Final-state interactions in the decays of charmed mesons

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Decays of (charmed) D mesons into two pseudoscalars are considered, with emphasis on final-state interactions. Uncertainties in the determination of final-state-interaction corrections that arise from theoretical ambiguities, and from incomplete meson-meson scattering data are discussed. Corrections for $\pi\pi$, $\bar{K}K$, and $\bar{K}\pi$ final states are evaluated. It is concluded that a quark-spectator model with final-state interactions cannot explain all of the unexpected rates for two-body decays of charmed pseudoscalars.

I. INTRODUCTION

Data on weak decays of charmed mesons have become available and have stimulated considerable theoretical activity over the last year. Even though the experimental results do not conflict directly with any fundamental principle, they contradict the expectations of the simplest models.

We have in mind the following empirical facts[1]:

(i) $\Gamma(D^0 \to \text{anything}) \approx 5\Gamma(D^+ \to \text{anything})$,

(ii) $\Gamma(D^0 \to \bar{K}^0\pi^0) \approx 0.7\Gamma(D^0 \to K^-\pi^+)$,

(iii) $\Gamma(D^0 \to K^-\pi^+) : \Gamma(D^0 \to K^-K^+) : \Gamma(D^0 \to \pi^+\pi^-)$
$= 1 : 0.11 \pm 0.03 : 0.033 \pm 0.015$.

Items (i) and (ii) disagree with the most naive spectator-quark model, which predicts (i) $\Gamma(D^0 \to \text{anything}) = \Gamma(D^+ \to \text{anything})$, and (ii), with color factors taken into account, $\Gamma(D^0 \to \bar{K}^0\pi^0) = \frac{1}{18}\Gamma(D^0 \to K^-\pi^+)$. It is also distressing that, if this spectator-quark model is "improved" to include perturbative quantum-chromodynamics (QCD) effects, matters are made worse, e.g., $\Gamma(D^0 \to \bar{K}^0\pi^0)$ is predicted to be approximately $\frac{1}{40}$ of $\Gamma(D^0 \to K^-\pi^+)$. Given the encouraging qualitative (if not quantitative) success of perturbative QCD in dealing with weak decays of *strange* particles,[2] this is perhaps surprising.

The results of item (iii) refer to Cabibbo-suppressed decays and, irrespective of the detailed dynamics of charm decay, are in contradiction with SU_3 symmetry and a four-quark Cabibbo mixing scheme, which predict $1 : \tan^2\theta_C : \tan^2\theta_C \approx 1 : 0.05 : 0.05$ for the ratios (iii).

Various mechanisms have been proposed to account for the above-mentioned surprises.[3] The possibility of extracting information about angles in the Kobayashi-Maskawa matrix from the above discrepancies has also been raised.[4]

It has been recognized[5] that final-state interactions of the hadronic decay products could be playing an important role. In this article we will investigate in detail this possibility for the results of items (ii) and (iii), for which a satisfactory theoretical framework is available. We shall review the ambiguities inherent to the theory of final-state interactions as well as the limitations imposed by currently available data on $\pi\pi$ and $\bar{K}\pi$ scattering. We shall give our best estimate of what the various enhancement factors are, and use them to compute final-state-interaction corrections to the spectator-quark-model amplitudes.

The rest of the paper is organized as follows. Section II discusses theoretical and experimental uncertainties in the determination of final-state-interaction corrections. Section III gives our parametrizations of meson scattering data, and the enhancement factors implied by it. Section IV discusses the results and attempts to draw conclusions.

II. UNCERTAINTIES IN THE DETERMINATION OF FINAL-STATE-INTERACTION CORRECTIONS

The basic equations of final-state-interaction theory are as follows. Let T be a vector formed by the amplitudes for the various decay processes, corrected for final-state interactions, and let V be a similar vector formed by the decay amplitudes in the absence of strong interactions. The relation between the two is

$$T = DV . \tag{1}$$

The elements of the matrix D are analytic functions which contain the information about final-state interactions. Their values $D^{\pm}$ on either side of the unitarity cut are required to satisfy the equation

$$D^+ = SD^- , \tag{2}$$

where S is related to the S matrix $\tilde{S}$ for the strong interactions in the following way:

$$\begin{aligned} \tilde{S} &= 1 + 2i\rho^{1/2}A\rho^{1/2} , \\ S &= 1 + 2iA\rho . \end{aligned} \tag{3}$$

The diagonal matrix ρ contains the relativistic phase-space factors for the various two-body channels

$$\rho_i = \frac{2q_i^{\text{c.m.}}}{\sqrt{s}}\,\theta(s - s_i^{\text{th}}),$$

$q_i^{\text{c.m.}}$ = center-of-mass momentum for channel i,

s_i^{th} = threshold s value for channel i. (4)

A complete discussion of the mathematics of the solution of Eq. (1) is given in Ref. 6 and reviewed in Ref. 7.

