Higher-order corrections in the cut-vertex formalism

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An algorithm is given for the calculation of higher-order corrections to the cut vertices and their coefficient functions of deep-inelastic and annihilation processes in the model field theory $(\Phi^3)_6$. The introduction of covariant phase-space variables renders the calculation simple and elucidates the relations between the deep-inelastic and annihilation structure functions.

I. INTRODUCTION

In the past few years, perturbative field theories were used to describe deep-inelastic lepton-hadron and totally inclusive electron-positron annihilation processes. Recently, an extension of the light-cone expansion method, the so-called cut-vertex formalism,[1] extended the applicability of these theories to other hard-scattering processes. Therefore, we think that it is worth trying to develop simple techniques for the calculation of cut vertices and the relevant coefficient functions. In this paper we present such a technique.

In Sec. II we present our renormalization techniques. In Sec. III we define phase-space variables, which are simply related to light-cone variables. They are useful in the study of various relations between the spacelike and timelike regions, while at the same time they simplify the calculation. In Sec. IV we give characteristic examples for the calculation of spacelike and timelike cut vertices as well as for their coefficient functions. In Sec. V we present the results of the cut vertices up to the order g^4 and the coefficient functions up to the order g^2. We also discuss the relations between the deep-inelastic and annihilation structure functions. In this paper we work in $(\Phi^3)_6$ theory because of the formal simplicity of that theory; the extension to quantum chromodynamics is postponed to a future paper.

A summary of the results of this paper appeared in Ref. 2. In the meantime a similar work had been done by Kubota.[3] In the later reference the coefficient functions up to order g^2 and the anomalous dimensions of the spacelike and timelike cut vertices up to order g^4 have been calculated in $(\Phi^3)_6$ theory. The calculation is carried out in the Mellin moment space and in the minimal-subtraction renormalization scheme, while ours is made in the momentum-subtraction scheme and in the inverse Mellin transform space. We agree as to the structure functions and the dimensionally regularized form of the cut vertices in order g^4 as they are presented in Ref. 3.

II. RENORMALIZATION ALGORITHM FOR THE CUT VERTICES

In this section we develop the conventions and notations for the renormalization algorithm which is applied to multiplicatively renormalizable Feynman amplitudes in a massless field theory. We will use the dimensional regularization going to $n=6-2\epsilon$ dimensions, and the renormalization conditions will be taken at a fixed point $4\pi\mu^2$, where μ^2 is the scale appearing in the dimensional regularization.

The prototype theory that we will use is $(\Phi^3)_6$ theory which is defined by the bare Lagrangian[4]

$$\mathcal{L}_0=\tfrac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi)+(g_0/3!)\Phi^3 \tag{2.1}$$

with the following renormalization conditions for the two- and three-point one-particle-irreducible (1PI) Green's functions:

$$\Gamma^{(2)}(p^2)\big|_{p^2=0}=0\,, \tag{2.2a}$$

$$\frac{d}{dp^2}\Gamma^{(2)}(p^2)\bigg|_{p^2=-4\pi\mu^2}=-i\,, \tag{2.2b}$$

$$\Gamma^{(3)}(p^2,\ k^2,(p-k)^2)\big|_{p^2=k^2=k\cdot p=-4\pi\mu^2}=-ig\,. \tag{2.2c}$$

g is the renormalized coupling constant. We also define

$$\alpha_0=g_0{}^2/(4\pi)^{n/2}\,,\quad \alpha=g^2/(4\pi)^3\,. \tag{2.3}$$

Renormalization conditions (2.2a)–(2.2c) determine the anomalous dimension of the field Φ and the β function,

$$\alpha\beta(\alpha,\epsilon)=\mu^2\frac{\partial}{\partial\mu^2}\alpha\bigg|_{\epsilon,\,\alpha_0\text{fixed}}. \tag{2.4}$$

As shown by Mueller, in order to analyze the deep-inelastic processes one has to introduce the spacelike and timelike cut vertices (SLCV, TLCV). The SLCV are closely related to the minimal-twist Wilson operators, and the TLCV can be formally considered as an "analytic continuation" of the SLCV. These quantities are multiplicatively renormalizable and we give their definition in what follows.

(i) The amputated minimal-twist Wilson operators (WO) [Fig. 1(a)]:

$$\Gamma_{O_\sigma}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)=\left\langle 0\left|\tilde\phi(p):\phi(0)\left(\frac{-i\Delta\cdot\partial}{\Delta\cdot p}\right)^\sigma\phi(0):\tilde\phi(p)\right|0\right\rangle. \tag{2.5a}$$

(ii) The spacelike cut vertex (SLCV) [Fig. 1(b)]:

$$A^\sigma_{\rm SL}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)=\int\frac{d^6k}{(2\pi)^6}\left(\frac{\Delta\cdot k}{\Delta\cdot p}\right)^\sigma\theta(\Delta\cdot k)T_{\rm SL}(p,k). \tag{2.5b}$$

(iii) The timelike cut vertex (TLCV) [Fig. 1(c)]:

$$A^\sigma_{\rm TL}\left(\frac{p^2}{4\pi\mu^2}\alpha\right)=\int\frac{d^6k}{(2\pi)^6}\left(\frac{\Delta\cdot k}{\Delta\cdot p}\right)^{-\sigma}T_{\rm TL}(p,k). \tag{2.5c}$$

In the former equations Δ is a lightlike vector

$$\Delta^2=0,\quad \Delta\cdot p>0. \tag{2.6}$$

p^2 is spacelike for WO and SLCV and timelike for TLCV; σ is any even integer for WO, and any complex number with $\mathrm{Re}\sigma>0$ for SLCV and TLCV,

$$T_{\rm SL,\,TL}(p,k)=\sum_X\langle 0|\overline{T}(\tilde\phi(\pm p)\tilde\phi(\mp k))|X\rangle\langle X|T(\tilde\phi(\pm k)\tilde\phi(\mp p))|0\rangle\,|G^{(2)}(k^2+i\epsilon)|^2, \tag{2.7}$$

where $G^{(2)}$ is the renormalized propagator, and T and $\overline{T}$ are the symbols for the time- and antitime-ordered products. Note that the formulas (2.5b), (2.5c), and (2.7) suggest a parton-model interpretation of the cut-vertex formalism.[5]

We choose the following renormalization conditions for Γ_{O_σ}, $A^\sigma_{\rm SL}$, and $A^\sigma_{\rm TL}$:

$$\Gamma_{O_\sigma}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)\bigg|_{p^2=-4\pi\mu^2}=1, \tag{2.8a}$$

$$A^\sigma_{\rm SL}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)\bigg|_{p^2=-4\pi\mu^2}=1, \tag{2.8b}$$

$$A^\sigma_{\rm TL}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)\bigg|_{p^2=-4\pi\mu^2}=1. \tag{2.8c}$$

In the case where O_σ is a conserved current, the renormalization conditions (2.2a), (2.2b), and (2.8a), valid already in the free-field theory, do not spoil the corresponding Ward identity. [In $(\Phi^3)_6$ theory, there is only one conserved current, the energy-momentum tensor O_2.] This point is essential for the parton-model interpretation of the cut-vertex formalism.

Being multiplicatively renormalizable, the WO, SLCV, and TLCV satisfy renormalization-group equations (RGE). In all that follows we will denote by $A(p^2/4\pi\mu^2,\alpha)$ any of these three quantities. One has

$$\left[\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha)\alpha\frac{\partial}{\partial\alpha}+\gamma_A-2\gamma_\Phi\right]A=0, \tag{2.9}$$

where γ_A is the anomalous dimension of A and γ_Φ the anomalous dimension of the field Φ. Using Eq. (2.9) (RGE), one greatly simplifies the computation of A. Indeed, since we work in a massless theory, and since A is a function of only one momentum, it is obvious by power counting that up to the Nth order in α we have

$$A\left(\frac{p^2}{4\pi\mu^2},\alpha\right)=\sum_{m=0}^{n}a_m(\alpha)\ln^m\frac{|p^2|}{4\pi\mu^2} \tag{2.10}$$

with

$$a_m(\alpha)=\sum_{k=m}^{n}\alpha^k c_{m,k}. \tag{2.11}$$

Therefore, by applying the RGE (2.9) to the expression (2.10) for A, one gets a recursion relation between the a_m's which can be expressed as

$$a_m(\alpha)=\frac{1}{m!}\left[\beta(\alpha)\alpha\frac{\partial}{\partial\alpha}+\gamma_A(\alpha)-2\gamma_\Phi(\alpha)\right]^m a_0(\alpha). \tag{2.12}$$

The relation (2.12) shows that, given $\beta(\alpha)$ and $\gamma_\Phi(\alpha)$, A is completely determined by its renormalization condition $a_0(\alpha)\equiv A(1,\alpha)$ and its anomalous dimension $\gamma_A(\alpha)$, or alternatively by $a_0(\alpha)$ and the simple logarithmic coefficient $a_1(\alpha)$, in the

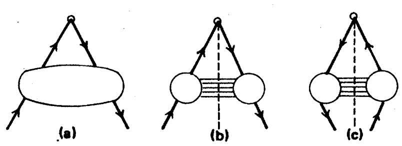


FIG. 1. (a) Graph for the matrix element of the minimal-twist Wilson operators; $p^2<0$, $\Delta^2=0$. (b) Spacelike cut vertices; $p^2<0$, $\Delta^2=0$, $\Delta\cdot p>0$. (c) Timelike cut vertices; $p^2>0$, $\Delta^2=0$, $\Delta\cdot p<0$.

