Dilepton production from collisions of polarized spin-1/2 hadrons. II. Parton-model predictions

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Parton-model predictions for lepton-pair production from polarized spin-1/2 particles are considered within the context of a formalism explained in a companion paper. The polarization effects in the quantum-chromodynamic 2→2 subprocesses which contribute to lepton-pair production are found to be quite simple. Following the work of Ralston and Soper, polarized distribution functions are defined for gluons as well as quarks. Finally, a detailed discusson of polarization and the hard-scattering formalism is presented for the two cases of intrinsic parton transverse momentum allowed or forbidden.

I. INTRODUCTION

This paper is concerned with the spin dependence of lepton-pair production in hadronic collisions. There should soon be the possibility of measuring such effects at Fermilab where a polarized proton beam will be built.[1] It may also be possible to do experiments at ISABELLE with polarized beams.[2]

A companion paper[3] contains a systematic treatment of lepton-pair production by spin-$\frac{1}{2}$ hadrons based on rotational covariance and the Jacob-Wick helicity formalism.[4] Recently, Ralston and Soper[5] treated this process emphasizing Lorentz covariance.

Our approach uses the helicity formalism, and although similar to that of Ralston and Soper, differs in some details. Here we are concerned with parton-model predictions for the process, and how the observable quantities for the polarized inclusive reaction can be expressed in terms of convolutions of subprocess cross sections with polarized distribution functions. We make the assumption that constituents are on the mass shell, with zero mass both for quarks and gluons.

There have been two explanations for the surprisingly high average transverse momentum of the lepton pair which has been observed experimentally. We explain how each of these may be generalized to the polarized case. Perhaps the additional information afforded by spin will be of value in deciding the relative importance of the two suggested mechanisms.

The first explanation has been based upon the inclusion of the $2\to 2$ processes, which are allowed in quantum chromodynamics (QCD), but which are higher order in the strong coupling constant than simple $q\bar{q}$ annihilation.[6] In these processes, the transverse momentum of the virtual photon is balanced by that of a quark or a gluon. We show that the spin dependence of the $2\to 2$ processes is quite simple when expressed in the proper way.

The second explanation of the lepton-pair transverse momentum has been on the basis of the intrinsic transverse momentum of the partons within the parent hadron.[7-9] We develop the formalism for the inclusion of the effects of intrinsic transverse momentum when both parton and hadron are polarized.

This paper is organized as follows.

Section II deals with polarization effects in the QCD corrections to the naive Drell-Yan mechanism. In the annihilation process, a quark and antiquark annihilate to produce a gluon in addition to the virtual photon, whereas in the gluon Compton process, an initial-state quark and gluon scatter to produce the virtual photon (see Fig. 1). A

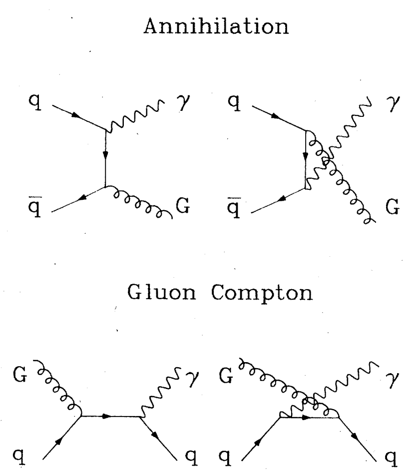

FIG. 1. The Feynman diagrams which contribute to the $2\to 2$ processes, quark-antiquark annihilation and gluon Compton scattering.

very simple picture of the spin effects in these processes is outlined. Of special interest is the fact that the Compton process has spin effects if the gluon alone has transverse polarization. How one might search for polarized gluons via this effect is discussed.

Section III explains how polarized distribution functions are to be defined. Ralston and Soper had defined distribution functions for quarks in a spin-$\frac{1}{2}$ particle. We use their four distribution functions h^{LL}, h^{TT}, h^{LT}, and h^{TL} in a way which differs slightly from their approach, the modifications being imposed by our use of the helicity formalism with massless constituents. Next, polarized distribution functions for gluons within spin-$\frac{1}{2}$ hadrons are defined. The relation between the longitudinal gluon polarization and parent polarization is quite analogous to that between longitudinal quark polarization and parent polarization. However, the transverse gluon polarization and parent polarization cannot be linearly related because they transform oppositely under PT. However, this does not rule out a transverse polarization which is independent of the spin-$\frac{1}{2}$ parent polarization.

It may be possible to find evidence for transverse gluon polarization when the parent is spin 1 and has tensorial polarization.

The QCD spin predictions for the basic subprocesses are very simple; however, the actual observables are convolutions of distribution functions with subprocess cross sections. It is necessary to relate the quantization axes for the subprocess, which are unobservable, to axes related to the hadrons themselves. This is accomplished in Secs. IV A and IV B. Section IV A is restricted to the case of no parton intrinsic transverse momentum. In this case, there are great simplifications. For the annihilation process, the only nonvanishing structure functions are R_{L00}^{M00}, R_{L11}^{M00}, and R_{L11}^{M1-1}. (The $R_{LK_1K_2}^{MN_1N_2}$ are defined in Ref. 3.) Because of the way in which the $R_{LK_1K_2}^{MN_1N_2}$ enter the expressions for $d\sigma t_L^M$, the normalized moments are independent of the longitudinal polarization. In contrast, transverse polarization of both quark and antiquark leads to a Φ-dependent cross section and affects the normalized t_L^M.

For the gluon Compton process, only R_{L00}^{M00} and R_{L11}^{M00} contribute, but the longitudinal-polarization effects do affect the normalized t_L^M. Since there are no transverse-polarization effects, there is no Φ dependence.

Section IV B contains a detailed treatment of the effects of intrinsic parton transverse momentum. In this case, all $R_{LK_1K_2}^{MN_1N_2}$ are allowed. The numerical importance of these terms is dependent on the details of the distribution functions, so that no definite prediction can be made without reliable knowledge of them. At this point, the best that one may do is to try simple models.

Section V contains a summary and our concluding remarks.

In the Appendix we present a comparison of our results for the standard Drell-Yan process at $Q_\perp = 0$ with those of Ralston and Soper. The two approaches agree to lowest order in $\langle k_\perp^2\rangle$, but differ in higher orders. In particular our approach allows for nonzero effects with one hadron transversely polarized, and the other longitudinally polarized.

II. POLARIZATION EFFECTS IN HARD SUBPROCESSES

The usual parton model relates the inclusive reaction to an incoherent sum of "hard" subprocesses, each weighted by the probability of finding the constituents having the desired momenta in the initial hadrons. Ralston and Soper have indicated how to generalize this idea to polarized cross sections by introducing new distribution functions which relate the polarization state of the (quark) constituent to that of the parent hadron. They considered the usual Drell-Yan annihilation of a quark from one hadron with an antiquark from the other, and proposed four new distribution functions $h_{a/A}^{LL}$, $h_{a/A}^{LT}$, $h_{a/A}^{TL}$, and $h_{a/A}^{TT}$ which relate the longitudinal or transverse polarization of quark a to the longitudinal or transverse polarization of spin-$\frac{1}{2}$ hadron A. These new distribution functions depend on x and $k_T{}^2$ of the constituent, and $\vec{k}_T$ appears explicitly in conjunction with $h_{a/A}^{LT}$ and $h_{a/A}^{TL}$ in order to maintain azimuthal covariance. Ralston and Soper have considered the process

$$q+\bar{q}\to\text{"}\gamma\text{"}$$

for which the polarization effects are rather simple. Neglecting quark masses, one finds that annihilation can only occur from states of opposite helicity, producing massive photons with

$$\lambda_\gamma = 2\lambda_q ,$$

where λ_γ is the spin projection along the quark direction. The spin-1 density matrix is related to the spin-$\frac{1}{2}$ density matrices of the initial-state particles, ρ^q and $\rho^{\bar{q}}$, by

$$\sigma\rho \propto \begin{pmatrix} \rho^q_{++}\rho^{\bar{q}}_{--} & 0 & \rho^q_{+-}\rho^{\bar{q}}_{-+} \\ 0 & 0 & 0 \\ \rho^q_{-+}\rho^{\bar{q}}_{+-} & 0 & \rho^q_{--}\rho^{\bar{q}}_{++} \end{pmatrix}. \quad (1)$$

In terms of even-L statistical tensors,

$$\sigma t_0^0 = (1-P_z^q P_z^{\bar{q}})\sigma_0 , \quad (2a)$$

$$\sigma t_2^0 = (1 - P_z^q P_z^{\bar{q}})\sigma_0/\sqrt{10}, \tag{2b}$$

$$\sigma t_2^2 = [\tfrac{1}{2}(\tfrac{3}{5})^{1/2}] |\vec{P}_\perp^q| \, |\vec{P}_\perp^{\bar{q}}| e^{i(\alpha_q - \alpha_{\bar{q}})}\sigma_0, \tag{2c}$$

where σ_0 is the cross section for unpolarized quarks, and where $\vec{P}^q$ and $\vec{P}^{\bar{q}}$ denote the usual polarization vectors. The normalized t_2^0 has the value $1/\sqrt{10}$, corresponding to the usual $1+\cos^2\theta$ distribution, whereas the t_2^2 term corresponds to a decay distribution containing

$$\sin^2\theta \cos(2\phi - \alpha_q + \alpha_{\bar{q}}),$$

and occurs only if both quark and antiquark have some transverse polarization. In terms of the $R_{LK_1K_2}^{MN_1N_2}$ formalism,[3] the only ones different from zero are R_{000}^{000}, R_{200}^{000}, R_{011}^{000}, R_{211}^{000}, and R_{211}^{2-11}, and these are related among themselves by

$$R_{200}^{000} = \frac{1}{\sqrt{10}} R_{000}^{000}, \tag{3a}$$

$$R_{011}^{000} = -\tfrac{1}{3} R_{000}^{000}, \tag{3b}$$

$$R_{211}^{000} = \frac{-1}{3\sqrt{10}} R_{000}^{000}, \tag{3c}$$

$$R_{211}^{2-11} = -\frac{1}{\sqrt{15}} R_{000}^{000}, \tag{3d}$$

where $4\pi R_{000}^{000}$ is just the unpolarized cross section.

