PHYSICAL REVIEW D VOLUME 23, NUMBER 11 1 JUNE 1981

Problem for theories with spontaneous *CP* violation and natural flavor conservation

A. I. Sanda
Rockefeller University, New York, New York 10021
(Received 19 January 1981)

Using a vacuum-saturation approximation, Vainshtein, Zakharov, and Shifman have shown that $L = L_{\rm QCD} + L_{\rm EW}$ can explain the $\Delta I = 1/2$ rule of strange-particle decays. Requiring $L_{\rm EW}$ to possess spontaneous CP violation and natural flavor conservation, we estimate ϵ'/ϵ using a similar approximation. We show that a very crude computation results in a very stringent limit $0.050 > |\epsilon'/\epsilon| > 0.048$. This estimate is in conflict with the experimental measurement $|\epsilon'/\epsilon| = 0.003 \pm 0.015$. This is a problem for theories with spontaneous CP violation and natural flavor conservation if the above understanding of the $\Delta I = 1/2$ rule is correct.

I. INTRODUCTION

Understanding the source of CP violation has been a challenge for nearly two decades. Even within the context of the $SU(2)_L \times U(1)$ gauge theory of weak interaction, there is more then one equally appealing way of introducing CP violation to the theory.[1-3]

(a) Spontaneous CP violation is introduced in a system of at least three doublets of Higgs bosons. If discrete symmetry is imposed to guarantee natural flavor conservation[2] (NFC), an observable complex phase appears only in the couplings of Higgs bosons to quarks.[4] In particular, the Kobayashi-Maskawa (KM) matrix[3] can be chosen to be real in this scheme.

(b) CP violation is introduced in a system of at least six quark flavors in a way that a complex KM matrix is needed to diagonalize the mass matrix.

In one scheme, the complex phase appears *only* in the Higgs-boson coupling to quarks, and in the other, the complex phase appears in the gauge-boson couplings to quarks. It should be possible to differentiate these two possibilities.

In this paper, we report on an investigation which shows a possible difficulty for scenario (a) in correctly describing the experimental result for ϵ'/ϵ, a ratio of CP-violating parameters for K decay. This result suggests a need for either a nontrivial complex phase in the KM matrix or an extension of the gauge group to include superweak interactions.[5] It is interesting to speculate that the complex phase in the KM matrix appears spontaneously [scheme (b) + spontaneous CP violation]. Then a strangeness-changing neutral-Higgs-boson interaction must exist at some level.

In Sec. II, we state the notations for reference. In Sec. III, we discuss the theoretical framework with which we compute ϵ'/ϵ. In Secs. IV and V, we discuss the computation of ϵ'/ϵ. In Sec. VI, we discuss uncertainties in our computation. In Sec. VII, we summarize our results and give some conjectures about CP violation. In Appendices A and B, we give details of our computations.

II. NOTATIONS

For completeness and for reference,[6] we state here some well-known conventions for parametrizing CP violations in K-meson decays. The mass matrix is given by

$$\begin{bmatrix} \langle K^0|M|K^0\rangle & \langle K^0|M|\bar{K}^0\rangle \\ \langle \bar{K}^0|M|K^0\rangle & \langle \bar{K}^0|M|\bar{K}^0\rangle \end{bmatrix} = \begin{bmatrix} M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & M-\frac{i}{2}\Gamma \end{bmatrix}. \quad (1)$$

The states of definite mass $M_{1,2}$ and widths $\Gamma_{1,2}$ are

$$|K_1\rangle = p|K^0\rangle + q|\bar{K}^0\rangle,$$
$$|K_2\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \quad (2)$$

where

$$\frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} = \left(\frac{M_{12}^*-\frac{i}{2}\Gamma_{12}^*}{M_{12}-\frac{i}{2}\Gamma_{12}}\right)^{1/2}.$$

Solving for ϵ and ignoring terms of order $(\mathrm{Im}M_{12}/M_{12})^2$, and $(\mathrm{Im}\Gamma_{12}/\Gamma_{12})^2$ we obtain

$$\epsilon = \frac{i\,\mathrm{Im}M_{12} + \frac{1}{2}\mathrm{Im}\Gamma_{12}}{\Delta m - i\frac{1}{2}\Delta\Gamma}, \quad (3)$$

where

$$\Delta\Gamma = \Gamma_1 - \Gamma_2 = -4\,\mathrm{Im}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2},$$
$$\Delta m = m_1 - m_2 = 2\,\mathrm{Re}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}. \quad (4)$$

