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Scalar mesons and nonleptonic decays

Eugene Golowich
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
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The role of scalar mesons in nonleptonic kaon decay is analyzed. Two kinds of scalar mesons are considered, S-wave $q^2\bar{q}^2$ and P-wave $q\bar{q}$ composites. In particular, the $\epsilon(700)$ $q^2\bar{q}^2$ composite cannot explain kaon decay.

I. INTRODUCTION

It has been recently suggested that an "exotic" quark configuration with the quantum numbers of an isoscalar spinless meson can play a significant role in the decay $\eta\to\pi^+\pi^-\pi^0$.[1] That is, one can use PCAC (partial conservation of axial-vector current) to relate the $\eta\to\pi^+\pi^-\pi^0$ amplitude to the matrix element $\langle\pi^+\pi^-|\bar{u}u+\bar{d}d|\eta\rangle$. The η meson is then assumed to undergo a transition to a relatively-low-mass $q^2\bar{q}^2$ state [$u\bar{u}d\bar{d}$(680 MeV)] which decays into two pions. It is our primary purpose in the following both to examine an analogous mechanism for nonleptonic kaon decay, and also to present a broader study of the role of scalar mesons in kaon decays as well as comment on their possible significance in other nonleptonic transitions.

There are several good reasons for this. What could be more natural than for a neutral kaon to undergo a parity-violating transition to a neutral scalar meson which then falls apart into a pion pair? If such a mechanism were to dominate other possible contributions, the *isoscalar* nature of the lightest available scalar mesons could lead naturally to a $\Delta I=\frac{1}{2}$ rule for kaons. In hyperon decay, scalar-meson poles contribute only to the parity-violating amplitudes. Seven such amplitudes have been measured experimentally, of which one ($\Sigma^+\to n\pi^+$) is nearly zero. Interestingly, the only hyperon transition for which the contribution of scalar-meson poles is absent is also $\Sigma^+\to n\pi^+$. Finally, the importance of final-state interactions in nonleptonic decays of charmed mesons has recently been emphasized.[2] Such final-state effects are naturally taken into account in the dynamical framework considered here.

Given that there is ample motivation for our study we are confronted with two immediate problems. These are our lack of knowledge of the parity-violating transition amplitudes between 0^- and 0^+ mesons, and also the rather uncertain experimental status of the 0^+ mesons themselves. We address the first problem by performing bag-model computations of the relevant amplitudes.[3] The latter problem requires us to make certain judgments regarding the current status of scalar mesons. According to the most recent data compilation,[4] there exist four reasonably well-founded types of scalar mesons. They are the isovector $\delta(980)$, the isoscalars $S^*(980)$ and $\epsilon(1400)$, and the strangeness-bearing $\kappa(1500)$. Taken together these constitute a standard mesonic nonet. However, Jaffe[5] has suggested that the spectrum of 0^+ states is far richer than this. In the bag model, many S-wave $q^2\bar{q}^2$ configurations are actually lighter than the P-wave $q\bar{q}$ modes. These $q^2\bar{q}^2$ states, dubbed *primitives* in Ref. 6, are atypical hadronic states in that, although confined in the bag model, they are color-singlet entities which in Nature can "fall apart" into any two-hadron system which has both appropriate quantum numbers and available phase space. For example, the lightest primitive $C_0(9)$ is an isoscalar system with mass around 680 MeV which can dissociate into a pair of pions. Can primitives be detected experimentally? According to Ref. 6, the answer is yes. However, methods which transcend the usual S-matrix data analysis must be employed. It is gratifying that the "P-matrix" analysis of Ref. 6 yields results in reasonable accord with the bag-model predictions, and thus lends credence to the concept of light $q^2\bar{q}^2$ configurations.

In view of the preceding discussion, we proceed as follows. We first consider the contributions of the "nonexotic" P-wave $q\bar{q}$ states $\epsilon(1400)$ and $\kappa(1500)$. In addition, we *assume* that light $q^2\bar{q}^2$ states exist with the wave functions and masses given in Ref. 5, and that such configurations can contribute to our quantum-mechanical amplitudes as intermediate states. It surely is of interest to see whether, even in principle, the $q^2\bar{q}^2$ primitives can play a significant role in nonleptonic processes.

In the matrix-element calculations we must be sure to employ meson wave functions of the correct spin and charge-conjugation property. Because these wave functions are generally the coherent sum of a number of contributions, the relative phase between individual terms is a matter of considerable importance. Therefore, a careful analysis of this subject is presented in Sec. II. In Sec. III, we compute the relevant matrix elements associated with the scalar mesons, and then fold in appropriate propagation functions and coupling constants to construct the full nonleptonic amplitudes. Finally, we discuss our results and comment on extensions of the model in Sec. IV. An Appendix contains certain details associated with the calculation.

II. CONSTRUCTION OF MESON STATES

We wish to construct meson states of the correct charge conjugation and spin properties. Therefore, we must determine the transformation properties of the quark creation operators under the charge-conjugation operation as well as write down a spin-lowering operator. The phase conventions we encounter throughout our analysis ultimately rest upon our choice of bag wave functions.

