successful relation

$$M_W = M_Z \, |\cos\theta_W| \tag{68}$$

will be destroyed. The remaining neutral Higgs fields carry $I=\frac{1}{2}$, or 0. Although the neutral Higgs fields with $I=0$ do not alter the relation (68), we will not include them in the first approximation of assigning vacuum expectation values (VEV's). (We will discuss the other possibility later.) Since H^5 and $H^{\alpha\beta}_{5\alpha\beta}$ (and $H^{\alpha}_{5\alpha}$) carry the opposite quantum numbers, we can choose one of them as an independent neutral component. Also, we choose one out of H^{45}_{467} and H^{5}_{67} since they carry identical quantum numbers:

	X	Y	Y_a	Y_b	
H^5	$-\frac{2}{7}$	$\frac{1}{2}$	$\frac{1}{5}$	$-\frac{3}{5}$	
H^{567}	$-\frac{6}{7}$	$\frac{1}{2}$	$-\frac{4}{5}$	$-\frac{3}{5}$	(69)
H^{57}_{abc}	$\frac{2}{7}$	$\frac{1}{2}$	$-\frac{9}{10}$	$-\frac{23}{10}$	
H^{45}_{467}	$\frac{2}{7}$	$\frac{1}{2}$	$\frac{6}{5}$	$-\frac{3}{5}$	

The Γ symmetry will be preserved in the course of the spontaneous symmetry breaking as long as the neutral Higgs components developing VEV's carry vanishing Γ quantum numbers. This condition together with (64) and (69) gives four homogeneous linear equations with four unknowns. These have a nontrivial solution except for an overall normalization if the determinant of the coefficients is zero. From (69), the determinant is shown to be zero. We fix the overall normalization by fixing $x=1$. The solution is then

$$\Gamma = X + \tfrac{28}{17} Y - \tfrac{4}{7} Y_a + \tfrac{12}{17} Y_b \, . \tag{70}$$

As required, the Higgs components which determined (70) have vanishing Γ numbers. These are also shown in Table I together with the Γ numbers of other neutral Higgs fields.

In Table II the Γ numbers are given for the fermion representation $\psi_\alpha + \psi^{\alpha\beta} + \psi_{\alpha\beta\gamma}$. From the $SU(3)_c \times SU(2) \times U(1)$ quantum numbers and the Γ

TABLE II. Fermion assignments.