Using the experimental information

$$\Delta m \cong \tfrac{1}{2}\,\Delta\Gamma \tag{5}$$

and ignoring terms proportional to

$$\frac{\Delta m - \frac{1}{2}\Delta\Gamma}{\Delta m}\times(\mathrm{Im}M_{12}\ \text{or}\ \mathrm{Im}\Gamma_{12})\,, \tag{6}$$

we have

$$\epsilon \approx \frac{e^{i\phi}}{\sqrt{2}}\left(i\frac{\mathrm{Im}M_{12}}{\Delta m}+\frac{\mathrm{Im}\Gamma_{12}}{\Delta\Gamma}\right), \tag{7}$$

$$\phi = \tan^{-1}\left(\frac{-\Delta\Gamma}{2\,\Delta m}\right). \tag{8}$$

Denote

$$\begin{aligned} A_I e^{i\delta_I} &= \langle(2\pi)_I|H|K^0\rangle\,,\\ \bar{A}_I e^{i\delta_I} &= \langle(2\pi)_I|H|\bar{K}^0\rangle\,, \end{aligned} \tag{9}$$

where δ_I is the $\pi\pi$ phase shift in the isospin-I state. CPT invariance implies $\bar{A}_I = A_I^*$. Since $|K^0\rangle$ and $|\bar{K}^0\rangle$ states are defined to be the eigenstates of the strong interaction, and the strong interaction cannot transform $|K^0\rangle$ into $|\bar{K}^0\rangle$, the relative phase of these states is left undetermined by this definition. We use the popular phase convention

$$\mathrm{Im}A_0 = 0\,.$$

With this convention

$$\begin{aligned} \eta_{+-} &\equiv \frac{\langle\pi^+\pi^-|H|K_2\rangle}{\langle\pi^+\pi^-|H|K_1\rangle} = \epsilon+\epsilon'\,,\\ \eta_{00} &\equiv \frac{\langle\pi^0\pi^0|H|K_2\rangle}{\langle\pi^0\pi^0|H|K_1\rangle} = \epsilon-2\epsilon'\,, \end{aligned} \tag{10}$$

where

$$\epsilon' = \frac{i}{\sqrt{2}}\,\frac{\mathrm{Im}A_2}{A_0}\,e^{i(\delta_2-\delta_0)}\,.$$

As we shall see below, computation using the KM matrix and free-quark model leads[6,7] to a complex A_0,

$$(A_0)_{\mathrm{KM}} = |A_0|\,e^{i\xi}\,. \tag{11}$$

In accordance with our phase convention, we make an adjustment $|K^0\rangle \to e^{-i\xi}|K^0\rangle$ to make A_0 real. This leads to

$$\begin{aligned} \frac{\mathrm{Im}M_{12}}{\Delta m} &\to \frac{\mathrm{Im}[(M_{12})_{\mathrm{KM}}e^{2i\xi}]}{2\,\mathrm{Re}[(M_{12})_{\mathrm{KM}}e^{2i\xi}]}\\ &= \frac{\mathrm{Im}(M_{12})_{\mathrm{KM}}}{2\,\mathrm{Re}(M_{12})_{\mathrm{KM}}}+\xi \end{aligned} \tag{12}$$

and

$$\frac{\mathrm{Im}A_2}{A_0} \to \frac{\mathrm{Im}(A_2)_{\mathrm{KM}}-\xi\,\mathrm{Re}(A_2)_{\mathrm{KM}}}{A_0}\,.$$

Since we expect $\mathrm{Im}(A_2)_{\mathrm{KM}} \ll \xi\,\mathrm{Re}(A_2)_{\mathrm{KM}}$, we obtain

$$\frac{\epsilon'}{\epsilon} = -e^{i(\delta_2-\delta_0-\phi)}\,\frac{2\xi}{\epsilon_m+2\xi}\,\frac{\mathrm{Re}(A_2)_{\mathrm{KM}}}{A_0}\,, \tag{13}$$

where $\epsilon_m = \mathrm{Im}(M_{12})_{\mathrm{KM}}/\mathrm{Re}(M_{12})_{\mathrm{KM}}$. We have ignored $\mathrm{Im}\Gamma_{12}$. Note that Γ_{12} is dominated by the $(2\pi)_{I=0}$ channel and this amplitude is defined to be real. If $|\epsilon_m| \ll |2\xi|$, we have

$$\left|\frac{\epsilon'}{\epsilon}\right| \approx \left[\frac{\Gamma(K\to(\pi\pi)_{I=2})}{\Gamma(K\to(\pi\pi)_{I=0})}\right]^{1/2} \approx 0.05\,.$$

It will be shown below that indeed this is the case.