To start, we note that the effect of charge conjugation on a fermion field operator is given by $C\psi C^{-1}=i\gamma^2\psi^*$, where the symbol C refers to a unitary operator acting in the Hilbert space of quark creation and annihilation operators. Upon acting on the bag-model field operator which destroys quarks and creates antiquarks of flavor q in the S-wave mode, we find

$$Cb(\lambda)C^{-1}\begin{bmatrix} \text{if }\chi(\lambda) \\ -g\vec{\sigma}\cdot\hat{x}\chi(\lambda)\end{bmatrix}+Cd^\dagger(\lambda)C^{-1}\begin{bmatrix} -ig\vec{\sigma}\cdot\hat{x}\overline{\chi}(\lambda) \\ f\overline{\chi}(\lambda)\end{bmatrix} =b^\dagger(\lambda)\begin{bmatrix} g\vec{\sigma}\cdot\hat{x}\chi_c(\lambda) \\ \text{if }\chi_c(\lambda)\end{bmatrix}+d(\lambda)\begin{bmatrix} f\overline{\chi}_c(\lambda) \\ ig\vec{\sigma}\cdot\hat{x}\overline{\chi}_c(\lambda)\end{bmatrix}, \quad (1)$$

where we temporarily suppress color indices, λ is a spin-projection label, χ and $\overline{\chi}$ are two-component spinors for quarks and antiquarks, respectively, and $\chi_c\equiv i\sigma_2\chi$, $\overline{\chi}_c\equiv i\sigma_2\overline{\chi}$. Also, in Eq. (1) we employ $f\equiv j_0(pr/R)$, $g\equiv[(\omega-mR)/(\omega+mR)]^{1/2}j_1(pr/R)$ for quarks of mass m contained in a spherical bag of radius R, p is the mode frequency, and $\omega=(p^2+m^2R^2)^{1/2}$. From Eq. (1) we deduce the relations

$$\begin{aligned} &Cq(\uparrow)C^{-1}=-i\overline{q}(\uparrow),\\ &Cq(\downarrow)C^{-1}=i\overline{q}(\downarrow),\\ &C\overline{q}^\dagger(\uparrow)C^{-1}=-iq^\dagger(\uparrow),\\ &C\overline{q}^\dagger(\downarrow)C^{-1}=iq^\dagger(\downarrow), \end{aligned} \quad (2a)$$

and from the unitary nature of C,

$$\begin{aligned} &C\overline{q}(\uparrow)C^{-1}=iq(\uparrow),\\ &C\overline{q}(\downarrow)C^{-1}=-iq(\downarrow),\\ &Cq^\dagger(\uparrow)C^{-1}=i\overline{q}^\dagger(\uparrow),\\ &Cq^\dagger(\downarrow)C^{-1}=-i\overline{q}^\dagger(\downarrow). \end{aligned} \quad (2b)$$

The above phase relations, although perhaps unconventional to the reader, are nonetheless the ones which hold in our bag-model analysis. Next we consider P-wave quarks and antiquarks confined within a bag. We use primes to distinguish all P-wave quantities from their S-wave counterparts. The relations for P waves corresponding to Eqs. (2a) and (2b) are

$$\begin{aligned} &Cq'(\uparrow)C^{-1}=i\overline{q}'(\uparrow),\\ &Cq'(\downarrow)C^{-1}=-i\overline{q}'(\downarrow),\\ &C\overline{q}'^\dagger(\uparrow)C^{-1}=iq'^\dagger(\uparrow),\\ &C\overline{q}'^\dagger(\downarrow)C^{-1}=-iq'^\dagger(\downarrow) \end{aligned} \quad (3a)$$

and

$$\begin{aligned} &C\overline{q}'(\uparrow)C^{-1}=-iq'(\uparrow),\\ &C\overline{q}'(\downarrow)C^{-1}=iq'(\downarrow),\\ &Cq'^\dagger(\uparrow)C^{-1}=-i\overline{q}'^\dagger(\uparrow),\\ &Cq'^\dagger(\downarrow)C^{-1}=i\overline{q}'^\dagger(\downarrow). \end{aligned} \quad (3b)$$

We now have the apparatus to begin the construction of meson states. Consider a positive ρ meson with spin alignment +1 along some axis,

$$|\rho_1^+\rangle=\frac{1}{\sqrt{3}}u_a^\dagger(\uparrow)\overline{d}_a^\dagger(\uparrow)|0\rangle, \quad (4)$$

where the subscripts on the creation operators are color indices. Upon obtaining the negatively charged ρ from charge conjugation, we find

$$|\rho_1^-\rangle\equiv C|\rho_1^+\rangle =-\frac{1}{\sqrt{3}}d_a^\dagger(\uparrow)\overline{u}_a^\dagger(\uparrow)|0\rangle \quad (5)$$

from which we see that the negative intrinsic charge-conjugation property of the ρ occurs naturally in our convention.