These results have been obtained by Ralston and Soper for the case of production at $Q_\perp = 0$, or integrated over $Q_\perp$. If one wishes to obtain results for modest but nonvanishing $Q_\perp$, it would be necessary to suppose that the constituents have nonzero transverse momenta relative to their parent hadrons. Since this involves a two-dimensional (or three-dimensional, if one integrates over the azimuthal angle Φ) integration, the simple polarization predictions given above will be "smeared" because the parton and hadron axes do not coincide. Collins and Soper[7] have suggested that smearing effects will be smallest if their choice of z axis is made, at least in the unpolarized case. In Ref. 8 it was shown that the integration domain could be considered to be the surface of a sphere, with the Euler angles of the rotation from parton to hadron axes as variables. The fact that the $R_{LK_1K_2}^{MN_1N_2}$ transform irreducibly under this rotation simplifies their calculation. However, in the absence of reliable estimates of the $k_\perp$ dependence of the four distribution functions, it seems impossible to make quantitative estimates of the smearing effects.

For large values of $Q_\perp$, i.e. ≥ 1 GeV/c, suggestions have been made to include the subprocesses which can yield large $Q_\perp$ photons. They are

annihilation: $q + \bar{q} \to$ "γ" $+ G$,

gluon Compton: $G + q \to$ "γ" $+ q$.

These processes have been calculated in the tree (or zero-loop) approximation, and results are available in the literature.[10-12] The s-channel helicity amplitudes for these reactions may be computed using standard formulas. In doing so we found that for both processes extremely simple results emerged, provided the spin-quantization axis for the massive photon was chosen appropriately. Since we neglect quark masses, the annihilation process can occur only when quark and antiquark have opposite helicity, whereas in the gluon Compton process (henceforth called simply Compton process) the final and initial quark helicities are equal. There then follows a remarkable correlation among gluon helicity and massive-photon spin.

For the annihilation process, with initial quark helicity $\frac{1}{2}$: If the final gluon has helicity 1, the massive photon has $J_z = 1$ along the incident quark direction, with amplitude

$$A_1^q = -4q \cdot Q/\sqrt{\hat{u}\hat{t}}\,. \tag{4}$$

If the final gluon has helicity -1, then the massive photon has $J_z = -1$ along antiquark direction, with amplitude

$$A_{-1}^{\bar{q}} = 4\bar{q} \cdot Q/\sqrt{\hat{u}\hat{t}}\,, \tag{5}$$

where $\hat{u} = (Q - \bar{q})^2$, $\hat{t} = (Q - q)^2$, with q, $\bar{q}$, Q denoting quark, antiquark, and massive-photon four-momenta, respectively. If the initial quark helicity is $-\frac{1}{2}$, then the same results hold with signs of gluon helicity and massive-photon spin projection reversed, as well as an overall change of sign for the amplitudes.

For the Compton process, with initial quark helicity $\frac{1}{2}$: If the initial gluon has helicity 1, the massive photon has s-channel helicity +1 ($J_z = -1$ along final-quark direction), with amplitude

$$A_1^s = \frac{2(\hat{s} - Q^2)}{\sqrt{-\hat{s}\hat{u}}}\,. \tag{6}$$

If the initial gluon has helicity -1, the massive photon has u-channel helicity -1 (i.e., $J_z = 1$ along initial quark direction) with amplitude

$$A_{-1}^u = \frac{2(Q^2 - \hat{u})}{\sqrt{-\hat{s}\hat{u}}}\,, \tag{7}$$

where $\hat{s} = (Q + q_f)^2$, $\hat{u} = (Q - q_i)^2$, q_i, q_f, and Q being momenta of initial quark, final quark, and masssive photon, respectively. Once again, if quark helicity is reversed, the same amplitudes apply provided gluon helicity and massive-photon spin projection are reversed.

These results show a very tight relation among gluon, massive-photon, and quark spin projections, which would give rise to very precise tests of the model provided one could prepare and detect the polarized constituents. Realistically, it seems necessary to sum over unobserved final-state polarizations, obtaining thereby somewhat weaker tests of the model. Using standard methods, one can derive the even-L part of the massive-photon density matrix multiplied by the differential cross section, namely, for annihilation

$$\left(\frac{d\sigma\,\rho_E}{dM^2 d\hat{t}\, d\hat{\Phi}}\right)^A = \frac{1}{3\pi}\left(\frac{4}{9}\right)\left(\frac{\alpha^2\alpha_s}{\hat{s}^2\hat{u}\hat{t}M^2}\right)\Big\{[1-(\rho^q_{++}-\rho^q_{--})(\rho^{\bar q}_{++}-\rho^{\bar q}_{--})] \times\left[(\hat{t}-M^2)^2\left(\frac{|1^q\rangle\langle 1^q|+|-1^q\rangle\langle -1^q|}{2}\right)+(\hat{u}-M^2)^2\left(\frac{|1^{\bar q}\rangle\langle 1^{\bar q}|+|-1^{\bar q}\rangle\langle -1^{\bar q}|}{2}\right)\right] + 2(\hat{t}-M^2)(\hat{u}-M^2)[\rho^q_{+-}\rho^{\bar q}_{-+}e^{2i\hat{\Phi}}(|1^q\rangle\langle 1^{\bar q}|+|-1^{\bar q}\rangle\langle -1^q|) + \rho^q_{-+}\rho^{\bar q}_{+-}e^{-2i\hat{\Phi}}(|1^{\bar q}\rangle\langle 1^q|+|-1^q\rangle\langle -1^{\bar q}|)]\Big\}, \qquad (8)$$

where ρ^q, $\rho^{\bar q}$ are the quark and antiquark helicity density matrices, $\hat{\Phi}$ is the azimuthal angle of the dilepton momentum, $\hat{s}=(q+\bar{q})^2$, $\hat{t}=(Q-q)^2$, $\hat{u}=(Q-\bar{q})^2$, $M^2=Q^2$ is the dilepton mass, and α_s is the strong-interaction coupling. The notation $|m^q\rangle$ indicates a dilepton having spin projection m along the direction of the quark momentum in the dilepton rest frame. The relation between $|m^q\rangle$ and $|m^{\bar q}\rangle$ is then

$$|m^q\rangle = \sum_{m'=-1}^{1} d^1_{m'm}(\chi)|m'^{\bar q}\rangle \qquad (9)$$

with $\chi \geq 0$, and

$$\frac{1-\cos\chi}{2} = \frac{\hat{s}M^2}{(M^2-\hat{u})(M^2-\hat{t})}. \qquad (10)$$

The $d^1_{m'm}$ are the usual rotation functions. From these formulas one can obtain the density matrix referred to either the q axis or $\bar{q}$ axis rather simply. If constituent transverse momenta and hadron masses are negligible compared to the hadron center-of-mass energy, the q and $\bar{q}$ directions in the dilepton frame coincide with those of their hadron parents.

If there is no beam or target polarization, our results reduce to those found by Kajantie, Lindfors, and Raitio.[10] If only longitudinal quark polarization is involved, the sole effect is to multiply the cross section by $[1-(\rho^q_{++}-\rho^q_{--})(\rho^{\bar q}_{++}-\rho^{\bar q}_{--})]$ leaving the normalized density matrix unchanged. In contrast, transverse quark polarization yields new observable effects, mainly a $\hat{\Phi}$ dependence in the differential cross section as well as a modification of the normalized density matrix. Note that $(d\sigma/d\hat{t}\,dM^2d\hat{\Phi})\rho_{00}$ is independent of transverse polarization when measured along either quark or antiquark directions. Finally, we note that the nonvanishing $R^{MN_1N_2}_{LK_1K_2}$ of Eq. (10) are of the form R^{M00}_{L00}, R^{M00}_{L11}, R^{M-11}_{L11}.

For $G+q\to$ "γ" $+q$, the differential cross section multiplied by the even-L part of the density matrix may be written as

$$\left(\frac{d\sigma\,\rho_E}{dM^2 d\hat{t}\, d\hat{\Phi}}\right)^C = \frac{1}{3\pi}\left(\frac{\alpha^2\alpha_s}{-\hat{s}^3\hat{u}M^2}\right)\left(\frac{1}{6}\right)\Big[(\hat{s}-M^2)^2\left(\frac{|1^s\rangle\langle 1^s|+|-1^s\rangle\langle -1^s|}{2}\right)[1+(\rho^G_{11}-\rho^G_{-1-1})(\rho^q_{++}-\rho^q_{--})] + (M^2-\hat{u})^2\left(\frac{|1^u\rangle\langle 1^u|+|-1^u\rangle\langle -1^u|}{2}\right)[1-(\rho^G_{11}-\rho^G_{-1-1})(\rho^q_{++}-\rho^q_{--})] + (\hat{s}-M^2)(M^2-\hat{u})[\rho^G_{1-1}e^{2i\hat{\Phi}}(|1^s\rangle\langle -1^u|+|1^u\rangle\langle -1^s|) + \rho^G_{-11}e^{-2i\hat{\Phi}}(|-1^u\rangle\langle 1^s|+|-1^s\rangle\langle 1^u|)]\Big]. \qquad (11)$$

In this expression, $\hat{s}=(G+q_i)^2$, $\hat{t}=(G-Q)^2$, $\hat{u}=(Q-q_i)^2$, where G is the gluon momentum, q_i is the initial-state quark momentum, $\hat{\Phi}$ is the azimuthal angle of the dilepton (measured with respect to transverse axes used to define ρ^G_{1-1}, the off-diagonal gluon density-matrix element), and ρ^G and ρ^q are gluon and quark helicity density matrices, respectively. The states $|m^s\rangle$ refer to axes in the dilepton rest frame such that the final-quark three-momentum is along $-\hat{z}$, while the states $|m^u\rangle$ refer to axes such that the initial-quark three-momentum is along $-\hat{z}$. (The y axis is along $\vec{q}_i\times\vec{q}_f$ in either case.) If the gluons are unpolarized, we again recover the results of

Kajantie, Lindfors, and Raitio. However, in contrast to the annihilation reaction, transverse polarization of the quarks yields no observable effects, and longitudinal quark polarization is unobservable unless there is also longitudinal gluon polarization, i.e., $\rho^G_{11} \neq \rho^G_{-1-1}$. In contrast, transverse gluon polarization can yield observable effects, independent of the initial-quark polarization. The quantity ρ^G_{1-1}, after a suitable choice of transverse axes, can be chosen real and is proportional to the difference between intensities of linear polarization along the two principal directions,

$$\rho^G_{1-1} \propto \rho^G_{yy} - \rho^G_{xx} \,,$$

where ρ^G_{yy} denotes the probability that a gluon has linear polarization along the principal direction y. The fact that $\rho^G_{1-1} e^{2i\Phi}$ occurs is required by invariance under arbitrary choice of transverse axes.