III. THEORETICAL FRAMEWORK

Recent progress[8] in understanding the nonleptonic decays makes it possible to give an estimate for ϵ'/ϵ. In particular, the authors of Ref. 8 demonstrated that the $\Delta I = \frac{1}{2}$ rule can be understood from the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}}+\mathcal{L}_{\mathrm{EW}}\,, \tag{14}$$

where $\mathcal{L}_{\mathrm{QCD}}$ and $\mathcal{L}_{\mathrm{EW}}$ are the quantum-chromodynamics (QCD) Lagrangian and the Weinberg-Salam electroweak Lagrangian, respectively. An effective Hamiltonian can be derived from the above Lagrangian,

$$H_{\mathrm{eff}} = H_0 + H_{\alpha_s}\,, \tag{15}$$

$$\begin{aligned} H_0 &= \frac{G_F}{\sqrt{2}}\,J_\mu J^\mu\,,\\ H_{\alpha_s} &= \frac{4G_F}{\sqrt{2}}\,\frac{\alpha_s}{6\pi}\ln\left(\frac{M_u^2}{M_c^2}\right)\bar{d}\gamma_\mu\gamma_-\lambda^a s\\ &\quad\times(\bar{u}\gamma^\mu\lambda^a u+\bar{d}\gamma^\mu\lambda^a d+\bar{s}\gamma^\mu\lambda^a s+\cdots)\,, \end{aligned} \tag{16}$$

where J_μ is the usual weak current and the second term is an effective interaction term derived from Fig. 1. The second term transforms like an octet and because of its $(V-A)V$ structure, it dominates the $K\to(2\pi)_{I=0}$ amplitude. The Hamiltonian H_{eff} with its QCD correction gives amplitudes for $K^+\to\pi^+\pi^0$ ($\Delta I = \frac{3}{2}$ transition) and $K_S\to\pi^+\pi^-$ in agreement with experiment. The parameter ϵ'/ϵ will be computed within this framework.

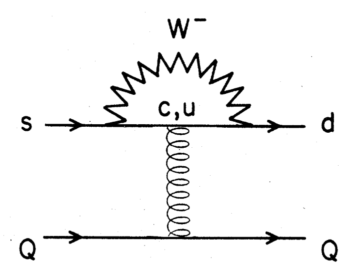

FIG. 1. The diagram which gives an imaginary part to the $K\to 2\pi$ amplitude if the KM matrix is complex. This diagram also gives a dominating real part to the $K\to 2\pi$ amplitude.

For a complex KM matrix, H_{α_s} introduces a phase in $K \to (2\pi)_{I=0}$ amplitude. This leads to a nontrivial prediction for ϵ'/ϵ. This has been investigated in Ref. (7). In this paper, we investigate a class of theories which has an additional interaction term in $H_{\rm eff}$ from additional Higgs particles.

In scenario (a) discussed in the Introduction, the spontaneous *CP* violation and natural flavor conservation greatly restrict the possible form of the Higgs couplings to quarks.[4] For example, if the Yukawa coupling constants Γ_{ij} are chosen to be real, the Lagrangian

$$\mathcal{L}_\phi = \sum_{ij} (\bar{\psi}_{iL} \Gamma^1_{ij} \varphi_1 d_{jR} + \bar{\psi}_{iL} \Gamma^2_{ij} \tilde{\varphi}_2 u_{jR}) \tag{17}$$

satisfies the above conditions. $\psi^\dagger_{iL} = (u^\dagger_i, d^\dagger_i)_L$, ϕ_1, and ϕ_2 are doublets of Higgs bosons and $\tilde{\phi}_2 = i\sigma_2 \phi_2^*$. A set of discrete symmetries can be imposed to guarantee the form of $\mathcal{L}_\phi$ given above. It is then immediately obvious that any phase in the mass matrix introduced by the vacuum expectation value of ϕ_i,

$$\langle \varphi^0_i \rangle = v_i e^{i\theta_i}, \tag{18}$$

can be rotated away by redefining the quark phases. In this way we see that the KM matrix for this scheme is real and a complex phase appears only in the charged-Higgs-boson couplings to quarks. A class of models which falls into the above category has a *CP*-violating charged-Higgs-boson interaction represented by a general form

$$\mathcal{L}_I = \sum_i \left(\frac{g}{\sqrt{2}\, M_W} \right) (\alpha_i \bar{U}_R M_u K D_L H^\dagger_i + \beta_i \bar{U}_L M_d K D_R H^\dagger_i), \tag{19}$$

where $H^\dagger_i$ is a charged physical Higgs-boson field,[9] $D^\dagger_{L,R} = (d^\dagger, s^\dagger, b^\dagger)_{L,R}$, $U^\dagger_{L,R} = (u^\dagger, c^\dagger, t^\dagger)_{L,R}$, M_u and M_d are diagonal quark mass matrices, K is a real KM matrix, and α_i and β_i are complex numbers.

