Dilepton production from collisions of polarized spin-1/2 hadrons. I. General kinematic analysis

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The kinematics for the production of lepton pairs from collisions of polarized spin-1/2 particles is discussed on the basis of rotational covariance and the Jacob-Wick helicity formalism. It is assumed that the leptons come from the decay of a single vector particle.

I. INTRODUCTION

We present a systematic treatment of lepton-pair production by spin-$\frac{1}{2}$ particles based on rotational covariance and the Jacob-Wick helicity formalism.[1] Recently, Ralston and Soper[2] treated this process emphasizing Lorentz covariance. Certain simplifications in the partially integrated cross section are more easily seen in the present treatment. Also, because the treatment is more systematic, it is easier to find all of the structure functions and to list the functions of angles and spins by which they are multiplied. In fact, Ralston and Soper identified 48 independent structure functions but they did not list all the Lorentz tensors which would multiply the structure functions. Our Eq. (10) explicitly shows how the cross section depends on each of the structure functions.

Ralston and Soper identified 9 structure functions which determine the cross section integrated over the dilepton transverse momentum. We find that there are, in fact, 11 independent structure functions for that special case.[3] Also, it is not necessary to integrate over $Q_\perp$; an integration over the dilepton azimuthal angle is sufficient to reduce the number of structure functions from 48 to 11.

Section II presents the formalism for lepton-pair production from spin-$\frac{1}{2}$ beam and target particles. The kinematic variables are explained, as are the most commonly used reference frames. In particular, the Collins-Soper[4] axes and the Ralston-Soper[2] axes are compared. The relationship of the latter to the Jacob-Wick conventions is explained.

A compact formula for the cross section is presented through the introduction of the structure functions, $R^{M\ N_1N_2}_{L\ K_1K_2}$. After outlining their properties, various moments of the cross section integrated over some of the lepton angles are introduced. Finally, our formalism is compared to that of Ralston and Soper. Two additional structure functions are found when the cross section is integrated over the azimuthal angle of the virtual-photon momentum.

Section III contains a short summary.

II. FORMALISM FOR MASSIVE-PHOTON PRODUCTION AND DECAY WITH POLARIZED SPIN-½ BEAM AND TARGET PARTICLES

Let the reaction be denoted by

$$A+B\to(l^+l^-)+X$$

and let the four-momentum of the dilepton be Q. The beam and target fermions have density matrices ρ^A and ρ^B, respectively. The decay angles of one of the leptons in the l^+l^- rest frame will be denoted by (θ,ϕ). The measurement of an event provides two three-momenta, and one may define the cross section for production and decay of the massive photon as

$$\frac{d\sigma}{d^4Q\,d\Omega}\,.$$

The six kinematic variables may be chosen as Q^2, Q_z, and $Q_\perp$, together with (θ,ϕ) and an azimuthal angle Φ. For the angular variables, the dependence of the cross section can be made explicit in terms of known functions which are multiplied by unknown structure functions that depend only on Q^2, Q_z, and $Q_\perp$. Clearly the choice of angles, angular functions, and structure functions can be made in many different ways, and some discussions are available in the literature. The formalism we present here relies heavily on a rotation-group approach, and provides results which are relatively compact.

A. Choice of axes

Since the formalism we use is based on the Jacob-Wick[1] helicity amplitudes, we present here a brief sketch of the axes used to define angles in the four different reference frames we use, namely the c.m. frame, the rest frames of particles A and B, and the dilepton rest frame. In the c.m. frame, the z axis is chosen along the beam-target line of flight, with particle A having $p_z^A > 0$. The choice of transverse axes x and y is then made arbitrarily. In the rest frame of particle A the x, y, and z axes are obtained by boosting the c.m. axes. In the particle-B rest frame, in accord with the Jacob-Wick "particle-2" convention, the z axis is antiparallel to the c.m. z axis, while the x axis is parallel. The situation is depicted in Fig. 1. In the c.m. frame the dilepton emerges with four-momentum