Representation	Assignment	X	Y	Y_a	Y_b	Γ [Eq. (70)]
$(3^*,1,\frac{1}{3})^0 = \Psi_a$	s^c_L	$-\frac{5}{7}$	$\frac{1}{3}$	$-\frac{1}{5}$	$-\frac{2}{5}$	$-\frac{1}{3}$
$(1,2,-\frac{1}{2}) = \Psi_i$	$(\nu_e, e)_L$	$-\frac{5}{7}$	$-\frac{1}{2}$	$-\frac{1}{5}$	$\frac{3}{5}$	-1
$(1,1,-1)^0 = \Psi_6$	M_L	$-\frac{5}{7}$	-1	$\frac{1}{2}$	$-\frac{1}{2}$	-3
$(1,1,1) = \Psi_7$	τ^c_L	$-\frac{5}{7}$	1	$\frac{1}{2}$	$\frac{1}{2}$	1
$(3^*,1,-\frac{2}{3})^0 = \Psi^{ab}$	u^c_L	$\frac{3}{7}$	$-\frac{2}{3}$	$\frac{2}{5}$	$\frac{4}{5}$	$-\frac{1}{3}$
$(3,2,\frac{1}{6}) = \Psi^{ai}$	$(c,s)_L$	$\frac{3}{7}$	$\frac{1}{6}$	$\frac{2}{5}$	$-\frac{1}{5}$	$\frac{1}{3}$
$(3,1,\frac{2}{3})^0 = \Psi^{a6}$	t_L	$\frac{3}{7}$	$\frac{2}{3}$	$-\frac{3}{10}$	$\frac{9}{10}$	$\frac{7}{3}$
$(3,1,-\frac{4}{3}) = \Psi^{a7}$	x_L	$\frac{3}{7}$	$-\frac{4}{3}$	$-\frac{3}{10}$	$-\frac{1}{10}$	$-\frac{5}{3}$
$(1,1,1) = \Psi^{45}$	e^c_L	$\frac{3}{7}$	1	$\frac{2}{5}$	$-\frac{6}{5}$	1
$(1,2,\frac{3}{2}) = \Psi^{i6}$	$(T^c, M^c)_L$	$\frac{3}{7}$	$\frac{3}{2}$	$-\frac{3}{10}$	$-\frac{1}{10}$	3
$(1,2,-\frac{1}{2}) = \Psi^{i7}$	$(\nu_\tau, \tau)_L$	$\frac{3}{7}$	$-\frac{1}{2}$	$-\frac{3}{10}$	$-\frac{11}{10}$	-1
$(1,1,0)^0 = \Psi^{67}$	N_L	$\frac{3}{7}$	0	-1	0	1
$(1,1,1)^0 = \Psi_{abc}$	μ^c_L	$-\frac{1}{7}$	1	$-\frac{3}{5}$	$-\frac{6}{5}$	1
$(3,2,\frac{1}{6}) = \Psi_{abi}$	$(u,d)_L$	$-\frac{1}{7}$	$\frac{1}{6}$	$-\frac{3}{5}$	$-\frac{1}{5}$	$\frac{1}{3}$
$(3,1,-\frac{1}{3})^0 = \Psi_{ab6}$	b_L	$-\frac{1}{7}$	$-\frac{1}{3}$	$\frac{1}{10}$	$-\frac{13}{10}$	$-\frac{5}{3}$
$(3,1,\frac{5}{3}) = \Psi_{ab7}$	y_L	$-\frac{1}{7}$	$\frac{5}{3}$	$\frac{1}{10}$	$-\frac{3}{10}$	$\frac{7}{3}$
$(3^*,1,-\frac{2}{3}) = \Psi_{a45}$	c^c_L	$-\frac{1}{7}$	$-\frac{2}{3}$	$-\frac{3}{5}$	$\frac{4}{5}$	$-\frac{1}{3}$
$(3^*,2,-\frac{7}{6}) = \Psi_{ai6}$	$(t^c, y^c)_L$	$-\frac{1}{7}$	$-\frac{7}{6}$	$\frac{1}{10}$	$-\frac{3}{10}$	$-\frac{7}{3}$
$(3^*,2,\frac{5}{6}) = \Psi_{ai7}$	$(x^c, b^c)_L$	$-\frac{1}{7}$	$\frac{5}{6}$	$\frac{1}{10}$	$\frac{7}{10}$	$\frac{5}{3}$
$(3^*,1,\frac{1}{3}) = \Psi_{a67}$	d^c_L	$-\frac{1}{7}$	$\frac{1}{3}$	$\frac{4}{5}$	$-\frac{2}{5}$	$-\frac{1}{3}$
$(1,1,-2) = \Psi_{456}$	T_L	$-\frac{1}{7}$	-2	$\frac{1}{10}$	$\frac{7}{10}$	-3
$(1,1,0)^0 = \Psi_{457}$	N^c_L	$-\frac{1}{7}$	0	$\frac{1}{10}$	$\frac{17}{10}$	1
$(1,2,-\frac{1}{2}) = \Psi_{i67}$	$(\nu_\mu, \mu)_L$	$-\frac{1}{7}$	$-\frac{1}{2}$	$\frac{4}{5}$	$\frac{3}{5}$	-1

numbers, we distinguish light quarks u, d, c, s and heavy quarks b and t.

We note the following.

(i) The VEV's with vanishing Γ numbers can give mass terms between fermion-antifermion pairs of opposite Γ numbers. Therefore, the neutral components H^5, H^{567}, H^{57}_{abc}, $H^{\alpha\beta}_{5\alpha\beta}$, H^{45}_{467}, H^{a5}_{a67}, H^{5}_{67}, and $H^{\alpha}_{\alpha 5}$ can allow d-s and u-c mixings.[34] However, the b quark cannot mix with the d and s quarks by these VEV's. If the Γ quantum number is not broken, the b quark cannot decay by a W-boson or a Z-boson exchange. It can decay via Higgs-boson exchange or by gauge bosons at the mass scale M_1. Since the Γ of three light quarks cannot form the Γ number of the b quark, the hadronic decay of the b quark is forbidden at the mass scale M_W.[35] The b-quark decay will be discussed later.

(ii) There are five neutral leptons, $\nu_e(-1)$, $\nu_\mu(-1)$, $\nu_\tau(-1)$, $N_L(1)$, and $N^c_L(1)$, where the Γ numbers are shown in the brackets. Therefore, one neutrino is exactly massless. Let us assume that this massless neutrino is ν_e. However, the ν_μ and the ν_τ can have additional components N^L and N^c_L to generate Dirac masses. We wish to forbid the mass generation of ν_μ and ν_τ at tree level. This can be achieved if N_L and N^c_L are removed at a higher mass scale. Then, the Γ invariance should necessarily be broken by two units since $N^c_L N_L$ carries $\Gamma = 2$. The neutral Higgs which has $\Gamma = 2$ should be an SU(2) $\times$ U(1) singlet so that a generation of N_L mass would not affect the SU(2) $\times$ U(1) gauge symmetry. A good choice is H^{67}_{457}. We can show that this component does not mix the b quark with the d and s quarks at tree level.

