

Kaonic mass by critical absorption of kaonic-atom x rays

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The energy of x rays from the transition $6h \rightarrow 5g$ in kaonic atoms of potassium falls on the K absorption edge of erbium. Measurement of the kaonic-x-ray attenuation in a precisely calibrated set of Er foils yields the x-ray energy $57\,458.8 \pm 6.3$ eV. The kaon mass is related to energy through the Klein-Gordon equation plus corrections for radiative effects, electron screening, and other effects. The negative-kaon mass was found to be 493.640 ± 0.054 MeV/ c^2 in agreement with the currently accepted value 493.669 ± 0.018 MeV/ c^2 which was determined from x rays emitted by high- Z atoms where the corrections were larger than for $Z = 19$.

INTRODUCTION

Accurate values of the negative-kaon mass have come from experiments by Columbia-Yale and CERN groups,^{1,2} in which kaons were stopped in Pb and other heavy elements. When brought to rest through energy loss by ionization, kaons are captured in the Coulomb field of the nuclei of the stopping material and form hydrogenlike atoms that de-excite by emitting x rays. The energies of the emitted x rays are related to the mass of the orbiting kaons by the Klein-Gordon equation plus corrections for various orders of vacuum polarization (radiative effects), electron screening of the nucleus, finite nuclear charge distribution, center-of-mass motion of the nucleus, strong interactions, and nuclear polarization. Another accurate value has been reported by a group at Novosibirsk using the reaction $e^+ + e^- \rightarrow \phi \rightarrow K^+ + K^-$.³

A significant reason for doing the experiment was that we used a fairly light element, potassium ($Z = 19$), in atomic states where the high-order radiative corrections and electron shielding effects are not so important as those for heavy atoms.

In previous experiments x-ray energies were obtained by pulse-height measurements of the output of semiconductor detector systems. Energies were determined by comparing the pulse heights of kaonic x rays to the pulse heights of radiations emitted by calibrated radioactive sources. The conversion from pulse height (analyzer channel number) to energy must be known to high accuracy. Even though semiconductor detectors have very good energy resolution, the instrumental width of typical kaonic x-ray lines from heavy elements amounts to about 1000 eV. The energies of the recorded lines must be determined to an accuracy of

a few eV which is difficult, especially if unknown weak lines almost buried in the background could shift the energy. Our experimental method employed a different technique, namely critical absorption of kaonic x rays. The use of atomic absorption edges to measure x-ray energies is a well-known procedure. It was applied to muonic atoms to determine the mass of the muon.⁴ In phosphorus the x-ray energy of the muonic-atom transitions $3d_{5/2} \rightarrow 2p_{3/2}$ was found to coincide with the K -edge absorption in Pb. At the time of the muon experiment (1960), NaI(Tl) detectors were in use, and they could not resolve the Pb fluorescent lines from the muonic-atom x rays. The currently described experiment employed ultrapure Ge detectors that were able to separate the intense fluorescent peaks from the mesonic-atom x rays.

We found that the energy of the kaonic potassium transition, $n, l = 6h \rightarrow 5g$, coincided with the K -absorption edge of erbium. At the Lawrence Berkeley Laboratory (LBL) we measured the transmission of the kaonic x rays of natural potassium through a set of Er foils. At the National Bureau of Standards (NBS) the energy versus transmission of the same foils was determined to very high precision by a double-axis crystal spectrometer. We determined the energy of the kaonic x rays to an accuracy of about one part in nine thousand.

KAONIC-X-RAY MEASUREMENTS

At the LBL Bevatron an external beam of 5.3-GeV protons produced negative kaons in a tungsten target. Kaons and pions that were projected in the forward direction were accepted by a symmetrical double-bend spectrometer tuned to momentum 450

GeV/c. Although the spectrometer included a mass separator to reject pions, the kaon beam that was focused on the potassium target contained about 1000 pions per stopped kaon. Kaons were identified by time of flight, and pion counts were vetoed by a Cherenkov counter. Furthermore, pions have a longer range than kaons and most did not stop in the target.

The arrangement of the apparatus is outlined by Fig. 1. The principal component was a single cryostat that contained four ultrapure Ge semiconductor detectors. Each detector was 3 cm in diameter by approximately 1.2 cm thick. Each had its own bias supply and amplifier system. Data were recorded by routing the outputs of each system to a separate section (1024 channels) of a pulse-height analyzer.

For the part of the experiment with kaons stopping in natural potassium, the target was pure metallic potassium (6 cm diameter by 1.75 g cm^{-2} thick) housed in an Al container. No kaonic x rays are expected from Al in the energy region of 57 keV.⁸ Three Er absorbers 5 cm by 5 cm were placed above the detectors as indicated in Fig. 2. The absorbers were uniform in thickness to better than ± 1 percent (two of 0.123 g/cm^2 and one of 0.237 g/cm^2 thickness) and were mounted on a rotatable wheel that was accurately positioned above the

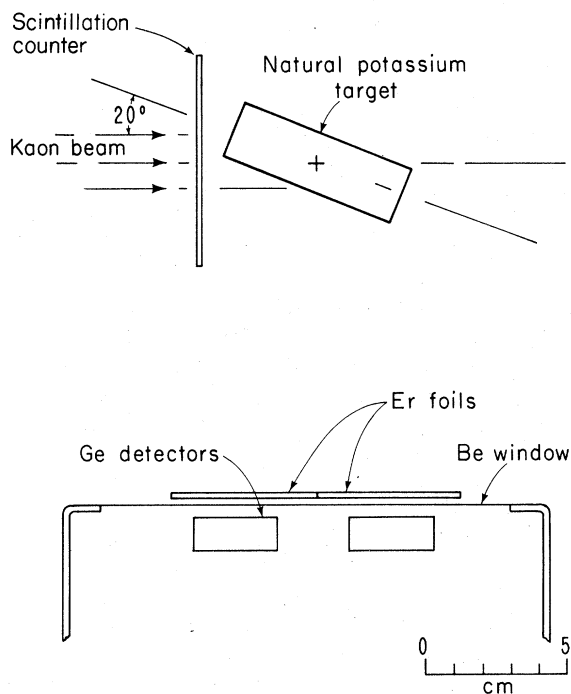


FIG. 1. Side view of the experimental arrangement. The center of the potassium target was above the center of a square at whose corners were positioned the four Ge detectors.