The relation between the states $|m^s\rangle$ and $|m^u\rangle$ is given by

$$|m^s\rangle = \sum_{m'=-1}^{1} d^1_{m'm}(\chi)\,|m'^u\rangle \,, \tag{12}$$

where $\chi \geq 0$ and

$$\frac{1-\cos\chi}{2} = \frac{-\hat{t}M^2}{(\hat{s}-M^2)(M^2-\hat{u})} \,. \tag{13}$$

Once again, if one neglects hadron masses and quark transverse momentum, the directions of the initial quark and its parent hadron coincide. In contrast, the axes used to define $|m^s\rangle$ do not correspond to any definite hadron direction, and it would be judicious to replace the $|m^s\rangle$ of Eq. (11) via Eq. (12) in carrying out practical calculations.

An interesting aspect of the Compton subprocess compared to annihilation is that the longitudinal-polarization effects do not factor out, but rather change the normalized density matrix. This property might be useful in isolating effects associated with the gluon-induced reaction. Finally we note that the transversely-polarized-gluon effects do not require any target-quark polarization, and could thereby yield observable polarization effects for dilepton pair production using only a polarized target or a polarized beam (but not both). As we shall see in Sec. III, it is impossible for a spin-$\frac{1}{2}$ polarized hadron to transmit transverse polarization to a gluon constituent. The same need not be true of a polarized deuteron (provided it has some tensorial polarization), hence one might imagine isolating polarized gluon effects in a reaction such as $\pi + D \rightarrow \mu^+\mu^- + X$ with a polarized deuterium target. The nonzero $R^{MN_1N_2}_{LK_1K_2}$ are of the form R^{M00}_{L00}, R^{M00}_{L11}, and R^{M20}_{L20}. For spin-$\frac{1}{2}$ parents R^{M20}_{L20} is not allowed, and does not contribute to any polarization effects.

III. RELATIONS AMONG CONSTITUENT AND HADRON POLARIZATIONS

The calculations of the previous section show that substantial polarization effects are to be expected at the level of the hard subprocesses. However, the extension of these calculations to the hadron inclusive process requires knowledge of the distribution functions for polarized constituents in polarized hadrons. Ralston and Soper[5] have discussed the relation between quark polarization and spin-$\frac{1}{2}$ parent polarization. They introduced four kinds of polarized distribution functions, called $h^{LL}_{a/A}$, $h^{LT}_{a/A}$, $h^{TL}_{a/A}$, and $h^{TT}_{a/A}$, where L and T denote longitudinal and transverse, respectively, a and A representing quark and hadron. These are functions of x_A, the fractional (light-cone) momentum of the quark, and $\vec{k}^{T2}_A$, its squared transverse momentum. The method proposed by Ralston and Soper is to use a polarized-quark propagator which is written as

$$\not{K}_A(1-\gamma_5\lambda^a - \gamma_5 \vec{S}^T_a\cdot\vec{\gamma}) \,,$$

where K_A is a lightlike vector, $K^+_A = Q^+$, $K^-_A = |\vec{K}_T| = 0$, λ^a and $\vec{S}^T_a$ are, respectively, twice the average quark helicity and the transverse (to $\vec{K}_A$) polarization. Using infinite-momentum-frame arguments, Ralston and Soper relate λ^a and $\vec{S}^T_a$ to the parent-hadron polarization via

$$\lambda^a = h^{LL}_{a/A}(x_A,\vec{k}^{T2}_A)\lambda^A + h^{LT}_{a/A}(x_A,\vec{k}^{T2}_A)\vec{k}^T\cdot\vec{S}^T_A \,, \tag{14a}$$

$$\vec{S}^T_a = h^{LT}_{a/A}(x_A,\vec{k}^{T2}_A)\lambda^A + h^{TT}_{a/A}(x_A,\vec{k}^{T2}_A)\vec{S}^T_A \,. \tag{14b}$$

In terms of the hadron helicity density matrix ρ^A,

$$\lambda^A = \rho^A_{++} - \rho^A_{--} \,, \tag{15a}$$

$$\tfrac{1}{2}[(\vec{S}^T_A)_x + i(\vec{S}^T_A)y] = \rho^A_{-+} \,. \tag{15b}$$

If $\vec{k}^T_A$ is zero, then there are only two relevant functions, $h^{LL}_{a/A}$ and $h^{TT}_{a/A}$, whereas for nonzero $\vec{k}^T_A$, longitudinally polarized quarks can be found in transversely polarized hadrons (provided $h^{LT}_{a/A} \neq 0$). Although the theoretical status of quark transverse momentum is far from clear, the presence (or absence) of distribution functions like $h^{TL}_{a/A}$ and $h^{LT}_{a/A}$ is subject to very simple experimental tests (at least in the context of the Drell-Yan mechanism). If one hadron has only transverse polarization, while the other has only longitudinal polarization, the observation of any effect implies either h^{TL} or $h^{LT} \neq 0$.

Although the approach of Ralston and Soper leads to definite predictions for the standard $q + \bar{q} \to \gamma$ process, it requires some modification in order to be used with our helicity formalism, which assumed massless constituents. Let us recall first some general properties of polarization for massless particles, as discussed, for example, by Michel and Wightman.[13] From the work of Wigner[14] it is known that for massless particles, a basis for a one-dimensional irreducible unitary representation of the proper Lorentz group is formed by one-particle states with definite helicity s or $-s$, s being the spin. Consequently, the only effect of a Lorentz transformation on such a state, apart from changing the four-momentum, is to multiply the state vector by a helicity-dependent phase, $e^{\pm is\varphi}$. Therefore, the normalized helicity density matrix is characterized by two Lorentz-invariant quantities, $(\rho_{ss} - \rho_{-s-s})$ and $|\rho_{s-s}|$. An arbitrary Lorentz transformation can modify only the phase of the off-diagonal matrix element ρ_{s-s}. This offers the enormous advantage that the helicity density matrix, once specified in any given frame, is then determined in all frames, up to the phase of ρ_{s-s}. In particular, if in the hadron c.m. frame the helicity density matrices for massless constituents are specified, then in the parton c.m. frame the same helicity density matrices apply, except for the phase of ρ_{s-s}. Of course, the phase of ρ_{s-s} depends on the choice of transverse axes in the parton c.m. frame, which can be made arbitrarily. In order to remove this arbitrariness, one may consider expressions such as $\rho^q_{+-} e^{i\hat{\Phi}}$ or $\rho^G_{1-1} e^{2i\hat{\Phi}}$, where $\hat{\Phi}$ denotes the azimuthal angle of the produced dilepton in the parton c.m. frame. It is expressions like these which occur naturally in the cross sections for the subprocesses (as can be seen from the previous section).

Our task is thus to propose expressions relating the helicity-density-matrix elements $(\rho_{ss} - \rho_{-s-s})$ and $\rho_{s-s} e^{2is\hat{\Phi}}$ in the parton c.m. frame to the parent-hadron polarization. Noting that these quantities are invariant under longitudinal boosts and rotations about the longitudinal axis in the parton c.m. frame, we propose they be related to quantities which display the same invariance in the hadron c.m. frame (longitudinal boosts which reverse the direction of either hadron are not permitted). Following the P- and T-invariance arguments of Ralston and Soper, we are led to suggest, for the quark helicity density matrix,

$$\lambda^a = (\rho^a_{++} - \rho^a_{--}) \tag{16a}$$
$$= h^{LL}\lambda^A + h^{LT} k^T_A (\rho^A_{+-} e^{i\Phi} e^{i(\varphi_1 - \Phi)} + \rho^A_{-+} e^{-i\Phi} e^{-i(\varphi_1 - \Phi)}),$$

$$\rho^a_{+-} e^{i\hat{\Phi}} = \rho^a_{+-} e^{i\hat{\varphi}_A} e^{i(\hat{\Phi} - \hat{\varphi}_A)}$$
$$= (h^{TT}\rho^A_{+-} e^{i\Phi} e^{i(\varphi_1 - \Phi)} + \tfrac{1}{2} h^{TL} k^T_A \lambda^A) e^{i(\hat{\Phi} - \hat{\varphi}_A)}, \tag{16b}$$

where we have dropped subscripts a/A and arguments $(x_A, \vec{k}^{T2}_A)$ of the distribution functions. The angles Φ, φ_1 refer to the azimuths of the produced dilepton and the quark in the hadron c.m. frame, whereas $\hat{\varphi}_A$ is the azimuthal angle of the parent hadron as seen in the parton c.m. frame. We use k^T_A as for $(\vec{k}^T_A \cdot \vec{k}^T_A)^{1/2}$. The off-diagonal hadron density matrix appears in combinations like $\rho^A_{+-} e^{i\Phi}$, while the angle φ_1 appears only in the combination $\varphi_1 - \Phi$. Since φ_1 is an integration variable, there is no real Φ dependence associated with such expressions. Similarly only $(\hat{\Phi} - \hat{\varphi}_A)$, which is determined kinematically by $\vec{k}^T_A$, x_A, and Φ, enters in these expressions. Our proposal is thus manifestly invariant under z boosts and rotations about the z axis in both hadron and parton c.m. frames.

For a quark in hadron B, we write

$$\lambda^b = h^{LL}\lambda^B + h^{LT} k^T_B (\rho^B_{-+} e^{i\Phi} e^{i(\varphi_2 - \Phi)} + \rho^B_{+-} e^{-i\Phi} e^{-i(\varphi_2 - \Phi)}), \tag{17a}$$

$$\rho^b_{-+} e^{i\hat{\Phi}} = \left(h^{TT}\rho^B_{-+} e^{i\Phi} e^{i(\varphi_2 - \Phi)} + \frac{h^{TL}}{2} k^T_B \lambda^B\right) e^{i(\hat{\Phi} - \hat{\varphi}_B)}. \tag{17b}$$

Here φ_2 and $\hat{\varphi}_B$ represent the azimuthal angles of $\vec{k}^T_B$ in the hadron c.m. frame and of the parent-hadron momentum in the parton c.m. frame, respectively. The choice of phases is dictated by our use of the Jacob-Wick "particle-2" convention.

At this point we compare our approach with that of Ralston and Soper. First we remark that our use of the helicity formalism with massless constituents is only an approximation, since off-shell constituents are to be expected as well. However, given our assumptions, the requirements that our parton-model cross sections, when convoluted with our polarized structure functions, yield results consistent with the general formalism for polarized hadroproduction of leptons, forces us to use Eqs. (16). This will become apparent in the next section, where we show that our approach indeed yields results consistent with the general formalism, once all smearing effects (including final-state rotations from parton-based axes to hadron-based axes) are included. This kinematical complexity is compensated, to some extent, by the simplicity of helicity amplitudes for the subprocesses.