The next step is to apply an appropriate lowering operator to the state of Eq. (4) and then by orthogonality to construct the spinless charged-pion state. We employ the lowering operator

$$J_-=u_i^\dagger(\downarrow)u_i(\uparrow)-\overline{d}_i^\dagger(\downarrow)\overline{d}_i(\uparrow)+d_i^\dagger(\downarrow)d_i(\uparrow)-\overline{u}_i^\dagger(\downarrow)\overline{u}_i(\uparrow). \quad (6)$$

The spin-lowered ρ state is thus

$$|\rho_0^+\rangle=\frac{1}{\sqrt{6}}[u_a^\dagger(\uparrow)\overline{d}_a^\dagger(\downarrow)-u_a^\dagger(\downarrow)\overline{d}_a^\dagger(\uparrow)]|0\rangle \quad (7)$$

from which the charged-pion state is

$$|\pi^+\rangle=\frac{1}{\sqrt{6}}[u_a^\dagger(\uparrow)\overline{d}_a^\dagger(\downarrow)+u_a^\dagger(\downarrow)\overline{d}_a^\dagger(\uparrow)]|0\rangle. \quad (8)$$

Again, the phases in Eqs. (7) and (8) are perhaps unexpected. How can we test the validity of these formulas? One answer is to study an appropriate matrix element of the vector current $V_i^\mu = \bar{q}\gamma^\mu \lambda_i q/2$, where we employ the SU(3) notation $q = u, d, s$. In particular, we find $\langle 0|V_-^3|\rho_0^+\rangle \neq 0$ and $\langle 0|V_-^3|\pi^+\rangle = 0$, where $V_-^3 \equiv V_1^3 - iV_2^3$. Moreover, the spin-lowering operator annihilates the pion state. Our phase relations are therefore correct, so we now turn to the construction of scalar-meson states.

First we consider the P-wave $q\bar{q}$ states. By now our methods should be clear so we proceed immediately to the charge-conjugation-positive $\kappa^+(1500)$,

$$|\kappa^+\rangle = \frac{1}{\sqrt{12}}[u_a^\dagger(\sigma)\bar{s}_a'^\dagger(-\sigma) - u_a'^\dagger(\sigma)\bar{s}_a^\dagger(-\sigma)]|0\rangle , \tag{9}$$

where σ is a spin-projection label summed over its two possible values. We identify the $\epsilon(1400)$ as a P-wave $q\bar{q}$ state which, through mixing, has had essentially all its strange-quark component removed,

$$|\epsilon\rangle = \frac{1}{\sqrt{24}}[u_a^\dagger(\sigma)\bar{u}_a'^\dagger(-\sigma) - u_a'^\dagger(\sigma)\bar{u}_a^\dagger(-\sigma) + (u \to d)]|0\rangle. \tag{10}$$

We do not require the $\delta(980)$, $S^*(980)$ states in our analysis.

According to Ref. 4 the lightest $q^2\bar{q}^2$ configuration, called $C_0(9)$, is an isoscalar state described as follows. First, one constructs diquark and antidiquark systems which are symmetric in the spin and color labels but antisymmetric in flavor. The $C_0(9)$ wave function is then formed as a $q^2\bar{q}^2$ singlet in spin, color, and flavor.[7,8] However, mixing is assumed to remove the strange quarks and we find

$$\begin{aligned}|C_0(9)\rangle = \frac{1}{6\sqrt{2}}\sum_\sigma \{&u_a^\dagger(\sigma)d_b^\dagger(\sigma)\bar{u}_a^\dagger(-\sigma)\bar{d}_b^\dagger(-\sigma) + u_a^\dagger(\sigma)d_b^\dagger(\sigma)\bar{u}_b^\dagger(-\sigma)\bar{d}_a^\dagger(-\sigma)\\ &+\tfrac{1}{2}[u_a^\dagger(\sigma)d_b^\dagger(-\sigma)u_a^\dagger(\sigma)\bar{d}_b^\dagger(-\sigma) + u_a^\dagger(\sigma)d_b^\dagger(-\sigma)\bar{u}_a^\dagger(-\sigma)\bar{d}_b^\dagger(\sigma)\\ &+u_a^\dagger(\sigma)d_b^\dagger(-\sigma)u_b^\dagger(\sigma)\bar{d}_a^\dagger(-\sigma) + u_a^\dagger(\sigma)d_b^\dagger(-\sigma)\bar{u}_b^\dagger(-\sigma)\bar{d}_a^\dagger(\sigma)]\}|0\rangle .\end{aligned} \tag{11}$$

The reader may wish to check that the above state is indeed charge-conjugation positive and also is annihilated by the spin-lowering operator. The next lightest $q^2\bar{q}^2$ state is the strange-particle state $C_K(9)$ which carries isospin $\frac{1}{2}$. Just as the $C^0(9)$ state of Eq. (11) can be considered roughly as "$ud\bar{u}\bar{d}$," then so can the positively charged $C_{K^+}(9)$ be considered as "$ud\bar{d}\bar{s}$." Any of the $C_K(9)$ states can be constructed from Eq. (11) by employing appropriate flavor substitutions.