IV. COMPUTATION OF ϵ_m

The major contributions to $\mathrm{Im}\langle K^0 | H | \bar{K}^0 \rangle$ and $\mathrm{Re}\langle K^0 | H | \bar{K}^0 \rangle$ are given by the diagrams shown in Figs. 2 and 3, respectively. While a general computation including the effects of a virtual t quark in the loop integral has been made, for the purpose of illustration we shall present our results ignoring the t-quark contribution (i.e., setting $s_2, s_3 = 0$; the effects of the t quark will be discussed below). From Figs. 2 and 3 we obtain

$$\mathrm{Im}\langle K^0 | H | \bar{K}^0 \rangle = \mathrm{Im}(\alpha^* \beta) \frac{G_F^2}{4\pi^2} (\sin\theta_C \cos\theta_C)^2 \frac{m_c^2 m_K^2 M_d}{M_\phi^2 m_s} \times 2 \langle K^0 | \bar{d}\gamma_\mu \gamma_- s | 0 \rangle \langle 0 | \bar{d} \gamma^\mu \gamma_- s | \bar{K}^0 \rangle, \tag{20}$$

$$\mathrm{Re}\langle K^0 | H | \bar{K}^0 \rangle = \frac{G_F^2}{4\pi^2} (\sin\theta_C \cos\theta_C)^2 m_c^2 \tfrac{8}{3} \times \langle K^0 | \bar{d}\gamma_\mu \gamma_- s | 0 \rangle \langle 0 | \bar{d} \gamma^\mu \gamma_- s | \bar{K}^0 \rangle. \tag{21}$$

Details of the computation are given in Appendix A. Here we stress that the vacuum-saturation approximation was used. The quark masses m_q and M_q are current and constituent masses of quark q, respectively. From (20) and (21), we obtain[10]

$$\epsilon_m = \frac{\mathrm{Im}\langle K^0 | H | \bar{K}^0 \rangle}{\mathrm{Re}\langle K^0 | H | \bar{K}^0 \rangle} = \mathrm{Im}(\alpha^* \beta) \frac{3}{4} \frac{m_K^2 M_d}{M_\phi^2 m_s}. \tag{22}$$

The *CP*-violation parameter for K decay, ϵ_K, is given by

$$\epsilon_K \approx \frac{i e^{i\phi}}{2\sqrt{2}} (\epsilon_m + 2\xi). \tag{23}$$

While our evaluation of ϵ_m agrees with a rough estimate made in Ref. 2, the dominating contribution of ϵ_K comes from ξ and we cannot obtain an upper limit for M_ϕ. If we arbitrarily set $2\xi/\epsilon_m = 30$, for example, we obtain $M_\phi \lesssim 100$ GeV for $m_s = 0.15$ GeV, $M_d = 0.3$ GeV, and $M_c = 2$ GeV.

V. COMPUTATION OF ξ

The diagram shown in Fig. 4 gives a dominating contribution to the imaginary part of the matrix element for $K \to 2\pi$. We show in Appendix B that the amplitude corresponding to the lowest-order term in the perturbation expansion in powers of $g_s(m_c^2)$ can be written in the form

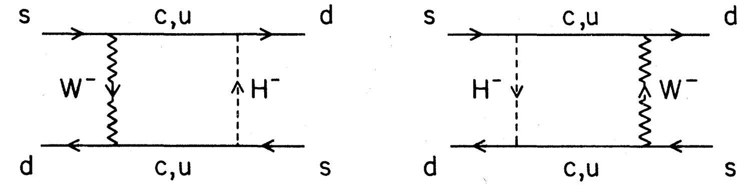

FIG. 2. Diagrams which give the dominating complex part to the $K^0-\bar{K}^0$ mixing amplitude.

$$\mathrm{Im}\langle\pi^+\pi^-|H|K^0\rangle = -\mathrm{Im}(\alpha\beta^*)\sin\theta_C\cos\theta_C\,\frac{4G_F}{\sqrt{2}}\,\frac{g_s(m_c^2)g_s(q^2)}{6\pi^2}\,\frac{m_c^2 m_s M_u}{M_\phi^2 q^2}\left(\tfrac{3}{2}+\ln\frac{M_c^2}{M_\phi^2}\right)\left[F_1(q^2)+\frac{q^2}{8M_u^2}F_2(q^2)\right]$$
$$\times\langle\pi^+\pi^-|(\bar d\gamma_- d\,\bar s\gamma_- d+\bar u\gamma_- d\,\bar s\gamma_- u)|K^0\rangle\,, \tag{24}$$

where we have set $M_u = M_d$. $F_1(q^2)$ and $F_2(q^2)$ are defined by a general gluon-quark vertex function

$$g_s(q^2)\bar{Q}\left[\gamma_\mu F_1(q^2)+i\frac{\sigma_{\mu\nu}q^\nu}{2M_u}F_2(q^2)\right]Q\,. \tag{25}$$

