Extended gauge theory and its mass spectrum

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Based on the recently proposed restricted gauge theory an extended gauge theory is constructed which takes into account not only the local structure but also the topological structure of a non-Abelian gauge symmetry into its dynamics. The gauge potential is made of two parts, the binding gluons which provide the color-confining force and the valence gluons which behave as a gauge-covariant colored vector source. The physical spectrum of the theory includes the magnetic glueballs which are the massive collective modes of the vacuum condensation, and the electric glueballs made of the valence gluons which form a set of linearly rising trajectories of their own. Based on the group SU(2) the slope parameter α'_X of the leading glueball trajectories is estimated to be $\alpha'_X \simeq 0.47\alpha'$ ($\alpha' \simeq 0.9$ GeV^2 is the Regge slope parameter of the mesons). Also, a crude estimate of the typical mass scale of the ground states of the trajectories is presented.

I. INTRODUCTION

In spite of its overwhelming popularity as the leading candidate for the theory of strong interaction, quantum chromodynamics (QCD), or non-Abelian gauge theory in general, has so far defied us to reveal its physical meaning, in particular its mass spectrum of the physical states. Clearly the problem of finding out the physical spectrum of the theory is intimately connected to the problem of color confinement. On the issue of confinement Nambu[1] has suggested that color confinement could occur in a similar way as magnetic flux confinement occurs in an ordinary superconductor due to the Meissner effect. The idea has further been elaborated by Mandelstam,[2] among others, who observed that the condensation of the non-Abelian magnetic monopoles of the QCD vacuum could produce a dual Meissner effect which may confine the color electric flux carried by the quarks. To realize this idea one has, of course, to show that magnetic condensation does occur in QCD, and at the same time show that the underlying dynamics does guarantee the dual Meissner effect. Recently, based on the group SU(2), it has been shown that[3,4] indeed one can construct out of QCD a dual gauge theory, called restricted chromodynamics (RCD), which could produce the magnetic condensation dynamically for its vacuum and at the same time exhibit the desired dual dynamics that guarantees us the dual Meissner effect. Thus RCD does provide us with the mechanism of magnetic confinement of color. Owing to the confinement mechanism the quarks as well as the monopoles disappear from the physical spectrum of the theory. At the same time the theory must contain two magnetic glueballs,[4] i.e., the massive scalar and axial-vector collective modes of the condensed vacuum, in its mass spectrum.

The restricted theory is extracted from the unrestricted QCD by imposing an extra internal symmetry called magnetic symmetry to its gauge potential. Consequently RCD does not contain the full dynamic degrees of QCD. Nonetheless, as we will show in the following, it does constitute a self-consistent subdynamics of QCD which is very interesting in its own right. Thus, with its confinement mechanism at hand, one may ask what is the complete physical spectrum of RCD. More importantly, when one removes the magnetic symmetry of RCD and reactivates the full dynamical degrees of QCD one would like to know whether the confinement mechanism will still work, and if so, what will be the full physical spectrum of QCD, and how one could obtain it. The purpose of this paper is to provide definite answers to these questions as far as possible. Based on the group SU(2) we obtain the following results in this paper. First, on the physical spectrum of RCD, we prove that the allowed physical states made of the quarks have to be color-singlet states of either $q\bar{q}$ mesons, or qq baryons. Any other exotic state is forbidden. Secondly, by dividing the QCD potential into *binding gluons* (the RCD piece) and *valence gluons* (the reactivated piece) we show that removing magnetic symmetry does not affect the confinement mechanism. Thus the confinement mechanism of RCD must still work in QCD. Third, we show that the physical spectrum of QCD must contain the color-singlet "electric glueballs" made of the symmetric combination of the valence-gluon pair. Similar to the mesons, these electric glueballs will have to form linearly rising trajectories. Finally we show how to calculate the slope parameter of the glueball trajectories. Also, for completeness we give a crude estimate of the typical mass scale of the ground states of the leading glueball trajectories.

To obtain the physical spectrum of RCD one must realize that the dual Meissner effect guarantees us only color screening, but *not* color confinement. To complete the proof of confinement it becomes crucial to observe that in our theory there exists a residual symmetry called *color reflection invariance*, under which any physical state has to be invariant after the confinement mechanism has set in. This reflection invariance, which originates from the fact that the non-Abelian magnetic charge can be well defined only up to the Weyl reflection, turns out to be precisely what one needs to establish that the physical spectrum of the theory is exclusively made of color singlets. Thus, for SU(2), the physical spectrum of RCD is made of the color-singlet $q\bar{q}$ mesons and qq baryons and the two magnetic glueballs, with no other exotic states.