$$Q^\mu = (Q^0,\ Q_\perp \cos\Phi,\ Q_\perp \sin\Phi,\ Q^z),$$

where

$$Q^2 = (Q^0)^2 - Q_\perp{}^2 - Q_z{}^2 .$$

The polarization vectors of the initial-state particles $\vec{P}^A$ and $\vec{P}^B$ refer to the axes of Fig. 1. We shall use a spherical description rather than a Cartesian one, hence we introduce angles β^A, α^A as follows:

$$\cos\beta^A = \vec{P}^A \cdot \hat{z}^A / P^A , \tag{1a}$$

$$\tan\alpha^A = \frac{\vec{P}^A \cdot \hat{y}^A}{\vec{P}^A \cdot \hat{x}^A} , \tag{1b}$$

where we have used

$$P^A = (\vec{P}^A \cdot \vec{P}^A)^{1/2} \tag{2}$$

and $\hat{x}^A$, $\hat{y}^A$, $\hat{z}^A$ denote unit vectors in the particle-A rest frame. Angles β^B and α^B may be introduced in a similar way. Note that since the transverse axes were chosen arbitrarily, Φ, α^A, and α^B reflect this arbitrariness. For a definite event, under a rotation about the z axis by ϵ,

$$\Phi \to \Phi - \epsilon, \quad \alpha^A \to \alpha^A - \epsilon, \quad \alpha^B \to \alpha^B + \epsilon .$$

Therefore, only $\Phi - \alpha^A$, $\Phi + \alpha^B$ have physical meaning.

In the rest frame of the dilepton, we choose the y axis normal to the production plane, which corresponds to the $R(\phi, \theta, 0)$ rotation convention of Jacob and Wick. In terms of the beam and target momenta, $\vec{p}^{\,A}$ and $\vec{p}^{\,B}$,

$$\hat{y} = -(\vec{p}^{\,A} \times \vec{p}^{\,B}) / |\vec{p}^{\,A} \times \vec{p}^{\,B}| . \tag{3}$$

The quantity Y^μ introduced by Collins and Soper[4] is antiparallel to our y direction. For the choice of the z axis, any direction in the plane formed by $\vec{p}_A$ and $\vec{p}_B$ may be used, although three choices are commonly used, s-channel axes, t-channel or Gottfried-Jackson axes, and Collins-Soper axes. For s-channel axes,

$$\hat{z}_s = -(\vec{p}_A + \vec{p}_B) / |\vec{p}_A + \vec{p}_B| ; \tag{4a}$$

for t-channel axes,

$$\hat{z}_t = \vec{p}_A / |\vec{p}_A| ; \tag{4b}$$

whereas for Collins-Soper axes,

$$\hat{z}_{\mathrm{CS}} = \frac{\vec{p}_A (p^B \cdot Q) - \vec{p}_B (p^A \cdot Q)}{|\vec{p}_A (p^B \cdot Q) - \vec{p}_B (p^A \cdot Q)|} . \tag{4c}$$

The orientation of these vectors is illustrated in Fig. 2.

If the beam and target masses are small compared to $\vec{p}^{\,A}$ and $\vec{p}^{\,B}$ (in limit $p_A{}^2 = p_B{}^2 = 0$), respectively, then the $\hat{z}_{\mathrm{CS}}$ makes angles of Ψ and $\pi - \Psi$ with the beam and target momenta. Once a particular choice for $\hat{z}$ is made, the angles θ and ϕ of the l^+ are then defined by

$$\cos\theta = \hat{l}^{\,+} \cdot \hat{z} , \tag{5a}$$

$$\sin\theta \sin\phi = \hat{l}^{\,+} \cdot \hat{y} . \tag{5b}$$

Ralston and Soper[2] have presented a formalism in which definite choices for z and y directions in the dilepton rest frame are made. The z axis is chosen along $\hat{z}_{\mathrm{CS}}$, while the y axis is chosen such that the common azimuthal angle of $\vec{p}_A$ and $\vec{p}_B$ is $(\pi + \Phi)$. (They obtain these axes by first boosting along the c.m. z direction to the frame where $Q_z = 0$, then boosting along the transverse direc-