III. NONLEPTONIC KAON TRANSITIONS

The weak Hamiltonian which induces nonleptonic transitions is currently believed to have the form[9]

$$H_W = \frac{G_F}{2\sqrt{2}}\cos\theta_C \sin\theta_C \sum_{i=1}^{6} c_i O_i + \text{H.c.}, \tag{12}$$

where

$$\begin{aligned}&O_1 = H_A - H_B , \quad O_2 = H_A + H_B + 2H_C + 2H_D ,\\ &O_3 = H_A + H_B + 2H_C - 3H_D , \quad O_4 = H_A + H_B - H_C ,\end{aligned} \tag{13}$$

where the $\{c_i\}$ are numerical coefficients and (the following operators are normal-ordered and color indices are suppressed)

$$\begin{aligned}&H_A = \bar{d}\Gamma_L^\mu u\bar{u}\Gamma_{L\mu}s , \quad H_B = \bar{u}\Gamma_L^\mu u\bar{d}\Gamma_{L\mu}s ,\\ &H_C = \bar{d}\Gamma_L^\mu s\bar{d}\Gamma_{L\mu}d , \quad H_D = \bar{d}\Gamma_L^\mu s\bar{s}\Gamma_{L\mu}s ,\end{aligned} \tag{14}$$

with

$$\begin{aligned}&O_5 = \bar{d}\Gamma_L^\mu t^A s\bar{q}\Gamma_{R\mu}t^A q ,\\ &O_6 = \bar{d}\Gamma_L^\mu s\bar{q}\Gamma_{R\mu}q .\end{aligned} \tag{15}$$

In Eqs. (14) and (15), we employ $\Gamma_L^\mu = \gamma^\mu(1+\gamma_5)$, $\Gamma_R^\mu = \gamma^\mu(1-\gamma_5)$, $\text{Tr}(t^At^B) = 2\delta^{AB}$, and as before the operator q in Eq. (15) is summed over quark flavors u, d, s. The structure of the operator H_W is heavily influenced by quantum-chromodynamics (QCD) radiative corrections and by the existence of heavy quarks.[10]

There are several aspects of the weak Hamiltonian which should be emphasized in view of the subsequent analysis. Of the six operators $O_1, \ldots, O_6$, the first four are "left-left" (LL), and hence form invariant under Fierz transformations, whereas the final two are "left-right" (LR). Also, the only $\Delta I = \frac{3}{2}$ operator is O_4. The others are $\Delta I = \frac{1}{2}$. Of particular importance are the operators O_1 and O_5. The former has an especially large coefficient ($c_1 \simeq 2.5$) whereas the latter has been suggested as having large matrix elements between hadron states.[10] Thus it appears plausible that any successful explanation of the $\Delta I = \frac{1}{2}$ rule will ultimately be associated with either or both of these operators. Finally, for completeness we mention that yet another nonleptonic operator, proportional to color-gluon electric and magnetic fields, can be induced by QCD radiative corrections. However, the coefficient function associated with this operator has been estimated to be extremely small,[11] and so we neglect it here.

The kaon-decay amplitudes associated with scalar-meson intermediate states are depicted in Figs. 1(a), 1(b), and 1(c). Observe that all three processes are necessary in order to provide us

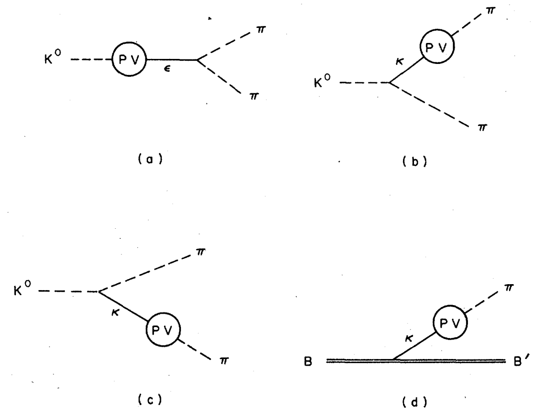

FIG. 1. Pole contributions to kaon and to hyperon nonleptonic decays.

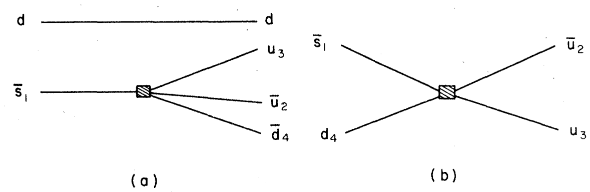

FIG. 2. Nonleptonic transition of a neutral kaon to a nonstrange scalar state. The interaction operator has quark content $\bar{s}_1 u_2 \bar{u}_3 d_4$, and the scalar state has configurations (a) S-wave $q^2\bar{q}^2$ and (b) P-wave $q\bar{q}$.

with a consistent crossing-symmetric model. The only parts of these amplitudes which explicitly involve quark dynamics are the matrix elements $\langle\epsilon|H_W(0)|K^0\rangle$ and $\langle\pi^\pm|H_W(0)|\kappa^\pm\rangle$. We use bag-model wave functions to compute these. Because we are interested in the dependence of these matrix elements upon the quark content of the scalar states, we divide the following discussion into two parts by considering separately the S-wave $q^2\bar{q}^2$ and P-wave $q\bar{q}$ configurations. There is one technical aspect of the calculation common to both cases worth pointing out here. As shown in Ref. 12, the bag states can be expressed as superpositions of plane-wave states. From this we learn that the proper meson normalization factor [given by $(4E_\pi E_\kappa)^{1/2}$ in the plane-wave case for a matrix element involving the π,κ mesons] for the transition between a heavy meson κ and a light meson π is approximately $(4E_\pi M_\kappa)^{1/2}$. Now we turn to the matrix-element analysis.

A. S-wave $q^2\bar{q}^2$ states

We first analyze the transition $K^0\to\epsilon$, where ϵ is the $q^2\bar{q}^2$ state whose wave function appears in Eq. (11). Recall that mixing has removed strange quarks from this state, and that the diquarks $q^2,\bar{q}^2$ are separately symmetric in color and spin but antisymmetric in flavor. This information is important. It allows us to make several exact statements about the matrix elements, i.e., statements independent of the spatial distributions of the quark wave functions.