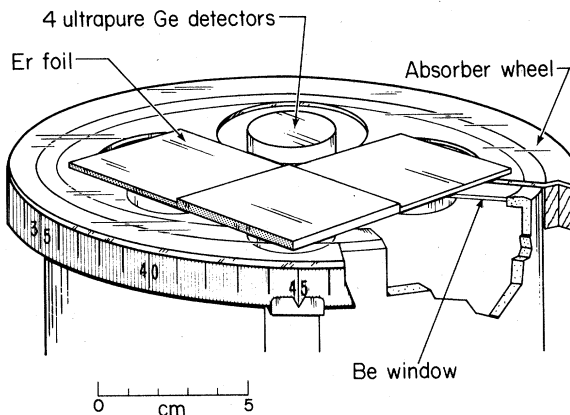


FIG. 2. Arrangement of the absorber wheel and detectors.

detectors. Tests were made to ensure that no radiation could reach the covered detectors directly from the target without passing through the absorbers. γ rays of energy 59.5 keV from ^{241}Am were used for the test by placing the source at positions corresponding to the periphery of the potassium target. A 2-g cm^{-2} -thick Pb absorber (transmission 1.8×10^{-4}) was then substituted for the Er foil of the detector under study. Plots of the 59.5-keV radiation received by the detector versus the wheel position were made for each detector and for several source positions at the location of the target. It was seen that there was a margin of safety of about ± 5 degrees in setting the wheel so that covered detectors were screened from the test source and therefore from the target.

The geometry of our source-absorber-detector arrangement was such that the detectors subtended almost half of the solid angle with respect to the absorbers as shown in Fig. 1. In this "poor" geometry the coherent scattering in the absorbers was not included in the absorption process because the elastically scattered x rays reached the detectors. Our measurements of the mass attenuation coefficient μ_m above and below the Er absorption edge agree with the values of McMaster *et al.*⁵ due to the photoelectric processes plus incoherent (Compton) scattering. We included only 90 percent of the incoherent cross section, because about 10 percent of the scattered photons lost less than 1 keV and were counted within the kaonic x-ray lines. Test measurements were made at 59.5 and 53.2 keV using γ -ray sources. The accuracy was sufficiently good in these cases to show that coherent scattering was excluded. For additional details see the thesis of Lum.⁶

In Fig. 3 we show the natural potassium spectra of one of the detectors: (a) no absorber, (b) Er, 0.124 g cm^{-2} . Our problem was to determine the

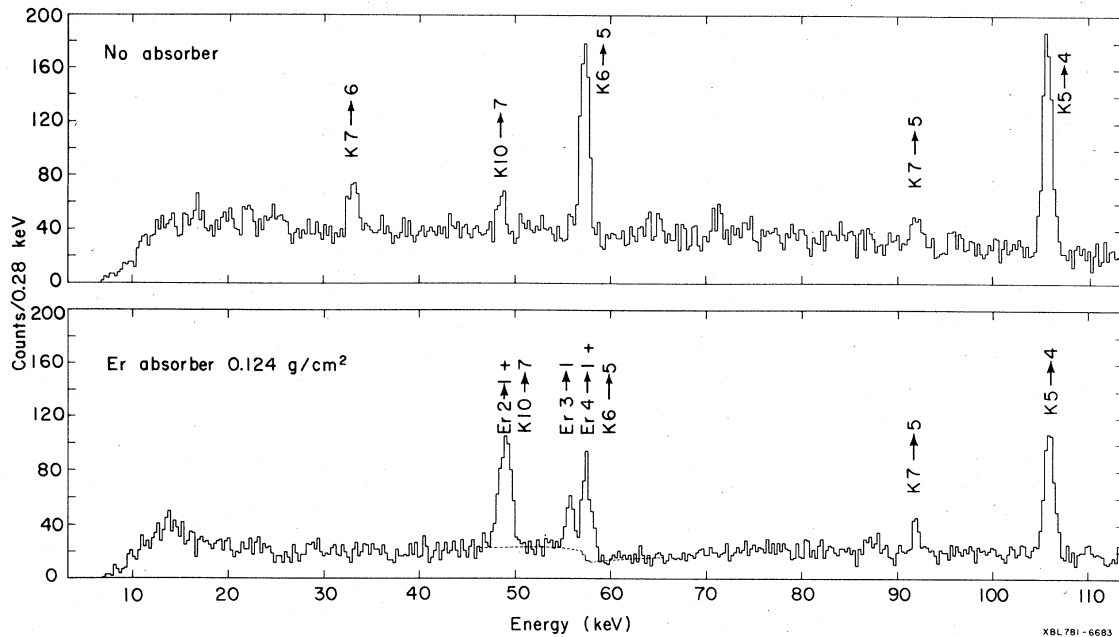


FIG. 3. Kaonic x-ray spectra of natural potassium with and without an erbium absorber. Each spectrum corresponds to about 1.4×10^6 stopped kaons.