(iii) Spontaneous breaking of the Γ symmetry implies either existence of a Goldstone boson or soft breaking of the Γ symmetry in the Higgs potential. Since a Goldstone boson is not discovered, we have the second alternative, which can be achieved by a term

$$M_\epsilon H^{\alpha\beta}_{\mu\nu\rho} H_{\alpha\beta\gamma} H_{\lambda\delta\sigma} \epsilon^{\mu\nu\rho\gamma\lambda\delta\sigma} . \tag{71}$$

The required vacuum expectation values are

$$\begin{aligned}
&\langle H^5\rangle = u,\quad \langle H^{567}\rangle = v/6,\quad \langle H^{57}_{123}\rangle = w/12,\quad \langle H^{ab}_{5ab}\rangle = -\tfrac{1}{12}(p+q+\tfrac{1}{3}r),\quad \langle H^{a4}_{5a4}\rangle = \frac{p}{12},\quad \langle H^{a6}_{5a6}\rangle = \frac{q}{12},\\
&\langle H^{a7}_{5a7}\rangle = \tfrac{1}{12}(p+q+\tfrac{2}{3}r),\quad \langle H^{46}_{546}\rangle = \frac{r}{12},\quad \langle H^{47}_{547}\rangle = -\tfrac{1}{12}(3p+r),\quad \langle H^{67}_{567}\rangle = -\tfrac{1}{12}(3q+r),\quad \langle H^{45}_{467}\rangle = \frac{s}{12},\\
&\langle H^{a5}_{a67}\rangle = -\frac{s}{36},\quad \langle H^{67}_{457}\rangle = \frac{\delta}{12},\quad \langle H^{6a}_{45a}\rangle = \frac{-\delta}{36},\quad \langle H^{a}_{a5}\rangle = \frac{\rho}{12},\quad \langle H^{4}_{45}\rangle = \frac{\sigma}{12},\quad \langle H^{6}_{65}\rangle = \frac{\lambda}{12},\quad \langle H^{7}_{75}\rangle = -\tfrac{1}{12}(3p+\sigma+\lambda),\\
&\langle H^{5}_{67}\rangle = \frac{\epsilon}{12},
\end{aligned} \tag{72}$$

which satisfy the traceless condition for the 588 ($H^{\alpha\beta}_{\mu\nu\rho}$) and 140 ($H^{\alpha}_{\beta\gamma}$) of Higgs fields,

$$\sum_\alpha H^{\alpha\beta}_{\alpha\mu\gamma} = 0 , \tag{73}$$

$$\sum_\alpha H^{\alpha}_{\alpha\beta} = 0 . \tag{74}$$

Then, the fermion contents in the SU(2) $\times$ U(1) are

$$\begin{aligned}
&\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L, \begin{pmatrix}M\\ T^{--}\end{pmatrix}_R, \quad e_R, \mu_R, \tau_R,\ M_L, N_L, T^{--}_L, N_R,\\
&\begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}b\\ x\end{pmatrix}_R \begin{pmatrix}y\\ t\end{pmatrix}_R, \quad b_L, y_L, u_R, c_R,\ x_L, t_L, d_R, s_R .
\end{aligned} \tag{75}$$

From Eqs. (61) and (72), one can work out the fermion masses and mixing angles. In particular, the mass matrix for neutral leptons is

$$M_\nu = \begin{array}{c c} & \begin{array}{ccccc} \nu_{eL} & \nu_{\mu L} & \nu_{\tau L} & N_L & N^c_L \end{array} \\ \begin{array}{c} \tilde{\nu}_{eL}C \\ \tilde{\nu}_{\mu L}C \\ \tilde{\nu}_{\tau L}C \\ \tilde{N}_L C \\ \tilde{N}^c_L C \end{array} & \begin{pmatrix} 0 & 0 & 0 & \frac{h_2}{12}\epsilon & 0 \\ 0 & 0 & 0 & -\frac{g_1}{2}(3q+r) - \frac{h_3}{12}(3\rho+\sigma) & 0 \\ 0 & 0 & 0 & 0 & \frac{g_1}{2}(3p+r) + \frac{h_3}{12}(3p+\lambda) \\ \frac{h_2}{12}\epsilon & -\frac{g_1}{2}(3q+r) - \frac{h_3}{12}(3\rho+\sigma) & 0 & 0 & \frac{1}{2}g_1\delta \\ 0 & 0 & \frac{g_1}{2}(3p+r) + \frac{h_3}{12}(3\rho+\lambda) & \frac{1}{2}g_1\delta & 0 \end{pmatrix} \end{array} \tag{76}$$