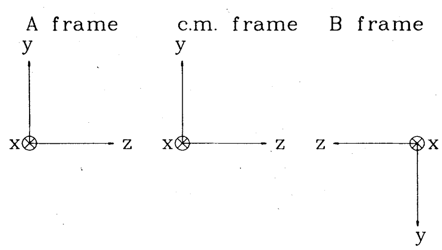

FIG. 1. The relative orientation of three sets of axes: the particle-A rest frame, the center-of-momentum frame, and the particle-B rest frame. The x axis points into the page for all three systems.

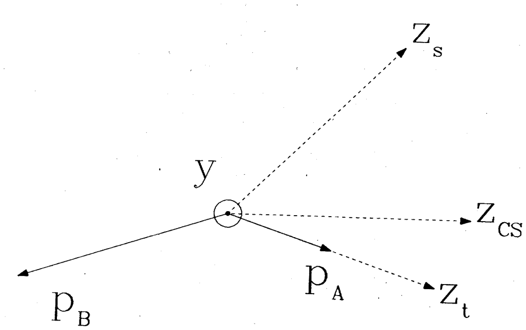

FIG. 2. The momenta of particles A and B in the dilepton rest frame are shown, along with the direction of three common choices of the z axis, namely, z_s, z_t, z_{CS}. The y axis points out of the page.

tion to the Q rest frame.) This is equivalent to using the Jacob-Wick $R(\phi, \theta, -\phi)$ convention, followed by a rotation about the normal to the plane of $(\vec{p}_A, \vec{p}_B)$ from the z_s axis to the $z_{\rm CS}$ direction. The situation is illustrated in Fig. 3. Obviously if ϕ is the azimuthal angle of $\hat{l}^+$, then the Ralston-Soper angle $\phi_{\rm RS}$ is related to our ϕ by

$$\phi_{\rm RS} = \phi + \Phi \,. \tag{6}$$

This relation is, of course, valid only when the $z_{\rm CS}$ direction is used to define θ.

B. Dependence of cross section on θ, ϕ, Φ

The quantity $d\sigma/d^4Q\, d\Omega$ may be written, assuming one-photon exchange for the lepton pair, as[5]

$$\frac{d\sigma}{d^4Q\, d\Omega} = \frac{1}{4\pi} \sum_{L=0,\text{even}}^{2} f(L) \left[\sum_{M=-L}^{L} \frac{d\sigma}{d^4Q}\, t_L^M D_{M0}^L(\phi, \theta, 0) \right], \tag{7}$$

where D_{M0}^L is a Wigner rotation matrix,

$$f(0) = 1 \,, \tag{8a}$$

$$f(2) = \frac{\sqrt{10}}{2} \left(\frac{1 - m^2/E^2}{1 + m^2/2E^2} \right), \tag{8b}$$

where m is the final-lepton mass and E is its energy. The quantities $(d\sigma/d^4Q)t_L^M$ transform irreducibly under rotations of the axes in the dilepton rest frame. We have adopted the convention that the spin density matrix and the statistical tensors are related by[6]

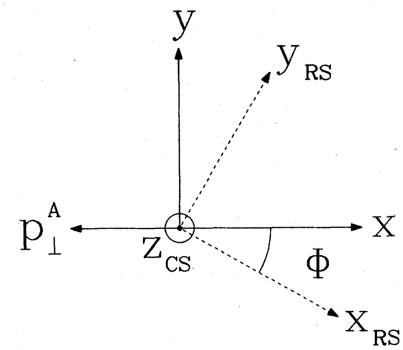

FIG. 3. Depiction of the Ralston-Soper axes and the Collins-Soper axes.