We estimate the right-hand side of (25) by setting

$$F_1(q^2)+\frac{q^2}{8M_u^2}F_2(q^2)\approx F_1(q^2) \tag{26}$$

and noting that

$$g_s(q^2)F_1(q^2)/q^2 \gg g_s(m_c^2)/m_K^2\,. \tag{27}$$

While there is no way of justifying (26), it should not mislead us in giving an estimate of a lower bound unless there is a miraculous cancellation between the two terms along with a sign change in the amplitude. With the above simplification, we obtain a bound

$$\mathrm{bound}(\mathrm{Im}\langle\pi^+\pi^-|H|K^0\rangle) = -\mathrm{Im}(\alpha\beta^*)\sin\theta_C\cos\theta_C\,\frac{4G_F}{\sqrt{2}}\,\frac{\alpha_s(m_c^2)}{4\pi}\,\frac{m_c^2 m_s M_Q}{M_\phi^2 m_K^2}\left(\tfrac{3}{2}+\ln\frac{M_c^2}{M_\phi^2}\right)$$
$$\times\tfrac{8}{3}\langle\pi^+\pi^-|(\bar d\gamma_- d\,\bar s\gamma_- d+\bar u\gamma_- d\,\bar s\gamma_- u)|K^0\rangle\,. \tag{28}$$

Since the sign of $\mathrm{Im}(\alpha\beta^*)$ is not specified we cannot, at this stage, specify the direction of the bound. The real part of the amplitude is given by Fig. 1.

$$\mathrm{Re}\langle\pi^+\pi^-|H|K^0\rangle = -\frac{4G_F}{\sqrt{2}}\,\frac{\alpha_s(m_c^2)}{6\pi}\sin\theta_C\cos\theta_C\ln\frac{M_u^2}{M_c^2}$$
$$\times\tfrac{8}{9}\langle\pi^+\pi^-|(\bar d\gamma_- d\,\bar s\gamma_+ d+\bar u\gamma_- d\,\bar s\gamma_+ d)|K^0\rangle\,. \tag{29}$$

Since we expect $|\mathrm{Re}\langle\pi^+\pi^-|H|K^0\rangle| \gg |\mathrm{Im}\langle\pi^+\pi^-|H|K^0\rangle|$, we write

$$\langle\pi^+\pi^-|H|K^0\rangle = \mathrm{Re}(\langle\pi^+\pi^-|H|K^0\rangle)e^{i\xi}\,,$$

where

$$\xi = \mathrm{Im}\langle\pi^+\pi^-|H|K^0\rangle/\mathrm{Re}\langle\pi^+\pi^-|H|K^0\rangle\,. \tag{30}$$

Using the vacuum-saturation approximation, and following the analysis of Ref. 8 which uses

$$\langle\pi^+|\bar u_R d_L|0\rangle = -i\frac{f_\pi m_\pi^2}{2(m_u+m_d)}\,,$$
$$\langle 0|\bar s_L d_R|K^0\rangle = i\frac{f_K m_K^2}{2(m_s+m_u)}\,, \tag{31}$$

$$\langle\pi^-|\bar s_L u_R|K^0\rangle = [f_+(m_K^2-m_\pi^2)+f_- m_\pi^2]/2(m_s-m_u)\,,$$
$$\langle\pi^+\pi^-|\bar d_R d_L|0\rangle = \frac{m_\pi^2}{2(m_u+m_d)}\,\frac{1}{1-m_K^2/m_\sigma^2}\,,$$

we obtain

$$\mathrm{bound}\,(\xi) = \mathrm{Im}(\alpha\beta^*)\,\frac{9}{2}\,\frac{m_c^2 m_s M_u}{M_\phi^2 m_K^2}\,\frac{(\frac{3}{2}+\ln M_c^2/M_\phi^2)}{\ln M_u^2/M_c^2}$$
$$\times\left[\frac{f_K+f_\pi f_+(1-m_K^2/m_\sigma^2)}{-f_K+f_\pi f_+(1-m_K^2/m_\sigma^2)}\right]. \tag{32}$$

Comparing ξ with ϵ_m given in (10) we obtain

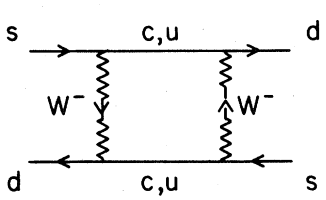

FIG. 3. The diagram which gives the dominating real part to the $K^0-\bar{K}^0$ mixing amplitude.