One could obtain the (unrestricted) QCD potential by removing the magnetic symmetry imposed on the RCD potential and by reactivating the suppressed degrees of freedom. Thus the QCD potential may be written as the sum of the dual potential of RCD and the newly reactivated piece. Then one can easily show that the RCD piece of the potential plays the same role as before to provide the binding force of color. For this reason we will call the RCD piece of the gluons the binding gluons. The reactivated piece, however, can be shown to form *a gauge-covariant multiplet* which behaves as a colored vector source. This means that removing magnetic symmetry merely amounts to having an additional colored vector source which forms an adjoint representation of the gauge group. For this reason we will call the reactivated gluons the valence gluons. Then QCD can be interpreted as nothing more than RCD which has the valence gluons as an additional source. As an immediate consequence the physical spectrum of QCD must contain the electric glueballs made of the valence-gluon pair. Naturally, owing to color reflection invariance only the color-singlet combination (i.e., the symmetric combination) of the gluon pair appears in the physical spectrum.

Clearly the electric-glueball spectrum will form another set of linearly rising trajectories, among which one would expect nine (3×3) leading ones. The existence of these glueball trajectories perhaps may not come as a surprise. What is more interesting to know are, of course, the slope parameters of the leading trajectories, and the masses of their ground states. The slope α'_X can easily be calculated by estimating the energy flux carried by the string solution of the valence-gluon pair. Again with SU(2) we find that $\alpha'_X \simeq 0.47\alpha'$ ($\alpha' = 0.9$ GeV^{-2} is the Regge slope of the meson trajectories). Now it is more difficult to estimate the masses of the ground states of the leading trajectories. However, with some simplifying technical assumptions we obtain a crude estimate of the typical mass scale of the ground states which turns out to be about twice as heavy (~1.6 GeV) as the ρ mass, the typical mass scale of the ground states of the leading meson trajectories.

The paper is organized as follows. In Sec. II the restricted gauge theory is briefly reviewed for later convenience. Throughout the paper we will use the group SU(2) as the color-gauge group to avoid unnecessary complications. The realistic color SU(3) gauge group will be treated in detail in another paper.[4] In Sec. III we show in detail how the masses of the magnetic glueballs can be estimated. The masses of the scalar and the vector modes are obtained to be around 2.4 GeV and 1.6 GeV, respectively. In Sec. IV we derive the criterion of color reflectance invariance for a physically acceptable state of RCD, and show that the mesons and the baryons of RCD are exclusively made of $q\bar{q}$ and qq color singlets. In Sec. V the concept of binding gluons and valence gluons are introduced and *the extended gauge theory*, or *QCD in the large*, is proposed. Then we show that the extended theory is nothing more than the restricted one but with the valence gluons as the additional colored source of the theory. As a consequence the extended theory must contain the glueball states made of the valence-gluon pair in its spectrum. In Sec. VI we show how to estimate the slope parameter of the linearly rising glueball trajectories as well as their ground-state masses. Finally, in Sec. VII, various aspects of the theory, in particular some unsettled problems, are discussed.

II. RESTRICTED GAUGE THEORY: A BRIEF REVIEW

The restricted gauge theory[3] describes an interesting dual dynamics between the color isocharge (i.e., the electric charge) and the topological charge (i.e., the magnetic charge) of a non-Abelian symmetry in which color confinement can be made manifest. More importantly, the restricted theory governs, as we will see later on, a subdynamics of the unrestricted theory which characterizes the vacuum structure of the theory. For these reasons we will start by briefly reviewing the restricted gauge theory in this section.

The mathematical foundation for the restricted gauge theory comes from the observation[3] that the non-Abelian gauge symmetry does allow an extra internal symmetry called magnetic symmetry which restricts and reduces the dynamical degrees of the theory while keeping the full gauge degrees of freedom intact. The magnetic symmetry may be imposed by insisting that the gauge potential $\vec{B}_\mu$ must satisfy the constraint

$$D_\mu\hat{m} = \partial_\mu\hat{m} + g\vec{B}_\mu \times \hat{m} = 0 \tag{1}$$

for an arbitrary scalar multiplet $\hat{m}$ which forms an adjoint representation of the gauge group G. Notice that the condition is a gauge-covariant constraint, so that the gauge symmetry remains unbroken by the constraint. Mathematically speaking, this means that the internal fiber space admits an additional Killing symmetry[3] characterized by $\hat{m}$ which commutes with the gauge symmetry itself. From the constraint it becomes obvious that one may always normalize $\hat{m}$ to be

$$\hat{m}^2 = 1 \tag{2}$$

without loss of generality, and we will do so in the following. For SU(2) the constraint restricts the potential to the form

$$\vec{B}_\mu = A_\mu\hat{m} - \frac{1}{g}\,\hat{m}\times\partial_\mu\hat{m}\,, \tag{3}$$

where A_μ is the electric component of $\vec{B}_\mu$ which is parallel to $\hat{m}$ and thus is not restricted by the constraint.