Consider the K^0-to-ϵ matrix element of $H_A^\dagger$. The attendant quark transition is depicted in Fig. 2(a). It can be seen that the effect of the interaction $H_A^\dagger$ is to convert the strange antiquark $\bar{s}_1$ into quarks $u_3,\bar{u}_2,\bar{d}_4$. There is also a spectator d quark. In the bag model this process is computed to be proportional to the factor $(f_1g_3+g_1f_3)(g_2f_4-f_2g_4)$, where f, g are the quark wave functions defined in the discussion following Eq. (11) and the indices $1,\ldots,4$ pertain to Fig. 2(a). For equal u,d quark mass, the matrix element vanishes. This result is due both to the symmetry of the ϵ wave function and to the chiral structure of H_A. The latter allows us to display the u,d flavor symmetry of H_A by means of a Fierz transformation. Combined with the flavor antisymmetry of the ϵ wave function, it implies the stated result. A similar analysis obtains for H_B, whose K^0-to-ϵ matrix element is one third that of H_A, as follows from the color structure of H_A and H_B.

Neither of the operators H_C and H_D contains u-flavored field operators. Nor does the initial state K^0. Because the final state ϵ does require the presence of $u,\bar{u}$ quarks, we deduce that $\langle K^0|H_{C,D}^\dagger|\epsilon\rangle=0$. Thus the K^0-to-ϵ matrix elements of all the left-left parts of H_W *vanish* if ϵ is the primitive, $C_0(9)$, and $m_u=m_d$.

A calculation of $\langle K^0|O_{5,6}^\dagger|\epsilon\rangle$ matrix elements yields a surprise when compared to their more familiar K-to-π and baryon-to-baryon counterparts. The latter two are known to obey the exact relations $\langle O_6\rangle/\langle O_5\rangle=\frac{3}{16}, -\frac{3}{8}$, respectively.[3] For these, the effect of O_6 is substantially reduced compared to that of O_5. However, we can derive the analogous relation $\langle K^0|O_6^\dagger|\epsilon\rangle/\langle K^0|O_5^\dagger|\epsilon\rangle=\frac{3}{4}$ which shows that the K^0-to-ϵ matrix elements of $O_{5,6}$ are comparable. This follows entirely from the relative color content of $O_5,O_6,C_0(9)$, and is independent of wave functions. For the O_5 matrix element we find in our model

$$\langle\epsilon|O_5^\dagger|K^0\rangle=-\frac{8i}{\sqrt{3}}R^{-3}\bar{N}^3\bar{N}'(4m_\epsilon m_K)^{1/2}I_1\,, \qquad (16)$$

where the bag radius R, normalization factors $\bar{N},\bar{N}'$, and overlap integral I_1 are defined in the Appendix. Primed quantities pertain to strange kinematics.

Considering, for example, the decay $K^0\to\pi^+\pi^-$, we can employ the above matrix elements to construct the ϵ-pole amplitude as in Fig. 1(a),

$$M(K^0\pi^+\pi^-) = g(\epsilon\pi^+\pi^-)\langle\epsilon|H_W|K^0\rangle/[m_\epsilon^{\,2} - m_K^{\,2} - i\Gamma_\epsilon(m_\epsilon m_K)^{1/2}]\,. \tag{17}$$

In Eq. (17), $\Gamma_\epsilon \simeq 0.5$ GeV is the total ϵ width, $m_\epsilon \simeq 0.68$ GeV is the mass of the primitive $C_0(9)$, and the coupling constant $g(\epsilon\pi^+\pi^-)$ is given by

$$g^2(\epsilon\pi^+\pi^-)/4\pi = 8\Gamma_\epsilon m_\epsilon^{\,2}/3(m_\epsilon^{\,2} - 4m_\pi^{\,2})^{1/2}\,. \tag{18}$$

To get a feeling for the magnitude of the ϵ-pole contribution, we can compare the expression in Eq. (17) with the $K^0 \to \pi^+\pi^-$ amplitude derived in Ref. 3 from current-algebra methods. Each of these two amplitudes can be expressed as a product of a matrix element of the weak Hamiltonian times a factor (with units of inverse energy) involving off-shell dynamics of the final state. For Eq. (17) this latter factor is given by $g(\epsilon\pi^+\pi^-)/[m_\epsilon^{\,2} - m_K^{\,2} - i\Gamma_\epsilon(m_\epsilon m_K)^{1/2}] \simeq 9.8$ GeV^{-1} whereas in Ref. 3, the analogous off-shell dynamics takes the form $\sqrt{2}/F_\pi \simeq 15.0$ GeV^{-1}. At this level there is not much difference between the two amplitudes. However, such is not the case for the respective matrix elements where we find $\langle\pi^+|O_5^\dagger|K^+\rangle/\langle\epsilon|O_5^\dagger|K^0\rangle \simeq 4.6$. This large ratio is produced primarily by the size of the bag-model overlap integral of $\langle\pi^+|O_5^\dagger|K^+\rangle$ which, as mentioned earlier, is especially large. It was concluded in Ref. 3 that despite the large $\langle\pi^+|O_5^\dagger|K^+\rangle$ matrix element, the coefficient c_5 is too small to give the experimental decay amplitude. The same is therefore *a fortiori* true of the $q^2\bar{q}^2$ ϵ-pole amplitude.