Schematically, Eq. (1), which is a consequence of linearized unitarity, determines the phase of D. Analyticity is then invoked to construct the complete "enhancement factor" D from its known phase. Various ambiguities arise in this process.

The strong amplitudes S must be known, in principle, from threshold to infinite energies. The problem is rendered manageable by truncating the unitarity relation and retaining only a finite number of channels. In practice it is assumed that only a few two-body or quasi-two-body channels are important at the energies of interest. The contribution from high energies and high inelasticities is ignored by having the S matrix for the two-body channels considered tend to 1 at infinity. In a sense this procedure amounts only to a minor generalization to several channels of final-state-interaction theory in potential scattering.

From the mathematical point of view there are further ambiguities. The phase of the enhancement factor is not changed if the latter is multiplied by a polynomial in s. The presence or absence of such a polynomial must be decided on physical grounds (boundary conditions). These may not be sufficient to determine the polynomial completely. A fairly common situation is that the overall normalization of the enhancement function remains arbitrary. One may adjust it to fit data at one energy and retain as prediction the effect of final-state interactions for varying energies. In the case at hand, D-meson decays, this is not useful because we are interested in the enhancement factors for one value of the energy, the mass of the D meson.

A solution to Eq. (2) can be written down immediately if a K-matrix parametrization of strong-interaction data is available.[7,8] Let the matrix A of Eq. (3) be given by

$$A = (1 - KC)^{-1}K, \qquad (5)$$

with $C = C(s)$, such that $C_j^+(s) - C_j^-(s) = 2i\rho_j(s)$, a diagonal matrix given by the kinematic Chew–Mandelstam functions for the various two-body channels, and K a matrix with no singularities

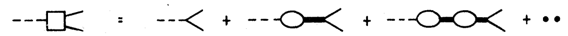

FIG. 1. The geometric series that sums to Eq. (6) in the text. The broken line represents the D meson, solid lines represent the final-state particles which are assumed to scatter via a resonance (thick line). Background terms could also be present.

aside from isolated poles. Then

$$D = (1 - KC)^{-1} \qquad (6)$$

solves Eq. (1).

As remarked before, there are ambiguities in the determination of D. The choice of Eq. (6) corresponds to the boundary condition $D \to 1$ as $s \to \infty$ if $K \to 0$ in the same limit. This choice of boundary condition is the natural one in potential scattering, where one expects that, as the energy becomes much larger than the depth of the potential, the outgoing hadrons will behave like free particles.

The correction factor of Eq. (6) also has the virtue of yielding the behavior expected in the presence of a resonance that couples to the two-body channels under consideration. If such a state is represented by a pole in the K matrix then the form (6) corresponds to summing the geometric series shown in Fig. 1. The attitude[8] behind such a treatment of resonances is that they are $q\bar{q}$ bound states coupled to meson-meson continuum states but not generated by the meson-meson potential (i.e., they correspond to what technically is known as a Castillejo-Dalitz-Dyson pole). It is a characteristic of the above approximation that it generates suppressions instead of enhancements for channels that are strongly coupled to a nearby resonance.

Such suppressions would disappear if direct couplings[8] through the weak Hamiltonian were allowed between the D meson and the resonances. (This is equivalent to adding another series like that in Fig. 1 but starting with the broken line coupled directly to the resonance.) A familiar example of this possibility is the enhancement due to the ρ meson in the electromagnetic form factor of the pion.