One of the virtues of magnetic symmetry is that it can be used to describe the topological (i.e., the magnetic) structure of the gauge symmetry. This is so since the multiplet $\hat{m}$ may be viewed to define the homotopy of the mapping $\Pi_2(S^2)$

$$\hat{m};\, S_R^2 \rightarrow S^2 = \mathrm{SU}(2)/\mathrm{U}(1)\,, \tag{4}$$

where S_R^2 is the two-dimensional sphere of the three-dimensional space and S^2 is the group coset space fixed by $\hat{m}$. So a topological singularity of $\hat{m}$ may be identified as a pointlike monopole of non-Abelian symmetry. Then it becomes clear that by imposing magnetic symmetry on the potential one may bring the topological structure into the dynamics explicitly. To illustrate this point let us make a gauge transformation and go to the magnetic gauge[3] by rotating $\hat{m}$ to the space-time independent $\hat{\xi}_3$,

$$\hat{m} \xrightarrow[U]{} \hat{\xi}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{5}$$

In this gauge the potential (3) may be written as

$$\vec{B}_\mu \longrightarrow (A_\mu + C_\mu^*)\hat{\xi}_3\,, \tag{6}$$

where C_μ^* is the magnetic potential of the monopole which is completely fixed by $\hat{m}$ up to the Abelian magnetic gauge degrees of freedom. This demonstrates the fact that in the magnetic gauge one may indeed bring the topological property of $\hat{m}$ down to a dynamical variable C_μ^*.

The restricted gauge theory may be written down explicitly using the potential (3). Let us consider the SU(2) QCD Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}\vec{G}_{\mu\nu}{}^2 + \bar{\Psi}i\gamma^\mu D_\mu\Psi - m_0\bar{\Psi}\Psi\,, \tag{7}$$

where $\vec{G}_{\mu\nu}$ is the gauge field strength and Ψ is the quark doublet. Then the Lagrangian for the restricted chromodynamics (RCD) could, in principle, be obtained by substituting the unrestricted potential by the restricted one (6) in the above Lagrangian (7). However, the Lagrangian obtained by this simple substitution will have a few undesirable features. First, the monopole appears as a pointlike object, but not as a regular field. Furthermore, C_μ^* describes the magnetic field of the monopole by a spacelike potential, and contains the well-known string singularity. One may get rid of these undesirable features by introducing the dual magnetic potential C_μ which can describe the magnetic field of the monopole with a regular timelike potential, and at the same time, by introducing a complex scalar field ϕ for the monopole. Thus one may obtain the following phenomenological Lagrangian[3,4] for RCD,

$$\begin{aligned}\mathcal{L}_{(R)} = {} & \bar{r}i\gamma^\mu\left(\partial_\mu + \frac{g}{2i}\,(A_\mu + C_\mu^*)\right)r \\ & + \bar{b}i\gamma^\mu\left(\partial_\mu - \frac{g}{2i}\,(A_\mu + C_\mu^*)\right)b \\ & + m_0(\bar{r}r + \bar{b}b) - \tfrac{1}{4}F_{\mu\nu}{}^2 - \tfrac{1}{2}F_{\mu\nu}H_{\mu\nu} - \tfrac{1}{4}H_{\mu\nu}^*{}^2 \\ & + \left|\left(\partial_\mu + i\,\frac{4\pi}{g}\,(A_\mu^* + C_\mu)\right)\phi\right|^2 - V(\phi^*\phi)\,,\end{aligned} \tag{8}$$

where r and b are the red and the blue quarks, A_μ and C_μ are the regular potentials that describe the electric and the magnetic charges with the ordinary timelike potentials

$$\begin{aligned}F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\,, \\ H_{\mu\nu}^* &= \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}H^{\rho\sigma} = \partial_\mu C_\nu - \partial_\nu C_\mu\,,\end{aligned} \tag{9}$$