Although our expressions involve h^{LL}, h^{LT}, h^{TL}, and h^{TT}, we use these distribution functions in a way which differs from that of Ralston and Soper. One may then ask whether our functions are to be identified with theirs. The answer is that if we examine our helicity density matrix in a frame where hadron A has very large momentum, then our effective polarized-quark propagator differs from that of Ralston and Soper by terms of order

$|\vec{k}_A^T|$, provided the four distribution functions are identified. Therefore, we will differ in our predictions for various observable quantities by terms typically of order $(\langle|\vec{k}^T|\rangle/M)$, where $\langle|\vec{k}^T|\rangle$ is some measure of the intrinsic transverse momentum. In the Appendix we show how our results differ from theirs in the special case of production at $Q_\perp = 0$.

The problem of gluon polarization was not discussed by Ralston and Soper, but it is not difficult to adapt their results to the helicity polarization of the gluons. The difference between intensities for positive- and negative-helicity gluons, $\rho_{11}^G - \rho_{-1-1}^G$, is similar to $\rho_{++}^q - \rho_{--}^q$ for quarks, and we may write

$$\rho_{11}^G - \rho_{-1-1}^G = h_{G/A}^{LL}\lambda^A + h_{G/A}^{LT}k_A^T(\rho_{+-}^A e^{i\Phi}e^{i(\phi_1-\Phi)} + \rho_{-+}^A e^{-i\Phi}e^{-i(\phi_1-\Phi)}) \tag{18}$$

and similarly for B, with $\phi_1 \to \phi_2$, $\rho_{+-}^A \to \rho_{-+}^B$. We have introduced polarized-gluon distribution functions $h_{G/A}^{LL}$ and $h_{G/A}^{LT}$, which depend on x and $(k^T)^2$. An estimate of $h_{G/A}^{LL}$ can be made using the results of Close and Sivers,[15] whereas $h_{G/A}^{LT}$ contributes only after integration over k^T, and may be expected to yield a smaller contribution.

The problem of transverse-gluon polarization is rather different. For a photon beam, it is well known that four real numbers specify the unnormalized density matrix. These may be chosen as the intensities of linear polarization along the (orthogonal) principal directions, the azimuthal angle of a principal direction, and the difference of circularly polarized intensities. The quantity ρ_{1-1}^G is related to the linear intensities, and is even under the operation PT, whereas the polarization vector for a spin-$\frac{1}{2}$ hadron is odd under PT. Thus there is no analog to Eq. (16b) relating the transverse polarization of the gluon to the polarization of a spin-$\frac{1}{2}$ hadron. However, this does not imply that ρ_{1-1}^G is zero. The relation

$$\rho_{1-1}^G e^{2i\hat{\Phi}} = h_{G/A}^{T0}(k_A^{T2})e^{2i(\hat{\Phi}-\hat{\phi}_A)} \tag{19}$$

is allowed, where $h_{G/A}^{T0}(x, k^{T2})$ represents the probability of finding linearly polarized gluons in an unpolarized spin-$\frac{1}{2}$ hadron. Although it may appear paradoxical that transverse gluons can be found in an unpolarized hadron, this is what occurs in the model of Altarelli and Parisi,[16] where the gluon radiated by a quark tends to have the electric vector mainly in the plane of its parent hadron. Since all azimuthal angles are equally likely there is no net transverse-gluon polarization, but there should be measurable effects due to the presence of $h_{G/A}^{T0}$.

Although the transverse gluon polarization cannot depend on the spin-$\frac{1}{2}$ hadron polarization, PT does allow a linear relation between transverse-gluon polarization and tensor polarization for a spin-1 parent hadron. By tensor polarization we mean that the quadrupole polarization moments of the hadron are different from zero. A simple but striking test of the presence of gluons in a deuteron would be dilepton production in

$$\pi + \mathrm{D}(\uparrow) \to l^+l^- + X\,,$$

with some tensorial deuteron polarization. The standard picture of the πD reaction would allow for no polarization effects, since the pion is spinless, and the basic quark subprocesses require both beam and target polarization. The gluon polarization in the deuteron might be related to the tensorial deuteron polarization by a relation of the form

$$\begin{aligned}\rho_{1-1}^G e^{2i\hat{\Phi}} &= \rho_{1-1}^G e^{2i\hat{\phi}_A}e^{2i(\hat{\Phi}-\hat{\phi}_A)} \\ &= [h^{T2}\rho_{1-1}^{\mathrm{D}}e^{2i(\phi_1-\Phi)}e^{2i\Phi} \\ &\quad + h^{T1}k^T\,\mathrm{Re}\rho_{10}^{\mathrm{D}}e^{i(\phi_1-\Phi)}e^{i\Phi} \\ &\quad + h^{T0}(k^T)^2(3\rho_{00}^{\mathrm{D}}-1)]e^{2i(\hat{\Phi}-\phi_A)}\,.\end{aligned} \tag{20}$$

In this expression $\rho_{mm'}^{\mathrm{D}}$ denotes the deuteron helicity density matrix. If suitable polarized deuterium targets were available, a measurement of dilepton production with a pion beam could indicate unambiguously the presence of spin-1 constituents.

It should be noted, however, that there is a good reason to expect these effects to be small for deuterium. Intuitively, the hard scattering involves the constituents of the deuteron. Since the proton and neutron will not contain transversely polarized gluons, any such gluons should come from the binding of the nucleons to form a deuteron. We expect such effects to be small.

Another difficult experiment which might find evidence for transversely polarized gluons involves a polarized hyperon beam. An Ω^- beam with tensor polarization could be a good source of gluons with transverse polarization.

IV. CONTRIBUTIONS TO HADRON-POLARIZATION EFFECTS

A. $k_\perp = 0$

If the constituents of a hadron are assumed to have no transverse momentum in the hadron c.m. frame, it is relatively simple to extend the formulas for the elementary processes to the inclusive reaction. The neglect of $k_\perp$ effects greatly simplifies the relations among polarization effects in the elementary and inclusive reactions. The basic technical difficulty is to relate quantization axes relevant for the subprocess to axes based on the hadron process. Standard parton-model formulas express inclusive cross sections as convolutions of cross sections for subprocesses with distribution functions. In order to describe the polarized inclusive cross sections, the usual formulas must be generalized to include rotations from parton-based axes to hadron-based axes. Such a generalization is readily obtained if one considers quantities which transform irreducibly under such rotations, namely $(d\sigma/d^4Q)t_L^M$.

Let us suppose that in the massive-photon rest frame, two sets of axes are defined, with corresponding states $|jm\rangle$ and $|\tilde{j}m\rangle$, related by

$$|\tilde{j}m\rangle = \sum_{m'} D^j_{m'm}(R)\,|jm'\rangle\,. \tag{21}$$

The moments $\tilde{t}_L^M$ and t_L^M will then be related by

$$\tilde{t}_L^M = \sum_{M'} D^L_{M'M}(R)\,t_L^{M'}\,, \tag{22}$$

with the inverse relation

$$t_L^M = \sum D^{L*}_{MM'}(R)\,\tilde{t}_L^{M'}\,. \tag{23}$$

In order to determine the rotation R, a simple procedure is to choose two vectors (typically momenta of some particles) and to find their spherical components a_μ, b_μ, $\tilde{a}_\mu$, $\tilde{b}_\mu$ in the two different systems. (Our convention is $p_0 = p_z$, $p_1 = -(p_x + ip_y)/\sqrt{2}$.) Knowing these quantities one can then solve the relations

$$\tilde{a}_\mu = \sum_\nu D^1_{\nu\mu}(R)\,a_\nu\,, \tag{24a}$$

$$\tilde{b}_\mu = \sum_\nu D^1_{\nu\mu}(R)\,b_\nu \tag{24b}$$

for the Euler angles (α,β,γ) of the rotation. In what follows, we shall use the $|\tilde{j}m\rangle$ to refer to parton-based axes, and $|jm\rangle$ to hadron-based axes.

In Sec. III we used a Dirac bracket notation to define the density matrix for the massive photon. The particular matrix, which we denote by ρ^1,

$$\rho^1 = \tfrac{1}{2}(|\tilde{1}\rangle\langle\tilde{1}| + |-\tilde{1}\rangle\langle-\tilde{1}|)\,, \tag{25}$$

in terms of $|\tilde{j}m\rangle$ states, occurred frequently. In terms of the t_L^M, referred to the $|jm\rangle$ basis,

$$\begin{aligned} t_L^M &= \sum C^{1L1}_{mMm'}\langle m|\rho^1|m'\rangle \\ &= \tfrac{1}{2}[3\delta_{L0}\delta_{M0} - C^{1L1}_{000}D^{L*}_{M0}(R)]\,, \end{aligned} \tag{26}$$

where R is the rotation defined by Eq. (21).

The results of Kajantie *et al.*[10] show that the relation between the cross sections for the hadronic inclusive reaction and the hard subprocesses $q+\bar{q} \to$ "γ" $+G$, $G+q\to$ "γ" $+q$ is

$$\frac{d\sigma}{dM^2dy\,dQ_\perp^2} = \sum_{i,j}\int_{x_1^{\min}}^1 dx_1\,\frac{x_1x_2}{(x_1-\frac{1}{2}\bar{x}_Te^y)} \times f_{i/A}(x_1)f_{j/B}(x_2)\,\frac{d\sigma_{ij}}{dM^2d\hat{t}}\,, \tag{27}$$

where $f_{i/A}(x_1)$, $f_{j/B}(x_2)$ are the constituent distribution functions. (The kinematic variables will be defined below.) We may generalize this formula to the polarized subprocesses noting that in the absence of $k_\perp$, only the distribution functions h^{LL} and h^{TT} are allowed, since h^{LT} and h^{TL} do not contribute in the absence of $k_\perp$. For annihilation, we find

$$\begin{aligned} \frac{d\sigma t_L^M}{dM^2dy\,dQ_\perp^2d\Phi} = \sum_i e_i^2\int_{x_1^{\min}}^1 dx_1\,&\frac{x_1x_2}{(x_1-\frac{1}{2}\bar{x}_Te^y)}\,\frac{1}{3\pi}\left(\frac{4}{9}\right)\left(\frac{\alpha^2\alpha_s}{\hat{s}^2\hat{u}\hat{t}M^2}\right) \\ &\times\Big\{[f_{i/A}\bar{f}_{i/B}(1-h^{LL}_{i/A}\bar{h}^{LL}_{i/B}\lambda^A\lambda^B)+\bar{f}_{i/A}f_{i/B}(1-\bar{h}^{LL}_{i/A}h^{LL}_{i/B}\lambda^A\lambda^B)] \\ &\quad\times\left[\frac{(\hat{t}-M^2)^2}{2}[3\delta_{L0}\delta_{M0}-C^{1L1}_{000}D^{L*}_{M0}(R_A)]+\frac{(\hat{u}-M^2)^2}{2}[3\delta_{L0}\delta_{M0}-C^{1L1}_{000}D^{L*}_{M0}(R_B)]\right] \\ &\quad+4(f_{i/A}\bar{f}_{i/B}h^{TT}_{i/A}\bar{h}^{TT}_{i/B}+\bar{f}_{i/A}f_{i/B}\bar{h}^{TT}_{i/A}h^{TT}_{i/B}) \\ &\quad\times(\hat{t}-M^2)(\hat{u}-M^2)\Big[\rho^A_{+-}\rho^B_{-+}e^{2i\Phi}\sum_{mm'}C^{1L1}_{mMm'}D^1_{m1}(R_A)D^{1*}_{m'-1}(R_B) \\ &\qquad+\rho^A_{-+}\rho^B_{+-}e^{-2i\Phi}\sum_{mm'}C^{1L1}_{mMm'}D^1_{m1}(R_B)D^{1*}_{m'-1}(R_A)\Big]\Big\}\,. \end{aligned} \tag{28}$$