from which we notice that there would exist neutrino oscillations between ν_e (or ν_μ) and ν_τ as shown in Fig. 1. As discussed, the natural mass scale criteria show that the neutrino oscillation arising from Fig. 1 is unacceptable if the mass of the neutral lepton N is about 1000 GeV. Therefore, it is good to forbid the neutrino oscillation completely at the level of the mass scale M_1. This is possible only if

$$\epsilon = 0, \quad 3q + r = 0, \quad 3\rho + \sigma = 0 . \tag{77}$$

V. PHENOMENOLOGICAL IMPLICATIONS

The most important phenomenological implication of the present model is certainly the proton decay. However, the rate is about 10 orders of magnitude smaller than the experimentally verifiable one at present. Hence, we discuss other phenomenologies.

A. Neutral currents

It is obvious that the neutral-current parameters in ν-e, ν-u (or ν-d), and e-u (or e-d) scattering are exactly the same as those of the SU(2) × U(1) electroweak theory in the limit of $M_1^2 \gg M_W^2$. [This can be easily recognized from the representation (75).] Therefore, the present model describes the neutral-current data as successfully as the standard SU(5) model. However, the neutral-current parameters for the b quark and the τ lepton are different from those of the SU(5) model. It is extremely important to probe the neutral-current parameters for these particles. Our calculation, Eqs. (30) and (30′), shows that M_1^2 is actually not smaller than 36 M_W^2. In other words, all the neutral-current parameters of the first-family fermions are in agreement with those of the SU(5) theory within 3%. Anticipating that high-precision experiments might distinguish the 3% level discrepancy in the future, it is worthwhile to derive corrections to the Weinberg-Salam-Glashow (WSG) theory,

$$\epsilon_{L,R}(u,d) = \epsilon_{L,R}(u,d)_{\rm WSG} + O\left(\frac{M_W^2}{M_1^2}\right),$$
$$C_{1,2}^{u,d} = (C_{1,2}^{u,d})_{\rm WSG} + O\left(\frac{M_W^2}{M_1^2}\right), \tag{78}$$

etc. This requires a treatment that the present SU(2) × U(1) electroweak theory is approximate, but an SU(3)$_w$ × U(1)′ electroweak theory at the mass scale M_1 is a covering electroweak theory. (See the Appendix and Fig. 2.) SU(3)$_w$ × U(1) models discussed in the literature[36] have been rejected from the determination of the neutral-current parameters[37] $\epsilon_L(u)$, $\epsilon_R(u)$, $\epsilon_L(d)$, $\epsilon_R(d)$, and $C_{1,2}^{u,d}$. The Lee-Weinberg and Langacker-Segre models do not reproduce the neutral currents of the WSG SU(2) × U(1) model. In the present case the SU(2) × U(1) is always a limit and hence a successful neutral-current phenomenology is guaranteed from the outset. Small corrections to the neutral-current parameters of the WSG model will be presented elsewhere.

B. Decay of the b quark

As discussed in Sec. IV, the b quark does not mix with the d and s quarks and hence the Γ invariance does not allow the hadronic decay mode of the b. Therefore, the Γ invariance should be violated at some level. Within the approximate Γ invariance, the allowed decay modes of the b quark by gauge bosons at the scale M_1 (refer to Fig. 2) are

$$b \to c \text{ (or } u) + \nu_e + \tau \text{ (or } \mu \text{ or } e) , \tag{79}$$

$$b \to c \text{ (or } u) + \nu_\tau + \tau \text{ (or } \mu \text{ or } e) , \tag{80}$$

$$b \to c \text{ (or } u) + d + \bar{c} \text{ (from mixing to } \bar{t}) . \tag{81}$$

By gauge bosons at the mass scale M_1, the Γ invariance is not violated but the separate lepton number is violated. From the specific $\nu_{\mu L}$ assignment, the decay product of the b does not include ν_μ in (79) and (80). The Higgs-particle-mediated decays can include additional processes

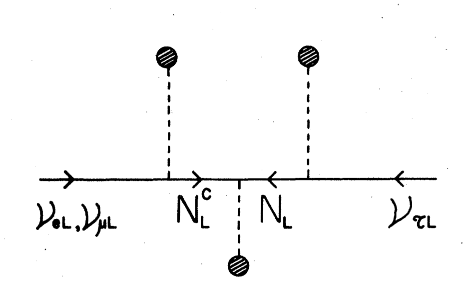

FIG. 1. Neutrino-oscillation diagram between ν_τ and ν_e (or ν_μ).