Our neglect of this possibility in the case of D decays should be considered primarily as a first exploratory step in the problem. The following heuristic arguments can be offered to support this approximation as an initial attempt. The simplest D-decay graph that leads to a $q\bar{q}$ state (we take the resonances at hand to be primarily $q\bar{q}$) is of the W-exchange type (Fig. 3). It has been argued to be negligible because of the $V-A$ nature[2] of the weak Hamiltonian. In addition, the relevant resonances for our case have spin-parity 0^{++}. In a $q\bar{q}$ description they would therefore be in a P wave, and the overlap with the pointlike W-exchange graph would be expected to be small. The case of the ρ meson and the electromagnetic current is very different in this respect because the ρ is a 1^- particle, and hence a $q\bar{q}$ S wave. The quark-decay graphs (Fig. 2) produce a $qq\bar{q}\bar{q}$ final state. They can be made to yield $q\bar{q}$ states by converting them into "penguin" graphs. A detailed investigation of this possibility is beyond the scope of this article, but we note in passing that such graphs would be suppressed by color counting and would, in any case, contribute only to the direct coupling of D mesons to nonstrange resonances, i.e., to Cabibbo-suppressed decay modes.

In what follows we shall adopt the prescription of Eq. (6) for the matrix D, and apply it to two-body decays of charmed pseudoscalars. The uncorrected ("Born") amplitudes will be those of the spectator-quark model, schematically shown in Fig. 2. Contributions from diagrams such as the one of Fig. 3, which were argued to be unimportant in earlier discussions of D-meson decays,[2] will be ignored.

In other words, we shall obtain K-matrix fits to meson-meson scattering data, and use them to compute the enhancement factors for various two-meson decays of charmed pseudoscalars.

Our main interest will be in $\bar{K}\pi$, $\bar{K}K$, and $\pi\pi$ decay modes. Final-state-interaction corrections for these modes can be estimated from hadron-hadron scattering data in the following sense. Experimental information[9,10] exists for center-of-mass energies of the meson-meson system up to approximately 1.6 GeV. The mass of the D mesons is 1.85 GeV/c^2. It will be assumed in this article that data can be extrapolated smoothly from 1.6 to 1.85 GeV.

The isospin-zero $\pi\pi$ and $\bar{K}K$ states appear to form a coupled system with very little residual inelasticity. We analyze it as such, and obtain the necessary K-matrix fit. The $\bar{K}\pi$ system with isospin $\frac{1}{2}$ shows some inelasticity. The threshold for the onset of this effect suggests it may be associated with the $\bar{K}\eta'$ channel. We therefore fit the $(\bar{K}\pi)_{1/2}$ data with a two-channel formalism assuming the other channel is $\bar{K}\eta'$.

The exotic channels $(\pi\pi)_2$ and $(\bar{K}\pi)_{3/2}$ have measured phase shifts up to 1.6 GeV. These are inferred from the magnitudes of amplitudes by assuming purely elastic scattering. Note, however, that for the 0^+ waves relevant to our problem there is no coupling to states with three pseudoscalars because of parity invariance of the strong interactions. Thus inelasticity would be associated only with production of four or more pseudoscalars and can be expected to be small. We have computed the enhancement factors for these assumed one-channel cases by means of the Omnès expression

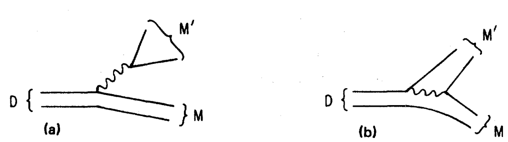

FIG. 2. Quark diagrams for the decay of a D meson in which the initial noncharmed quark is a spectator.

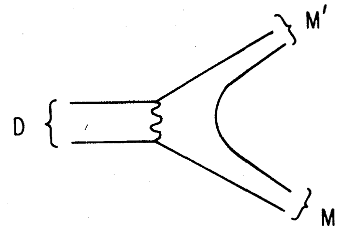

FIG. 3. W-exchange-type diagram. Both initial quarks participate in the decay.

$$D(s) = \exp\left[-\frac{1}{\pi}\int \frac{\delta(s')}{s'-s}ds'\right]. \qquad (7)$$

The phase shift $\delta(s)$ was assumed to go to zero as $1/s$ at infinity.