In this expression the distribution functions $f_{i/A}$, $h_{i/A}$ ($f_{i/B}$, $h_{i/B}$) depend on x_1 (x_2), the integration variable which is the fractional light-cone momentum of the quark in hadron A (B). The fractional charge of the quark is e_i, and the sum is over flavors. The distribution functions for antiquarks are denoted by $\bar{f}$ and $\bar{h}$. In terms of the dilepton four-momentum and x_1, the kinematic quantities are defined by

$$\tau = M^2/s\,, \tag{29a}$$

$$\bar{x}_T{}^2 = 4(Q_\perp{}^2 + M^2)/s\,, \tag{29b}$$

$$x_1^{\min} = (\bar{x}_T e^{y} - 2\tau)/(2 - \bar{x}_T e^{-y})\,, \tag{29c}$$

$$x_2 = (\bar{x}_T e^{-y} x_1 - 2\tau)/(2x_1 - \bar{x}_T e^{y})\,, \tag{29d}$$

$$\hat{s} = x_1 x_2 s\,, \tag{29e}$$

$$\hat{t} = s(\tau - \tfrac{1}{2}\bar{x}_T e^{-y} x_1)\,, \tag{29f}$$

$$\hat{u} = s(\tau - \tfrac{1}{2}\bar{x}_T e^{y} x_2)\,, \tag{29g}$$

$$\hat{t}\hat{u} = Q_\perp{}^2 \hat{s}\,. \tag{29h}$$

Rather than specifying a definite choice of axes in the dilepton frame, we have defined the t_L^M in terms of two rotations, R_A and R_B, which relate axes systems corresponding to the tilde systems of Eq. (21) for which the $\tilde{z}$ axis is along the momentum of hadrons A and B, respectively. (The $\tilde{y}$ axis is always parallel to $\vec{p}_B \times \vec{p}_A$.) Once a definite choice is made, the Euler angles of the rotations may be determined and predictions may be made. The rotations are independent of the integration variable x_1 and may be taken outside the integral. If the choice of axes is such that the y axis is normal to the production plane, then the Euler angles α and γ for rotations R_A and R_B are zero.

If Eq. (28) is compared with Eq. (10) of Ref. 3, it is seen that the only nonvanishing $R_{LK_1K_2}^{MN_1N_2}$ are R_{L00}^{M00}, R_{L11}^{M00}, and R_{L11}^{M1-1}. The R_{L11}^{M00} enter in the same way as the R_{L00}^{M00}, i.e., the normalized t_L^M $(d\sigma t_L^M/d\sigma t_0^0)$ are independent of longitudinal polarization. In contrast, the R_{L11}^{M1-1} term corresponds to a Φ dependence in the cross section, and it affects the normalized t_L^M as well. Note that transverse-polarization effects require that both beam and target be transversely polarized.

For the Compton process we find

$$\frac{d\sigma t_L^M}{dM^2 dy\, dQ_\perp{}^2 d\Phi} = \sum_i e_i{}^2 \int_{x_1^{\min}}^1 dx_1 \frac{x_1 x_2}{(x_1 - \frac{1}{2}\bar{x}_T e^{y})}\left(\frac{1}{3\pi}\right)\left(\frac{1}{6}\right)\left(\frac{-\alpha^2\alpha_s}{\hat{s}^3 M^2}\right)$$
$$\times\Bigg\{\frac{f_{G/A} f_{i/B}}{\hat{u}}\Bigg[\frac{(\hat{s}-M^2)^2}{2}(1 + h_{G/A}^{LL} h_{i/B}^{LL}\lambda^A\lambda^B)[3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L\,*}(R_{\hat{s}})]$$
$$+ \frac{(\hat{u}-M^2)^2}{2}(1 - h_{G/A}^{LL} h_{i/B}^{LL}\lambda^A\lambda^B)[3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L\,*}(R_{\hat{u}})]\Bigg]$$
$$+ \frac{f_{i/A} f_{G/B}}{\hat{t}}\{(\hat{s}-M^2)^2/2(1 + h_{i/A}^{LL} h_{G/B}^{LL}\lambda^A\lambda^B)[3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L\,*}(R_{\hat{s}})]$$
$$+ (\hat{t}-M^2)^2/2(1 - h_{i/A}^{LL} h_{G/B}^{LL}\lambda^A\lambda^B)[3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L\,*}(R_{\hat{t}})]\}\Bigg\}\,. \tag{30}$$

In this expression the kinematic quantities are those defined in Eq. (29), and the distribution functions for hadrons A and B depend on x_1 and x_2, respectively. There are three rotations $R_{\hat{s}}$, $R_{\hat{t}}$, and $R_{\hat{u}}$, defined as follows: For all axes $\tilde{y}$ along $(\vec{p}_B \times \vec{p}_A)$,

$R_{\hat{s}}$: $\tilde{z}$ along $-(x_1\vec{p}_A + x_2\vec{p}_B)$,

$R_{\hat{u}}$: $\tilde{z}$ along $-\vec{p}_B$,

$R_{\hat{t}}$: $\tilde{z}$ along $\vec{p}_A$.

Thus $R_{\hat{u}}$ and $R_{\hat{t}}$ do not depend on the integration variable, whereas $R_{\hat{s}}$ does. Since only $D_{M0}^{L\,*}$ appear, the Euler angle γ is irrelevant. Once again, if the axes are chosen with y normal to p_A and p_B, the three Euler angles $\alpha_{\hat{s}}$, $\alpha_{\hat{t}}$, $\alpha_{\hat{u}}$ are all zero as well.

The Compton process contributes terms of the form R_{L00}^{M00} and R_{L11}^{M00} only. In contrast to the annihilation process, the longitudinal-polarization effects do not factor out of the normalized density matrix. There are no transverse polarization effects, hence no Φ dependence.

B. Intrinsic $k_\perp$ and smearing

If there were no intrinsic $k_\perp$ (primordial or intrinsic parton transverse momentum), a number of simple observable consequences for hadron-polarization effects in dilepton production would follow. For example, with one initial hadron transversely polarized and the other longitudinally polarized no effects should be observed. When such an experiment is preformed it is likely that some (possibly small) effects will be observed. The simple predictions of the Drell-Yan process and its lowest-order QCD corrections are likely to be modified by some smearing in $k_\perp$. It is therefore of interest to set up a formalism in which $k_\perp$ effects may be estimated, if not rigorously computed. These effects may be grouped into several classes, namely: (1) The directions of initial-parton momenta do not coincide exactly with those of parent hadrons, and the parton reaction plane is not the same as that for the hadronic reaction. (2) The kinematic variables depend on $k_\perp$ as well as on x_1 and x_2. (3) The distribution functions $h^{LT}_{q/A}$ and $h^{TL}_{q/A}$ as well as the gluon distribution function $h^{TO}_{G/A}$ will, in general, contribute to polarization effects.

In writing explicit parton-model formulas, our aim is to respect the general formalism presented in Ref. 3. In Sec. III we have shown how the distribution functions of Ralston and Soper should be interpreted in terms of our helicity formalism, in order to obtain the correct φ dependence. Since the order of magnitude of $k_\perp$ is comparable to hadron masses, we shall include the initial and final hadron masses in our formulas.

In the hadron c.m. using light-cone momenta ($p^\pm = (p_0 \pm p_z)/\sqrt{2}$) the four-momenta of the beam, target, dilepton, beam constituent, and target constituents may be written as (order is p^+, p^-, p_x, p_y)

$$p_A = \frac{M_A}{\sqrt{2}}(e^{\xi}, e^{-\xi}, 0, 0)\,, \tag{31a}$$

$$p_B = \frac{M_B}{\sqrt{2}}(e^{-\eta}, e^{\eta}, 0, 0)\,, \tag{31b}$$

$$Q = \left(\frac{M_\perp e^{y}}{\sqrt{2}}, \frac{M_\perp e^{-y}}{\sqrt{2}}, Q_\perp \cos\Phi,\ Q_\perp \sin\Phi\right)\,, \tag{31c}$$

$$k_a = \left(x_1\frac{M_A e^{\xi}}{\sqrt{2}}, \frac{k_{1\perp}{}^2 e^{-\xi}}{\sqrt{2}x_1 M_A}, k_{1\perp}\sin\phi_1\right)\,, \tag{31d}$$

$$k_b = \left(\frac{k_{2\perp}{}^2 e^{-\eta}}{\sqrt{2}x_2 M_B}, \frac{x_2 M_B e^{\eta}}{\sqrt{2}}, k_{2\perp}\cos\phi_2, k_{2\perp}\sin\phi_2\right) \tag{31e}$$

where

$$\cosh\xi = (s + M_A{}^2 - M_B{}^2)/(2M_A\sqrt{s})\,, \tag{32a}$$

$$\cosh\eta = (s + M_B^2 - M_A{}^2)/(2M_B\sqrt{s})\,, \tag{32b}$$

$$M_\perp = (M^2 + Q_\perp{}^2)^{1/2}\,, \tag{32c}$$

$$\hat{s} = x_1 x_2 M_A M_B e^{\xi+\eta} + \frac{k_{1\perp}{}^2 k_{2\perp}{}^2 e^{-(\xi+\eta)}}{x_1 x_2 M_A M_B} - 2k_{1\perp}k_{2\perp}\cos(\phi_1 - \phi_2)\,, \tag{32d}$$

$$\hat{t} = M^2 - x_1 M_A M_\perp e^{\xi - y} - \frac{k_{1\perp}{}^2 M_\perp e^{-(\xi - y)}}{x_1 M_A} - 2k_{1\perp}Q_\perp\cos(\Phi - \phi_1)\,, \tag{32e}$$

$$\hat{u} = M^2 - x_2 M_B M_\perp e^{\eta + y} - \frac{k_{2\perp}{}^2 M_\perp e^{-(\eta + y)}}{x_2 M_B} - 2k_{2\perp}Q_\perp\cos(\Phi - \phi_2)\,. \tag{32f}$$

If these momenta are transformed into the parton c.m. frame, tha hadron momenta $\vec{p}_A$ and $\vec{p}_B$ will have azimuthal angles $\hat{\phi}_A$ and $\hat{\phi}_B$, respectively, while the dilepton will have $\hat{\Phi}$. The differences $\hat{\Phi} - \hat{\phi}_A$, $\hat{\Phi} - \hat{\phi}_B$ may then be expressed in terms of $\Phi - \phi_1$, $\Phi - \phi_2$, defined in the hadron c.m. frame. For example,

$$\frac{\sin(\hat{\Phi} - \hat{\phi}_A)}{\cos(\hat{\Phi} - \hat{\phi}_A)} = -\frac{\epsilon_{\mu\nu\lambda\sigma}p_A^\mu Q^\nu p_a^\lambda p_b^\sigma}{(p_A \cdot p_a Q \cdot p_b + p_A \cdot p_b Q \cdot p_a - \frac{1}{2}\hat{s} p_A \cdot Q)}\,, \tag{33}$$

where the scalar products and invariant pseudoscalar may be evaluated in the hadron c.m. A similar formula holds for $\hat{\Phi} - \hat{\phi}_B$, provided p_A is replaced by p_B on the right-hand side.