number of x rays in the unattenuated and the attenuated $n=6 \rightarrow 5$ lines. The unattenuated lines posed no problem because there was no fluorescent background except for a small contribution from Er foils on adjacent detectors for which a 1 percent correction was made. Our detectors had a resolution of about 1 keV full width at half maximum (FWHM) and could not completely resolve all the Er fluorescent x rays from the kaonic x rays. Er x rays of 48.2 keV ($2p_{1/2} \rightarrow 1s_{1/2}$) and 49.1 keV ($2p_{3/2} \rightarrow 1s_{1/2}$) were blended into a broadened peak. The peak around 55.6 keV contained the $3p_{1/2}$, $3p_{3/2}$, and $3d_{3/2}$ to $1s_{1/2}$ transitions. X rays from $n=4 \rightarrow 1$ at 57.2 keV were unresolved from the attenuated kaonic-x-ray peak at 57.5. However, the relative intensities of the fluorescent lines are known⁷; $(2p_{3/2} \rightarrow 1s_{1/2}) : (2p_{1/2} \rightarrow 1s_{1/2}) : (3 \rightarrow 1) : (4 \rightarrow 1) = 100 : 53 : 32 : 7$. To determine the number of fluorescent x rays under the kaonic-x-ray peak, we counted the number of $n=2 \rightarrow 1$ and $n=3 \rightarrow 1$ x rays and used the above ratios. To confirm our technique we used ^{241}Am γ rays to generate the fluorescence peaks. We measured their intensity ratios and found $(3 \rightarrow 1)/(2 \rightarrow 1) = 0.199 \pm 0.003$ compared to the reference value $32/(100 + 53) = 0.209$ and $(4 \rightarrow 1)/(2 \rightarrow 1) = 0.047 \pm 0.002$ compared to $7/(100 + 53) = 0.046$.

In addition to fluorescent x rays, we took into account kaonic x rays from the transition $n=8 \rightarrow 6$ that were 425 eV lower in energy than the $n=6 \rightarrow 5$ line. The number of $n=8 \rightarrow 6$ x rays was estimated from the intensities of other $\Delta n=2$ transitions that

were measured in K, Cl, and Ar.⁸ The average intensity of $n=7 \rightarrow 5$ and $n=9 \rightarrow 7$ transitions in potassium and its neighboring elements amounts to 10 ± 2 percent of the $n=6 \rightarrow 5$ intensity. We assume that there is no anomaly in the cascade at $n=8 \rightarrow 6$ and ascribe to it the same intensity ratio as $n=7 \rightarrow 5$ and $n=9 \rightarrow 7$.

Background radiation that was incident on the detectors underwent a discontinuity at the absorption edge. The dotted line under the (Er $4 \rightarrow 1$) + (K $6 \rightarrow 5$) peak in Fig. 3 illustrates how this problem was treated. The width of the background edge was taken to be the same as the instrumental peak width. For additional details of the data analysis, see Ref. 6.

Two series of measurements were made using the natural potassium target. Each series consisted of four positions of the wheel so that each detector was covered in turn by three absorbers plus a blank. This procedure helped to cancel some possible systematic errors. For example, the centroid of the stopped-kaon distribution might not have been constantly in the center of the target.

The target was positioned above the center of the cluster of detectors. Thus the angles of incidence of the x rays at the absorbers were different from normal by about 15 degrees. Therefore, the effective thicknesses of the absorbers were not the same as the actual thicknesses. By studying the geometry and experimenting with ^{241}Am γ rays of 59.5 keV energy, we determined the effective

absorber thicknesses. They averaged 3.7 percent larger than the actual thicknesses.

Gains of the four amplifier systems remained practically constant during the experiment so that corresponding spectra of the two series could be added channel by channel. The result was 16 spectra: four independent detectors each with three absorbers plus a blank.

It was necessary to know the relative number of stopped kaons in each of the runs. Therefore, we needed to know the relative electronic efficiencies of the four detector systems. A conventional counter telescope arrangement was used to signal the passage of energy-degraded kaons through the thin beam-defining counter immediately in front of the target. One out of each 100 of these kaon stop signals was used to trigger a pulse generator whose output was fed into a detector preamplifier. The trigger pulse was switched in succession among four pulse generators, one for each detector. The energies of the pulses were set to be higher than those of the highest-energy x-ray line to be recorded. The number of counts found in a pulse-generator peak divided by the number of triggers gave the electronic efficiency. If a system were busy, its pulser would not be recorded. The average efficiency of the four detectors stayed reasonably constant at about 0.76 which was an indication that the kaon beam was sufficiently steady.

In the planning stage of the experiment we were not certain which of the potassium isotopes, 39 or 41, would give the more sensitive measurement. Calculations indicated that the x-ray energy of potassium 39 should lie below the inflection point of the μ_m vs E curve and the energy of potassium 41 would be above the inflection point. The energy nearest the inflection point would give the least error in energy. To increase the chance of coming close to the inflection point, we obtained a target of isotopically pure (99.2%) potassium 41. Only 30 g of the isotope were available in the form of KF. Although the F x-ray lines did not interfere directly with the potassium lines, the background was higher than that from the natural-potassium target, and fewer kaons could be stopped in potassium. Only Er foils of 0.123 g cm^{-2} were used with the KF target. It turned out that the x rays of natural potassium fell at a point on the transmission-versus-energy curve that was steeper, $\Delta(I/I_0) = 0.004$ per eV, than that of potassium 41, $\Delta(I/I_0) = 0.001$ per eV. Although the potassium-41 measurement did not contribute to the accuracy of the kaon mass, it lent credibility to the experiment.