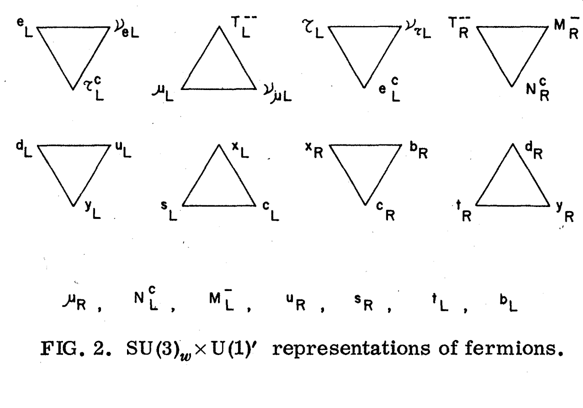

FIG. 2. SU(3)$_w$ × U(1)′ representations of fermions.

such as

$$b \to c + \tau + \nu_\mu ,$$

$$b \to c + \mu + \nu_\mu , \quad \text{etc.}$$

One important decay process of the b is (81) which does not include any lepton. This can describe the hadronic decay mode of the B meson

$$B \to J/\psi + K + \pi . \tag{82}$$

The decay process (81) is possible because the Γ invariance is violated by the vacuum expectation value[34] $\langle H^{6\alpha}_{45\alpha}\rangle$. (The $H^{6\alpha}_{45\alpha}$ carries two units of the Γ quantum number.) $\langle H^{6\alpha}_{45\alpha}\rangle$ mixes the c and the t quarks and hence the t_R is actually mixed with c_R. From Fig. 2 we can therefore see that process (81) is possible. If we neglect Higgs-particle-mediated decays, the processes (79), (80), and (81) give the hadronic branching ratio of the b quark as

$$B(b \to \text{hadrons}) = \frac{3\sin^2\theta_R}{2 + 3\sin^2\theta_R} , \tag{83}$$

where θ_R is the mixing angle between c_R and t_R.

VI. CONCLUSION

Based on the two principles that the fermion representation is complex under the subgroup $SU(3)_c \times SU(2) \times U(1)$ and is real under the subgroup $SU(3)_c \times U(1)_{em}$, we have found that the spinor representation of SU(7) can provide the simplest example of flavor unification. In the course of constructing the flavor-unity model, we have introduced a doubly charged lepton T^{--}, $Q = \frac{5}{3}$ quark y, and $Q = -\frac{4}{3}$ quark x. The fermion representation for each of these fields includes an $SU(2) \times U(1)$ doublet and a singlet. Therefore, these fermion masses should be less than 300 GeV as far as the current wisdom of the perturbation theory makes sense.

The specific symmetry-breaking pattern (model 1) determines the grand-unification mass scale $\tilde{M} \approx 10^{17}$ GeV and the intermediate mass scale $M_1 \approx (\frac{1}{2} \sim 1) \times 10^3$ GeV. The grand-unification mass scale $\tilde{M}$ is 3 orders of magnitude larger than that of the SU(5) model. Therefore, the proton decay rate is about 12 orders of magnitude smaller (i.e., $\tau_p \gtrsim 10^{42}$ yr). Unfortunately, the proton looks virtually stable in this model. If the proton decay is not observed at the level of 10^{33} yr and the exotic fermions T^{--}, x, and y are discovered around 100 GeV, the flavor-unity model in SU(7) discussed in this paper might hint at an ultimate theory of all elementary-particle forces. If the exotic fermions are not discovered below 300 GeV, the model presented in this paper should be ruled out unless it is substantially modified.

As in any other flavor-unification models, the decay mechanism of the b quark is a delicate problem. One scenario for the hadronic decay of the b quark has been discussed in Sec. V. Here, the gauge bosons of the mass scale M_1 are responsible for the hadronic decay of the b quark. For this, the Γ invariance has been violated by the $SU(2) \times U(1)$-singlet Higgs vacuum expectation value $\langle H^{6\alpha}_{45\alpha}\rangle$.

Since the intermediate mass scale M_1 is rather small we might see the effect by precision experiments on the weak neutral currents. Another important implication of the small M_1 is that the magnetic-monopole mass is small ($\approx 10^5$ GeV) and hence we avoid the potential conflict arising from a grand unified theory and the hot-big-bang cosmology.

In our representation of fermions, we produced two left-handed quark doublets and three left-handed lepton doublets. Therefore, we successfully produced the observed weak-interaction phenomenologies of the e, μ, τ, d, and s doublets. For right-handed doublets, the electroweak representation for the first family is the conventional one, not destroying the successful neutral-current phenomenology of the $SU(2)_L \times U(1)$ theory.