In order to describe the dilepton polarization, we choose the y axis along $\vec{p}_B \times \vec{p}_A$, but leave the choice of the z axis free. To be precise, we suppose that in the dilepton rest frame unit vectors $\hat{u}_A$ and $\hat{u}_B$ in the directions of $\vec{p}_A$ and $\vec{p}_B$ may be written

$$\hat{u}_A = \cos\Psi\, \hat{e}_z + \sin\Psi\, \hat{e}_x\,, \tag{34a}$$

$$\hat{u}_B = \cos(\Psi - \chi)\hat{e}_z + \sin(\Psi - \chi)\hat{e}_z \tag{34b}$$

where $\chi (0 \le \chi \le \pi)$ is the angle between $\vec{p}_A$ and $\vec{p}_B$, defined by

$$\cos\chi = \frac{-(p_A \cdot p_B - p_A \cdot Q p_B \cdot Q/M^2)}{\{[(p_A \cdot Q/M)^2 - M_A{}^2][(p_B \cdot Q/M)^2 - M_B{}^2]\}^{1/2}} \tag{35}$$

and where the parameter Ψ fixes the choice of the z axis relative to the hadron momenta. Let the spherical polar angles of the parton momenta $\vec{k}_a$, $\vec{k}_b$ in the dilepton rest frame be (θ_a, ϕ_a) and (θ_b, ϕ_b), respectively. Once more, a judicious choice of scalar products and pseudo scalars enables one to relate these angles to the kinematic quantities defined in the hadron c.m. frame.

As a first example of $k_\perp$ smearing, we consider the standard Drell-Yan process $q + \bar{q} \to$ "γ", which was also discussed by Ralston and Soper, for $Q_\perp = 0$. The relation between hadron and parton cross sections may be written, in the unpolarized case, as

$$\frac{d\sigma}{dM^2 dy\, dQ_\perp{}^2 d\Phi} = \frac{1}{4}\sum_{i,j}\int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} \times \delta^4(Q - k_1 - k_2) f_{i/A} f_{j/B} \sigma_{ij}\,, \tag{36}$$

where the distribution functions are now understood to depend on both x and $k_\perp{}^2$. The integration may be performed in the dilepton rest frame to yield

$$\frac{d\sigma}{dM^2 dy\, dQ_\perp{}^2 d\Phi} = \frac{1}{8}\sum_i \int d\cos\theta_a d\phi_a \times (f_{i/A}\bar{f}_{i/B} + \bar{f}_{i/A} f_{i/B})\left(\frac{4\pi\alpha^2}{3M^2}\right), \tag{37}$$

where (θ_a, ϕ_a) are the angles of parton a. In this case the momentum of parton b is opposite in direction to that of parton a, and the kinematic variables x_1, x_2, $k_{1_\perp}$, $k_{2_\perp}$, $\Phi-\phi_1$, and $\Phi-\phi_2$ may be expressed in terms of the quantities s, M^2, y, $Q_\perp$, Ψ, θ_a, ϕ_a. The arguments of the distribution functions may thus be determined, and the integral evaluated numerically. It is then relatively simple to extend the discussion to include polarization effects. The essential point is that the massive-photon density matrix is given by Eq. (1) relative to the tilde axes such that parton a. is along the $\tilde{z}$ axis. In this tilde system let $\hat{\phi}_A$, $\hat{\phi}_B$ denote the azimuthal angles of hadrons A and B, respectively. Then we may rewrite Eq. (2c), for example, as

$$\sigma\tilde{t}_2^2 = 2(\tfrac{3}{5})^{1/2}\sigma_0[\rho^q_{-+}e^{-i\hat{\phi}_A}\rho^{\bar{q}}_{+-}e^{-i\hat{\phi}_B}e^{i(\hat{\phi}_A+\hat{\phi}_B)}]. \tag{38}$$

Then via Eqs. (16b) and (17b), we express the parton density matrices in terms of hadron density matrices, to obtain

$$\sigma\tilde{t}_2^2 = 2(\tfrac{3}{5})^{1/2}\sigma_0\left(h^{TT}\rho^A_{-+}e^{-i\Phi}e^{-i(\phi_1-\Phi)} + \frac{h^{TL}}{2}k_{1\perp}\lambda^A\right) \times\left(\bar{h}^{TT}\rho^B_{+-}e^{-i\Phi}e^{-i(\phi_2-\Phi)} + \frac{\bar{h}^{TL}}{2}k_{2\perp}\lambda^B\right)e^{i(\hat{\phi}_A+\hat{\phi}_B)}. \tag{39}$$

In this expression, the Φ dependence is explicit, the physical angles $\phi_1-\Phi$ and $\phi_2-\Phi$ are related to the integration variables, and only $(\hat{\phi}_A+\hat{\phi}_B)$ is convention dependent. However, it is necessary to "rotate" the $\tilde{t}_L^M$ to the hadron-based axes, and this rotation will restore convention independence.

In the tilde system the $\tilde{z}$ axis is along the direction of the quark (which we take to be a constituent of hadron A), hence the rotation must have Euler angles $(\phi_a, \theta_a, \gamma)$, where γ is to be determined. The relation between the $\tilde{t}_L^M$ and t_L^M may be written

$$t_L^M = \sum_{M'} D^{L*}_{MM'}(\phi_a, \theta_a, \gamma)\tilde{t}_L^{M'}. \tag{40}$$

If we use the fact that the azimuthal angles of hadrons A and B are $\hat{\phi}_A$ and $\hat{\phi}_B$, we may form $(\tilde{p}_A)_1(\tilde{p}_B)_1/|p_A||p_B|$, to obtain

$$\frac{\sin\theta_{aA}\sin\theta_{aB}e^{i(\hat{\phi}_A+\hat{\phi}_B)}}{2} = \left(\sum_m D^1_{m1}(\phi_a,\theta_a,\gamma)d^1_{m0}(\Psi)\right)\left(\sum_{m'} D^1_{m'1}(\phi_a,\theta_a,\gamma)d^1_{m'0}(\Psi-\chi)\right), \tag{41}$$

from which it follows that

$$e^{i(\hat{\phi}_A+\hat{\phi}_B+2\gamma)} = \frac{2}{\sin\theta_{aA}\sin\theta_{aB}}\left(\sum_m D^1_{m1}(\phi_a,\theta_a,0)d^1_{m0}(\Psi)\right)\left(\sum_{m'} D^1_{m'1}(\phi_a,\theta_a,0)d^1_{m'0}(\Psi-\chi)\right), \tag{42}$$

where θ_{aA} is the angle $(0\le\theta_{aA}\le\pi)$ between the direction of quark a and hadron A in the dilepton frame. Expressions for θ_{aA} and θ_{aB} in terms of rotation matrices are

$$\cos\theta_{aA} = \sum_m D^1_{m0}(\phi_a,\theta_a,0)d^1_{m0}(\Psi), \tag{43a}$$

$$\cos\theta_{aB} = \sum_m D^1_{m0}(\phi_a,\theta_a,0)d^1_{m0}(\Psi-\chi). \tag{43b}$$

It is thus seen that the phase $e^{i(\hat{\phi}_A+\hat{\phi}_B+2\gamma)}$ can be expressed in terms of the angles θ_a, ϕ_a, Ψ, and χ, and is thus well defined. Proceeding in this manner, one obtains the explicit formula (for L even)

$$\begin{aligned}\frac{d\sigma t_L^M}{dM^2dy\,dQ_\perp{}^2d\Phi} = \frac{1}{8}\sum_i e_i{}^2\int d\Omega_a x_1x_2\left(\frac{4\pi\alpha^2}{3M^2}\right)(f_{i/A}\bar{f}_{i/B}\{(1-\tilde{\lambda}^a\tilde{\lambda}^b)\tfrac{1}{2}[3\delta_{L0}\delta_{M0} - C^{1L1}_{000}D^{L*}_{M0}(\phi_a,\theta_a,0)]\\ + 2(\tfrac{3}{5})^{1/2}[\tilde{\rho}^a_{-+}\tilde{\rho}^b_{+-}D^{L*}_{M2}(\phi_a,\theta_a,0)e^{i(\hat{\phi}_A+\hat{\phi}_B+2\gamma)}\\ + (\tilde{\rho}^a_{-+}\tilde{\rho}^b_{+-}e^{i(\hat{\phi}_A+\hat{\phi}_B+2\gamma)})^*D^{L*}_{M-2}(\phi_a,\theta_a,0)] + (f\leftrightarrow\bar{f}, h\leftrightarrow\bar{h})),\end{aligned} \tag{44}$$

where we have introduced the quantities $\tilde{\lambda}^a$, $\tilde{\lambda}^b$, ρ^a_{-+}, $\tilde{\rho}^b_{+-}$ in order to simplify the expression. These are defined by

$$\tilde{\lambda}^a = h^{LL}_{i/A}\lambda^A + h^{LT}_{i/A}k_{1\perp}(\rho^A_{+-}e^{i\Phi}e^{i\phi_1'} + \rho^A_{-+}e^{-i\Phi}e^{-i\phi_1'}), \tag{45}$$

$$\tilde{\lambda}^b = \bar{h}^{LL}_{i/B}\lambda^B + \bar{h}^{LT}_{i/B}k_{2\perp}(\rho^B_{-+}e^{i\Phi}e^{i\phi_2'} + \rho^B_{+-}e^{-i\Phi}e^{-i\phi_2'}), \tag{46}$$

$$\tilde{\rho}^a_{-+} = h^{TT}_{i/A}\rho^A_{-+}e^{-i\Phi}e^{-i\phi_1'} + h^{TL}_{i/A}k_{1\perp}\lambda^A/2, \tag{47}$$

$$\tilde{\rho}^b_{+-} = \bar{h}^{TT}_{i/B}\rho^B_{+-}e^{-i\Phi}e^{-i\phi_2'} + \bar{h}^{TL}k_{2\perp}\lambda^B/2, \tag{48}$$

where $\phi_1' = \phi_1 - \Phi$, $\phi_2' = \phi_2 - \Phi$, and all distribution functions are understood to depend on x and $k_\perp^2$. Under the substitution $\phi_a \to -\phi_a$, x_1, x_2, $k_{1\perp}$, and $k_{2\perp}$ are invariant, but ϕ_1', ϕ_2', and $(\hat{\phi}_A + \hat{\phi}_B + 2\gamma)$ all change sign, from which it follows that all $R_{L1}^{MN_1N_2}$ are real, as is required on general grounds. Although we have not specified the integration domain, the usual parton-model requirement that a parton should not have longitudinal moment opposite to its parent hadron provides a cutoff in the $\theta_a \varphi_a$ integration. We note also that the inclusion of $k_\perp$ smearing has produced all the terms $R_{L1}^{MN_1N_2}$ permitted by general considerations, in particular, effects with one hadron polarized longitudinally and the other transversely are now present. Detailed numerical investigation of this expression (assuming simple forms for the distribution functions) will enable one to estimate the importance of such effects.