One piece of information not obtainable through $\pi\pi$ or $K\pi$ scattering data is the behavior of the $(\bar{K}K)$ system with isospin one. Identical particle symmetry decouples this state from S-wave $\pi\pi$ systems. Scattering off a virtual kaon does not possess a sufficiently clear signature to allow experimental studies of $\bar{K}K$ scattering. Some information on the behavior of $(\bar{K}K)$ with isospin one has been extracted from studies of the reactions $\pi^-p \to K^-K^+n$ and $\pi^+n \to K^-K^+p$.[10]

III. FITS TO THE DATA AND MAGNITUDE OF THE FINAL-STATE-INTERACTION CORRECTIONS

We find that the following parametrizations produce reasonable global fits to the data, as shown in Figs. 4 and 5. For the $(\bar{K}\pi, \bar{K}\eta')$ system with isospin $\frac{1}{2}$,

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + a_{ij} \qquad (8)$$

with

$$g_1 = 2.09\,, \quad g_2 = 3.70\,, \quad m^2 = 2.37\,,$$
$$a_{11} = -2.08\,, \quad a_{12} = -7.4\,, \quad a_{22} = -20\,, \qquad (9)$$

where 1 denotes $\bar{K}\pi$ and 2 denotes $\bar{K}\eta'$ (with units such that energies are in GeV).

For the $(\pi\pi, \bar{K}K)$ system with isospin zero,

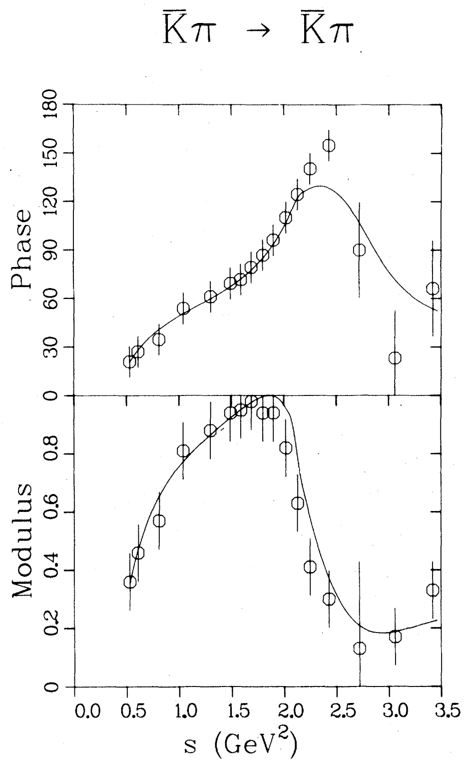

FIG. 4. The fit to the $(\bar{K}\pi)_{1/2}$ scattering amplitude provided by the parameters in Eqs. (8) and (9).

$$K_{ij}=\sum_{l}\frac{g_i^{(l)}g_j^{(l)}}{m_l^{\,2}-s}+a_{ij} \tag{10}$$

with

$$\begin{aligned} &g_1^{(1)}=0.75\,,\quad g_2^{(1)}=0.66\,,\quad m_1^{\,2}=0.60\,,\\ &g_1^{(2)}=0.15\,,\quad g_2^{(2)}=-0.58\,,\quad m_2^{\,2}=2.00\,,\\ &g_1^{(3)}=3.10\,,\quad g_2^{(3)}=-0.10\,,\quad m_3^{\,3}=6.37\,,\\ &a_{11}=-2.32\,,\quad a_{12}=a_{21}=0.15\,,\quad a_{22}=1.73\,. \end{aligned} \tag{11}$$

We remark that the positions of the poles in the K matrix do not coincide exactly with the energies where (in the absence of background) phase shifts would go through 90°. This is because the Chew-Mandelstam functions $C_i(s)$ have real parts which cause the phase shifts to go through 90° when $m^2-s-g^2\,\mathrm{Re}C(s)=0$. We have chosen[11] their normalization so that $C_i(0)=0$. The constant part of the K matrix in Eqs. (8) and (10) is to be interpreted as an approximation to distant "left-hand-cut" singularities of the partial-wave scattering amplitude that give an approximately constant contribution over the limited energy range considered here.