$$\rho = \frac{1}{(2J+1)} \sum_{L=0}^{2J} (2L+1) \sum_{M=-L}^{L} t_L^{M*} T_L^M \,, \tag{9}$$

where the T_L^M are matrices with elements which are Clebsch-Gordan coefficients,

$$(T_L^M)_{mm'} = C_{m'Mm}^{JLJ}$$

We can then obtain a relatively compact form for the dependence of the quantities $(d\sigma/d^4Q)t_L^M$ on Φ and $\vec{P}^A$, $\vec{P}^B$. The result may be written as[7]

$$\begin{aligned} \frac{d\sigma}{d^4Q}\, t_L^M = \Big\{ & R_{L00}^{M00} + \sqrt{3} \left[P^A \sum_{N_1=-1}^{1} R_{L10}^{MN_10} D_{N_10}^1(\alpha^A - \Phi, \beta^A, 0) + P^B \sum_{N_2=-1}^{1} R_{L01}^{M0N_2} D_{N_20}^1(\alpha^B + \Phi, \beta^B, 0) \right] \\ & + 3P^AP^B \sum_{N_1N_2} R_{L11}^{MN_1N_2} D_{N_10}^1(\alpha^A - \Phi, \beta^A, 0)\, D_{N_20}^1(\alpha^B + \Phi, \beta^B, 0) \Big\} \,, \end{aligned} \tag{10}$$

where we have introduced the structure functions $R_{LK_1K_2}^{MN_1N_2}$ which depend only on Q^2, Q_z, and $Q_\perp$. The requirement of Hermiticity implies

$$R_{LK_1K_2}^{MN_1N_2} = (-1)^{M+N_1+N_2} (R_{LK_1K_2}^{-M-N_1-N_2})^* \,, \tag{11}$$

whereas the requirement of parity conservation (reflection in the plane of production) implies (provided the y axis is chosen normal to the production plane in the dilepton rest frame)

$$R_{LK_1K_2}^{MN_1N_2} = \begin{cases} \text{real} & \text{if } L + K_1 + K_2 \text{ even}, \\ \text{imaginary} & \text{if } L + K_1 + K_2 \text{ odd}. \end{cases}$$

Since only the even-L moments are observable (unless one measures the final-lepton polarization as well), it follows that there are 48 independent structure functions, 4 with $K_1 = K_2 = 0$, 16 with either $K_1 = 0$ or $K_2 = 0$ (but not both), and 28 with $K_1 = K_2 = 1$.

In order to extract relatively simple combinations of these structure functions, moments of the quantity $d\sigma/d^4Q\, d\Omega$ may be taken as

$$\Theta(L, M, N) = \int \frac{d\sigma}{d^4Q\, d\Omega} D_{M0}^{L*}(\phi, \theta, 0) e^{iN\Phi} d\Omega\, d\Phi \,.$$

Performing the integration, one finds

$$\begin{aligned} \Theta(L, M, N) = \frac{2\pi}{2L+1} f(L) \Big\{ & R_{L00}^{M00} \delta_{N0} + \sqrt{3} [P^A R_{L10}^{M-N0} D_{-N0}^1(\alpha^A, \beta^A, 0) + P^B R_{L01}^{M0N} D_{N0}^1(\alpha^B, \beta^B, 0)] \\ & + 3P^AP^B \sum_{N_1N_2} \delta_{N,N_2-N_1} R_{L11}^{MN_1N_2} D_{N_10}^1(\alpha^A, \beta^A, 0) D_{N_20}^1(\alpha^B, \beta^B, 0) \Big\} \,. \end{aligned} \tag{12}$$

C. Comparison with formalism of Ralston and Soper

Ralston and Soper[2] have provided an explicit formula for the dilepton production in the limit $Q_\perp = 0$, or after integration over all $Q_\perp$ and Φ, which involves 9 structure functions. We shall now relate our results to theirs, noting that the integration over $Q_\perp$ is unnecessary; Φ integration alone reduces the structure functions from 48 to 11 (two appear to have been overlooked by Ralston and Soper).[3] First we note that

$$\frac{d\sigma}{d^4Q\,d\Omega'}\bigg|_{\phi_{RS}} = \frac{d\sigma}{d^4Q\,d\Omega}\bigg|_{\phi=\phi_{RS}-\Phi}. \tag{13}$$