We have thus far considered that kaonic x rays were emitted only by transitions $6h \rightarrow 5g$ (transi-

tions between the so-called circular orbits). Actually some of the kaons made transitions, $6g \rightarrow 5f$, etc. The energies of the "parallel" transitions were significantly different from the circular transitions: $6g \rightarrow 5f$ is 56 eV higher than $6h \rightarrow 5g$. The relative number of parallel transitions was obtained from a computer program that has an option of using either Ferrell's formula⁹ or the dipole perturbation theory for Auger processes. Radiative transition rates were computed by the dipole perturbation theory and the cascade was started at $n=30$. Various initial distributions over angular-momentum states were tried in order to get the best fit to the observed x-ray intensities.⁸ Cascade schedules that gave satisfactory intensities indicated that the ratio of the number of transitions $(6g \rightarrow 5f)/[(6h \rightarrow 5g) + (6g \rightarrow 5f)] = 0.09 \pm 0.03$. The cascade calculations showed that a negligible number of kaons made transitions from states of angular momenta lower than $l=4$.

An alternative approach to the determination of the number of noncircular transitions uses measured intensities. We have from experiment⁸ $(6g \rightarrow 4f)/[(6g \rightarrow 5f) + (6h \rightarrow 5g)] = 0.081 \pm 0.014$. Assuming that the kaonic potassium atoms were hydrogenic and that strong interaction effects were negligible for $n=6$ states near maximum angular momentum, we can use the ratio $(6g \rightarrow 5f)/(6g \rightarrow 4f) = 0.8029$ given by Bethe and Salpeter¹⁰ provided that we also assume negligible Auger transitions. The product of these ratios, 0.07 ± 0.01 , is the ratio of noncircular to circular transitions and is in good agreement with the number found by the cascade program. It will be seen later that the error in energy due to the error in the above ratio is small compared to the statistical error due to the x-ray counts.

The experimental results of the kaonic x-ray measurements are summarized in Table I where the transmissions I/I_0 of the Er absorbers are given as determined by the four Ge detectors.

ERBIUM-K-EDGE MEASUREMENT

Interpretation of the above-described attenuation measurements requires a detailed profile of the attenuation of the erbium foils on a precise energy scale around the K edge. The three erbium foils used in the absorption measurement of the x-rays emitted from kaonic atoms of natural potassium were transferred to NBS for measurement of the K absorption edge. The measurements were made using flat silicon crystals in conjunction with a two-axis spectrometer. Our procedure has two unique features. The lattice spacing of the crystals has been measured in terms of the wavelength of a visible laser closely related to R_∞ (Rydberg)

TABLE I. Results of attenuation of kaonic x rays emitted by natural potassium. η was the intrinsic detector efficiency including the solid angle. ϵ was the electronic counting efficiency ($1 - \epsilon = \text{dead time}$). Transmission of x rays out of the target was 0.636.

Detector	Absorber g/cm ²	K stops in units of 10 ⁶	η in units of 10 ⁻³	ϵ	$n=6 \rightarrow 5$	
					x rays counted per 10 ⁶ K stops	$I/I_0 \pm \Delta(I/I_0)$
C	blank	1.34	2.95	0.76	486 ± 35	
	0.123	1.20	2.95	0.71	299 ± 41	0.617 ± 0.100
	0.124	1.45	2.95	0.72	299 ± 35	0.617 ± 0.088
	0.237	0.76	2.95	0.83	148 ± 38	0.305 ± 0.083
D	blank	1.42	2.95	0.74	486 ± 35	
	0.124	1.16	2.95	0.73	232 ± 42	0.477 ± 0.096
	0.123	1.33	2.95	0.80	166 ± 31	0.341 ± 0.071
	0.237	1.31	2.95	0.80	93 ± 34	0.184 ± 0.067
E	blank	1.18	2.94	0.79	491 ± 37	
	0.124	1.35	2.94	0.82	261 ± 29	0.532 ± 0.076
	0.123	1.33	2.94	0.83	235 ± 29	0.479 ± 0.072
	0.237	1.45	2.94	0.86	177 ± 24	0.361 ± 0.057
F	blank	0.59	3.05	0.81	503 ± 49	
	0.124	1.05	3.05	0.84	282 ± 41	0.558 ± 0.104
	0.123	1.14	3.05	0.76	273 ± 40	0.539 ± 0.100
	0.237	0.93	3.05	0.78	136 ± 41	0.270 ± 0.088

by x-ray/optical interferometry. The double-axis spectrometer is equipped with a sensitive angle interferometer on each axis whose angle calibrations are established from first principles. More detailed descriptions of this work are given in Refs. 11 and 12.

The continuum radiation source was a 150-keV industrial tungsten x-ray tube with a current of 20 mA and a focal spot of 4×4 mm². Radiation in the neighborhood of the erbium K edge (57.5 keV) has approximately constant intensity as a function of energy. In addition to bremsstrahlung, the nearby tungsten $K\alpha_2$ line (57.98 keV), which is approximately Lorentzian in shape, contributes significantly.

The source, foil, and spectrometer arrangement is shown in Fig. 4. The foils were placed at the entrance to the source collimator, which has an angular acceptance of approximately 20 min. Foil thicknesses were 0.1238, 0.237, and 0.3608 g/cm², the last being obtained by using the first two foils together. Three different types of measurements are needed to determine the absorption profile. The nondispersive (parallel) position of the two crystals must be located to set the zero of the angle scale. The continuum intensity as a function of energy in the dispersive (antiparallel) position must then be measured with and without the absorber present.