expansion (2.10). If $a_0(\alpha)\equiv 1$ as in Eqs. (2.8a), (2.8b), and (2.9c) one has simply that

$$\gamma_A - 2\gamma_\phi = a_1(\alpha). \tag{2.13}$$

We now turn to the computation technique itself, which will be illustrated up to two loops. First, we must define the wave-function and coupling normalization constants Z_ϕ and Z_α,

$$\phi_0 = Z_\phi{}^{1/2}(\alpha,\epsilon)\phi\,, \tag{2.14}$$

$$\alpha_0 = (4\pi\mu^2)^\epsilon Z_\alpha(\alpha,\epsilon)\alpha\,. \tag{2.15}$$

Z_ϕ and Z_α can be computed in perturbation theory from the following conditions:

$$\Gamma^{(2)}(p^2, 4\pi\mu^2,\alpha) = \lim_{\epsilon\to 0} Z_\phi(\alpha,\epsilon)\Gamma_0^{(2)}(p^2,\alpha_0,\epsilon)\,, \tag{2.16a}$$

$$\Gamma^{(3)}(p,q,4\pi\mu^2,\alpha) = \lim_{\epsilon\to 0} Z_\phi{}^{3/2}(\alpha,\epsilon)\Gamma_0^{(3)}(p,q,\alpha_0,\epsilon)\,. \tag{2.16b}$$

Equation (2.16b) is equivalent to

$$\Gamma^{(3)}(p,q,4\pi\mu^2,\alpha)/-ig = \lim_{\epsilon\to 0} Z_\Gamma(\alpha,\epsilon)\Gamma_0^{(3)}(p,q,\alpha_0,\epsilon)/-ig_0 \tag{2.16c}$$

with

$$Z_\Gamma(\alpha,\epsilon) \equiv Z_\phi{}^{3/2}(\alpha,\epsilon) Z_\alpha{}^{1/2}(\alpha,\epsilon)\,. \tag{2.16d}$$

The subscript zero denotes the bare quantities. In any finite order in α, say N, Z_ϕ and Z_α can be written in any renormalization scheme as

$$Z_{\phi(\alpha)}(\alpha,\epsilon) = \sum_{n=0}^{N} \frac{\alpha^n}{\epsilon^n} Z_{\phi(\alpha)}^{(n)}(\epsilon)\,, \quad Z_{\phi(\alpha)}^{(0)}(\epsilon)=1\,. \tag{2.17}$$

$Z_{\phi(\alpha)}^{(n)}$ is a polynomial in ϵ of a maximal degree n. On the other hand, the renormalizability of A implies the relation

$$A\left(\frac{P^2}{4\pi\mu^2},\alpha\right) = \lim_{\epsilon\to 0} Z_A(\alpha,\epsilon) A_0(P^2,\alpha_0,\epsilon)\,. \tag{2.18}$$

In this equation the renormalization constant Z_A has a similar expansion to that of Z_ϕ and Z_α,

$$Z_\alpha(\alpha,\epsilon) = \sum_{n=0}^{N} \frac{\alpha^n}{\epsilon^n} Z_A^{(n)}(\epsilon)\,, \quad Z_A^{(0)}(\epsilon)=1\,. \tag{2.17$'$}$$

We observe, using dimensional analysis, that the general form of the bare amplitude $A_0(P^2,\alpha_0,\epsilon)$ is

$$A_0(P^2,\alpha_0,\epsilon) = 1 + \sum_{k=1}^{N} \frac{\alpha_0{}^k}{\epsilon^k}(P^2)^{-k\epsilon} A^{(k)}(\epsilon)\,. \tag{2.19}$$

In Eq. (2.19), the $A^{(k)}(\epsilon)$ are analytic functions of ϵ. (We will take the convention $P^2=-p^2$ for the WO and $P^2=|p^2|$ for the SLCV and TLCV.) Then we combine Eqs. (2.19), (2.15), (2.17), and (2.17′) to get

$$A(t,\alpha) = \lim_{\epsilon\to 0}\left[1 + \sum_{n=1}^{N} \frac{\alpha^n}{\epsilon^n}\left(Z_A^{(n)}(\epsilon) + \sum_{k=0}^{n-1} e^{\epsilon(n-k)t} A^{(n-k)}(\epsilon) Z_{n-k}^{(k)}(\epsilon)\right)\right], \tag{2.20}$$

where we have defined $t=\ln(4\pi\mu^2/P^2)$ and the polynomial in ϵ, $Z_{n-k}^{(k)}(\epsilon)$, as

$$Z_A(\alpha,\epsilon)[Z_\alpha(\alpha,\epsilon)]^m = \sum_{k=0}^{N} \frac{\alpha^k}{\epsilon^k} Z_m^{(k)}(\epsilon), m=0,1,2,\ldots,N-1\,. \tag{2.21}$$

In Eq. (2.20), whatever the renormalization conditions are, the $Z_A^{(n)}$ must satisfy the following constraint coming from the finiteness of A as $\epsilon\to 0$:

$$\frac{d^\lambda}{d\epsilon^\lambda}\left[Z_A^{(n)}(\epsilon) + \sum_{k=0}^{n-1} e^{\epsilon(n-k)t} A^{(n-k)}(\epsilon) Z_{n-k}^{(k)}(\epsilon)\right]\Bigg|_{\epsilon=0} = 0 \tag{2.22}$$

with $\lambda = 0,1,2,\ldots,n-1$. We illustrate that up to two loops $(N=2)$

$$\begin{aligned} A(t,\alpha) = \lim_{\epsilon\to 0}\Big(1 &+ \frac{\alpha}{\epsilon}[A^{(1)}(\epsilon)(e^{\epsilon t}-1) + A^{(1)}(\epsilon) + Z_A^{(1)}(\epsilon)] \\ &+ \frac{\alpha^2}{\epsilon^2}\{A^{(2)}(\epsilon)(e^{\epsilon t}-1)^2 \\ &\quad + [2A^{(2)}(\epsilon) + A^{(1)}(\epsilon)Z_1^{(1)}(\epsilon)](e^{\epsilon t}-1)\} \\ &+ \frac{\alpha^2}{\epsilon^2}[A^{(2)}(\epsilon) + Z_A^{(2)}(\epsilon) + Z_1^{(1)}(\epsilon)A^{(1)}(\epsilon)]\Big). \end{aligned} \tag{2.23}$$

We note that Eq. (2.21) gives for $n=1$

$$Z_1^{(1)}(\epsilon) = Z_A^{(1)} + Z_\alpha^{(1)}\,. \tag{2.24}$$

Therefore, the conditions (2.22) for $N=2$ [i.e., the fact that there are no poles in ϵ in the expression (2.23)] read

$$A^{(1)}(\epsilon) + Z_A^{(1)}(\epsilon)\big|_{\epsilon=0} = 0\,, \tag{2.25a}$$

$$2A^{(2)}(\epsilon) + A^{(1)}(\epsilon)[Z_A^{(1)}(\epsilon) + Z_\alpha^{(1)}]\big|_{\epsilon=0} = 0\,, \tag{2.25b}$$

$$A^{(2)}(\epsilon) + A^{(1)}(\epsilon)[Z_A^{(1)}(\epsilon) + Z_\alpha^{(1)}(\epsilon)] + Z_A^{(2)}(\epsilon)\big|_{\epsilon=0} = 0\,, \tag{2.25c}$$

$$\frac{d}{d\epsilon}\{A^{(2)}(\epsilon) + A^{(1)}(\epsilon)[Z_A^{(1)}(\epsilon) + Z_\alpha^{(1)}(\epsilon)] + Z_A^{(2)}(\epsilon)\}\Big|_{\epsilon=0} = 0\,. \tag{2.25d}$$

Using these constraints and Eq. (2.23) we obtain for A the following expression valid in any renormalization scheme:

$$
\begin{aligned}
A(t,\alpha) = 1 &+ \alpha[A^{(1)}t + (A^{(1)} + Z_A^{(1)})'] + \alpha^2 A^{(2)} t^2 \\
&+ \alpha^2 t[2A^{(2)\prime} + A^{(1)\prime}(Z_\alpha^{(1)} - A^{(1)}) \\
&\quad + A^{(1)}(Z_A^{(1)\prime} + Z_\alpha^{(1)\prime})] \\
&+ \alpha^2 \tfrac{1}{2}[A^{(2)\prime\prime} + Z_A^{(2)\prime\prime} + A^{(1)\prime\prime}(Z_\alpha^{(1)} - A^{(1)}) \\
&\quad + 2A^{(1)\prime}(Z_\alpha^{(1)\prime} + Z_A^{(1)\prime})]\,, \qquad (2.26)
\end{aligned}
$$

where

$$
A' = \frac{d}{d\epsilon} A(\epsilon)\Big|_{\epsilon=0}, \quad A'' = \frac{d^2}{d\epsilon^2} A(\epsilon)\Big|_{\epsilon=0}.
$$

We suppose now that Z_α has already been determined and before specifying the renormalization scheme for A we observe the following well-known results.

(i) The leading-logarithm coefficient is independent of the renormalization conditions for the Green's functions of the theory and for the operators SLCV and TLCV.

(ii) The coefficient of the simple logarithm at the two-loop level depends on the renormalization conditions of the operators through $Z_A^{(1)\prime}$ which appears in the terms of order α; as soon as one fixes the renormalization conditions $Z_A^{(1)\prime}$ can be written as a function of $A^{(1)\prime}$.