A_μ^* and C_μ^* are the singular "dual potentials"[2,3] of the fields $F_{\mu\nu}^*$ and $H_{\mu\nu}$. Classically these dual potentials could be identified as the singular potentials which describe the corresponding charges with a spacelike potential with the well-known string singularity,

$$\begin{aligned}F_{\mu\nu}^* &= \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} = \partial_\mu A_\nu^* - \partial_\nu A_\mu^*\,, \\ H_{\mu\nu} &= \partial_\mu C_\nu^* - \partial_\nu C_\mu^*\,.\end{aligned}$$

In spite of the well-known fact that in field-theoretic formulation such naive identification may raise a serious difficulty,[3,5] we have used the singular potentials in the above Lagrangian as a mnemonic means to represent the dual interaction that exists between the quarks and the monopoles. The Lagrangian may then be viewed as an effective Lagrangian that can describe the dual dynamics of RCD at the phenomenological level, just as the Ginsburg-Landau Lagrangian is regarded as an effective Lagrangian for the theory of a superconductor. With the Lagrangian (8) at hand one may now pursue its far-reaching consequences.

III. QUARK CONFINEMENT AND MAGNETIC GLUEBALLS

One of the virtues of the Lagrangian (8) is that the theory could be argued to have two phases,[3] the normal phase in which both the quarks and the monopoles appear as physical particles and the confinement phase in which both disappear from the physical spectrum. Furthermore, in the confinement phase the theory contains two magnetic glueballs, a scalar mode and an axial-vector mode, whose masses could be estimated to be of the order of a few GeV. In this section we briefly review the confinement mechanism and show how one can estimate the masses of the predicted glueballs.

The confinement mechanism of RCD can easily be understood.[3] In the absence of quarks the Lagrangian (8) may be reduced to

$$\mathcal{L}^{(m)}_{(R)} = -\tfrac{1}{4} H^{*2}_{\mu\nu} + \left| \left(\partial_\mu + i \frac{4\pi}{g} C_\mu \right) \phi \right|^2 - V(\phi^*\phi) \,. \tag{10}$$

Now one immediately notices that the above Lagrangian looks exactly like the Ginsburg-Landau Lagrangian of the theory of superconductivity, except that here the role of the order parameter is played by the monopole field and the role of the electric potential by the magnetic one. Thus, so far as the effective potential $V_{\mathrm{eff}}(\phi^*\phi)$ of the theory could be shown to break the magnetic symmetry dynamically, the theory will obviously explain the phenomenon of electric flux confinement, i.e., the dual Meissner effect. In other words, *color confinement in* RCD *will be enforced by the dynamical breaking of magnetic symmetry.* Now to obtain the desired dynamical symmetry breaking, notice that it is natural for us to require that the Lagrangian (10) must be both ultraviolet finite and infrared unstable since the QCD Lagrangian (7) bears the same feature. This fixes the form of the potential in (10) as

$$V(\phi^*\phi) = \frac{\lambda_0}{2} (\phi^*\phi)^2 \,. \tag{11}$$

Then, by requiring infrared instability by keeping the renormalized mass of the ϕ field vanishing, one may indeed show that the effective potential V_{eff} of the theory obtained by the loop-expansion technique could break the magnetic symmetry dynamically in the strong-coupling limit, as has been argued by Coleman and Weinberg.[6] In one-loop approximation they found that

$$V_{\mathrm{eff}} = \frac{24\pi^2}{g^4} \left[\phi_0{}^4 + (\phi^*\phi)^2 \left(2 \ln \frac{\phi^*\phi}{\phi_0{}^2} - 1 \right) \right] , \tag{12}$$

where ϕ_0 is the vacuum expectation value of the ϕ field,

$$\phi_0 = \langle \phi^*\phi \rangle_0^{1/2} \,. \tag{13}$$

Once the magnetic condensation of the vacuum is established, the confinement of any colored flux becomes unavoidable.

An immediate consequence of the magnetic condensation is the existence of two massive modes, a scalar and an axial vector, that characterize the vacuum. *The mass of the scalar mode μ determines how fast the perturbative vacuum around a colored source reaches the condensation, whereas that of the vector mode m determines the penetration length of the colored flux.* The effective potential (12) fixes the ratio of these mass scales κ,

$$\kappa^2 = \left(\frac{\mu}{m} \right)^2 = \frac{3}{2\pi\alpha_s} \,, \tag{14}$$

where $\alpha_s = g^2/4\pi$ is the fine-structure constant of RCD. The masses of the glueballs can be estimated by evaluating the string tension of the classical string solutions of the quark pair. Let us denote by ρ, φ, and z the cylindrical coordinates of the space and choose the string axis as the z axis. Then, with the string ansatz