Since we have used the spinor representation in SU(7), the present model can trivially be extended to an O(14) gauge theory with a spinor representation, as the simplest SU(5) model is extended to an O(10) theory.[38]

ACKNOWLEDGMENTS

It is a pleasure to acknowledge useful discussions with Stephen Barr, Henry Primakoff, Gino Segre, Arthur Weldon, and Anthony Zee. Most of the work was done when the author was at the University of Pennsylvania. This research was supported in part by the grants from the Ministry of Education, through the Research Institute of Basic Sciences, Seoul National University and from Korean Science and Engineering Foundation.

APPENDIX

When we want to see the effect of the intermediate mass scale M_1, we should consider the covering electroweak theory $SU(3) \times U(1)'$. Treating 4, 5, and 7 as the $SU(3)_w$ indices, we obtain the $SU(3) \times U(1)'$ property of fermions, which are shown in Fig. 2. The electroweak charge Q is

$$Q = \tfrac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8) + Y' , \tag{A1}$$

where

$$\begin{aligned}
&Y'=0 \ \text{ for } e_L, \tau_L \text{ triplets},\\
&Y'=-1 \ \text{ for } \mu_L, M_R \text{ triplets},\\
&Y'=\tfrac{2}{3} \ \text{ for } d_L \text{ triplet},\\
&Y'=\tfrac{1}{3} \ \text{ for } s_L \text{ triplet},\\
&Y'=-\tfrac{1}{3} \ \text{ for } b_R \text{ triplet},\\
&Y'=-\tfrac{2}{3} \ \text{ for } t_R \text{ triplet},\\
&Y'=-1 \ \text{ for } \mu_R,\\
&Y'=0 \ \text{ for } N_L^c,\\
&Y'=-1 \ \text{ for } M_L^-,\\
&Y'=\tfrac{2}{3} \ \text{ for } u_R,\\
&Y'=-\tfrac{1}{3} \ \text{ for } s_R,\\
&Y'=\tfrac{2}{3} \ \text{ for } t_R,\\
&Y'=-\tfrac{1}{3} \ \text{ for } b_L.
\end{aligned} \qquad \text{(A2)}$$

We know that the fermion representation Fig. 2 in $SU(3) \times U(1)'$ does not have the Adler-Bell-Jackiw anomaly since we have started from an anomaly-free representation in SU(7) and no fermion is excluded above M_1. One result of the absence of the anomaly is

$$\mathrm{Tr}Y'_L - \mathrm{Tr}Y'_R = 0, \qquad \text{(A3)}$$

where L and R refer to the left-handed and right-handed representations, respectively. Equation (A3) is satisfied for the Y' assignment of (A2).

If we consider the $SU(3) \times U(1)'$ theory instead of the $SU(2) \times U(1)$, we will have trilepton events which motivated consideration of the former theory.[36] From Fig. 2, we note that a high-energy ν_μ beam will produce T^{--} and y which subsequently decay

$$\begin{array}{l}
\nu_\mu + d \to T^{--} + y \\
\qquad\qquad\quad \searrow \begin{cases} u+\tau^+ +\bar{\nu}_e \\ u+e^+ +\bar{\nu}_\tau \\ \text{etc.} \end{cases} \\
\qquad \downarrow \begin{cases} \mu+\tau+\bar{\nu}_e \\ \mu+e+\bar{\nu}_\tau \\ \mu+b+\bar{c} \\ \text{etc.} \end{cases}
\end{array} \qquad \text{(A4)}$$

The threshold ν_μ energy for the trilepton events is roughtly $(m_T+m_y)^2/2m_p > 800$ GeV, since m_T and m_y are larger than 20 GeV. However, it is very difficult to prove the $SU(3)_w \times U(1)'$ structure by observing trileptons since the process (A4) competes with backgrounds coming from $\nu_\mu N \to \mu^- + \gamma + \cdots \to \mu^- + \mu^+ + \mu^- + \cdots$ because of large $M_1^2 \approx 40\, M_W^2$. Detailed information on the trilepton kinematics is necessary to distinguish the elementary processes.

[1]H. Weyl, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. KL., 465 (1918); Ann. Phys. (Leipzig) 59, 101 (1919); Z. Phys. 56, 330 (1921). The history of the gauge principle has been excellently reviewed by C. N. Yang, in *Five Decades of Weak Interaction Theory,* proceedings of the Symposium, New York, 1977, edited by N. P. Chang (New York Academy of Sciences, New York, 1977) [Ann. N. Y. Acad. Sci. 294, 86 (1977)].

