Difficulty for the Weinberg model of *CP* nonconservation through Higgs-boson exchange

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We evaluate the *CP*-violation parameter ϵ'/ϵ in the Weinberg model of *CP* nonconservation. When gluon-exchange effects are included, we find $\epsilon'/\epsilon \sim -0.045$, which is in conflict with the experimental measurement $\epsilon'/\epsilon = -0.003 \pm 0.015$.

Two alternate models of *CP* nonconservation seem equally compatible with observed phenomena. The standard model with (at least) three quark doublets leads to the general Kobayashi-Maskawa[1] mixing matrix which contains a *CP*-violating phase δ. In this model *CP* is never an approximate symmetry, the smallness of the effect arising solely due to the value of the mixing angles. In contrast, the *CP* violation arising from the Higgs bosons, as in the model of Weinberg,[2] is naturally suppressed at least for light quarks by a factor $(m_q/m_H)^2$ in the amplitude. The model further allows for the possibility that *CP* is a spontaneously broken symmetry. In such a case, for an arbitrary number of quark doublets, provided one imposes natural flavor conservation as done by Weinberg, the quark mixing matrix is real, and *CP* violation resides exclusively in the Higgs sector.[3] This kind of soft *CP* violation has been advocated recently as a solution to the strong *CP* problem.[4] It is the purpose of this paper to show that careful calculations in the above model lead to an unacceptably large value for the *CP* parameters ϵ'/ϵ in the *K* system, and hence the model can in all probability be ruled out.

The interaction of Higgs bosons with quarks, assuming arbitrary number of quark doublets $(u_i, d_i)_L$, in the standard model is

$$\mathcal{L} = \frac{\phi_1^-}{\lambda_1^*} \bar{d}_{iR} M_i^d V_{ij}^\dagger u_{jL} - \frac{\phi_2^+}{\lambda_2} \bar{u}_{iR} M_i^u V_{ij} d_{jL} + \frac{\bar{\phi}_1^0}{\lambda_1^*} \bar{d}_{iR} M_i^d d_{iL} + \frac{\phi_2^0}{\lambda_2} \bar{u}_{iR} M_i^u u_{iL} + \text{H.c.} \quad (1)$$

Here $M_i^d = (m_d, m_s, m_b, \ldots)$ and $M_i^u = (m_u, m_c, m_t, \ldots)$. The matrix V is a Kobayashi-Maskawa–type with real elements. λ_1 and λ_2 are expectation values of ϕ_1^0 and ϕ_2^0, respectively. The exchange of Higgs bosons leads to *CP* violation because the transition propagator

$$A = (\text{FT}\langle 0 | \phi_1^- \phi_2^+ | 0 \rangle)_{q^2=0} / \lambda_1^* \lambda_2$$

is a complex quantity, where FT denotes Fourier transform.[5] We now proceed to calculate $\epsilon_m = \text{Im}M_{12}/\text{Re}M_{12}$, where M is the K-$\bar{K}$ mass matrix, and

$$\xi = \text{Im}\langle 2\pi(I=0) | H_{\text{wk}} | K^0 \rangle / \text{Re}\langle 2\pi(I=0) | H_{\text{wk}} | K^0 \rangle .$$

Both the calculation and the final conclusion are not dependent on the heavy-quark sector, and we shall therefore consider only u, d, s, and c quarks. We also use the standard current-algebra quark masses m_u, $m_d \ll m_s$. The dominant interactions relevant to the K system are

$$\mathcal{L} = \frac{\phi_1^-}{\lambda_1^*} m_s (\sin\theta\, \bar{s}_R u_L + \cos\theta\, \bar{s}_R c_L) + \frac{\phi_2^+}{\lambda_2} m_c (\sin\theta\, \bar{c}_R d_L - \cos\theta\, \bar{c}_R s_L) + \text{H.c.} \quad (2)$$

The relevant diagram for calculations of ϵ_m is given in Fig. 1. Explicit calculation shows that we get, assuming $M_W > M_H > m_c$,

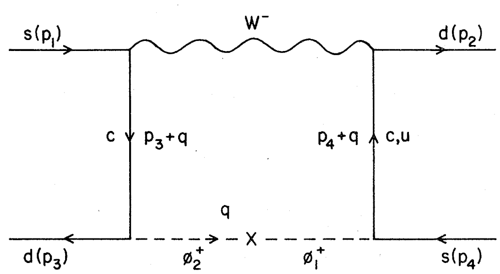

FIG. 1. Dominant diagram contributing to $\text{Im}M_{12}$. Note the Glashow-Iliopoulos-Maiani cancellation on one side only.