(iii) $Z^{(2)\prime\prime}$ can also be eliminated from the formula (2.26) in the same way as $Z_A^{(1)\prime}$ and can be written as a function of $A^{(2)\prime\prime}$, $A^{(1)}$, $A^{(1)\prime}$, $Z_\alpha^{(1)}$, and $Z_\alpha^{(1)\prime}$ after fixing the renormalization conditions for A. So we have only to compute the contribution of the bare diagrams of A, $A^{(1)}(0)$, $A^{(1)\prime}(0)$, $A^{(2)}(0)$, and $A^{(2)\prime}(0)$, i.e., the coefficients of simple and double poles in ϵ of the bare two-loop diagrams and the coefficient of the simple pole and constant term in ϵ of the one-loop bare diagram. In order to go further we need to compute Z_α and Z_ϕ. Therefore, we expand the 1PI Green's functions $\Gamma_0^{(2)}/-ip^2$ and $\Gamma^{(3)}/ig_0|_{p^2=q^2=pq}$ as

$$
\Gamma_0^{(2)}(p^2,\alpha_0)/-ip^2 = 1 + \sum_{k=1}^{N} \frac{\alpha_0^k}{\epsilon^k}(P^2)^{-k\epsilon} D^{(k)}(\epsilon)\,, \qquad (2.27a)
$$

$$
\frac{\Gamma_0^{(3)}(p,q,\alpha_0)}{-ig_0}\Bigg|_{p^2=q^2=pq} = 1 + \sum_{k=1}^{N} \frac{\alpha_0^k}{\epsilon^k}(P^2)^{-k\epsilon} R^{(k)}(\epsilon)\,. \qquad (2.27b)
$$

These expressions are of the same type as that of Eq. (2.19) when we change $A \to D$ and $A \to R$, respectively. Then one can write the analogous equations to (2.26) for the renormalized $\Gamma^{(2)}/-ip^2$ and $\Gamma^{(3)}/-ig|_{p^2=q^2=pq}$:

$$
\Gamma^{(2)}(p^2, 4\pi\mu^2, \alpha)/-ip^2 = [\text{as Eq. (2.19)}, A \to D\,, Z_A \to Z_\phi]\,, \qquad (2.28a)
$$

$$
\Gamma^{(3)}(p,q,4\pi\mu^2,\alpha)/-ig|_{p^2=q^2=pq} = [\text{as Eq. (2.19)}, A \to R\,, Z_A \to Z_\Gamma]\,. \qquad (2.28b)
$$

Taking into account the renormalization conditions (2.2a), (2.2b), (2.2c), and (2.8a), (2.8b), and (2.8c) we can now eliminate all the Z_A, Z_ϕ, and Z_α dependence in the quantities A, $\Gamma^{(2)}$, and $\Gamma^{(3)}|_{p^2=q^2=pq}$:

$$
A(t,\alpha) = 1 + \alpha A^{(1)}t + \alpha^2 A^{(2)}t^2 + \alpha^2 t[2A^{(2)} - A^{(1)}A^{(1)} - 2A^{(1)}R^{(1)} + 3A^{(1)}D^{(1)}(1-\epsilon)]'_{\epsilon=0}\,, \qquad (2.29a)
$$

$$
\Gamma^{(3)}(p,q,t,\alpha)|_{p^2=q^2=pq} = -ig\{1 + \alpha R^{(1)}t + \alpha^2 R^{(2)}t^2 + \alpha^2 t[2R^{(2)} - 3R^{(1)}R^{(1)} + 3R^{(1)}D^{(1)}(1-\epsilon)]'_{\epsilon=0}\}\,, \qquad (2.29b)
$$

$$
\begin{aligned}
\Gamma^{(2)}(p^2,t,\alpha) = -ip^2\{1 &+ \alpha D^{(1)}(t+1) + \alpha^2 D^{(2)}t^2 \\
&+ \alpha^2(t+1)[2D^{(2)}(1-2\epsilon) - 2D^{(1)}(1-\epsilon)R^{(1)} + 2D^{(1)}D^{(1)}(1-2\epsilon]'_{\epsilon=0}\}\,. \qquad (2.29c)
\end{aligned}
$$

The expressions (2.29a), (2.29b), and (2.29c) give the renormalized quantities as functions of bare ones. (The renormalization conditions are therefore automatically taken into account.) Using Eq. (2.19) we can now easily determine the anomalous dimension γ_A and γ_ϕ as functions of bare quantities:

$$
\begin{aligned}
2\gamma_\phi = \alpha D^{(1)} &+ \alpha^2[2D^{(2)}(1-2\epsilon) \\
&- 2D^{(1)}(1-\epsilon)(R^{(1)} - D^{(1)}(1-\epsilon))]'_{\epsilon=0}\,, \qquad (2.30a)
\end{aligned}
$$

$$
\begin{aligned}
-\gamma_A + 2\gamma_\phi = \alpha A^{(1)} &+ \alpha^2[2A^{(2)} - A^{(1)}A^{(1)} - 2A^{(1)}R^{(1)} \\
&+ 3A^{(1)}D^{(1)}(1-\epsilon)]'_{\epsilon=0}\,. \qquad (2.30b)
\end{aligned}
$$

We are now at the point where the renormalized quantities have been expressed in terms of regularized quantities. Section III will be devoted to the phase-space analysis necessary to compute them.

III. PHASE-SPACE PARAMETRIZATION

In this section we study the phase-space domain which is necessary for the computation of the diagrams contributing to the SLCV, the TLCV, and their coefficient functions.

Considering the processes shown in Figs. 2 (a) and 2 (b) we choose a frame where

$$
q = (q_0, q_3, \vec{0})\,, \qquad (3.1a)
$$

$$
p = (p_0, p_3, \vec{0})\,. \qquad (3.1b)
$$

In the SL case we have [Fig. 2 (a)]

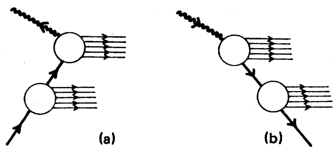


FIG. 2. (a) One-virtual-particle (k^μ) configuration of the process $J(-q)+\varphi(p)\to X$; $p^2, q^2, 2p\cdot q<0$. (b) One-virtual-particle (k^μ) configuration of the process $J(q)\to\varphi(p)+X$; p^2, q^2, $2p\cdot q>0$.

$$q^2<0;\quad q_+=q_0+q_3>0,\quad q_-=q_0-q_3<0, \tag{3.2a}$$

$$p^2<0;\quad p_+=p_0+p_3>0,\quad p_-=p_0-p_3<0 \tag{3.2b}$$

and the condition that the particles produced in the upper and lower parts of Fig. 2 (a) are real particles implies the following inequalities:

$$(p-k)_+\geq 0,\quad (p-k)_-\geq 0, \tag{3.3a}$$

$$(k-q)_+\geq 0,\quad (k-q)_-\geq 0, \tag{3.3b}$$

$$(p-k)_+(p-k)_-\geq (p-k)^2\geq 0, \tag{3.3c}$$

$$(k-q)_+(k-q)_-\geq (k-q)^2\geq 0. \tag{3.3d}$$

In the TL case [Fig. 2 (b)] one has

$$q^2>0;\quad q_+>0,\quad q_->0, \tag{3.4a}$$

$$p^2>0;\quad p_+>0,\quad p_->0 \tag{3.4b}$$

and also

$$(k-p)_+\geq 0,\quad (k-p)_-\geq 0, \tag{3.5a}$$

$$(q-k)_+\geq 0,\quad (q-k)_-\geq 0, \tag{3.5b}$$

$$(k-p)_+(k-p)_-\geq (k-p)^2\geq 0, \tag{3.5c}$$

$$(q-k)_+(q-k)_-\geq (q-k)^2\geq 0. \tag{3.5d}$$

We introduce the following set of dimensionless variables:

$$x=\frac{q_+}{p^+},\quad z=\frac{k^2}{k_-p_+},\quad \rho=\frac{k_+}{p_+} \tag{3.6a}$$

and

$$\beta=\frac{p^2}{k^2}z\frac{1-\rho}{1-z}. \tag{3.6b}$$

Then in terms of these variables the phase-space domains for the processes of Figs. 2 (a) and 2 (b) are defined by the following sets of inequalities:

(i) In the SL case

$$0\leq x\leq\rho\leq z\leq 1, \tag{3.7a}$$

$$0\leq\beta_{\min}\leq\beta\leq 1. \tag{3.7b}$$

(ii) In the TL case

$$\infty\geq x\geq\rho\geq z\geq 1, \tag{3.8a}$$

$$0\leq\beta_{\min}\leq\beta\leq 1, \tag{3.8b}$$

where

$$\beta_{\min}=\frac{p^2}{q^2}x\frac{(z-x)(1-\rho)}{(\rho-x)(1-z)}. \tag{3.9}$$

We note that the variables ρ, z, β, and x can be expressed in terms of Lorentz invariants, since one has

$$k^2=p^2\frac{z(1-\rho)}{(1-z)}\frac{1}{\beta}, \tag{3.10a}$$

$$(p-k)^2=-p^2(1-\rho)\frac{(1-\beta)}{\beta}, \tag{3.10b}$$

$$(k-q)^2=-q^2\frac{\rho-x}{x}\frac{(\beta-\beta_{\min})}{\beta}, \tag{3.10c}$$

$$(p-q)^2=-(q^2-xp^2)\frac{1-x}{x}. \tag{3.10d}$$

We now consider the phase-space domains associated with the SLCV and TLCV shown in Figs. 1 (b) and 1 (c).