[2]Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. KL, 966 (1921); A. Einstein and P. Bergmann, Ann. Math. 39, 683 (1938). A history can be found in P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Englewood, New Jersey, 1942), p. 245; S. Weinberg, Rev. Mod. Phys. 52, 515 (1980).

[3]S. L. Glashow, Nucl. Phys. 22, 579 (1961); A. Salam and J. C. Ward, Phys. Lett. 13, 168 (1964); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1969), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[4]L. M. Sehgal, Phys. Lett. 71B, 99 (1977); P. Q. Hung and J. J. Sakurai, *ibid.* 72B, 208 (1977); P. Langacker and D. P. Sidhu, *ibid.* 74B, 233 (1978); L. F. Abbott and R. M. Barnett, Phys. Rev. Lett. 40, 1303 (1978); C. Y. Prescott *et al.*, Phys. Lett. 77B, 347 (1978); J. E. Kim, P. Langacker, M. Levine, and H. H. Williams, Rev. Mod. Phys. 53, 211 (1981); C. Baltay, Comments Nucl. Part. Phys. 8, 157 (1979); K. Winter, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab*, edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1980), p. 258.

[5]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. Quinn, and S. Weinberg, *ibid.* 33, 451 (1974); A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978). Though not in a simple group, the first specific grand unified model which has baryon nonconservation was proposed by J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); 10, 275 (1974). In E_6, see F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. 60B, 177 (1976). For a review, see, M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).

[6]G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977); P. Hut, *ibid.* 69B, 85 (1977); S. Weinberg, in *Neutrinos-78,* edited by E. C. Fowler (Purdue University Press, W. Lafayette, Indiana, 1978). For a review on charged-current processes, see D. N. Schramm and R. V. Wagoner, Ann. Rev. Nucl. Sci. 27, 34 (1977).

[7]S. Weinberg, in *Lectures on Particles and Field Theory*, edited by S. Deser and K. Ford (Prentice-Hall, Englewood, New Jersey, 1964), p. 482; A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 5, 32 (1967) [JETP Lett. 5, 24 (1967)]; M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978); 42, 746(E) (1979); S. Dimopoulos and L. Susskind, Phys. Rev. D 18, 4500 (1978); D. Toussaint, S. B. Treiman, F. Wilczek, and A. Zee, *ibid.* 19, 1036 (1979); S. Weinberg, Phys. Rev. Lett. 42, 859 (1979); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. 86B, 360 (1979); A. Yildiz and P. Cox, Phys. Rev. D 21, 906 (1980); S. Barr, G. Segre, and H. A. Weldon, *ibid.* 20, 2494 (1979); D. V. Nanopoulos and S. Weinberg, *ibid.* 20, 2484 (1979); G. Segre and H. A. Weldon, Phys. Rev. Lett. 44, 1737 (1980); A. D. Sakharov, Zh. Eksp. Teor. Fiz. 76, 1172 (1979) [Sov. Phys.—JETP 49, 594 (1979)].

[8]S. L. Glashow, Harvard University Report No. HUTP-78/015, 1978 (unpublished).

[9]F. Wilczek and A. Zee, Phys. Rev. Lett. 42, 421 (1979).

[10]H. Georgi, Nucl. Phys. B156, 126 (1979).

[11]In this paper, we use chiral representations for fermion fields. Right-handed fermions can be represented as left-handed antifermions.

[12]J. E. Kim, J. Kim, K. S. Soh, and H. S. Song, Nucl. Phys. B (to be published).

[13]P. Frampton, Phys. Lett. 88B, 299 (1979); S. M. Barr, Phys. Rev. D 21, 1424 (1980).

[14]L. Susskind, Phys. Rev. D 20, 2619 (1979); S. Dimopoulos and L. Susskind, Nucl. Phys. B155, 237 (1979).

[15]M. A. B. Beg, H. D. Politzer, and P. Ramond, Phys. Rev. Lett. 43, 1701 (1979).

[16]S. Weinberg, Phys. Rev. D 13, 974 (1976); Rev. Mod. Phys. 46, 255 (1974).

[17]E. Eichten and K. Lane, Phys. Lett. 90B, 125 (1980).

[18]We thank Stephen Barr and Arthur Weldon for clarifying this point.

[19]Such models indeed exist even in SU(7); K. Kang, J. Kim, J. E. Kim, K. S. Soh, and H. S. Song, Phys. Rev. D 22, 2869 (1980); M. Claudson, A. Yildiz, and P. H. Cox, Phys. Lett. 97B, 224 (1980).

[20]J. E. Kim, Phys. Rev. Lett. 45, 1916 (1980).

[21]Z. Ma, T. Tu, P. Xue, and Z. Yue, Chinese Institute of High Energy Report, 1980 (unpublished).