We then integrate our formula (10) over Φ, at fixed ϕ_{RS}, to obtain

$$\begin{aligned}4\pi\int d\Phi \frac{d\sigma}{d^4Q\,d\Omega'} &= \int d\Phi \sum f(L) \sum_{M=-L}^{L} \frac{d\sigma}{d^4Q} t_L^M D^L_{M0}(\phi_{RS},\theta,0)e^{iM\Phi} \\ &= 2\pi \sum_{L=0}^{2} f(L) \sum_{M=-L}^{L} D^L_{M0}(\phi_{RS},\theta,0)\Big\{R^{M00}_{L00}\delta_{M0} + \sqrt{3}\,[P^A R^{M\;-M0}_{L\;\;10} D^1_{M0}(\alpha^A,\beta^A,0) + P^B R^{M0\;M}_{L01} D^1_{M0}(\alpha^B,\beta^B 0)] \\ &\quad + 3P^AP^B \sum_{N_1N_2} \delta_{M,N_2-N_1} R^{MN_1N_2}_{L11} D^1_{N_10}(\alpha^A,\beta^A,0) D^1_{N_20}(\alpha^B,\beta^B,0)\Big\}.\end{aligned} \tag{14}$$

This expression depends on the 9 real structure functions R^{000}_{000}, R^{000}_{200}, R^{000}_{011}, R^{011}_{011}, R^{000}_{211}, R^{011}_{211}, R^{101}_{211}, $R^{1-10}_{2\;\;11}$, and $R^{2-11}_{2\;\;11}$ in just the way Ralston and Soper find. However, there are two additional terms, involving the imaginary quantities $R^{1-10}_{2\;\;10}$ and R^{101}_{201}. The contribution may be written explicitly as

$$\begin{aligned}2\pi f(2)\tfrac{3}{2}\sin\theta\cos\theta[&iR^{1-10}_{2\;\;10}P^A\sin\beta^A\sin(\phi_{RS}-\alpha^A) \\ &-iR^{101}_{201}P^B\sin\beta^B\sin(\phi_{RS}+\alpha^B)].\end{aligned} \tag{15}$$

Note that since we use the Jacob-Wick "particle-2" convention our α^B is opposite in sign to that of Ralston and Soper, hence our formulas differ from theirs by this sign.

The quantities $R^{1-10}_{2\;\;10}$ and R^{101}_{201} can be measured using either a polarized beam alone (for the former) or a polarized target (for the latter). Since they are imaginary, they require that some helicity amplitudes be out of phase, and are zero in any parton-model calculations where no loops are involved. If experiment were to indicate their presence, standard parton models would require substantial modification.

III. SUMMARY

We have presented a compact formalism for the production of lepton pairs from polarized spin-$\frac{1}{2}$ particles. The kinematic dependence of the cross section on the 48 structure functions is explicitly displayed.

In a companion paper[8] we discuss the parton-model predictions for lepton-pair production within the present formalism.

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[7] The spin formalism for the inclusive reaction $A+B \to Q+X$ is the same as that for a quasi-two-body reaction where X is treated as a spin-0 particle. The $R^{MN_1N_2}_{LK_1K_2}$ in the latter case are defined by

$$R^{MN_1N_2}_{LK_1K_2} = \sum F_{\lambda\mu\nu} F^*_{\lambda'\mu'\nu'} C^{1L1}_{\lambda M\lambda'} C^{1/2\,K_1\,1/2}_{\mu'\;N_1\;\mu} C^{1/2\,K_1\,1/2}_{\nu'\;N\;\nu},$$

where the $F_{\lambda\mu\nu}$ denote Jacob-Wick helicity amplitudes at $\Phi=0$. In the inclusive reaction there are no such amplitudes, since the relevant $R^{MN_1N_2}_{LK_1K_2}$ are obtained by a sum over all unobserved final spins and momenta. However, the parity properties derived for the spinless quasi-two-body reaction remain valid.

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