For the SLCV we have,

$$p^2<0;\quad p_+>0,\quad p_-<0 \tag{3.11}$$

and the reality of the produced particles together with the presence of the function $\theta(k^+)$ in the Fig. 1 (b) imply the inequalities

$$(p-k)_+\geq 0,\quad (p-k)_-\geq 0, \tag{3.12a}$$

$$(p-k)_+(p-k)_-\geq (p-k)^2\geq 0, \tag{3.12b}$$

$$k^+\geq 0. \tag{3.12c}$$

For the TLCV we have

$$p^2>0;\quad p_+>0,\quad p_->0 \tag{3.13}$$

and also

$$(k-p)_+\geq 0,\quad (k-p)_-\geq 0, \tag{3.14a}$$

$$(k-p)_+(k-p)_-\geq (k-p)^2\geq 0. \tag{3.14b}$$

[In the TLCV there is no $\theta(k^+)$.] The phase-space domain defined by Eqs. (3.12a) and (3.14b) can be expressed in terms of the variables defined in Eqs. (3.6a) and (3.6b). One obtains the following set of inequalities:

(i) SLCV:

$$0\leq\rho\leq z\leq 1, \tag{3.15a}$$

$$0\leq\beta\leq 1. \tag{3.15b}$$

(ii) TLCV:

$$\infty\geq\rho\geq z\geq 1, \tag{3.16a}$$

$$0\leq\beta\leq 1. \tag{3.16b}$$

Introducing a lightlike vector Δ^μ,

$$\Delta^2=0 \text{ and } \Delta\cdot p=p_+>0, \tag{3.17}$$

we can express k^2, $(p-k)^2$, and $\Delta\cdot k$ as functions of ρ, z, and β:

$$k^2=p^2\, z\frac{(1-\rho)}{(1-z)}\frac{1}{\beta}, \tag{3.18a}$$

$$(p-k)^2=-p^2(1-\rho)\frac{(1-\beta)}{\beta}, \tag{3.18b}$$

$$\frac{\Delta\cdot k}{\Delta\cdot p}=\rho. \tag{3.18c}$$

These equations show that z, ρ, and β are invariant under a Lorentz transformation.

For the calculation of the TL cut diagrams we shall use the variables $\bar{x}$, $\bar{\rho}$, $\bar{z}$, and $\bar{\beta}$,

$$\bar{x}=\frac{1}{x},\ \bar{\rho}=\frac{1}{\rho},\ \bar{z}=\frac{1}{z},\ \bar{\beta}=\beta, \tag{3.19}$$

rather than x, ρ, z, and β. It is worth noticing that the inequalities (3.8a), and (3.8b) and (3.16a) and (3.16b) written in terms of these variables,

$$0\leq\bar{x}\leq\bar{\rho}\leq\bar{z}\leq 1, \tag{3.20a}$$

$$0\leq\beta_{\min}\leq\bar{\beta}\leq 1, \tag{3.20b}$$

are the same as the inequalities (3.7a) and (3.7b) and (3.15b) corresponding to the SL case. We think that this symmetry between the TL and SL phase spaces is a key point for the existence of crossing relations between spacelike and timelike quantities.

We now express the measure d^nk ($n=6-2\epsilon$) in terms of the variables ρ, z, β or $\bar{\rho}$, $\bar{z}$, and $\bar{\beta}$. First we write

$$d^nk=\tfrac{1}{2}\,dk_+dk_-d^{4-2\epsilon}k_\perp \tag{3.21}$$

and we find the following.

(i) In the SL case:

$$\left(\frac{2\pi}{-p^2}\right)\int\frac{d^nk(\mu^2)^\epsilon}{(2\pi)^n(k^2)^2}=\frac{e^{\epsilon t}}{(4\pi)^3\Gamma(2-\epsilon)}\int_{x\text{ or }0}^{1}d\rho\int_{z}^{1}\frac{dz}{z}z^{-\epsilon}(1-\rho/z)^{1-\epsilon}(1-\rho)^{1-\epsilon}(1-z)^{-1+\epsilon}\int_{\beta_{\min}\text{ or }0}^{1}\frac{d\beta}{\beta^{2-\epsilon}}. \tag{3.22a}$$

(ii) In the TL case:

$$\left(\frac{2\pi}{p^2}\right)\int\frac{d^nk(\mu^2)^\epsilon}{(2\pi)^n(k^2)^2}=\frac{e^{\epsilon t}}{(4\pi)^3\Gamma(2-\epsilon)}\int_{x\text{ or }0}^{1}\frac{d\bar{\rho}}{\bar{\rho}^2}\int_{\bar{z}}^{1}\frac{d\bar{z}}{\bar{z}}\bar{z}^{\epsilon}\left(\frac{\bar{z}}{\bar{\rho}}-1\right)^{1-\epsilon}\left(\frac{1-\bar{\rho}}{\bar{\rho}}\right)^{1-\epsilon}\left(\frac{1-\bar{z}}{\bar{z}}\right)^{-1+\epsilon}\int_{\bar{\beta}_{\min}\text{ or }0}^{1}\frac{d\bar{\beta}}{\bar{\beta}^{2-\epsilon}}, \tag{3.22b}$$

where $t\equiv\ln(4\pi\mu^2/|p^2|)$.

The expressions (3.22a) and (3.22b) allow us to compute in a systematic and straightforward way the bare cut diagrams up to two loops. We will also use the following identities[6]:

$$x^{-1+\epsilon}=\frac{1}{\epsilon}\left[\delta(x)+\epsilon\frac{1}{x_+}+O(\epsilon^2)\right], \tag{3.23a}$$

$$\text{disc}\frac{1}{(-x-i0)^{1-\epsilon}}=2\pi i\left[\delta(x)+\epsilon\frac{1}{x_+}+O(\epsilon^2)\right] \tag{3.23b}$$

and

$$(x+i0)^\epsilon=x^\epsilon\theta(x)+e^{i\pi\epsilon}(-x)^\epsilon\theta(-x), \tag{3.23c}$$

where

$$\int_0^1\frac{dx}{x_+}\varphi(x)=\int_0^1\frac{dx}{x}[\varphi(x)-\varphi(0)]. \tag{3.23d}$$

These identities will allow us to compute in a very simple way the Laurent expansion in ϵ of the diagrams. Explicit examples will be given in Sec. IV.

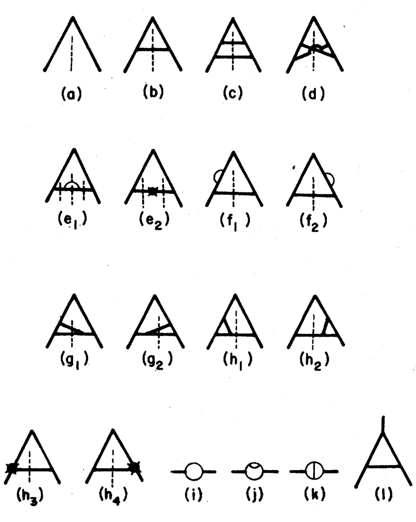


FIG. 3. (a)–(h). Relevant diagrams for both spacelike and timelike cut vertices, up to the two-loop approximation. (i)–(k). The two-point function diagrams. (l) The three-point function at the one-loop level.

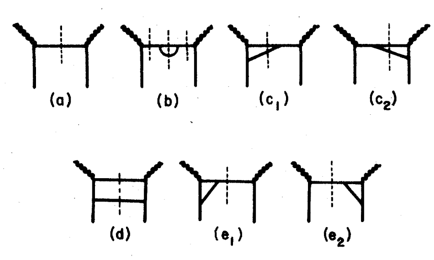


FIG. 4. (a)–(e). Diagrams related to the spacelike and timelike structure functions of the processes $J(q)+\varphi(p)\to X$ and $J(q)\to\varphi(p)+X$ up to the one-loop approximation.

IV. TWO-LOOP CALCULATION OF SLCV AND TLCV AND THEIR COEFFICIENT FUNCTIONS

We now describe a technique to compute two-loop diagrams contributing to the SLCV, TLCV, and their coefficient functions for the case of the current $J(X)=:\Phi^2(X):$. The renormalization algorithm is the one described in Sec. II.

We show in detail the calculation of the diagrams presented in Figs. 3(a)–3(e) contributing to the SLCV and TLCV, and of the diagrams of Figs. 4(a)–4(e) contributing to the SL and TL coefficient functions. We start with the sum of the diagrams $3(h_1)$ and $3(h_2)$ and of their relevant counterterms $3(h_3)$ and $3(h_4)$. The phase-space analysis we gave in Sec. III allows us to write directly the value of the sum of the diagrams $3(h_1)$ and $3(h_2)$:

$$A_\sigma=\alpha\frac{e^{\epsilon t}}{\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\int_\rho^1\frac{dz}{z}z^{-\epsilon}(1-\rho_z)^{1-\epsilon}(1-p)^{1-\epsilon}(1-z)^{-1+\epsilon}\int_0^1\frac{d\beta}{\beta^{2-\epsilon}}G(p,k)\,, \tag{4.1}$$

where the variables ρ, z, and β are defined in Eqs. (3.2), (3.5), or (3.17a), (3.17b), and (3.17c),

$$t=\ln\frac{4\pi\mu^2}{|p^2|}$$

and

$$G(p,k)=\frac{|p^2|}{2\pi}\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\operatorname{disc}\frac{i}{(p-k)^2+i0}=\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\delta\left(\frac{(p-k)^2}{|p^2|}\right), \tag{4.2}$$

where $\Gamma_\pm^{(3)}$ is the one-loop bare three-point function [Fig. 3(e)]

$$\frac{1}{-ig_0}\Gamma_\pm^{(3)}=(-ig_0)^2\int\frac{d^nqi^3}{(2\pi)^n(q^2\pm i0)[(p-q)^2\pm i0][(q-k)^2\pm i0]}$$

$$=\alpha\frac{e^{\epsilon t}\Gamma(1+\epsilon)}{\epsilon}\int_0^1 dx\int_0^1 dy\,y^{1-2}\left\{xy(1-x)\frac{(p-k)^2}{|p^2|}+(1-y)\left[x\frac{-p^2}{|p^2|}+(1-x)\frac{-k^2}{|p^2|}\mp i0\right]\right\}^{-\epsilon} \tag{4.3}$$