[22]J. E. Kim and H. S. Song (unpublished).

[23]Ya. B. Zel'dovich and M. Y. Khlopov, Phys. Lett. 79B, 239 (1979); J. P. Preskill, Phys. Rev. Lett. 43, 1365 (1979); A. H. Guth and S.-H. H. Tye, *ibid.* 44, 631 (1980); 44, 963(E) (1980); M. B. Einhorn, D. Toussaint, and M. B. Stein, University of Michigan report, 1980 (unpublished).

[24]J. E. Kim and H. S. Song, Phys. Rev. D 22, 1753 (1980). See also, I. Umemura and K. Yamamoto, Prog. Theor. Phys. 64, 278 (1980).

[25]The coupling constants are at the mass scale M_W. W. J. Marciano [Phys. Rev. D 20, 274 (1979)] considered the fermion (three generations) contribution to extrapolate the electromagnetic fine-structure constant from $\alpha_{\rm em}(9)=\frac{1}{137}$ to $\alpha(M_W)=1/128.5$. We used this electroweak fine-structure constant. For $\alpha_c(M_W)$, the experiments are not accurate enough. For illustration, we took $\alpha_c(M_W)=0.23$. See S. Ting, talk presented at Coral Gables conference, 1980 (unpublished). We take $\sin^2\theta_W(M_W)\approx 0.20$ which is the standard SU(5) prediction.

[26]We thank Professor Gino Segre for emphasizing this point in grand-unification schemes at a low energy.

[27]G. Lazarides and Q. Shafi, Phys. Lett. 94B, 149 (1980).

[28]P. Langacker and S.-Y. Pi, Phys. Rev. Lett. 45, 1 (1980); P. Langacker, talk presented at the First Workshop on Grand Unifications, University of New Hampshire, 1980 (unpublished).

[29]Professor Henry Primakoff exphasizes this point.

[30]We introduce enough Higgs fields so that there do not appear accidentally massless charged fermions or accidental degeneracy of fermion masses.

[31]If strangeness-changing neutral-current processes are allowed by gauge bosons of the mass scale M, the observed mass difference of the K_L and K_S mesons ($\Delta m\cong 1.6\times 10^{-14} m_K$) gives a lower bound on the mass $M\gtrsim 10^5$—10^6 GeV.

[32]K. Kang and J. E. Kim, Phys. Lett. 64B, 93 (1976); S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977); E. A. Paschos, *ibid.* 15, 1966 (1977); K. Kang, J. E. Kim, and R. Nabavi, *ibid.* 16, 911 (1977).

[33]Decay of the b quark by W-boson exchange is forbidden. The violation of the Γ invariance by $\langle H^{67}_{457}\rangle$ does not allow W-mediated decays.

[34]The Greek indices α, β, ... stand for SU(7) indices 1, 2, ... , 7, and the Roman indices a, b, and c stand for $SU(3)_c$ indices 1, 2, and 3.

[35]E. Derman, Phys. Rev. D 19, 317 (1979); S. Barr and P. Langacker, Phys. Rev. Lett. 42, 1654 (1979); G. Kane, University of Michigan Report No. UM-HE-80-18, 1980 (unpublished).

[36]B. W. Lee and S. Weinberg, Phys. Rev. Lett. 38, 1237 (1977); P. Langacker and G. Segre, *ibid.* 39, 259 (1977); P. Langacker, G. Segre, and M. Golshani, Phys. Rev. D 17, 1402 (1978); B. W. Lee and R. E. Shrock, *ibid.* 17, 2410 (1978); R. M. Barnett and L. N. Chang, Phys. Lett. 72B, 233 (1977).

[37]L. M. Sehgal, Phys. Lett. 71B, 99 (1977); P. Q. Hung and J. J. Sakurai, *ibid.* 72B, 208 (1977); G. Ecker, *ibid.* 72B, 450 (1978); Nucl. Phys. B151, 147 (1979); P. Langacker and D. P. Sidhu, Phys. Lett. 74B, 233 (1978); Phys. Rev. Lett. 41, 732 (1978); L. F. Abbott and R. M. Barnett, *ibid.* 40, 1303 (1978); Phys. Rev. D 18, 3214 (1978); E. Monsay, *ibid.* 18, 2277 (1978); Kim *et al.*, Ref. 4.

[38]H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975); H. Georgi, in *Particles and Fields—1974*, proceedings of the Williamsburg Meeting of the Division of Particles and Fields of the American Physical Society, edited by Carl Carlson (AIP, New York, 1975); H. Georgi and D. V. Nanopoulos, Nucl. Phys. B155, 52 (1979).