With the above choices of parameters, Eq. (6) yields the following set of enhancement factors

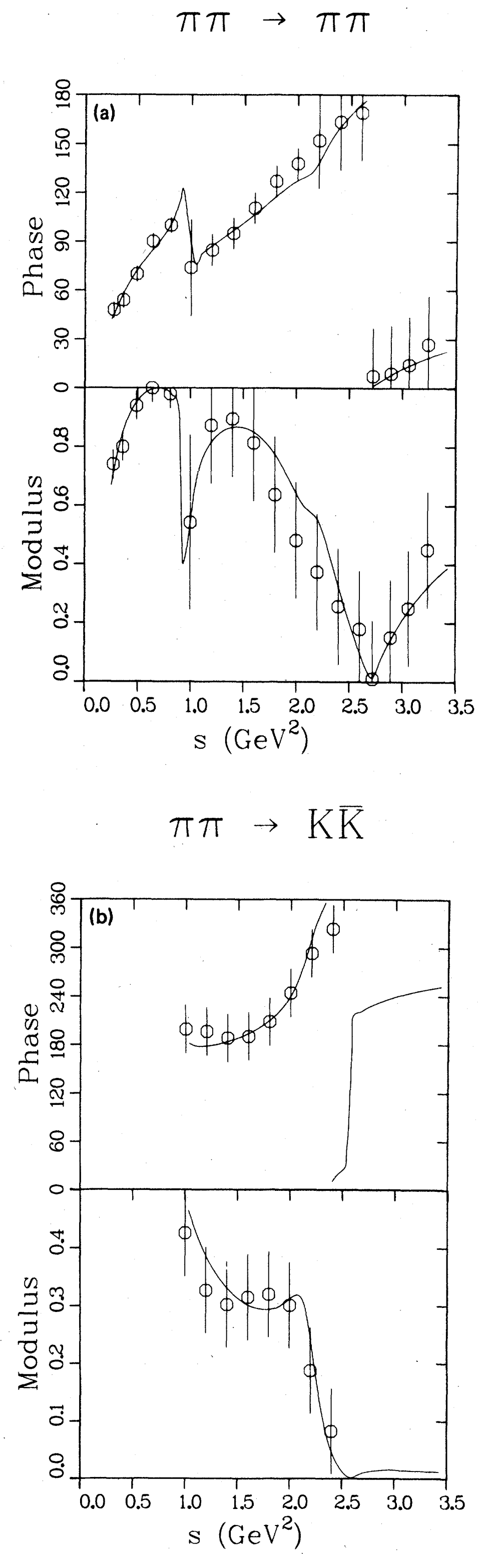

FIG. 5. The fit to the amplitudes with isospin zero for $\pi\pi\to\pi\pi$ and $\pi\pi\to\bar{K}K$ provided by Eqs. (10) and (11).

at $s = m_D^2 = 3.47\ \mathrm{GeV}^2$:

$$D^{(0)}_{\pi\pi-\pi\pi} = 0.52\, e^{i23.6^\circ}\,,\quad D^{(0)}_{\bar{K}K-\pi\pi} = 0.02\, e^{-i163.4^\circ}\,,$$

$$D^{(0)}_{\pi\pi-\bar{K}K} = 0.03\, e^{-i154.4^\circ}\,,\quad D^{(0)}_{\bar{K}K-\bar{K}K} = 0.65\, e^{i47.4^\circ}\,,$$

$$D^{(1/2)}_{\bar{K}\pi-\bar{K}\pi} = 0.77\, e^{i5.2^\circ}\,,\quad D^{(1/2)}_{\bar{K}\eta'-\bar{K}\pi} = 0.48\, e^{i58.4^\circ}\,,$$

$$D^{(1/2)}_{\bar{K}\pi-\bar{K}\eta'} = 0.33\, e^{i7.3^\circ}\,,\quad D^{(1/2)}_{\bar{K}\eta'-\bar{K}\eta'} = 0.19\, e^{i49.3^\circ}\,.$$

The phase shifts given in Ref. 9 together with Eq. (7) give the following enhancement factors for the exotic channels:

$$D^{(2)}_{\pi\pi-\pi\pi} = 1.26\, e^{-i22^\circ}\,,$$

$$D^{(3/2)}_{\bar{K}\pi-\bar{K}\pi} = 1.04\, e^{-i15^\circ}\,.$$

The following remarks are in order. The off-diagonal enhancement factors coupling $\pi\pi$ to $\bar{K}K$ are very small. This is not surprising given that the amplitude for $\pi\pi$-$\bar{K}K$ is known to be small in this energy range. Off-diagonal elements in the $(\bar{K}\pi, \bar{K}\eta')$ complex are larger, as expected from the sizable inelasticity observed in $\bar{K}\pi$ scattering with isospin $\frac{1}{2}$.