The crystals were cut for symmetric Laue-case (transmission) diffraction from the (111) planes. The instrumental width, determined from the par-

allel-position linewidth of 1.96 arc sec, corresponded to 8 eV; this contributed noticeable broadening of the erbium-K-edge profile.

The intercrystal collimator had a vertical slit height of 8.5 mm. This slit height along with the 4-mm focal-spot height and the separation between the focal spot and the intercrystal collimator (1.51 m) limited the vertical divergence of the continuum radiation. The detector for most of the measurements was a 12-mm-thick NaI(Tl) crystal. However, an ultrapure Ge detector was also used for a limited number of measurements to explore geometric influences on the measured intensity transmitted by the erbium foils.

The measurement sequence for each foil was a parallel-crystal-position measurement, and then antiparallel measurements of the incident intensity (no absorber in the beam) and the transmitted intensity through the erbium absorber. In order to ensure that there were no serious drifts, this sequence was repeated several times for each foil.

A parallel-position measurement took approximately 10 min to record and resulted in a Lorentzian profile corresponding to a dispersive FWHM of 8 eV and a peak counting rate of 30 000/sec. The continuum curve was recorded in 35 min and resulted in a nearly constant counting rate of 500/sec at each energy recorded. The points were fit with a straight line to obtain a mathematical representation of the continuum radiation. Finally, the intensity transmitted by the foil was recorded in about 2 h, resulting in a

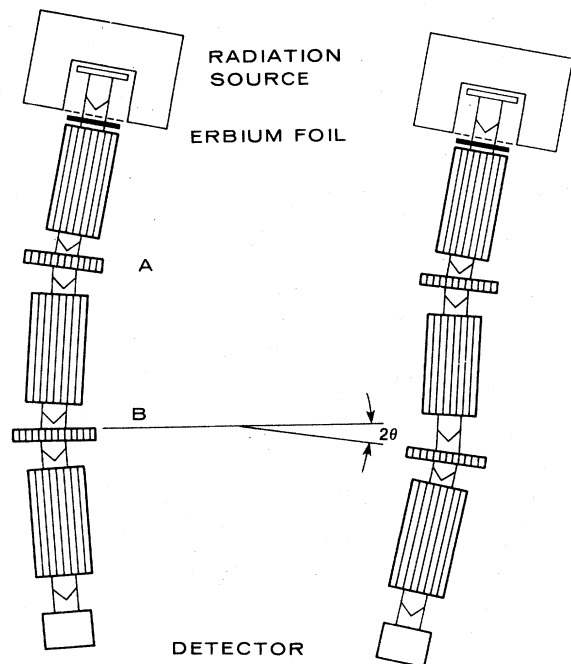


FIG. 4. Experimental arrangement for erbium K -edge measurement. The Er foils were placed in front of the source collimator. The angle of the first crystal A was set for Bragg diffraction. The second crystal B was alternately set parallel and antiparallel to the first crystal. The angle through which the second crystal was rotated is 2θ , where θ is the Bragg angle.

curve of transmitted intensity versus energy.

Absorption measurements are quite sensitive to the background intensity, which is here taken as independent of spectrometer setting. When radiation was blocked from entering the source collimator, a room background of <0.1 counts/sec was measured. By using three different foil thicknesses and requiring that the measured absorption curves be in agreement, a background component associated with stray radiation in the instrument was also determined. The transmitted-intensity curve is most sensitive to the background in the high-absorption side of the edge. Also, the assumed shape of the curve has little influence if points that are relatively far from the edge are used. For each measurement of the three foil thicknesses a background was determined and was found to be approximately 0.7 counts/sec.

In order to establish a precise energy scale for the K -edge profile, the angle interferometers were calibrated by using them to measure the external angles of a 24-sided optical polygon. The calibration constant follows from the constraint that the sum of the angles must equal 360° and leads in this case to a value with an uncertainty

<0.5 ppm. Because this measurement was made on a newly constructed spectrometer, a few measurements of the well established $W K\alpha_1$ x ray¹³ were made as a consistency test. Within statistical limits agreement with previous work was obtained.

Linearity in intensity measurements was checked by measurements on Au at the energy of the erbium edge which is well removed from any absorption edge in Au. Good agreement with the tabulated values from McMaster *et al.*⁵ was obtained.

The transmitted-intensity curves were computer fit with the customary exponential-arctangent function convoluted with a Lorentzian instrument window. The mathematical model used is given by

$$I - b = (I_0 - b)e^{-\mu_m t} \frac{1}{1 + [E/(\omega/2)]^2}, \quad (1)$$

where

I = recorded intensity,
 I_0 = continuum intensity,
 b = background,
 t = foil thickness (g/cm^2),
 ω = FWHM instrument function,
 E = energy,

$$\mu_m = B_1 + B_2 \left[\frac{1}{2} - \frac{1}{\pi} \tan^{-1} \left(\frac{\bar{E} - E}{W/2} \right) \right]$$

= attenuation coefficient (cm^2/g),

B_1 = low-energy asymptotic absorption value,
 B_2 = magnitude of the jump in absorption value across the edge,
 W = width of the edge,
 \bar{E} = energy of the edge.

The smooth behavior of the mass-attenuation coefficient at the K edge following a simple arctangent function was found to be quite general for the rare-earth elements.¹⁴ The above convolution is not readily integrated in closed form, so that a numerical integration was performed. The parameters B_1 , B_2 , W , and \bar{E} were obtained from a least-squares fit to the data.