The effects of $k_\perp$ smearing may also be included in the $q+\bar{q} \to$ "γ" $+G$ and Compton processes. All that is needed is a method of determining the rotation parameters needed to pass from constituent-based axes to hadron-based axes. In contrast to the preceding example, it appears to be simpler to work in the hadron c.m. frame, using as integration variables x_1, x_2, $\vec{k}_{1\perp}$, $\vec{k}_{2\perp}$ (subject to one constraint) and to find the necessary rotation parameters as functions of these. Let us first note that the usual relation between cross sections for "hard" subprocesses and the inclusive reaction is

$$\frac{d\sigma}{dM^2 dy\, dQ_\perp^2 d\Phi} = \sum_{ij} \int dx_1 dx_2 d^2k_{1\perp} d^2k_{2\perp} f_{i/A} f_{j/B} \hat{s}\, \delta(\hat{s} + \hat{t} + \hat{u} - M^2) \frac{d\sigma_{ij}}{d\hat{t}\, dM^2 d\hat{\Phi}}, \tag{49}$$

where the kinematic variables $\hat{s}$, $\hat{t}$, $\hat{u}$ have been defined in Eq. (32). We then use our explicit formula for the parton cross sections to obtain the even-L t_L^M multiplied by the differential cross section.

For annihilation, we have

$$\begin{aligned}\left(\frac{d\sigma t_L^M}{dM^2 dQ_\perp^2 dy\, d\Phi}\right)^A = \tfrac{1}{4} \sum e_i^2 \int dx_1 dx_2 dk_{1\perp}^2 dk_{2\perp}^2 d\phi_1' d\phi_2' \hat{s}\, \delta(\hat{s} + \hat{t} + \hat{u} - M^2) \left[\frac{1}{3\pi}\left(\frac{4}{9}\right) \frac{\alpha^2 \alpha_s}{\hat{s}^2 \hat{u} \hat{t} M^2}\right] \\ \times \Bigg(f_{i/A} \bar{f}_{i/B} \Bigg\{ (1 - \tilde{\lambda}^a \tilde{\lambda}^b) \left[\frac{(t - M^2)^2}{2} [3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L*}(R_a)] + \frac{(\hat{u} - M^2)^2}{2} [3\delta_{L0}\delta_{M0} - C_{000}^{1L1} D_{M0}^{L*}(R_b)] \right] \\ + 4(\hat{t} - M^2)(\hat{u} - M^2) \Bigg[\tilde{\rho}_{-+}^a \tilde{\rho}_{+-}^b e^{-i(\hat{\Phi} - \hat{\phi}_A + \hat{\Phi} - \hat{\phi}_B)} \sum_{m,m'} C_{mMm'}^{1L1} D_{m1}^1(R_b) D_{m'1}^{1*}(R_a) \\ + (\tilde{\rho}_{-+}^a \tilde{\rho}_{+-}^b e^{-i(\hat{\Phi} - \hat{\phi}_A + \hat{\Phi} - \hat{\phi}_B)})^* \sum_{m,m'} C_{mMm'}^{1L1} D_{m1}^1(R_a) D_{m'1}^{1*}(R_b) \Bigg] \Bigg\} \\ + (f \leftrightarrow \bar{f}\, h \leftrightarrow \bar{h}) \Bigg)\end{aligned} \tag{50}$$

The quantities $\tilde{\lambda}^a$, $\tilde{\lambda}^b$, $\tilde{\rho}_{-+}^a$, $\tilde{\rho}_{+-}^b$, have been introduced in Eqs. (49)–(52), $\phi_1' = \phi_1 - \Phi$, $\phi_2' = \phi_2 - \Phi$, and the rotations R_a and R_b are defined as follows: Using the axes in the dilepton frame defined implicitly by Eqs. (34a) and (34b), the spherical polar angles of partons a and b, namely (θ_a, ϕ_a) and $(\theta_b, \phi_b,)$, may be determined by evaluating scalar products and invariant pseudoscalars. The rotations R_a and R_b are then described by Euler angles $(\phi_a, \theta_a, \gamma_a)$ and $(\phi_b, \theta_b, \gamma_b)$, respectively. From the form of Eq. (50), we see that only the combination $\gamma_a - \gamma_b$ enters, for which the following relation holds:

$$e^{i(\gamma_b - \gamma_a)} = 2\left(\sum_\nu D_{\nu 1}^1(\phi_b, \theta_b, 0) D_{\nu 0}^{1*}(\phi_a, \theta_a, 0)\right)\left(\sum_\tau D_{\tau -1}^1(\phi_a, \theta_a, 0) D_{\tau 0}^{1*}(\phi_b, \theta_b, 0)\right) \Big/ \sin^2\theta_{ab}, \tag{51}$$

where

$$\sin^2\theta_{ab} = 4\hat{s}\hat{u}\hat{t} M^2 / ((M^2 - \hat{u})(M^2 - \hat{t}))^2. \tag{52}$$

We have thus shown that all factors in Eq. (50) are well-defined functions of the kinematic variables which characterize the final state and the integration variables.

Proceeding in the same manner, one derives the following expression for the contribution of the Compton scattering process:

$$\left(\frac{d\sigma\, t_L^M}{dM^2 dy\, dQ_\perp{}^2 d\Phi}\right)^c = \frac{1}{4}\sum_i e_i{}^2 \int dx_1 dx_2 dk_{1_\perp}{}^2 dk_{2_\perp}{}^2 d\phi_1' d\phi_2' \hat{s}\,\delta(\hat{s}+\hat{t}+\hat{u}-M^2)\left(\frac{1}{3\pi}\right)\left(\frac{1}{6}\right)\left(\frac{-\alpha^2\alpha_s}{\hat{s}^3 M^2}\right)$$
$$\times\Bigg\{ f_{G/A} f_{i/B}\Bigg[(1+\tilde{\lambda}_A^G\tilde{\lambda}^b)\,\frac{(\hat{s}-M^2)^2}{2\hat{u}}\,[3\delta_{L0}\delta_{M0} - C^{1L1}_{000} D^{L\,*}_{M0}(R_{\hat{s}})]$$
$$+(1-\tilde{\lambda}_A^G\tilde{\lambda}^b)\,\frac{(\hat{u}-M^2)^2}{2\hat{u}}\,[3\delta_{L0}\delta_{M0} - C^{1L1}_{000} D^{L*}_{M0}(R_b)]$$
$$+8\,\frac{(M^2-\hat{u}^2)^2}{\hat{u}}\, h^{T0}_{G/A} k_{1_\perp}{}^2\, \mathrm{Re}\left(e^{2i(\hat{\Phi}-\hat{\phi}_A)}\sum C^{1L1}_{mMm'} D^1_{m-1}(R_{\hat{s}}) D^{1*}_{m'1}(R_b)\right)\Bigg]$$
$$+ f_{i/A} f_{G/B}\Bigg[(1+\tilde{\lambda}_B^G\tilde{\lambda}^a)\,\frac{(\hat{s}-M^2)^2}{2\hat{t}}\,[3\delta_{L0}\delta_{M0} - C^{1L1}_{000} D^{L*}_{M0}(R_{\hat{s}})]$$
$$+(1-\tilde{\lambda}_B^G\tilde{\lambda}^a)\,\frac{(\hat{t}-M^2)^2}{2\hat{t}}\,[3\delta_{L0}\delta_{M0} - C^{1L1}_{000} D^{L*}_{M0}(R_a)]$$
$$+8\,\frac{(M^2-\hat{t})^2}{\hat{t}}\, h^{T0}_{G/B} k_{2\perp}{}^2\, \mathrm{Re}\left(e^{2i(\hat{\Phi}-\hat{\phi}_B)}\sum C^{1L1}_{mMm'} D^1_{m-1}(R_{\hat{s}}) D^{1*}_{m'1}(R_a)\right)\Bigg]\Bigg\}, \quad (53)$$

where we have used the substitutions $\tilde{\lambda}^a$, $\tilde{\lambda}^b$ and the rotations R_a, R_b defined previously. The additional substitutions are

$$\tilde{\lambda}_A^G = h^{LL}_{G/A}\lambda^A + h^{LT}_{G/A} k_{1\perp}(\rho^A_{+-}e^{i\Phi}e^{i\phi_1'} + \rho^A_{-+}e^{-i\Phi}e^{-i\phi_1'}), \quad (54)$$

$$\tilde{\lambda}_B^G = h^{LL}_{G/B}\lambda^B + h^{LT}_{G/B} k_{2\perp}(\rho^B_{-+}e^{i\Phi}e^{i\phi_2'} + \rho^B_{+-}e^{-i\Phi}e^{-i\phi_2'}), \quad (55)$$

where the distribution functions again depend on x and $k_\perp{}^2$. The Euler angles of the rotation $R_{\hat{s}}$ are defined as follows: Let $(\theta_{\hat{s}}, \phi_{\hat{s}})$ be the spherical polar angles of the recoil quark momentum $\vec{k}_a + \vec{k}_b$, as seen in the dilepton rest frame with respect to the conventionally chosen axes. Then the Euler angles are $(\phi_{\hat{s}}, \theta_{\hat{s}}, \gamma_{\hat{s}})$, where $\gamma_{\hat{s}}$ remains to be defined. Noting that only the combinations $\gamma_{\hat{s}}+\gamma_a$ and $\gamma_{\hat{s}}+\gamma_b$ are needed in Eq. (53), it is sufficient to find an expression for these in terms of $\phi_{\hat{s}}$, $\theta_{\hat{s}}$, ϕ_a, θ_a, ϕ_b, θ_b:

$$e^{i(\gamma_{\hat{s}}+\gamma_b)} = \frac{-2}{\sin^2\theta_{\hat{s}b}}\left(\sum_v D^1_{v1}(\phi_{\hat{s}}, \theta_{\hat{s}}, 0) D^{1*}_{v0}(\phi_b, \theta_b, 0)\right)$$
$$\times\left(\sum_T D^1_{T1}(\phi_b, \theta_b, 0) D^{1*}_{T0}(\phi_{\hat{s}}, \theta_{\hat{s}}, 0)\right), \quad (56)$$

$$e^{i(\gamma_{\hat{s}}+\gamma_a)} = \frac{-2}{\sin^2\theta_{\hat{s}a}}\left(\sum_v D^1_{v1}(\phi_{\hat{s}}, \theta_{\hat{s}}, 0) D^{1*}_{v0}(\phi_a, \theta_a, 0)\right)$$
$$\times\left(\sum_T D^1_{T1}(\phi_a, \theta_a, 0) D^{1*}_{T0}(\phi_{\hat{s}}, \theta_{\hat{s}}, 0)\right), \quad (57)$$

where

$$\sin^2\theta_{\hat{s}b} = 4\hat{s}\hat{t}\hat{u}M^2/[(\hat{s}-M^2)(M^2-\hat{u})]^2, \quad (58)$$

$$\sin^2\theta_{\hat{s}a} = 4\hat{s}\hat{t}\hat{u}M^2/[(\hat{s}-M^2)(M^2-\hat{t})]^2. \quad (59)$$

The explicit forms of Eqs. (50) and (53) are fully consistent with the formalism developed in Ref. 3. In particular, the fact that the kinematic variables are invariant under reflection of parton three-momenta in the production plane defined by the momenta of the initial-state hadrons and the massive photon means that all R^{M00}_{L00} and $R^{MN_1N_2}_{L11}$ are real, as they should be. As one might expect, $k_\perp$ smearing has produced expressions for which all $R^{MN_1N_2}_{L11}$ are potentially different from zero. The question of which effects are likely to be most important requires detailed numerical study, but in the absence of reliable estimates for distribution functions (especially the $k_\perp{}^2$ dependence) all one can do is try various simple models.

V. CONCLUSIONS

Following the work of Ralston and Soper,[5] we have made some generalizations motivated by the observed relatively high transverse momentum of the lepton pair produced in hadronic collisions. We investigated the effects of intrinsic parton transverse momentum and the QCD $2\to 2$ subprocesses. The $2\to 2$ processes exhibit substantial spin correlations. Of special note is the fact that gluon transverse polarization by itself may be observed with a polarized deuteron or hyperon beam.

The QCD $2\to 2$ subprocesses dominate the QCD contributions in the limit $s\to\infty$, $Q_\perp{}^2/s$ fixed; however, there is not very much data in this region even in the unpolarized case. Considerations of intrinsic parton transverse momentum hold in the region $s\to\infty$, $Q_\perp{}^2/s\to 0$.[17] Again, there is not much data in this region. Most of the data lies between the two regions so we have included both effects in our treatment. Since the effects of intrinsic parton transverse momentum are dependent upon the actual $k_\perp$ dependence of the distribution functions, one needs to have a model of the distribution functions. It would be valuable to do some numerical work as it would be especially interesting to see if spin may shed additional light onto the issue of the relative importance of $k_\perp$ vs the $2\to 2$ subprocesses.

It should be possible within the next few years to study muon-pair production with polarized beams and targets, and to measure in detail the multiply differential cross sections discussed here. There have also been some discussions of the longitudinal spin assymmetry (which should be easier to measure), notably by Hidaka,[17] and by Mani and Noman.[18] It will be quite interesting to study all these spin effects.

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APPENDIX

The production of lepton pairs at $Q_\perp = 0$ has been studied in detail by Ralston and Soper,[5] who have given explicit formulas for the observable quantities which can be determined with polarized beam and target. We asserted in Sec. III that our approach employing massless constituents and the helicity formalism produces results which, while similar to those of Ralston and Soper, contain extra terms proportional to higher powers of some average intrinsic momentum. In order to show this, we evaluate the contribution of the standard Drell-Yan process, $q+\bar{q}\to\gamma$ in the configuration $Q_\perp = 0$, using Eq. (44). In this special case, the kinematics simplifies somewhat, and the axes in the lepton-pair rest frame may be chosen parallel to those in the hadron c.m. frame, since only a z boost is involved. It then follows that $k_\perp = \frac{1}{2}M\sin\theta_a$, $\varphi_1 = \varphi_2 + \pi = \varphi_a$, $\Psi = 0$, $\chi = \pi$, and $(\hat{\varphi}_A + \hat{\varphi}_B + 2\gamma) = \pi$. The kinematic variables are then independent of φ_a, and the φ_a integration in Eq. (44) may be explicitly performed, provided one uses the φ_a dependence of $\bar{\lambda}^a$, $\bar{\lambda}^b$, $\bar{\rho}^a_{-+}$, and $\bar{\rho}^b_{+-}$ shown in Eqs. (45)–(48). [Since the azimuthal variable of the massive lepton pair is undefined at $Q = 0$, one may simply set $\Phi = 0$ in eqs. (45)–(48).] If we express the results of the φ_a integration in the form

$$\frac{d\sigma\, t_L^M}{dM^2 dy\, dQ_\perp d\Phi} = \frac{2\pi}{8}\sum e_i^{\,2}\int d(\cos\theta_a)x_1x_2\left(\frac{4\pi\alpha^2}{3M^2}\right)[f_{i/A}\bar{f}_{i/B}I_L^M + (f\to\bar{f},\, h\to\bar{h})], \tag{A1}$$

then the I_L^M may be written as follows:

$$I_L^0 = [1 - h_{i/A}^{LL}h_{i/B}^{LL}\lambda^A\lambda^B + k_\perp{}^2 h_{i/A}^{LT}h_{i/B}^{LT}(\rho_{+-}^A\rho_{+-}^B + \rho_{-+}^A\rho_{-+}^B)]\left\{\begin{matrix}1\\ D_{00}^2(0,\theta_a,0)/\sqrt{10}\end{matrix}\right\} - (\tfrac{3}{5})^{1/2}k_\perp{}^2 h_{i/A}^{TL}\bar{h}_{i/B}^{TL}\lambda^A\lambda^B\left\{\begin{matrix}0\\ D_{02}^2(0,\theta_a,0)\end{matrix}\right\}, \tag{A2}$$

where the upper term in braces is for $L=0$, the lower for $L=2$,

$$I_2^2 = [-2(\tfrac{3}{5})^{1/2}h_{i/A}^{TT}\bar{h}_{i/B}^{TT}D_{22}^2(0,\theta_a,0) + h^{LT}\bar{h}^{\,LT}k_\perp{}^2 D_{20}^2(0,\theta_a,0)]\,\rho_{-+}^A\rho_{+-}^B\,, \tag{A3}$$

$$I_2^1 = k_\perp(h_{i/A}^{LL}\bar{h}_{i/B}^{LT}\lambda^A\rho_{+-}^B - h_{i/A}^{LT}\bar{h}^{\,LL}\lambda^B\rho_{-+}^A)D_{10}^2(0,\theta_a,0)/\sqrt{10} - (\tfrac{3}{5})^{1/2}k_\perp(h_{i/A}^{TL}\bar{h}_{i/B}^{TT}\lambda^A\rho_{+-}^B - h_{i/A}^{TT}\bar{h}^{\,TL}\lambda^B\rho_{-+}^A)D_{12}^2(0,\theta_a,0). \tag{A4}$$

Following the conventions established in Ref. 3, we see that all nine nonvanishing (at $Q_\perp = 0$) $R_{LK_1K_2}^{MN_1N_2}$ are present, namely R_{000}^{000}, R_{200}^{000}, R_{011}^{000}, R_{211}^{000}, R_{011}^{011}, R_{211}^{011}, R_{211}^{2-11}, R_{211}^{101}, and R_{211}^{1-10}. (Note that with our Jacob-Wick convention for particle 2, $\rho_{+-}^A\rho_{+-}^B + \rho_{-+}^A\rho_{-+}^B = \frac{1}{2}\vec{S}_A^T\cdot\vec{S}_B^T$ in the notation of Ralston and Soper.) If we compare our results to Eq. (2.10) of Ralston and Soper, we find the same terms as they do, except that the sign of the $\vec{S}_A^T\cdot\vec{S}_B^T$, terms must be changed to read $+\vec{S}_A^T\cdot\vec{S}_B^T V_{00}^T$ and $+S_A^T\cdot S_B^T V_{20}^T$.[19] Furthermore, Ralston and Soper propose five linear relations [Eqs. (3.19)] among the nine coefficients appearing in Eq. (2.10). Expressed in terms of our notation, these five relations have the form

$$R_{011}^{000} - \sqrt{10}\,R_{200}^{000} = 0, \tag{A5}$$

$$R_{011}^{011} - \sqrt{10}\,R_{211}^{011} = 0\,, \tag{A6}$$

$$R_{211}^{011} - \sqrt{10}\,R_{011}^{011} = 0, \tag{A7}$$

$$R_{211}^{101} = 0, \tag{A8}$$

$$R_{211}^{1-10} = 0. \tag{A9}$$

If we examine Eqs. (A2), (A3), and (A4) we find that these relations can be obtained if one replaces the rotations functions $D^2_{NN'}(0, \theta_a, 0)$ by $D^2_{NN'}(0, 0, 0)$. However, our approach implies that the right-hand sides of (A5)–(A9), instead of being zero, are $O(\langle|k_\perp{}^2|\rangle)$ compared to the left-hand side. [The quantities on the left-hand of (A7) are already $O(k_\perp{}^2)$, hence the right-hand side is $O(k_\perp{}^4)$.] Furthermore, our apporach allows for small effects when one hadron is longitudinally polarized, the other having transverse polarization. Such effects are absent in the Ralston-Soper approach. It should of course be realized that Ralston and Soper deliberately omitted effects which do not appear in the limit of heavy dilepton mass ($M^2 \to \infty$, hence $\langle k_\perp{}^2\rangle/M^2 \to 0$). However, at modest masses (~ 4 GeV) it may be possible to use polarization effects in order to extract information on $k_\perp$ distributions of partons.

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