The final-state-interaction corrections for exotic channels suggest a very slight enhancement. This may be understood as follows. In potential scattering, repulsive forces (i.e., negative phase shifts) cause suppressions instead of enhancements, but this is so only at low energies. If the energy is greater than the height of the potential there can be an enhancement instead of a suppression.

In the spectator-quark model the amplitudes for various two-meson decays of charmed pseudoscalars are given by the following expressions:

$$\begin{aligned} V(D^0 \to K^-\pi^+) &= A\cos^2\theta\,,\\ V(D^0 \to \bar{K}^0\pi^0) &= \frac{B}{\sqrt{2}}\cos^2\theta\,,\\ V(D^0 \to \pi^-\pi^+) &= A\sin\theta\cos\theta\,,\\ V(D^0 \to K^-K^+) &= A\sin\theta\cos\theta\,,\\ V(D^0 \to \bar{K}^0\eta') &= \tfrac{1}{2}B\cos^2\theta\,. \end{aligned} \tag{12}$$

In the above θ is the Cabibbo angle. The physical η' has been defined as $\frac{1}{2}(u\bar{u}+d\bar{d}+\sqrt{2}\,s\bar{s})$. A and B denote the contributions of the diagrams of Figs. 2(a) and 2(b), respectively. The ratio A/B is equal to $3r$ with $r=1$ in the absence of perturbative QCD corrections. An estimate of these suggests $r \approx -1.5$. The factor-3 suppression in the absence of higher-order QCD corrections is a consequence of color counting.

Use of the values given in Eqs. (12) for the various matrix elements of D yields the following relative rates (these are uncorrected for phase-space differences between $\pi\pi$ and $\bar{K}\pi$ final states, which are small at the mass of the D meson):

$\Gamma(D^0 \to K^-\pi^+)/\cos^4\theta$	$\Gamma(D^0 \to \bar{K}^0\pi^0)/\cos^4\theta$
0.69	0.17
0.70	0.11

$\Gamma(D^0 \to \pi^-\pi^+)/\sin^2\theta\cos^2\theta$	$\Gamma(D^0 \to \bar{K}K^+)/\sin^2\theta\cos^2\theta$
0.55	0.54
0.53	0.54

The first line corresponds to the spectator-quark model corrected for final-state interactions. The second line corresponds to spectator-quark model + short-distance QCD corrections + final-state interactions. The normalization is such that the spectator-quark model without any corrections would give the ratios $1 : \frac{1}{18} : 1 : 1$ for the above quantities.

An estimate of the rate for $D^0 \to K^-K^+$ is included in the table above. As remarked before, calculation of this rate requires a knowledge of $\bar{K}K$ interactions in the isospin-one configuration, which is not easily available experimentally. To obtain the numbers given above, we assumed that the enhancement factor for $\bar{K}K$ with isospin one was unity. This is not grossly inconsistent with behavior of the $(\bar{K}K)_1$ phase shown in Ref. 10.

IV. DISCUSSION

We have reviewed the procedure to be followed in order to compute final-state-interaction corrections in hadronic D decays and stressed some of the uncertainties. These arise from theoretical (polynomial) ambiguities in the determination of the relevant enhancement factors for two-meson final states, and from incomplete data for meson-meson scattering.

Concerning the latter point the following experimental information would be especially useful: studies of $\pi\pi \to K\bar{K}$ for center-of-mass energies up to 2 GeV, determination of the nature of the inelastic channels coupled to $(\bar{K}\pi)_{1/2}$, determination of the inelasticity in the exotic channels $(K\pi)_{3/2}$ and $(\pi\pi)_2$, and in $(\bar{K}K)_1$.

The uncertainties induced by theoretical ambiguities are at least as serious. We have resolved them following an approach that has met some success in the past.[8]

If the ambiguities are treated in this way, with the assumption of no direct coupling between D^0 and $q\bar{q}$ resonances, the corrections to $D^0 \to K^-\pi^+$, $D^0 \to \bar{K}^0\pi^0$, $D^0 \to \pi^-\pi^+$, and $D^0 \to K^-K^+$ may be estimated. If the spectator-quark model is used as input, the ratio of corrected rates turns out to be $\Gamma(K^-\pi^+) : \Gamma(\bar{K}^0\pi^0) : \Gamma(\pi^-\pi^+) : \Gamma(K^-K^+)$

$=1:0.24:0.04:0.04$. (We give here the more favorable case, i.e., we ignore QCD corrections.) This is to be compared with the experimental ratios: $1:0.75\pm0.38:0.030\pm0.015:0.11\pm0.03$.