(x and y are some Feynman parameters). Using the constraint $(p-k)^2=0$ we have

$$\left.\frac{1}{-ig_0}\Gamma_\pm^{(3)}\right|_{(p-k)^2=0}=\alpha\frac{e^{\epsilon t}\Gamma(1+\epsilon)B(2-\epsilon,1-\epsilon)}{\epsilon}\int_0^1 dx\left[x\frac{-p^2}{|p^2|}+(1-x)\frac{-k^2}{|p^2|}\mp i0\right]^{-\epsilon}. \tag{4.4}$$

Taking into account the identity (3.23c), we have

$$\left.\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\right|_{\substack{p^2,k^2<0\\(p-k)^2=0}}=\alpha\frac{e^{\epsilon t}R^{(1)}(\epsilon)}{\epsilon}\int_0^1 dx\,2\left(x\frac{-p^2}{|p^2|}+(1-x)\frac{-k^2}{|p^2|}\right)^{-\epsilon}, \tag{4.5}$$

where

$$R^{(1)}(\epsilon)=\Gamma(1+\epsilon)B(2-\epsilon,1-\epsilon)\,. \tag{4.6}$$

Combining Eqs. (4.1), (4.2), (4.5), and (4.6) one gets

$$A_\sigma=\alpha^2\frac{e^{2\epsilon t}R^{(1)}(\epsilon)}{\epsilon\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\int_\rho^1\frac{dz}{z}z^{-2\epsilon}(1-\rho/z)^{1-\epsilon}(1-\rho)^{-2\epsilon}(1-z)^{-1+2\epsilon}$$

$$\times\int_0^1\frac{d\beta}{\beta}\delta(1-\beta)\int_0^1 dx\left[\frac{(1-z)x}{(1-p)z}+(1-x)\right]^{-\epsilon}. \tag{4.7}$$

We can expand $(1-z)^{-1+2\epsilon}$ in powers of ϵ [see Eq. (3.23a)], and the expression (4.7) is written as

$$A_\sigma=2\alpha^2\frac{e^{2\epsilon t}R^{(1)}(\epsilon)}{2\epsilon^2\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\left\{\int_\rho^1\frac{dz}{z}z^{-2\epsilon}\left(1-\frac{\rho}{z}\right)^{1-\epsilon}(1-\rho)^{-\epsilon}\delta(1-z)\int_0^1 dx(1-x)^{-\epsilon}\right.$$

$$\left.+2\epsilon\int_\rho^1\frac{dz}{z}(1-\rho/z)\frac{1}{(1-z)_+}\int_0^1 dx+O(\epsilon^2)\right\} \tag{4.8a}$$

or equivalently

$$A_\sigma=2\alpha^2\frac{e^{2\epsilon t}R^{(1)}(\epsilon)}{2\epsilon^2\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\left((1-\rho)+\epsilon\left\{(1-\rho)[(1-3\ln(1-\rho)]+2\left[(1-y)\otimes\frac{1}{(1-y)_+}\right]_\rho\right\}+O(\epsilon^2)\right), \tag{4.8b}$$

where the convolution product $\otimes$ is defined as follows:

$$(\varphi(y)\otimes f(y))_\rho=\int_\rho^1\frac{dy}{y}\varphi(y)f(\rho/y)=\int_\rho^1\frac{dy}{y}\varphi(\rho/y)f(y)\,. \tag{4.9}$$

Note that the inverse Mellin transform of the diagrams appears directly in Eq. (4.8b). We now compute the contribution of the diagrams which contain the three-point function counterterms $Z_\Gamma^{(1)}$. Since we have chosen the renormalization condition (2.2c) for the three-point function we have [see Eq. (4.4)]

$$Z_\Gamma^{(1)}(\epsilon)=-R^{(1)}(\epsilon)=\Gamma(1+\epsilon)B(2-\epsilon,1-\epsilon)\,. \tag{4.10}$$

Proceeding exactly in the same way as for Eq. (4.8a), we obtain

$$-2\alpha^2\frac{e^{\epsilon t}R^{(1)}(\epsilon)}{\epsilon^2\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\left((1-\rho)+\epsilon\left\{-2(1-\rho)\ln(1-\rho)+\left[(1-y)\otimes\frac{1}{(1-y)_+}\right]_\rho\right\}+O(\epsilon^2)\right). \tag{4.11}$$

From Eqs. (4.8b) and (4.11) we extract $A_\sigma^{(2)}$ and $-A_\sigma^{(1)}R^{(1)}$:

$$A_\sigma^{(2)}=2\frac{R^{(1)}(\epsilon)}{2\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\left\{(1-\rho)+\epsilon\left[(1-\rho)[(1-3\ln(1-\rho)]+(1-y)\otimes\frac{1}{(1-y)_+}\right]+O(\epsilon^2)\right\}, \tag{4.12a}$$

$$-A_\sigma^{(1)}R^{(1)}=2\frac{R^{(1)}(\epsilon)}{\Gamma(2-\epsilon)}\int_0^1 d\rho\,\rho^\sigma\left\{(1-\rho)+\epsilon\left[-2(1-\rho)\ln(1-\rho)+(1-y)\otimes\frac{1}{(1-y)_+}\right]+O(\epsilon^2)\right\}. \tag{4.12b}$$

Then the contribution to the anomalous dimension γ^σ is [see Eq. (2.31b)]

$$\gamma^\sigma=\alpha^2(2A_\sigma^{(2)}-A_\sigma^{(1)}R^{(1)})'_{\epsilon=0}\,, \tag{4.13a}$$

$$\gamma^\sigma=\alpha^2\int_0^1 d\rho\,\rho^\sigma(1-\rho)\ln\frac{1}{\rho}\,. \tag{4.13b}$$

The inverse Mellin transform of γ^σ, $P(\rho)$, is

$$P(\rho)=\alpha^2\rho(1-\rho)\ln\frac{1}{\rho}\,. \tag{4.14}$$

We now turn to the TLCV, with the example of diagrams $3(h_1)$ and $3(h_2)$. The computation is very similar to the case of the SLCV, and we will only outline the differences which are of interest for the investigation of the crossing relations between SLCV and TLCV.

Indeed from the phase-space analysis of Sec. III, the sum of these diagrams is given as

$$A_\sigma=\alpha\frac{e^{\epsilon t}}{\Gamma(2-\epsilon)}\int_0^1\frac{d\bar\rho}{\bar\rho^2}\bar\rho^\sigma\int_{\bar\rho}^1\frac{d\bar z}{\bar z}\bar z^\epsilon(\bar z/\bar\rho-1)^{1-\epsilon}\left(\frac{1-\bar\rho}{\bar\rho}\right)^{1-\epsilon}\left(\frac{1-\bar z}{\bar z}\right)^{-1+\epsilon}\int_0^1\frac{d\beta}{\beta^{2-\epsilon}}\bar G(p,k)\,, \tag{4.15}$$

where

$$\bar G(p,k)=\frac{|p^2|}{2\pi}\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\operatorname{disc}\frac{i}{(p-k)^2+i0}=\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\delta\left(\frac{(p-k)^2}{|p^2|}\right). \tag{4.16}$$

The expression for the sum $\Gamma_+^{(3)}+\Gamma_-^{(3)}$ becomes after Eqs. (4.4), and (4.5)

$$\left.\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-ig_0}\right|_{\substack{p^2,k^2>0\\(p-k)^2=0}}=\alpha\frac{e^{\epsilon t}R^{(1)}(\epsilon)}{\epsilon}\times\int_0^1 dx\,2\cos\pi\epsilon\left(x\frac{p^2}{|p^2|}+(1-x)\frac{k^2}{|p^2|}\right)_+^{-\epsilon}. \tag{4.17}$$

However, since we need only the $1/\epsilon$ and ϵ^0 terms in the expression (4.17) we can take $\cos\epsilon\pi\equiv1$.

The rest of the computation of TLCV is made using the same procedure as in the spacelike case, and the contribution of the diagrams $3(h_1)$ and $3(h_2)$ and $3(h_3)$ and $3(h_4)$ to the anomalous dimensions $\tilde\gamma^\sigma$ is

$$\tilde\gamma^\sigma=\alpha^2\int_0^1\frac{d\bar\rho}{\bar\rho^2}\bar\rho^\sigma\frac{1-\bar\rho}{\bar\rho}\ln\bar\rho \tag{4.18a}$$

$$=\alpha^2\int_1^\infty d\rho\,\rho^{-\sigma}(\rho-1)\ln\frac{1}{\rho} \tag{4.19a}$$

and its inverse Mellin transform:

$$\bar{P}(\rho)=\alpha\rho(\rho-1)\ln\frac{1}{\rho}\,. \tag{4.19b}$$

The relation satisfied by $P(\rho)$ and $\bar{P}(\rho)$

$$P(\rho)=-\bar{P}(\rho) \tag{4.20}$$

will be discussed in Sec. V. Let us give also some interesting points appearing in the calculation of the diagram of Fig. 3(e). Since we work in a massless theory the particular discontinuities of the diagram 3(e) are without meaning, so one has first to calculate the one-loop bare propagator and then take the discontinuity. We find for the regularized bare propagator [Fig. 3(i)]

$$-\alpha\frac{e^{\epsilon t}}{\epsilon}D^{(1)}(\epsilon)\frac{1}{|p^2|}\left(\frac{\beta}{1-\rho}\right)^{1+\epsilon}\frac{i}{[-(1-\beta)-i0]^{1+\epsilon}}\,, \tag{4.21a}$$

where

$$D^{(1)}(\epsilon)=-\frac{1}{12}\left[1+\epsilon\left(\tfrac{8}{3}-\gamma\right)\right]. \tag{4.21b}$$

Using the distribution identity (3.23b), we find the discontinuity of the regularized one-loop bare propagator,

$$\frac{|p^2|}{2\pi}\operatorname{disc}G_0^{(2)}((p-k)^2+i0)$$
$$=\alpha\frac{e^{\epsilon t}D^{(1)}(\epsilon)}{\epsilon}\left(\frac{\beta}{1-\rho}\right)^{1+\epsilon}\left[\delta(1-\beta)-\frac{\epsilon}{(1-\beta)_+}+O(\epsilon^2)\right]. \tag{4.22}$$

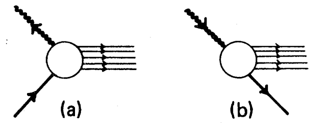


FIG. 5. (a) $J(-q)+\varphi(p)\to X$; q^2, p^2, $2p\cdot q<0$. (b) $J(q)\to\varphi(p)+X$; $q^2, p^2, 2p\cdot q>0$.