There remain the $\kappa^\pm$-pole amplitudes of Figs. 1(b) and 1(c). It follows from charge conjugation that $\langle\pi^-|H_W|\kappa^-\rangle = \langle\pi^+|H_W|\kappa^+\rangle$ so that only the latter process need be analyzed. Using arguments analogous to those employed above, it is easy to show that $\langle\pi^+|H_{A,B,D}^\dagger|\kappa^+\rangle = 0$ and $\langle\pi^+|O_6^\dagger|\kappa^+\rangle/\langle\pi^+|O_5^\dagger|\kappa^+\rangle = \frac{3}{4}$. Explicit calculation yields the additional results

$$\langle\pi^+|H_C^\dagger|\kappa^+\rangle = \frac{8i}{\sqrt{3}}R^{-3}\overline{N}^3\overline{N}'(4m_\kappa E_\pi)^{1/2}I_2\,, \tag{19a}$$

$$\langle\pi^+|O_5^\dagger|\kappa^+\rangle = -\frac{8i}{\sqrt{3}}R^{-3}\overline{N}^3\overline{N}'(4m_\kappa E_\pi)^{1/2}(I_1 + 2I_2/3)\,, \tag{19b}$$

where I_2 is given in the Appendix. Numerically, these matrix elements are expressible in terms of the K^0-to-ϵ transition as $\langle\pi^+|H_C^\dagger|\kappa^+\rangle = 0.09\langle\epsilon|O_5^\dagger|K^0\rangle$ and $\langle\pi^+|O_5^\dagger|\kappa^+\rangle = 0.94\langle\epsilon|O_5^\dagger|K^0\rangle$. Thus the κ-to-π matrix elements are not large. Moreover, the propagators corresponding to the $\kappa^\pm$ primitives are suppressed relative to that of the ϵ due to the larger $\kappa^\pm$ mass ($m_\kappa \simeq 0.96$ GeV vs $m_\epsilon \simeq 0.68$ GeV). We conclude that the $\kappa^\pm$-pole contributions are even less significant than the ϵ-pole amplitude.

B. *P*-wave $q\bar{q}$ states

The *P*-wave $q\bar{q}$ configurations certainly appear as physical states, and as such contribute pole terms to the kaon-decay amplitudes. However, the scale masses occurring in the propagator functions are well in excess of 1 GeV. Given this, and in view of the preceding analysis, it perhaps appears unlikely that such amplitudes can contribute significantly to kaon decay. Yet, before reaching this conclusion we should be sure that the relevant matrix elements contain no substantial enhancements. We now turn to these.

The state ϵ is given by the $q\bar{q}$ state of Eq. (10). Since the $s\bar{s}$ component is assumed to have been removed from ϵ by mixing, we immediately conclude $\langle\epsilon|H_D^\dagger|K^0\rangle = 0$ for the same reason as in Sec. III A. Moreover, it follows from color and flavor considerations that $\langle\epsilon|H_C^\dagger|K^0\rangle = 4\langle\epsilon|H_A^\dagger|K^0\rangle$ and $\langle\epsilon|H_B^\dagger|K^0\rangle = \langle\epsilon|H_A^\dagger|K^0\rangle/3$. Both relations are independent of spatial wave functions. Thus the problem of enumerating the K^0-to-ϵ matrix element of the left-left parts of H_W reduces to that of determining $\langle\epsilon|H_A^\dagger|K^0\rangle$, and explicit calculation yields the value zero. This is the same result as that occurring for the respective $q^2\bar{q}^2$ matrix element. The underlying reason is similar, involving the cancellation of wave-function overlap integrals. However, now it is an antisymmetry regarding whether the quark or antiquark in the $q\bar{q}$ model of the ϵ occupies a *P*-wave bag mode which is partially responsible for the zero amplitude. The chiral structure of H_A is the other significant factor.

Because ϵ is of the "usual" $q\bar{q}$ type, the relationship between matrix elements of O_5 and O_6 is computed to obey the standard mesonic relation $\langle\epsilon|O_6^\dagger|K^0\rangle = 3\langle\epsilon|O_6^\dagger|K^0\rangle/16$. We obtain for the latter matrix element

$$\langle\epsilon|O_5|K^0\rangle = -\tfrac{64}{3}\,iR^{-3}\overline{N}^2\overline{N}'\tilde{N}(4m_\epsilon m_K)^{1/2}I_3\,, \tag{20}$$

where a tilde signifies the *P*-wave bag mode and the integral I_3 is given in the Appendix. This amplitude is the largest one encountered thus far in our analysis. Indeed, it compares favorably with that of Ref. 3, $\langle\pi^+|O_5^\dagger|K^+\rangle/\langle\pi^+|O_5^\dagger|\kappa^+\rangle \simeq 0.5$. However, the large mass and relatively modest width of $\epsilon(1400)$ suppress the propagator and coupling-constant contributions to such an extent that the *total* ϵ-pole term is a factor 4.9 smaller than the current-algebra amplitude of Ref. 3.