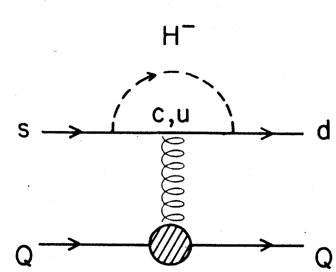

FIG. 4. The diagram which gives the dominant contribution to the imaginary part of the $K\to 2\pi$ amplitude.

$$\frac{2\xi}{\epsilon_m} \gtrsim 12\left(\frac{m_c m_s}{m_K^2}\right)^2 \frac{(\frac{3}{2}+\ln M_c^2/M_\phi^2)}{\ln M_u^2/M_c^2} \times\left[\frac{f_K+f_\pi f_+(1-m_K^2/m_\sigma^2)}{f_K-f_\pi f_+(1-m_K^2/m_\sigma^2)}\right]. \qquad (33)$$

For $m_c = 1.5$ GeV, $m_s = 0.15$ GeV, $m_K = 0.5$ GeV, $M_u = 0.3$ GeV, $m_\sigma = 0.7$ GeV, $f_K = f_\pi$, and $f_+ \approx 1$, we have

$$\frac{2\xi}{\epsilon_m} \gtrsim 30. \qquad (34)$$

Setting

$$\left|\frac{\mathrm{Re}(A_2)_{\mathrm{KM}}}{A_0}\right| \approx 0.05, \quad \cos(\delta_2-\delta_0-\phi)\approx 1$$

we obtain the final result from (13),

$$0.050 \gtrsim |\mathrm{Re}(\epsilon'/\epsilon)| \gtrsim 0.048, \qquad (35)$$

The bound (35) is inconsistent with the experimental measurement

$$|\mathrm{Re}(\epsilon'/\epsilon)| = 0.003 \pm 0.015. \qquad (36)$$

VI. UNCERTAINTIES

There are two major sources of uncertainties. One is the vacuum-saturation approximation which seems to, at least qualitatively, give a correct order of magnitude for the K_L-K_S mass difference, $K \to 2\pi$ decay rates, etc. Hopefully, some of the uncertainties cancel out for $2\xi/\epsilon_K$ which is a ratio of ratios. More serious uncertainties arise from the nonlocal nature of the interaction represented by Fig. 4. This forces us to an equality (26) and an inequality (27). While we are comfortable with (27) which should be a gross underestimate, there is no argument to justify (26).

We note, however, that the lower bound on ϵ'/ϵ is quite insensitive to any uncertainities in $2\xi/\epsilon_m$. The only way to invalidate the estimate for $|\mathrm{Re}(\epsilon'/\epsilon)$ given in (35) is to have an almost exact cancellation between $\langle F_1(q^2)\rangle$ and $\langle (q^2/8M_u^2)F_2(q^2)\rangle$, where the bracket implies their values folding in the wave function and integrating over q^2.

In our actual computation, we have included the effect of a top quark. We found that the bound for $2\xi/\epsilon_m$ is not modified very much when m_t is varied in the range 15–50 GeV. This is because both ξ and ϵ_m have similar dependence on m_t. This is in sharp contrast with a case in which ϵ'/ϵ is computed with a complex KM matrix. In this case,[7] the t-quark contributions behave as

$$\epsilon_m \sim m_t^2, \quad \xi \sim \ln m_t^2/m_q^2,$$

and ξ/ϵ_m is strongly dependent on the value of m_t.

We have also relaxed the inequality $M_H \ll M_W$ used in our presentation. Since only the bound is known for ξ/ϵ_m, ϵ_K does not fix M_H. The value of ϵ'/ϵ presented for the case $M_H \ll M_W$ remains unaltered.

Finally, we note that the generalization of our computation to include cases in which there are more than one Higgs boson introduces further parameters and thus more freedom. The possibility of having a cancellation between terms arising from different Higgs bosons seems to be a very unattractive way of avoiding the conflict with the experimental measurement.

VII. SUMMARY

We have computed the CP-violation parameter ϵ'/ϵ for $K \to 2\pi$ decay for the class of models in which CP is violated spontaneously and which have natural flavor conservation for the Higgs-boson-exchange interaction. Our result is considerably larger than a previous estimate.[10] Barring some unusual and unattractive cancellations, our result (35) suggests that the class of models considered here is ruled out.