Another interesting technical point in the calculation of Fig. 3(e) is the integration over β which leaves us with an integral of the form

$$I=\int_0^1\frac{d\beta}{\beta^{1-2\epsilon}}\frac{1}{(1-\beta)_+}=\frac{1}{2\epsilon}\int_0^1 d\beta\left[\delta(\beta)+2\epsilon\frac{1}{\beta_+(1-\beta)_+}+O(\epsilon^2)\right] \tag{4.23a}$$

or

$$I=\frac{1}{2\epsilon}\left\{1+\epsilon\int_0^1 d\beta\left[\frac{1}{\beta_+}+\frac{1}{(1-\beta)_+}\right]+O(\epsilon^2)\right\}=\frac{1}{2\epsilon}[1+O(\epsilon^2)]\,. \tag{4.23b}$$

We now compute the coefficient functions of the structure functions W and $\bar{W}$ of the deep-inelastic and semi-inclusive annihilation processes

$$J(-q)+\tilde{\phi}(p)\to X$$

[see Fig. 5(a)] and

$$J(q)\to\tilde{\phi}(p)+X$$

[see Fig. 5(b)] in the particular case of the current $J(x)=:\Phi^2:$

$$W(q,p)\underset{\substack{-q^2\to\infty\\ q^2/2pq\ \text{fixed}}}{=}R(q^2)\sum_X\langle 0|\bar{T}(J(-q)\tilde{\phi}(p))|X\rangle\langle X|T(\tilde{\phi}(-p)J(q))|0\rangle_{\text{AMP}}\,, \tag{4.24a}$$

$$\bar{W}(q,p)\underset{\substack{q^2\to\infty\\ q^2/2pq\ \text{fixed}}}{=}R(q^2)\sum_X\langle 0|\bar{T}(J(q)\tilde{\phi}(-p))|X\rangle\langle X|T(\tilde{\phi}(p)J(-q))|0\rangle_{\text{AMP}}\,, \tag{4.24b}$$

where $R(q^2)$ is a normalization factor, such that W and $\bar{W}$ are dimensionless and also with anomalous dimension equal to zero,

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)W=0 \tag{4.25a}$$

and

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)\bar{W}=0\,. \tag{4.25b}$$

Explicitly,

$$R(q^2)=\frac{|q^2|}{2\pi}\exp\left\{\int_\alpha^{\bar{\alpha}}\frac{d\alpha'}{\alpha'\beta(\alpha')}[2\gamma_J(\alpha')-2\gamma_\Phi(\alpha')]\right\}, \tag{4.26}$$

where $\bar{\alpha}(q^2)$ is defined as follows:

$$\ln\frac{|q^2|}{4\pi\mu^2}=\int_\alpha^{\bar{\alpha}(q^2)}\frac{d\alpha'}{\alpha'\beta(\alpha')}\,. \tag{4.27}$$

γ_J and γ_Φ are the anomalous dimension of the current J and the field operator Φ, respectively.

As has been shown[1] the moments of the structure function are factorized as follows:

$$\int_0^1 dx\,x^{\sigma-1}W(x,q^2,p^2)=C^\sigma\left(\frac{-q^2}{4\pi\mu^2},\alpha\right)A^\sigma_{\text{SL}}\left(\frac{-p^2}{4\pi\mu^2},\alpha\right), \tag{4.28a}$$

$$\int_1^\infty dx\,x^{-\sigma-1}\bar{W}(x,q^2,p^2)=\bar{C}^\sigma\left(\frac{q^2}{4\pi\mu^2},\alpha\right)A^\sigma_{\text{TL}}\left(\frac{p^2}{4\pi\mu^2},\alpha\right), \tag{4.28b}$$

where c^σ and $\bar{c}^\sigma$ are the SL and TL coefficient functions, and A^σ_{SL} and A^σ_{TL} the SLCV and TLCV.

Since we have chosen the renormalization conditions (2.8b) and (2.8c) we obtain

$$C^\sigma\left(\frac{-q^2}{4\pi\mu^2},\alpha\right)=\int_0^1 dx\,x^{\sigma-1}W(x,q^2,p^2)\Bigg|_{|p^2|=4\pi\mu^2} \tag{4.29a}$$

and

$$\overline{C}^\sigma\left(\frac{q^2}{4\pi\mu^2},\alpha\right)=\int_1^\infty dx\,x^{-\sigma-1}\overline{W}(x,q^2,p^2)\Bigg|_{|p^2|=4\pi\mu^2} \tag{4.29b}$$

or with inverse Mellin transformation

$$C\left(x,\frac{-q^2}{4\pi\mu^2},\alpha\right)=W(x,q^2,p^2)\Bigg|_{|p^2|=4\pi\mu^2}, \tag{4.30a}$$

$$\overline{C}\left(x,\frac{q^2}{4\pi\mu^2},\alpha\right)=\overline{W}(x,q^2,p^2)\Bigg|_{|p^2|=4\pi\mu^2}. \tag{4.30b}$$

For a next-to-leading-logarithm analysis of the scaling violation in W and $\overline{W}$ one has to compute the coefficient functions $C(x,1,\alpha)$ and $\overline{C}(x,1,\alpha)$ up to one loop. So one has to compute the one-loop diagrams shown in Figs. 4(a)–4(e) in the Bjorken limit, and after that we put $|p^2|=|q^2|=4\pi\mu^2$. We define the effective charges $e(x,\overline{\alpha}(q^2))$ and $e(x,\overline{\alpha}(q^2))$,

$$e(x,\overline{\alpha}(q^2))\equiv C(x,1,\overline{\alpha}(q^2)), \tag{4.31a}$$

$$e(x,\overline{\alpha}(q^2))\equiv \overline{C}(x,1,\overline{\alpha}(q^2)), \tag{4.31b}$$

and using Eqs. (4.30a) and (4.30b) we have

$$e(x,\overline{\alpha}(q^2))=W\left(\frac{-q^2}{4\pi\mu^2},\frac{-p^2}{4\pi\mu^2},\alpha\right)\Bigg|_{\substack{p^2=q^2=-4\pi\mu^2\\ \alpha=\overline{\alpha}(q^2)}}, \tag{4.32a}$$

$$e(x,\overline{\alpha}(q^2))=\overline{W}\left(\frac{q^2}{4\pi\mu^2},\frac{p^2}{4\pi\mu^2},\alpha\right)\Bigg|_{\substack{p^2=q^2=4\pi\mu^2\\ \alpha=\overline{\alpha}(q^2)}}, \tag{4.32b}$$

We will show the explicit computation of Fig. 4(d) in the SL and TL cases. Before taking the Bjorken limit the contributions of this diagram to W and $\overline{W}$ are

$$W=\alpha\frac{|q^2|(p-q)^2\rho_1}{\mathfrak{D}^3}\int_\lambda^1\frac{d\xi(1-\xi)(\xi-\lambda)}{\xi^2}, \tag{4.33a}$$

$$\overline{W}=\alpha\frac{|q^2|(p-q)^2\rho_2}{\mathfrak{D}^3}\int_\lambda^1\frac{d\xi(1-\xi)(\xi-\lambda)}{\xi^2}, \tag{4.33b}$$

where

$$\rho_{1,2}=\tfrac{1}{2}(|2pq|\pm\mathfrak{D}), \tag{4.34a}$$

$$\mathfrak{D}=+[(2pq)^2-4p^2q^2]^{1/2}, \tag{4.34b}$$

$$\lambda=\rho_1/\rho_2, \tag{4.34c}$$

$$\xi=|k^2|/\rho_2. \tag{4.34d}$$

After performing the integration over ξ and neglecting the terms of the order p^2/q^2 we obtain

$$W=\alpha x(1-x)\left[\ln\left(\frac{q^2}{p^2}\right)+2\ln\left(\frac{1}{x}\right)-2\right], \tag{4.35a}$$

$$\overline{W}=\alpha x(1-x)\left[\ln\left(\frac{q^2}{p^2}\right)+2\ln\left(\frac{1}{x}\right)-2\right], \tag{4.35b}$$

with

$$x=\frac{2|q^2|}{|2pq|+[(2pq)^2-4p^2q^2]^{1/2}}=x_F+O\left(\frac{p^2}{q^2}\right). \tag{4.36}$$

We note that among the diagrams 4(b)–4(e) only the diagram 4(d) which is non-2PI in the $(p-q)$ channel, contains a $\log|p^2|$ term. It is now straightforward to extract from the expressions (4.31a), (4.31b), (4.32a), and (4.32b) the corresponding contribution to the coefficient function and thus to the effective charges:

$$C\left(x,\frac{-q^2}{4\pi\mu^2},\alpha\right)=\alpha x(1-x)\left[\ln\frac{-q^2}{4\pi\mu^2}+2\ln\left(\frac{1}{x}\right)-2\right], \tag{4.37a}$$

$$\overline{C}\left(x,\frac{q^2}{4\pi\mu^2},\alpha\right)=\alpha x(x-1)\left[\ln\frac{q^2}{4\pi\mu^2}+2\ln\left(\frac{1}{x}\right)-2\right] \tag{4.37b}$$

and

$$e(x,\overline{\alpha}(q^2))=\overline{\alpha}(q^2)x(1-x)\left(2\ln\frac{1}{x}-2\right), \tag{4.38a}$$

$$\overline{e}(x,\overline{\alpha}(q^2))=\overline{\alpha}(q^2)x(x-1)\left(2\ln\frac{1}{x}-2\right). \tag{4.38b}$$

In Table I we have displayed the contribution to e and $\overline{e}$ of all the diagrams 4(a)–4(e).