However, some of the scans show evidence of a small deviation from the above arctangent model in the energy region 25 eV lower than the edge position. The data suggest that the absorption coefficient is slightly increased in this region. A possible explanation for this phenomenon is absorption which causes an electron transition from the $1s$ level to the $5p$ levels. This transition occurs approximately 24 eV below the edge position¹⁵ and has a width of 28.4 eV.¹⁶ To mathematically model this explanation, a Lorentzian component whose position was fixed at 24 eV below the edge position, whose width was 28.4 eV, and whose in-

tensity was variable, was added to the above model for μ_m . The transmitted intensity scans with the best statistics were fit with this model and an average value for the intensity of the added Lorentzian was determined to be $0.181 \text{ cm}^2/\text{g}$. All transmitted intensity scans were then refit with μ_m assumed to be the sum of the arctangent form above and a small Lorentzian correction with the parameters given above.

Four complete sets of measurements were recorded over a period of three weeks. Each set included measurements on all three foil thicknesses. For the last two sets the counting time was doubled to improve the statistics. Values for the coefficients B_1 , B_2 , W , and \bar{E} were determined for each of the four data sets. The four curves for μ_m were then weighted according to the observation time and again least-squares fitted to obtain a final estimate for the parameters. The absorption-curve model and the measured parameters are

$$\mu_m = B_1 + B_2 \left[\frac{1}{2} - \frac{1}{\pi} \tan^{-1} \left(\frac{\bar{E} - E}{W/2} \right) \right] + \frac{0.181 \text{ cm}^2/\text{g}}{1 + \left(\frac{E - 24 - \bar{E}}{28.4/2} \right)^2}, \quad (2)$$

where

$$B_1 = 3.138 \text{ cm}^2/\text{g}, \quad B_2 = 10.883 \text{ cm}^2/\text{g}, \\ W = 28.23 \text{ eV}, \quad \bar{E} = 57485.6 \text{ eV}.$$

A plot of this curve is shown in Fig. 5.

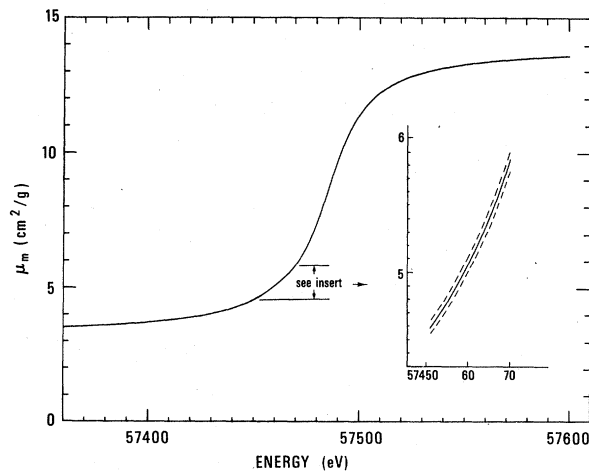


FIG. 5. Mass-attenuation coefficient μ_m in cm^2/g versus energy for Er. The absorption coefficient includes photoelectric effect, coherent scattering, and incoherent scattering. The insert shows the one σ uncertainty in the region of interest.

ERBIUM-K-EDGE PROFILE ANALYSIS

Use of the above mass-attenuation curve for estimating the energy of the kaonic x rays from potassium requires modification due to experimental geometry and detector resolution. In the absorption-edge measurements, the foils were positioned in front of the source collimator and thus yielded "good geometry" values. Photons which were photoelectrically absorbed, coherently or incoherently scattered by the foil or produced by fluorescence were not seen by the detector.

As described above, the geometry which was used in the kaonic x-ray experiment permitted photons which were coherently and incoherently scattered to enter the detector. However, only 10 percent of the incoherently scattered photons were counted because they lost less than 1 keV and fell within the detector window. In order to correct for these geometrical considerations, the coherent-scattering component and 0.1 of the incoherent-scattering component given by McMaster *et al.*⁵ were subtracted from B_1 , the low-energy asymptotic absorption value.

As a partial check on the effect of geometry and detector, a limited number of measurements on the erbium absorption curve were taken using an ultrapure Ge detector of resolution $\sim 0.7 \text{ keV}$ in place of the NaI(Tl) detector. The transmitted intensity was recorded with the foils in front of the detector. By comparing values for μ_m obtained with the foil in the two locations agreement with the coherent-elastic-scattering coefficient given by McMaster *et al.*⁵ was found.

ENERGY OF KAONIC X-RAYS

If the target had been isotopically pure and if there had been only one x-ray energy, we could have taken the weighted average of the absorption coefficients given by $\mu_m = (-1/t) \ln(I/I_0)$, applied directly to the μ_m -versus-energy curve and found the energy of the x rays. To include the effects of the noncircular transitions and isotopic composition of the target, it was convenient to express I/I_0 as a function of E and t through $\mu_m = f(E)$, Eq. (2),

$$I/I_0 = A_1 \{ C_1 \exp[-f(E)t] + C_2 \exp[-f(E + \Delta E_1)t] \} \\ + A_2 \{ C_1 \exp[-f(E + \Delta E_2)t] \\ + C_2 \exp[-f(E + \Delta E_1 + \Delta E_2)t] \}, \quad (3)$$

where E is the energy of the $6h \rightarrow 5g$ transition in pure potassium 39, t is the effective absorber thickness, and the constants are