The final step in our enumeration of scalar-pole contributions to kaon decay is to consider the effect of *P*-wave $q\bar{q}$ $\kappa(1500)$ mesons. Some aspects of this case are repetitious of those analyzed previously. For example, we find $\langle \pi^+ | H^\dagger_{C,D} | \kappa^+ \rangle = 0$ because the operators $H^\dagger_{C,D}$ contain too many d-quark and s-quark fields, respectively, to allow the $\kappa^+ \to \pi^+$ transition to occur. The main new feature associated with the κ^+-pole terms is that the $\langle \pi^+ | H^\dagger_{A,B} | \kappa^+ \rangle$ matrix elements are nonzero. Recall that H_A, H_B are especially important operators because they appear with the large coefficient c_1. In particular, we find $\langle \pi^+ | H^\dagger_B | \kappa^+ \rangle = \langle \pi^+ | H^\dagger_A | \kappa^+ \rangle /3$ independent of quark wave functions and

$$\langle \pi^+ | H^\dagger_A | \kappa^+ \rangle = \frac{6i}{\sqrt{2}} R^{-3} \overline{N}^2 (4m_\kappa E_\pi)^{1/2} (\overline{N}\tilde{N}' I_4 - \overline{N}'\tilde{N} I_5). \tag{21}$$

A quick inspection of the integrals I_4, I_5 (given in the Appendix) shows that the above matrix element vanishes in the SU(3) limit. The mechanism is the same as that encountered previously for the K^0-to-ϵ (the latter is a P-wave $q\overline{q}$ composite here) matrix element of H_A. Hence it is not surprising that this contribution to kaon decay is suppressed by about an order of magnitude relative to the amplitude of Ref. 3. The only remaining independent matrix element is $\langle \pi^+ | O^\dagger_5 | \kappa^+ \rangle$ for which we find

$$\langle \pi^+ | O^\dagger_5 | \kappa^+ \rangle = -\frac{64i}{3\sqrt{2}} R^{-3} (4m_\kappa E_\pi)^{1/2} \overline{N}^2 (\overline{N}\tilde{N}' I_6 + N'\tilde{N} I_7) . \tag{22}$$

Like the other contributions studied here, its magnitude is rather modest in that although the matrix element is comparable to $\langle \pi^+ | O^\dagger_5 | \kappa^+ \rangle$ the large κ^+ mass reduces the propagator contribution and the full pole contribution is down by a factor of 5 relative to our bench mark, the corresponding current-algebra analysis of Ref. 3.

IV. SUMMARY AND CONCLUSIONS

It is worthwhile to see whether novel configurations of quarks and gluons can explain seemingly anomalous phenomena.[13,14] In this context the possibility raised in Ref. 1 that a scalar, isoscalar $q^2\overline{q}^2$ $C_0(9)$ provides an explanation of η decay deserves serious consideration. Perhaps this configuration also figures importantly in kaon decay.

Such is our motivation for studying the role $C_0(9)$ and its partners in kaon decay. The conclusion consequently reached is negative—the mechanism simply does not work. Although for definiteness we worked within the framework of the bag model, this result is much less model dependent than it might appear. Our explanation for the failure of the lowest-mass $q^2\overline{q}^2$ states to explain kaon decay lies with the color, flavor, and spin structure of these states. They are seen to clash with the available weak Hamiltonian transition operators and as a result, the computed decay amplitudes are either zero or too small to account for the observed decay rates.

A somewhat troubling aspect of the suggested dynamical importance of the $q^2\overline{q}^2$ primitives is their multiplicity. According to Ref. 6, there exist seven families of primitives with mass under 1.6 GeV. Although those singularities with the lowest mass are assumed to be dominant in dispersion-theoretic calculations, the presence here of a number of possible additional contributing primitives of slightly higher mass is a matter of concern which might undo the success of the η-decay calculation. This issue deserves further study. However, the calculations are likely to be a good deal more tedious than those performed thus far because the wave functions of the higher-mass primitives have a more complicated color, flavor, and spin structure.

Vis a vis the subject of kaon decay itself, our negative conclusion regarding the role of primitives still leaves one with the problem of providing an appropriate mechanism. Our present feeling is that the current-algebra estimate of Ref. 3 involving the operator O_5 gives a reasonable, although somewhat small, $\Delta I = \frac{1}{2}$ amplitude. The numerical effects of the various operators in the weak Hamiltonian are neatly displayed in Ref. 15. Given the amount of off-shell extrapolation involved in a current-algebra analysis of kaon decay, it is perhaps the case that any computation of this type contains uncertainties as large as a factor of 2. This could explain the discrepancy between theory and experiment.