As an alternative to the above scenario, it is attractive to consider spontaneous CP violation without natural flavor conservation for the Higgs-boson-exchange interaction. This allows for the KM matrix to be complex. The neutral Higgs boson which mediates the flavor changing interaction in this new scenerio must be quite massive in order to be consistent with the K_L-K_S mass difference. If the contribution from the Higgs-boson-exchange diagrams to CP-violation parameters is negligible, then CP violation arises only from the KM phases and the new scheme falls into the category of scheme (b).

Experimental studies of CP violation in heavy-quark systems[11] to detect a possible phase in the KM matrix are extremely important in further understanding of the origin of CP violation.

Note added. After this work was completed, the author found that a similar investigation by N. G. Deshpande and E. Takasugi was in progress.

ACKNOWLEDGMENTS

This work was supported in part by the Department of Energy under Contract No. DE-AC02-76ER0223B.00. It is a pleasure to acknowledge a fruitful discussion with M. Wise.

APPENDIX A

The amplitude for the diagram shown in Fig. 2 is given by

$$\mathcal{A} = -(K_{id}K_{is})(K_{jd}K_{js})\beta\alpha^* \frac{g^4}{8}\frac{m_s}{16\pi^2 i M_W^4}\int \frac{dZ_1 dZ_2 dZ_3 \delta(1-Z_1-Z_2-Z_3)}{m_i^2 Z_3 + m_j^2 Z_2 + M_\Phi^2 Z_1}$$

$$\times\{m_j^2 \bar{d}(p_2)\gamma_\mu[\not{p}_1(1-Z_2)-\not{p}_3 Z_3]\gamma_+ s(p_1)\bar{d}(p_3)\gamma^\mu\gamma_- s(p_4) + m_i^2\bar{d}(p_2)\gamma_\mu\gamma_- s(p_1)\bar{d}(p_3)\gamma^\mu[\not{p}_4(1-Z_2)-\not{p}_2 Z_4]\gamma_+ s(p_4)\},$$

where we have assumed m_i, m_j, $M_\Phi \ll M_W$. Using an identity

$$2\bar{d}(p_2)\gamma^\mu\gamma_- s(p_1)\bar{d}(p_3)\gamma_\mu\not{p}_2\gamma_+ s(p_4) = 2M_d\bar{d}(p_2)\gamma_- s(p_1)\bar{d}(p_3)\gamma_+ s(p_4) + M_d\bar{d}(p_2)\sigma^{\mu\nu}\gamma_- s(p_1)\bar{d}(p_3)\sigma_{\mu\nu}\gamma_+ s(p_4) + \epsilon^{\gamma\delta\mu\lambda}p_{2\delta}\bar{d}(p_2)\gamma_\lambda\gamma_- s(p_4)\bar{d}(p_3)\sigma_{\mu\nu}\gamma_+ s(p_4) \tag{A1}$$

and noting that

$$M_d\langle K^0|\bar{d}(p_2)\gamma_- s(p_1)|0\rangle\langle 0|\bar{d}(p_3)\gamma_+ s(p_4)|\bar{K}^0\rangle \gg M_s\langle K^0|\bar{d}(p_2)\gamma^\mu\gamma_- s(p_1)|0\rangle\langle 0|\bar{d}(p_3)\gamma_\mu\gamma_- s(p_4)|\bar{K}^0\rangle$$

together with the fact that the second and third terms on the right-hand side of (A1) are negligible in the vacuum-saturation approximation, we obtain

$$\mathcal{A} = -(\sin\theta_C\cos\theta_C)^2\beta\alpha^* G_F^2\frac{m_c^2 m_s}{8\pi^2 i M_\Phi^2}[\bar{d}(p_2)\gamma^\mu\gamma_- s(p_1)\bar{d}(p_3)\gamma_\mu\not{p}_2\gamma_+ s(p_4) + \bar{d}(p_2)\gamma_\mu\not{p}_3\gamma_+ s(p_1)\bar{d}(p_3)\gamma^\mu\gamma_- s(p_4)],$$

and with $H = i\mathcal{A}$,

$$H = -(\sin\theta_C\cos\theta_C)^2\beta\alpha^*\frac{G_F^2}{4\pi^2}\frac{m_c^2 m_s M_d}{M_\Phi^2}\bar{d}\gamma_- s\bar{d}\gamma_+ s.$$

Taking the matrix element $\langle\bar{K}^0|H|K^0\rangle$ with the vacuum-saturation approximation and using (31) we obtain (20).