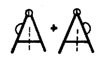
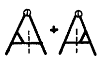
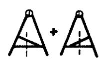
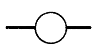
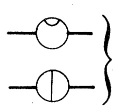
V. RESULTS AND RELATION FOR DEEP-INELASTIC AND ANNIHILATION STRUCTURE FUNCTIONS

In Ref. 5 we gave a parton-model interpretation to the moment equations

$$\int_0^1 dx\,x^{\sigma-1}W(x,p^2,q^2)=A^\sigma_{\rm SL}\left(\frac{-p^2}{4\pi\mu^2},\alpha\right)\exp\left[-\int_\alpha^{\overline{\alpha}(q^2)}\frac{d\alpha'\gamma^\sigma(\alpha')}{\alpha'\beta(\alpha')}\right]C^\sigma(1,\overline{\alpha}(q^2)) \tag{5.1}$$

and

TABLE I. Graph-by-graph contribution to deep-inelastic and semi-inclusive annihilation charge densities $e(x,\alpha)$ and $\bar{e}(x,\alpha)$.

Cut vertices	$P(x)$	$\bar{P}(x)$
	$\alpha x(1-x)$	$\alpha x(x-1)$
	$\frac{\alpha^2}{12}x(1-x)\ln(1-x)-\frac{\alpha^2}{12}x(1-x)$	$\frac{\alpha^2}{12}x(x-1)\ln(x-1)-\frac{\alpha^2}{12}x(x-1)$
	$\frac{\alpha^2}{6}x(1-x)\ln x$	$\frac{\alpha^2}{6}x(x-1)\ln x$
	$\alpha^2 x(1-x)\ln\frac{1}{x}$	$\alpha^2 x(x-1)\ln\frac{1}{x}$
	$\alpha^2 x(1-x)+\alpha^2 x\ln x$	$\alpha^2 x(x-1)-\alpha^2 x\ln x$
	$\alpha^2 x(1+x)\ln\frac{1}{x}-2\alpha^2 x(1-x)$	$\alpha^2 x(1+x)\ln x-2\alpha^2 x(x-1)$
	$-\alpha^2 x(1+x)\ln\frac{1}{x}+2\alpha^2 x(1-x)$ $+\frac{\alpha^2}{2}x(1+x)\ln^2 x+\alpha^2 x(1-x)\ln x$	$-\alpha^2 x(1+x)\ln x+2\alpha^2 x(x-1)$ $-\frac{\alpha^2}{2}x(1+x)\ln^2 x+\alpha^2 x(x-1)\ln x$
	$-\frac{\alpha}{12}x\delta(1-x)$	$-\frac{\alpha}{12}x\delta(x-1)$
	$-\frac{\alpha^2}{432}x\delta(1-x)$	$-\frac{\alpha^2}{432}x\delta(x-1)$

$$\int_1^\infty dx\, x^{-\sigma-1}\bar{W}(x,p^2,q^2)=A^\sigma_{\mathrm{TL}}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)\exp\left[-\int_\alpha^{\alpha(q^2)}\frac{d\alpha'\bar{\gamma}^\sigma(\alpha')}{\alpha'\beta(\alpha')}\right]\bar{C}^\sigma(1,\bar{\alpha}(q^2)) \quad (5.2)$$

as follows. Define the parton density and fragmentation functions such that

$$\int_0^1 dx\, x^{\sigma-1}q(x,q^2) =A^\sigma_{\mathrm{SL}}\left(\frac{-p^2}{4\pi\mu^2},\alpha\right)\exp\left[-\int_\alpha^{\bar{\alpha}(q^2)}\frac{d\alpha'\gamma^\sigma(\alpha')}{\alpha'\beta(\alpha')}\right] \quad (5.3)$$

and

$$\int_1^\infty dx\, x^{-\sigma-1}D(x,q^2) =A^\sigma_{\mathrm{TL}}\left(\frac{p^2}{4\pi\mu^2},\alpha\right)\exp\left[-\int_\alpha^{\alpha(q^2)}\frac{d\alpha'\bar{\gamma}^\sigma(\alpha')}{\alpha'\beta(x')}\right]. \quad (5.4)$$

Define also the effective charges $e(X,\bar{\alpha})$ and $\bar{e}(X,\bar{\alpha})$ such that

$$\int_0^1 dx\, x^{\sigma-1}e(x,\bar{\alpha}(q^2))=C^\sigma(1,\bar{\alpha}(q^2)) \quad (5.5)$$

and

$$\int_1^\infty dx\, x^{-\sigma-1}\bar{e}(x,\bar{\alpha}(q^2))=\bar{C}^\sigma(1,\bar{\alpha}(q^2)). \quad (5.6)$$

We can write now the moment Eqs. (5.1), and (5.2) in the form

$$W(x,p^2,q^2)=e(x,\bar{\alpha})\underset{\mathrm{SL}}{\otimes} q(x,q^2) \quad (5.7)$$

and

$$\bar{W}(x,p^2,q^2)=\bar{e}(x,\bar{\alpha})\underset{\mathrm{TL}}{\otimes} D(x,q^2). \quad (5.8)$$

Note that

$$f(x)\otimes g(x)=\int_x^1\frac{dy}{y}f\left(\frac{x}{y}\right)g(y)$$

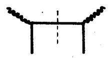
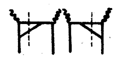
for the spacelike region and

$$f(x)\otimes g(x)=\int_1^x\frac{dy}{y}f\left(\frac{x}{y}\right)g(y)$$

for the timelike region. From (5.7) and (5.8) we obtain the following evolution equations for the structure functions W and $\bar{W}$:

$$q^2\frac{\partial}{\partial q^2}W(x,q^2)=\left[e^{-1}(x,\bar{\alpha})\underset{\mathrm{SL}}{\otimes} q^2\frac{\partial}{\partial q^2}e(x,\bar{\alpha})+P(x,\bar{\alpha})\right]\otimes W(x,q^2) \quad (5.9)$$

TABLE II. Graph-by-graph contribution to the inverse Mellin transform of the anomalous dimensions $p(x)$ and $\bar{p}(x)$ of the spacelike and timelike cut vertices.

	$e(x,\alpha)$	$\bar{e}(x,\alpha)$
	$x\delta(1-x)$	$x\delta(x-1)$
	$-\frac{\alpha}{12}x\delta(1-x)+\frac{\alpha}{12}\frac{x}{(1-x)_+}$	$-\frac{\alpha}{12}x\delta(x-1)+\frac{\alpha}{12}\frac{x}{(x-1)_+}$ *
	$\alpha x\delta(1-x)$	$\alpha x\delta(x-1)$
	$-\alpha x$	αx
	$\alpha x(1-x)$	$\alpha x(x-1)$
	$2\alpha x(1-x)\ln\frac{1}{x}$ $-2\alpha x(1-x)$.	$2\alpha x(x-1)\ln\frac{1}{x}$ $-2\alpha x(x-1)$.

and

$$q^2\frac{\partial}{\partial q^2}\bar{W}(x,q^2)=\left[\bar{e}^{-1}(x,\bar{\alpha})\underset{\mathrm{TL}}{\otimes}q^2\frac{\partial}{\partial q^2}\bar{e}(x,\alpha)+\bar{P}(x,\bar{\alpha})\right]\underset{\mathrm{TL}}{\otimes}\bar{W}(x,q^2)\,, \tag{5.10}$$

where

$$\int_0^1 dx\,x^{\sigma-1}P(x,\bar{\alpha})=-\gamma^{\sigma}(\bar{\alpha}) \tag{5.11}$$

and similarly for $\bar{P}(\bar{x},\bar{\alpha})$. In Table II, we give the contribution to the functions P and $\bar{P}$ diagram by diagram up to order α^2. The total contribution for P is

$$P(x,\alpha)=\alpha[x(1-x)-\tfrac{1}{12}\delta(1-x)]+\alpha^2[\tfrac{1}{2}x(1+x)\ln^2x+\tfrac{1}{6}x(7-x)\ln x+\tfrac{1}{12}x(1-x)\ln(1-x)+\tfrac{11}{12}x(1-x)-\tfrac{1}{432}x\delta(1-x)] \tag{5.12}$$

and for $\bar{P}$

$$\bar{P}(x,\alpha)=\alpha[x(x-1)-\tfrac{1}{12}\delta(1-x)]+\alpha^2[-\tfrac{1}{2}x(1+x)\ln^2x+\tfrac{1}{6}x(x-7)\ln x+\tfrac{1}{12}x(x-1)\ln(x-1)+\tfrac{11}{12}x(x-1)-\tfrac{1}{432}x\delta(x-1)]\,. \tag{5.13}$$