Despite the failure of the specific scalar-meson configurations studied here to explain kaon decay, the scalar channel still has some appealing features as regards nonleptonic transitions. Thus, we conclude with some comments on the nature of scalar contributions to hyperon and charmed-particle decays. For hyperon decays, the appropriate scalar-pole diagram is the one depicted in Fig. 1(d). A hyperon is seen to emit a κ meson which then undergoes a parity-violating transition to a pion. As mentioned in the Introduction this mechanism gives rise to a vanishing $\Sigma^+ n \pi^+$ amplitude (in agreement with experiment) because there exists no positively charged meson with negative strangeness. Moreover, if the κ-to-pion matrix element is predominantly $\Delta I = \frac{1}{2}$, and the neutral and charged κ particles couple to the baryons in an isospin invariant manner, the remaining Σ, Λ, and Ξ decay amplitudes have the correct $\Delta I = \frac{1}{2}$ property. To determine the relative size of the Σ, Λ, Ξ amplitudes we must consider the κ-baryon-baryon coupling constants in greater detail. Unfortunately not much is known experimentally about these quantities. Instead one must construct a theoretical model based on one's insights, e.g., an SU(3) σ model which has the virtue of chiral invariance, or some analogous approach. We have found it instructive to employ the phenomenological analysis of Carruthers who considers the relation between scalar mesons and the trace of the hadronic energy-momentum tensor.[16] In coupling an octet of scalar mesons to the baryon octet, he determines the F/D ratio to be $\alpha = -0.25$. A subsequent evaluation in Ref. 17 yields $\alpha = -0.44$. Now if we fit the κ-pole model to the hyperon-decay amplitudes, an optimal value is found to be $\alpha \simeq -0.7$. This is interestingly near the scalar-dominance value, especially upon realizing that in principle the range of the F/D ratio is $-\infty < \alpha < \infty$. There is at least a hint here of universality in the low-mass scalar-meson channel. Unfortunately we cannot say much more than this at present.

Reliable data on nonleptonic decays of charmed D mesons into two-body states has finally become available. Both pseudoscalar-pseudoscalar and vector-pseudoscalar final states have been observed. The data is clearly inconsistent with the "first-generation" of theoretical predictions, which tended to lean heavily upon quark diagrams evaluated in the vacuum-insertion approximation. Instead it appears that final-state interactions play a decisive role in these decays. If we analyze D decays with a scalar-pole model, diagrams analogous to those in Figs. 1(a)–1(c) appear. For definiteness, suppose it is the P-wave $q\bar{q}$ scalar mesons whose pole contributions we consider. This is sensible because the masses of these mesons are in the right neighborhood to have a large effect in charm decays. Two attractive features of this model are immediately apparent. There is no scalar meson to which the D^+ meson can make a parity-violating transition. It follows that the D^+ nonleptonic amplitude is thereby suppressed. This is consistent with the data. Also, if the $D^0 \to K^0$ parity-violating term dominates the D^0 decays, it follows from isospin considerations that the $D^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$ amplitudes differ in magnitude by $\sqrt{2}$. This too is in accordance with the data. Before we can proceed further with this promising line of thought, it is important to know something about the charm-bearing scalar mesons. These contribute to the pole amplitudes in crossed channels. This would unfortunately take us far beyond the context of the present work. However, research is continuing in this direction.

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APPENDIX

The purpose of this appendix is to define some notation employed in the main body of the paper. We remind the reader that primed quantities refer to strange quarks and that a tilde signifies a quark in a P-wave bag mode. Otherwise a quark is nonstrange and occupies an S-wave bag mode.

The reduced normalization factors which appear repeatedly in our matrix element formulas are given by

$$\bar{N}^2 = p^4/(2\omega^2 - 2\omega + mR)\sin^2 p\ ,$$
$$\tilde{N}^2 = p^4/(2\omega^2 + 2\omega + mR)\sin^2 p. \qquad \text{(A1)}$$

The wave-function overlap integrals I_i ($i = 1, \ldots, 7$) are defined as

$$I_1 = \frac{1}{4\pi}\int_0^1 u^2du(f^2+g^2)(ff'-gg') = 0.0049\ ,$$

$$I_2 = \frac{1}{4\pi}\int_0^1 u^2du\, 2fg(fg'-f'g) = 0.00044,$$

$$I_3 = \frac{1}{4\pi}\int_0^1 u^2du(ff'+gg')(f\tilde{f}+g\tilde{g}) = 0.0040\ ,$$

$$I_4 = \frac{1}{4\pi}\int_0^1 u^2du\,(f^3\tilde{f}'+g^3\tilde{g}'-3f\tilde{f}'g^2-3f^2\tilde{g}g')$$
$$= 0.0032\ , \qquad \text{(A2)}$$

$$I_5 = \frac{1}{4\pi}\int_0^1 u^2du(f^2f'\tilde{f}+g^2g'\tilde{g}-2ff'g\tilde{g}$$
$$-2f\tilde{f}gg'-f'\tilde{f}g^2-f^2g'\tilde{g})$$
$$= 0.0019\ ,$$

$$I_6 = \frac{1}{4\pi}\int_0^1 u^2du(f^2+g^2)(f\tilde{f}'+g\tilde{g}') = 0.0050\ ,$$

$$I_7 = \frac{1}{4\pi}\int_0^1 u^2du(f^2+g^2)(\tilde{f}f'+\tilde{g}g') = 0.0038\ .$$

Almost all of the numerical statements made in the text involved ratios. These turn out to be very weakly dependent upon quantities such as quark masses or the bag radius. However, in order to provide specific results we employed the values $m_u = m_d = 0$, $m_s = 0.3$ GeV, and to optimize comparison of our analysis with that of Ref. 1, we chose the bag radius used there, $R = 4.7\ \text{GeV}^{-1}$.

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$$I_- = \bar{u}_i^\dagger(\lambda)\,\bar{d}_i(\lambda) - d_i^\dagger(\lambda)u_i(\lambda),$$

where the spin label λ is summed over.

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