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$$\frac{G_F m_c m_s}{4\sqrt{2}\,\lambda_1\lambda_2^*}(\sin\theta\cos\theta)^2\int\frac{d^4q}{(2\pi)^4}[\bar d_\alpha(p_3)(1+\gamma_5)(\not p_3+\not q-m_c)^{-1}\gamma_\mu(1-\gamma_5)s_\alpha(p_1)]$$
$$\times(q^2-M_H^2)^{-1}\{\bar d_\beta(p_2)\gamma^\mu(1-\gamma_5)[(\not p_4+\not q-m_c)^{-1}-(\not p_4+\not q-m_u)^{-1}](1+\gamma_5)S_\beta(p_4)\}\;. \tag{3}$$

Note the Glashow-Iliopoulos-Maiani-type cancellation. Explicit evaluation of the integrals gives the value

$$\tilde g[\bar d_\alpha(p_3)\gamma^\mu(\not p_4-\not p_3)(1+\gamma_5)s_\beta(p_4)][\bar d_\beta(p_2)\gamma_\mu(1-\gamma_5)s_\alpha(p_1)]\,, \tag{4}$$

where

$$\tilde g=\frac{i}{64\pi^2}\frac{G_F}{\sqrt2}m_c^2m_s(\sin\theta\cos\theta)^2A\,. \tag{5}$$

Calculating both the s- and t-channel graphs as well as W–Higgs-boson–interchanged graphs, we find in x space, in agreement with Anselm and D'yaknov,[6]

$$=\tilde g[\bar d_\alpha(x)\gamma^\mu(1-\gamma_5)s_\beta(x)]\partial^\nu[\bar d_\beta(x)\gamma_\mu\gamma_\nu(1+\gamma_5)s_\alpha(x)]\,. \tag{6}$$

For the imaginary part of M_{12}, using vacuum saturation we obtain

$$\mathrm{Im}M_{12}\approx\frac{G_F}{32\pi^2\sqrt2}\left(\frac{m_c^2m_K^4}{2m_K}\right)f_K^2(\sin\theta\cos\theta)^2\,\mathrm{Im}A\,. \tag{7}$$

From Ref. 7, and including color we also have

$$\mathrm{Re}M_{12}=\frac{G_F^2m_c^2m_K^2}{6\pi^2(2m_K)}f_K^2(\sin\theta\cos\theta)^2\,. \tag{8}$$

Therefore, we find for ϵ_m

$$\epsilon_m=\frac{3}{16\sqrt2}\left(\frac{m_K^2\,\mathrm{Im}A}{G_F}\right)\,. \tag{9}$$

In calculating ξ, we note that the dominant mechanism for the CP-conserving amplitude $K_S\to2\pi$ is the gluon-exchange diagram, as first pointed out by Vainshtein, Zakharov, and Shifman.[8] A similar gluon-exchange diagram also contributes to the CP-violating process (see Fig. 2). However, an important difference is that the gluon pole is absent in the former case, while it is present in the latter. A reasonable procedure seems to be to use an effective mass for the gluon propagator comparable to the hadron mass (in this case m_K). The $s\to dg$ CP-conserving and CP-violating interactions are, respectively,

$$\mathcal{L}^+=\frac{G_F}{\sqrt2}(\sin\theta\cos\theta)\frac{g}{12\pi^2}\ln\left(\frac{m_c^2}{\mu^2}\right)\times\left[\bar s\gamma^\nu(1-\gamma_5)\frac{\lambda^a}{2}d\right]\partial^\mu G^a_{\mu\nu}+\mathrm{H.c.}\,, \tag{10}$$

$$\mathcal{L}^-=i(\mathrm{Im}A)(\sin\theta\cos\theta)\frac{g}{32\pi^2}\left[\ln\left(\frac{m_H^2}{m_c^2}\right)-\frac32\right]m_c^2m_s\times\left(\bar s\sigma_{\mu\nu}\frac{\lambda^a}{2}d\right)G^{\mu\nu}_a+\mathrm{H.c.} \tag{11}$$

Here μ is the typical external quark momentum in K decay, λ^a are Gell-Mann matrices normalized by $\mathrm{Tr}\lambda^a\lambda^b=2\delta^{ab}$, and $G^a_{\mu\nu}$ is the gluon field tensor. Effective four-quark interactions are then easily calculated by using equations of motion for the gluon field. These are

$$M^+=C^+[\bar s\gamma^\nu(1-\gamma_5)\lambda^ad](\bar u\gamma_\nu\lambda^au+\bar d\gamma_\nu\lambda^ad)\,, \tag{12}$$

$$M^-=C^-[\partial^\mu(\bar s\sigma_{\mu\nu}\gamma_5\lambda^ad)](\bar u\gamma^\nu\lambda^au+\bar d\gamma^\nu\lambda^ad)\,, \tag{13}$$

where

$$C^+=\frac{G_F}{\sqrt2}(\sin\theta\cos\theta)\frac{\alpha_s}{12\pi}\ln\left(\frac{m_c^2}{\mu^2}\right), \tag{14}$$

$$C^-=i(\mathrm{Im}A)(\sin\theta\cos\theta)\frac{\alpha_s}{32\pi}\left[\ln\left(\frac{m_H^2}{m_c^2}\right)-\frac32\right]\frac{m_c^2m_s}{m_G^2}\,. \tag{15}$$