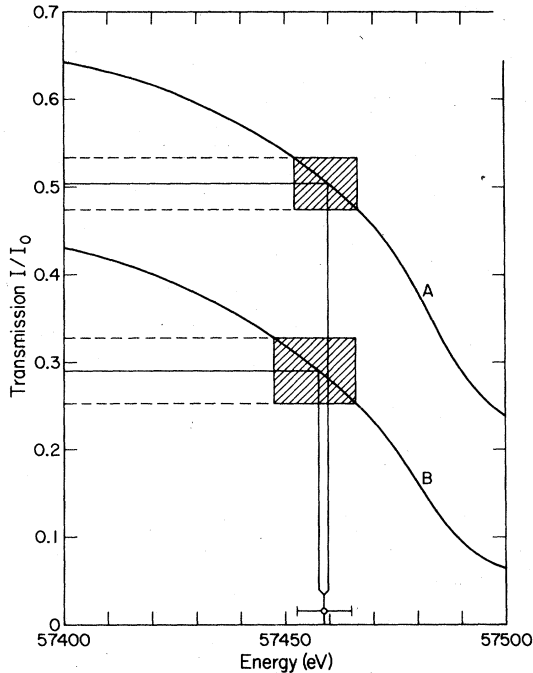


FIG. 6. Transmission of Er versus energy including the corrections for difference in energy due to noncircular transitions and isotopic abundance. The measured transmissions for potassium kaonic x rays, including uncertainties, are indicated by the cross-hatched areas which also indicate the corresponding energies. Curve A is for the average effective thickness of the 0.123 and 0.124 g/cm² absorbers and curve B is for the 0.237 g/cm² absorbers.

$$\begin{aligned} \Delta E_1 &= 56 \text{ eV, energy difference between } 6g \rightarrow 5f \\ &\text{and } 6h \rightarrow 5g \text{ transitions,} \\ \Delta E_2 &= 37 \text{ eV, energy increase due to additional} \\ &\text{mass of potassium 41 over potassium 39,} \\ A_1 &= 0.933, \text{ abundance of potassium 39,} \\ A_2 &= 0.067, \text{ abundance of potassium 41,} \\ C_1 &= 0.91 \pm 0.03, (6h \rightarrow 5g)/[(6h \rightarrow 5g) + (6g \rightarrow 5f)], \\ C_2 &= 0.09 \pm 0.03, (6g \rightarrow 5f)/[(6h \rightarrow 5g) + (6g \rightarrow 5f)]. \end{aligned}$$

Figure 6 is a plot of I/I_0 vs E for two Er absorber thicknesses; 0.1284 and 0.2458 g cm⁻² (average of the effective thicknesses of the two sets of foils). The cross-hatched regions indicate the weighted averages of the measurements and how they are related to the energy. Values for each measurement and absorber thickness given in Table I were combined to give 57 458.8 eV. The main source of error was due to kaonic x-ray counting statistics. Contributions to the errors in energy are as follows.

$$\text{x-ray counting statistics in } I/I_0 \quad 5.7 \text{ eV}$$

Number of noncircular transitions	2.3
Average effective absorber thickness including nonuniformity	
$t_{\text{eff}} = (1.037 \pm 0.01)t_0$	0.7
Fraction of incoherently scattered x rays counted	0.2
μ_m in Eq. (2)	0.9

Finally, we found that the experimentally determined energy of the $6h \rightarrow 5g$ transition in kaonic potassium 39 was 57 458.8 \pm 6.3 eV. Note that the natural linewidth was calculated to be 0.4 eV.

KAON MASS

To arrive at the kaon mass corresponding to the x-ray energy that we have measured, it is convenient to calculate the energy on the basis of the current value of the kaon mass given by the Particle Data Group¹⁷ (PDG), $m_K = 493.669 \pm 0.018$ MeV/ c^2 . From Atomic Data and Nuclear Data Tables,¹⁸ the atomic mass of potassium 39 is 38.967 amu which amounts to 36 297.823 MeV/ c^2 (1 amu = 931.5016 MeV/ c^2). Subtracting the mass of 19 electrons, we find the nuclear mass 36 288.114 MeV/ c^2 and the reduced mass 487.043 MeV/ c^2 . The $6h \rightarrow 5g$ energy was then calculated in the following way: We first obtained the Coulomb and Uehling (first order, $\alpha Z\alpha$) vacuum-polarization potentials¹⁹ modified by the finite distribution of nuclear charge (assumed to be the two-parameter Fermi distribution) as well as the phenomenological pion-nucleus strong-interaction potential.²⁰ A modified Klein-Gordon equation²¹ was then integrated numerically. Higher-order radiative corrections were computed by perturbation theory in the $\alpha^2(Z\alpha)$ (Ref. 19) and $\alpha(Z\alpha)^{3,5,7,\dots}$ (Ref. 22) orders as well as the finite-charge-distribution effect in the $\alpha(Z\alpha)^3$ order.²³ The electron-screening correction was calculated in perturbation theory using the empirical three-parameter potential²⁴ from the electron density obtained by the relativistic Hartree-Fock-Slater method. Also included were estimates of the dipole-nuclear-polarization²⁵ and nuclear-center-of-mass-motion effects.²⁶ We show the results in Table II. The total energy of the $6h \rightarrow 5g$ transition came out to be 57 462.1 eV ($\equiv E_0$) with a possible theoretical uncertainty of ± 0.5 eV.

Strictly speaking, all corrections to the point-Coulomb Klein-Gordon energy are not proportional to the reduced mass of the kaon μ_K . However, the largest correction, the Uehling term in the atomic states of our interest, contributes only a few tenths of a percent and was found numerically to be roughly proportional to $\mu_K^{3/2}$. Other corrections give smaller contributions by at least two

TABLE II. Calculated theoretical transition energies.