In Table I we give the contribution to the effective charges $e(X,\bar{\alpha})$ and $\bar{e}(x,\bar{\alpha})$ diagram by diagram up to order α for a current $J(x)=:\Phi^2(x):$. The total contribution for $e(x,\bar{\alpha})$ is

$$e(x,\bar{\alpha})=x\delta(1-x)+\bar{\alpha}\left[\tfrac{11}{12}x\delta(1-x)+\tfrac{1}{12}\frac{x}{(1-x)_+}-2x(1-x)\ln x+x(x-2)\right] \tag{5.14}$$

and for $\bar{e}(x,\bar{\alpha})$ is

$$\bar{e}(x,\bar{\alpha})=x\delta(x-1)+\bar{\alpha}\left[\tfrac{11}{12}x\delta(x-1)+\tfrac{1}{12}\frac{x}{(x-1)_+}-2x(x-1)\ln x+x(2-x)\right]\,. \tag{5.15}$$

In Ref. 5 calculating the renormalization-scheme independent quantities $S(x,\bar{\alpha})$ and $\bar{S}(x,\bar{\alpha})$ where

$$S(x,\bar{\alpha})=e^{-1}(x,\bar{\alpha})\otimes q^2\frac{\partial}{\partial q^2}e(x,\bar{\alpha})+P(x,\bar{\alpha}) \tag{5.16}$$

and similarly for $\bar{S}(X,\bar{\alpha})$, we found that in order $\bar{\alpha}^2$,

$$x^3\bar{S}\left(\frac{1}{x},\bar{\alpha}\right)\neq S(x,\bar{\alpha}) \tag{5.17}$$

and this implies considering (5.9) and (5.10) that the Gribov–Lipatov relation[17]

$$x^3\bar{W}\left(\frac{1}{x},q^2\right)=W(x,q^2) \tag{5.18}$$

is violated in the next-to-leading logarithmic approximation.

Let us turn now to the other interesting possible relation between deep-inelastic and annihilation processes, the Drell-Yan relation.[8] We can see from our results that in $(\Phi^3)_6$ theory in the next to-leading logarithmic approximation the following relation between the structure functions $W(x,q^2)$ and $\bar{W}(x,q^2)$ holds:

$$\bar{W}(x,q^2)=-W(x,q^2)\,. \tag{5.19}$$

This last equality has to be understood in the following way. It is known that passing from the $0 < x < 1$ to the $1 < x < \infty$ region we meet cuts of the function $W(x, q^2)$ expressing the opening of thresholds when one goes from a negative value of q^2 to a positive one.[9] Therefore, the exact meaning of the relation (5.19) is the following. As it stands for $x > 1$ the right-hand side is complex (and double valued from the appearance of cuts) and the left-hand side is real and positive. In order to arrive at the timelike structure function $\overline{W}(x, q^2)$, analytically continuing $W(x, q^2)$ in the region $x > 1$, we must start from the nonforward structure function $W(x_1, q_1, x_2, q_2)$ of the process $J(q_1) + \phi(p_1) \to J(q_2) + \phi(p_2)$ in the spacelike region

$$q_1^2, q_2^2, p_1^2, p_2^2 < 0, \quad 0 < x_1, x_2 < 1 .$$

In this region and in the limit $x_1 = x_2 = x$, $q_1^2 = q_2^2 = q^2$, $p_1^2 = p_2^2 = p^2$ the above structure function $W(x_1, q_1, x_2, q_2)$ becomes equal to the structure function $W(x, q)$.

In the timelike region $p_1^2, p_2^2, q_1^2, q_2^2 > 0$, $x_1, x_2 > 1$ the function $W(x_1, q_1, x_2, q_2)$ has additional discontinuities in these variables and we must give prescriptions for the values of these variables around their cuts in order to eliminate these discontinuities. In order to recover the annihilation structure function $\overline{W}(x, q)$ starting from the spacelike one $W(x, q)$ we must give the following definition to $W(x, q)$ Ref. (9):

$$W(x, q, p) = \lim_{\epsilon \to 0} W(x + i\epsilon, x - i\epsilon, q^2 - i\epsilon) \qquad (5.20)$$

with

$$p_1^2 = p^2 - i\epsilon, \quad p_2^2 = p^2 + i\epsilon .$$

The right-hand side of Eq. (5.19) in the physical region for annihilation has to be understood under definition (5.20).

Now we proceed to show that the relation (5.19) is true for $x \neq 1$. For this it is enough to show that the functions $e(x, \overline{\alpha})$, $\overline{e}(x, \overline{\alpha})$, $q(x)$, $D(x)$ obey the same type of relations:

$$\overline{e}(x, \alpha) = -e(x, \alpha) , \qquad (5.21)$$

$$D(x) = -q(x) . \qquad (5.22)$$

This is true because for $x > 1$,

$$\begin{aligned} W(x, q^2) &= e(x, \overline{\alpha}) \underset{SL}{\oplus} q(x) \\ &= -\overline{e}(x, \overline{\alpha}) \underset{TL}{\oplus} D(x) \\ &= -\overline{W}(x, q^2) . \end{aligned} \qquad (5.23)$$

The fact that Eq. (5.21) is true we find from relations (5.14), and (5.15). To prove that relation (5.22) is true one simply has to remember that they are determined by their evolution Eqs. (5.3) and (5.4):

$$q^2 \frac{\partial}{\partial q^2} q(x, q^2) = P(x, \overline{\alpha}) \underset{SL}{\oplus} q(x, q^2) , \qquad (5.24a)$$

$$q^2 \frac{\partial}{\partial q^2} D(x, q^2) = \overline{P}(x, \overline{\alpha}) \underset{TL}{\oplus} D(x, q^2) . \qquad (5.24b)$$

Now for $p^2 = q^2$ we can choose

$$q(x, q^2)\big|_{p^2 = q^2} = \delta(1 - x) , \qquad (5.25a)$$

$$D(x, q^2)\big|_{p^2 = q^2} = \delta(1 - x) . \qquad (5.25b)$$

so for $x \neq 1$, say, $x > 1$, we have

$$q(x, q^2)\big|_{p^2 = q^2} = -D(x, q^2)\big|_{p^2 = q^2} = 0 . \qquad (5.26)$$

On the other hand from Eq. (5.24a) and (5.25b) and the fact that $p(x, \overline{\alpha})$ and $\overline{p}(x, \overline{\alpha})$ satisfy relation (5.19) we find

$$q^2 \frac{\partial}{\partial q^2} q(x, q^2)\bigg|_{p^2 = q^2} = -q^2 \frac{\partial}{\partial q^2} D(x, q^2)\bigg|_{p^2 = q^2} \quad (5.27)$$

and so on for higher derivatives. The procedure we follow for the analytic continuation of the dangerous $\ln(1 - x)$ terms for $x < 1$ to $x > 1$ is

$$\begin{aligned} \ln(1 - x) &= \tfrac{1}{2}[\ln(1 - x + i0) + \ln(1 - x - i0)] \\ &\to \ln(x - 1) \quad \text{for } x > 1 . \end{aligned}$$

Thus by Taylor expansion of $q(x, q^2)$ and $D(x, q^2)$ around $p^2 = q^2$ we obtain the desired result $(x > 1)$:

$$q(x, q^2) = -D(x, q^2) . \qquad (5.28)$$

Finally, we would like to show the meaning of analytic continuation of the distributions $1/(1-x)_+$ of the spacelike region $(x < 1)$ and $1/(x-1)_+$ of the timelike region $(x > 1)$. The distribution $1/y_+$ is defined in general[6] as

$$\begin{aligned} \int_{-\infty}^{\infty} dy \frac{1}{y_+} \varphi(y) &= \int_0^1 dy \frac{\varphi(y) - \varphi(0)}{y} + \int_1^{\infty} dy \frac{\varphi(y)}{y} \\ &= \lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} dy \frac{\varphi(y)}{y} - \int_{\epsilon}^{1} dy \frac{\varphi(0)}{y} . \end{aligned}$$

In the spacelike case the domain of the functions $\varphi \leq x \leq 1$ and one has

$$\int_0^1 \frac{dx}{(1-x)_+} \varphi(x) = \int_0^1 dx \frac{\varphi(x) - \varphi(1)}{1 - x} .$$

In the timelike case where the corresponding domain is $\infty > x \geq 1$

$$\int_1^\infty \frac{dx}{(x-1)_+}\varphi(x) = \lim_{\epsilon\to 0}\int_{1+\epsilon}^\infty \frac{dx\,\varphi(x)}{(x-1)} - \int_{1+\epsilon}^2 \frac{dx\,\varphi(1)}{(x-1)}$$
$$= \lim_{\epsilon\to 0}\int_0^{1-\epsilon} \frac{d\overline{x}}{1-\overline{x}}\frac{1}{\overline{x}}\varphi\left(\frac{1}{\overline{x}}\right) - \int_0^{1-\epsilon}\frac{dt\,\varphi(1)}{1-t}$$
$$= \int_0^1 \frac{dx}{(1-\overline{x})_+}\left[\frac{1}{\overline{x}}\varphi\left(\frac{1}{\overline{x}}\right)\right].$$

VI. CONCLUSION

The calculations have been presented for the cut vertices (spacelike and timelike) in $(\Phi^3)_6$ theory up to order g^4 and for the coefficient functions of deep-inelastic and annihilation processes up to order g^2. The explicit formulation of the dimensional regularization procedure for cut vertices allows for the complete characterization of the renormalization-scheme dependence. Convenient phase-space variables render the calculations transparent and they illustrate the relation of timelike and spacelike regions. Explicit examples offer the ground for the demonstration of the necessary computational techniques which, we believe, simplify the calculation enough.

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