The matrix elements of these two operators between $|K^0\rangle$ and $|\pi^+\pi^-\rangle$ can be evaluated in the approximation that only vacuum and one-particle states contribute. The procedure for M^+ is given in detail in Ref. 8, and for M^- is similar though involved[9]:

$$\langle\pi^+\pi^-|M^+|K^0\rangle=C^+\frac{16\sqrt2}{9}\frac{f_\pi m_K^2m_\pi^2}{m_s(m_d+m_u)}\left(\frac{f_K}{f_\pi}-1+\frac{f_Km_K^2}{f_\pi m_\sigma^2}\right), \tag{16}$$

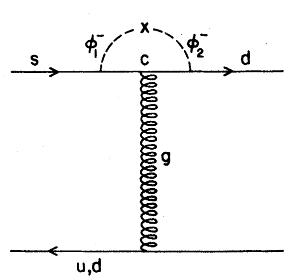

FIG. 2. "Higgs-boson–penguin"–type diagram contributing to the imaginary part of $K^0\to2\pi$ amplitude.

$$\langle\pi^+\pi^-|M^-|K^0\rangle = C^- \frac{8}{3}\frac{f_\pi m_K^2 m_\pi^2}{(m_d+m_u)}\left(\frac{f_K}{f_\pi}+1+\frac{f_K}{f_\pi}\frac{m_K^2}{m_\sigma^2}\right). \tag{17}$$

Here all the quark masses are current-algebra quark masses, $f_\pi \approx m_\pi$, and m_σ is the mass of 0^+ scalar meson, taken to be 700 MeV. We then find for ξ the expression

$$\xi = \frac{9(\mathrm{Im}A)[\ln(m_H^2/m_c^2)-\frac{3}{2}]m_c^2 m_s^2(f_K+f_\pi+f_K m_K^2/m_\sigma^2)}{16G_F\ln(m_c^2/\mu^2)m_G^2(f_K-f_\pi+f_K m_K^2/m_\sigma^2)}$$
$$= (1.7-0.7)\left(\frac{m_K^2\,\mathrm{Im}A}{G_F}\right). \tag{18}$$

Here we have assumed $m_H \sim 15$ GeV, $m_c = 1.2$ GeV, $m_s = 150$ MeV, $m_G = m_K = 0.5$ GeV, $f_K = 1.2 f_\pi$, $m_\sigma = 0.7$ GeV, and $\mu \sim m_K$ of m_π. The latter value of μ might be preferred to get enhancement of $K_S \to 2\pi$. Therefore, we obtain for the ratio ξ/ϵ_m values in the range

$$\frac{\xi}{\epsilon_m} = 12.8-5.2\,. \tag{19}$$

This implies for the ratio ϵ'/ϵ (Ref. 10)

$$\frac{\epsilon'}{\epsilon} = -\frac{1}{20}\left(\frac{2\xi}{\epsilon_m+2\xi}\right) = -0.045 \text{ to } -0.048\,. \tag{20}$$

This is in gross conflict with the present experimental value $\epsilon'/\epsilon = -0.003 \pm 0.015$.

We now discuss some of the theoretical uncertainties in our expression (18) for ξ. We have assumed that the CP-conserving process $K \to 2\pi$ is dominated by the gluon-exchange diagram. If this diagram contributes a fraction f only to the process, then $\xi \sim 1/f$ and this only serves to increase ϵ'/ϵ. We have not taken higher-order quantum-chromodynamic effects into account, as was done by Gilman and Wise[10] for the standard model. These corrections are much more difficult to calculate for the "Higgs-boson penguin" diagram. However, the ratio of CP-odd to the CP-even matrix element is less sensitive to these corrections as α_s is seen to cancel out. For the same reason, both ϵ_m and ξ are insensitive to the use of vacuum saturation approximation; if the bag model is employed the ratios would be nearly the same. Barring a remarkable cancellation of the single-gluon-exchange CP-odd operator with multiple-gluon exchanges, we see no way to decrease the value of ξ/ϵ_m from 5 to 0.3 as required by data.

The consideration of this paper serves to rule out models of CP violation through Higgs-boson exchange that have natural flavor conservation (NFC) built in them. Models without NFC are still allowed.[11] They have a rather unattractive requirement that Higgs bosons that lead to direct CP-nonconserving $K_L \to K_S$ transition be extremely heavy ($M_H \sim 250$ GeV). These models resemble superweak[12] theories of CP violation.

Note added. When this manuscript was being written up the author found that Professor A. Sanda had investigated the same problem with a similar conclusion.[13]

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