Transition Group	Pb $n=9 \rightarrow 8$		Pb $n=13 \rightarrow 12$		K $n=6 \rightarrow 5$
	Col-Yale	LBL-NBS	Col-Yale	LBL-NBS	LBL-NBS
Point Coulomb (keV)	290.081	290.055	90.697	90.689	57.2453
$\alpha(Z\alpha)$ (eV)	1569.5	1569.4	284.0	283.9	215.2
$\alpha(Z\alpha)$ iteration (eV)	3.9	13.7 ^c	0.5	1.2 ^c	0.6 ^c
$\alpha(Z\alpha)$ finite size (eV)	7.1		0.4		
$\alpha^2(Z\alpha)$ (eV)	10.9	10.8	2.0	2.0	1.5
$\alpha(Z\alpha)^3$ (eV)	-29.8	-29.6	-7.7	-8.0	-0.3
$\alpha(Z\alpha)^5$ (eV)	-4.3	-3.6	-1.1	-1.0	0.0
$\alpha(Z\alpha)^7$ (eV)		-0.7		-0.2	0.0
$\alpha(Z\alpha)^3$ finite size (eV)	3.6	4.0	0.5	0.7	0.0
Electron screening (eV)	-25.9	-27.6	-51.8	-59.6	-0.6
Nuclear polarization (eV)	5	5	0	0	0.1
Nuclear motion (eV)		1.8		0.3	0.3
Calculated total (keV)	291.625 ± 0.005^a	291.598 ± 0.005^b	90.924 ± 0.005^a	90.909 ± 0.005^b	57.4621 ± 0.0005^b

^aKaon mass used: 493.715 MeV/ c^2 .

^bKaon mass used: 493.669 MeV/ c^2 .

^cThe Klein-Gordon equation was numerically solved for the $\alpha(Z\alpha)$ -order potential corrected for the finite nuclear charge distribution.

orders of magnitude. Furthermore, the measured energy E_1 differs from the calculated energy E_0 by only 56 ppm. Because of these reasons we can safely obtain the kaon mass m'_K corresponding to E_1 by assuming that the reduced mass is proportional to the energy. Our value of the kaon mass is given by

$$m'_K = (\mu E_1 / E_0) / (1 - \mu E_1 / E_0 M),$$

where $\mu = 487.043$ MeV/ c^2 , reduced mass for $m_K = 493.669$ MeV/ c^2 ,

$E_1 = 57458.8 \pm 6.3$ eV, measured energy,

$E_0 = 57462.1 \pm 0.5$ eV, calculated,

$M = 36288.114$ MeV/ c^2 , nuclear mass.

We find for the negative-kaon mass 493.640 ± 0.054 MeV/ c^2 .

DISCUSSION

At the time our experiment was planned, there was some disagreement among theorists concerning the calculation of the $\alpha^2(Z\alpha)^2$ radiative correction to the energy levels of mesonic atoms. Wilets and Rinker²⁷ calculated that the correction should amount to < 3 eV for the transition $5g \rightarrow 4f$ in the muonic Pb atom, while Chen²⁸ calculated the correction for the same transition to be -35 eV. Two other independent calculations²⁹ were subsequently reported and their results agree with the Wilets and Rinker value.

The PDG value of the K^- mass is based mainly on the work of the Columbia-Yale group.¹ They found $m_K = 493.657 \pm 0.02$ MeV/ c^2 if the correction

of Chen were excluded and also reported that, if Chen's correction were taken into account, the mass would be raised to 493.696 ± 0.025 . Apparently the Columbia-Yale group had more confidence in the results without Chen's corrections because they used 493.657 in the abstract of their article and this is their mass as reported by PDG. Another accurate measurement was reported by a CERN group,² 493.691 ± 0.04 which carries less weight than the Columbia-Yale result. Both the Columbia-Yale and CERN groups studied kaonic x rays of high- Z elements.

In Table II we have listed the corrections that contribute to the transition energies of potassium 39. Also listed are the lowest and highest energies of the kaonic Pb lines used and reported by the Columbia-Yale group in Ref. 1. It is seen that the high-order $Z\alpha$ corrections and screening are less important for $Z=19$ than for $Z=82$. For Pb the $\alpha(Z\alpha)^3$ correction amounted to about 100 ppm, which is significant compared to the accuracy of the kaon mass (37 ppm), whereas for potassium the same correction amounted to 6 ppm. Electron screening corrections were especially significant for the high- n transitions in Pb (about 600 ppm for $n=13 \rightarrow 12$ and about 250 ppm for $n=11 \rightarrow 10$), whereas for potassium the correction was about 11 ppm.

Obviously much care is required in the estimate of the corrections in heavy atoms such as Pb. For example, if the Columbia-Yale group had included the electron shielding effect due to electrons other than 1s electrons, the contribution from the effect

would have been larger by about 10 percent. This change decreases the transition energies by about 1.7 eV for $n=9 \rightarrow 8$ and about 7.8 eV for $n=13 \rightarrow 12$. This change in the electron shielding effect would have increased their kaon mass. Though the precise amount of the increase is difficult for us to determine because of other errors involved in the final uncertainty of the kaon mass, it would certainly be well within the quoted error of 0.02 MeV/ c^2 .

We used a measurement technique and an energy standard different from the Columbia-Yale and CERN groups and found that the results are in good agreement. Since both high- Z and low- Z kaonic atoms were used, we conclude that there is no discernible discrepancy due to calculations of the high-order radiative terms. Although our work may have helped to dispel some possible lingering doubts about the high-order radiative corrections in exotic atoms, it tends to intensify wonderment over the results of the Columbia-Yale group's measurement of the antiproton mass.³⁰ The \bar{p} -mass experiment employed the same apparatus and energy standards as used in the kaon-

mass measurement, but the value obtained was about 1.7 standard deviations from the proton mass. Our comment on the electron-shielding effect is in the direction to reduce the difference but does not resolve the situation. For the remainder of the discrepancy, statistical fluctuation could be responsible as suggested in Ref. 30 as a possibility.

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