APPENDIX B

The amplitude for the diagram shown in Fig. 4 is given by

$$\mathcal{A} = -\frac{4G_F}{\sqrt{2}}\frac{g_s(m_c^2)g_s(q^2)}{16\pi^2}\frac{m_c^2 m_s}{q^2 M_\Phi^2}\alpha^*\beta\sin\theta_C\cos\theta_C\left[\tfrac{3}{2}+\ln\left(\frac{M_c^2}{M_\Phi^2}\right)\right]\bar{d}(p')\sigma_{\mu\nu}q^\nu\gamma_+\lambda^a s(p) \times\sum_{Q=U,d}\bar{Q}\left[F_1(q^2)\gamma^\mu + i\frac{\sigma^{\mu\lambda}q_\lambda}{2M_Q}F_2(q^2)\right]\lambda^a Q, \tag{B1}$$

where we have expanded the amplitude in powers of $g_s(m_c^2)$ and kept only the leading term. Since q^2 may be small, $g_s(q^2)$ can be large and we cannot write the gluon-quark vertex in a specific form. The vertex function is given in the general form in terms of function $F_1(q^2)$ and $F_2(q^2)$. In evaluating (B1), it is useful to note the identity

$$\bar{d}(p')\sigma_{\mu\nu}\gamma_+ s(p)q^\nu\bar{Q}(k')\gamma^\mu Q(k) = iM_Q\bar{d}(p')\sigma_{\mu\nu}\gamma_+ s(p)\bar{Q}(k')\sigma^{\mu\nu}Q(k) + \frac{i}{2}\epsilon^{\nu\lambda\mu\tau}\bar{d}(p')\sigma_{\mu\nu}\gamma_+ s(p)(k'+k)_\lambda\bar{Q}(k')\gamma_5\gamma_\tau Q(k)$$

and to note that the second term on the right-hand side gives negligible contributions to $K\to 2\pi$ decay in the vacuum-saturation approximation. Then setting $M_u = M_d$, we obtain

$$\mathcal{A} = \frac{4G_F}{\sqrt{2}}\frac{g_s(m_c^2)g_s(q^2)}{16\pi^2 i M_\Phi^2 q^2}\alpha^*\beta\sin\theta_C\cos\theta_C m_c^2 m_s M_u\left(\tfrac{3}{2}+\ln\frac{M_c^2}{M_\Phi^2}\right)$$

$$\times\sum_{Q=U,d}\left[F_1(q^2)\bar{d}(p')\sigma_{\mu\nu}\gamma_+\lambda^a s(p)\bar{Q}\sigma^{\mu\nu}\lambda^a Q + i\frac{F_2(q^2)}{2M_Q^2}\bar{d}(p')\sigma_{\mu\nu}q^\nu\gamma_+\lambda^a s(p)\bar{Q}\sigma^{\mu\lambda}q_\lambda\lambda^a Q\right].$$

Performing the Fierz transformation and keeping only the color-singlet term,

$$\mathfrak{A} = -\frac{4G_F}{\sqrt{2}}\frac{g_s(m_c^2)g_s(q^2)}{6\pi^2 i}\alpha^*\beta\sin\theta_c\cos\theta_c\frac{m_c^2 m_s M_u}{M_\phi^2 q^2}\left(\tfrac{3}{2}+\ln\frac{M_c^2}{M_\phi^2}\right)\left[F_1(q^2)+\frac{q^2}{8M_u^2}F_2(q^2)\right]\bar{d}\gamma_+ Q\bar{Q}\gamma_+ s\,.$$

A term of the form $\bar{d}\sigma_{\mu\nu}\gamma_+ Q\bar{Q}\sigma_{\mu\nu}\gamma_+ s$ was ignored since it does not contribute to $K\to 2\pi$ decay in the vacuum-saturation approximation.

[1]T. D. Lee, Phys. Rev. D 8, 1226 (1973); Phys. Rep. 96, 143 (1979).
[2]S. Weinberg, Phys. Rev. Lett. 31, 657 (1976).
[3]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[4]This point was emphasized by G. C. Branco, Phys. Rev. Lett. 44, 504 (1980).
[5]L. Wolfenstein, Phys. Rev. Lett. 13, 180 (1964).
[6]For details see T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 16, (1966).
[7]F. Gilman and M. B. Wise, Phys. Lett. 83B, 83 (1979).
[8]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. 72, 1275 (1977) [Sov. Phys.—JETP 45, 670 (1977)].
[9]C. H. Albright, J. Smith, and S.-H. H. Tye, Phys. Rev. D 21, 711 (1980).
[10]While this expression is numerically not very different from an estimate originally made in Ref. 2, this is in disagreement with that of A. A. Anselm and N. G. Uraltsev, Yad. Fiz. 30, 897 (1979) [Sov. J. Nucl. Phys. 30, 465 (1979)]. Also our result for ξ differs from those of the above authors. For this reason, we have provided Appendices in which we discuss our calculation in some detail.
[11]A. B. Carter and A. I. Sanda, Phys. Rev. Lett. 45, 952 (1980).