We see that final-state interactions can cause the $\bar{K}^0\pi^0$ decay mode to be approximately four times as large as what is predicted by the uncorrected spectator-quark model. This is, however, not quite enough to explain the observed branching ratio for $\bar{K}^0\pi^0/K^-\pi^+$. The $\pi^-\pi^+/K^-\pi^+$ mode comes out about right if final-state interactions are included. On the other hand, $K^-K^+/K^-\pi^+$ does not seem to be improved by final-state-interaction corrections, although it should be recalled that for K^-K^+ there may be effects in the $(\bar{K}K)_1$ final state which we have not included for lack of experimental information.

Since our final-state-interaction corrections contain SU_3-symmetry-breaking effects, a nonvanishing $D^0\to K^0\bar{K}^0$ decay rate is generated. This rate is zero in the SU_3-symmetric limit irrespective of the nature of the decay graphs at the quark level. With the spectator-quark model as input, we find $\Gamma(D^0\to\bar{K}^0K^0)/\Gamma(D^0\to K^-K^+)\approx 0.27$.

Lipkin has recently pointed out[12] that comparison of the $\bar{K}\eta$, $\bar{K}\eta'$, and $\bar{K}^0\pi^0$ decay modes of the D^0 would shed light on the nature of the dominant decay graph at the quark level. With the machinery developed above, we find that final-state-interaction corrections to the spectator-quark model change the ratio $\Gamma(D^0\to\bar{K}^0\pi^0)/\Gamma(D^0\to\bar{K}'\eta')$ from 2 to ~2.6 (no phase-space corrections included in either number). Thus, even with final-state interactions, a small measured value for the above ratio would signal strong contributions from nonspectator graphs.

When this work was in its final stages, we received a paper by Cooper and Kamal, Ref. 4, which adopts a point of view very similar to the one taken here. It thus seems proper to emphasize some of the differences between the two efforts. The question of polynomial ambiguities in the determination of the relevant enhancement factors has been given more prominence in our work. Comparison of the actual resolution of this difficulty in the two papers serves as a further illustrative example of the problem. Both articles take $D\to 1$ as $s\to\infty$ as a reasonable boundary condition. This implies that, for every Breit-Wigner-type resonance present in the hadronic amplitude, one has the freedom of multiplying by an arbitrary linear polynomial in s. Cooper and Kamal choose the position of the associated zero to be at $s\ll -m_D^2$. Our approach puts it at s approximately equal to the mass squared of the resonance. We do not know of any definitive argument to support either choice. In favor of ours we can point to the motivation of Fig. 1, to the physically appealing possibility of interpreting the absence of such a zero as an indication of direct weak coupling between charmed and noncharmed $q\bar{q}$ resonances, and to past phenomenological success of this point of view.[8]

Besides this contrast in the resolution of the ambiguities, there is considerable difference of detail. We have gone to greater extent in trying to fit actual data for meson-meson scattering. This is of relevance to the $\bar{K}\pi$ channel, for which data indicate considerable inelasticity which we have built into our coupled-channel analysis. Besides the inelasticity, phase-shift analyses for the $(\bar{K}\pi)_{1/2}$ channel suggest a phase of around 90° for $\sqrt{s}\approx M_D$. This should be contrasted with the assumption of Lipkin,[4] followed by Cooper and Kamal,[4] that the inelasticity may be ignored and that the phase is near 180° causing a cancellation between the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ decay amplitudes for $K^-\pi^+$. Our analysis, on the other hand, accounts more realistically for data, and fails to produce a sufficient suppression of $D^0\to K^-\pi^+$ compared to other two-body channels.

We summarize the conclusions of this work as follows: Investigation of final-state interactions in two-body decays of charmed mesons requires not only data on meson-meson scattering (of which there is barely enough), but also a definite prescription for resolving the ambiguities inherent in final-state-interaction theory. If these ambiguities are resolved in a way that has met some success in other applications, it has to be concluded that final-state interactions alone cannot restore agreement between the quark-spectator model and observed two-body decays of the D^0.

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