$$\phi = \phi_0 R(\zeta) e^{i\varphi} \,, \qquad (\zeta = m\rho) \qquad \vec{E} = \hat{z} m \phi_0 E(\zeta) \,, \tag{15}$$

one obtains, using the effective potential (12), the following equations of motion:

$$\ddot{R} + \frac{1}{\zeta} \dot{R} - \frac{3}{2\pi} \frac{1}{\alpha_s} R^3 \ln R = \frac{1}{2} \frac{\dot{E}^2}{R^3} \,, \qquad \ddot{E} + \left(\frac{1}{\zeta} - 2 \frac{\dot{R}}{R} \right) \dot{E} - R^2 E = 0 \,. \tag{16}$$

Of course any acceptable solution must satisfy the following boundary condition:

$$R(\infty)=1\,,\quad E(\infty)=0\,. \tag{17}$$

Furthermore, for the string made of the quark pair the solution must satisfy the electric flux quantization condition

$$\frac{1}{\sqrt{2}}\int_0^\infty \zeta d\zeta\, E(\zeta)=1\,. \tag{18}$$

Now one can easily calculate the string tension k of the solution, which is nothing more than the energy per unit length carried by the string. One finds

$$k=\frac{1}{2\pi\alpha'}=\gamma\phi_0{}^2=\frac{\alpha_s}{8\pi}\gamma m^2\,, \tag{19}$$

where α' is the Regge slope parameter ($\alpha'=0.9$ GeV^{-2}) and γ is a dimensionless parameter given by

$$\gamma=\int_0^\infty 2\pi\zeta\left(\frac{1}{2}E^2+\dot{R}^2+\frac{1}{2}\frac{\dot{E}^2}{R^2}+\frac{3}{16\pi\alpha_s}[1+R^4(4\ln R-1)]\right)d\zeta$$
$$=\int_0^\infty 2\pi\zeta\left(\frac{1}{2}E^2+\frac{3}{16\pi\alpha_s}[1-R^4(4\ln R+1)]\right)d\zeta\,. \tag{20}$$

Then the masses μ and m can be evaluated from (14) and (19) as functions of α_s. The numerical results of the masses μ and m obtained by computer[7] are shown in Table I. Of course, to determine the masses one must further know the right value of α_s of the theory. Remember that the theory has two parameters α_s and ϕ_0, so that the experimental input $\alpha'=0.9$ GeV^{-2} alone cannot fix the theory completely. We need one more experimental input. In principle the additional information should be provided by a direct measurement of α_s which is fixed by the coupling strength between the two magnetic glueballs as shown in Fig. 1. In practice, however, one has to find out a way

TABLE I. The masses μ and m of the magnetic glueballs as functions of α_s. The result is obtained using the Coleman-Weinberg potential (12).

$\kappa^2(=3/2\pi\alpha_s)$	α_s	γ	μ (GeV)	m (GeV)
10	0.048	9.75	9.745	3.082
4	0.119	7.80	4.376	2.188
2	0.239	6.75	2.347	1.659
1	0.477	5.90	1.256	1.256
0.5	0.955	5.05	0.679	0.960
0.25	1.910	4.30	0.368	0.736
0.1	4.775	3.04	0.175	0.553

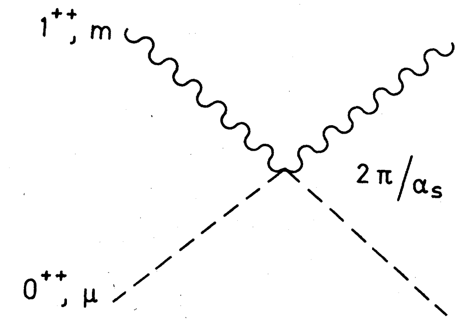

FIG. 1. The Feynman graph that determines the fine-structure constant α_s of the theory. The coupling strength between the two massive magnetic glueballs fixes α_s.

to estimate α_s from the existing experimental data. A natural way to do so seems to be to *assume* that the α_s of the theory is given by the running coupling constant $\bar{\alpha}(s)$ of QCD fixed by the renormalization-group equation at $s=\mu m$. This assumption leads us to $\alpha_s\simeq 0.22$ as shown in Fig. 2(I). From this we obtain $\mu\simeq 2.5$ GeV and $m\simeq 1.7$ GeV.

At this point one may reject the estimate as self-inconsistent. Indeed, the validity of the effective potential (12) obtained from one-loop approximation may be questionable when $\alpha_s\simeq 0.22$ since the one-loop approximation may be taken seriously[6] only in the strong-coupling limit when $\alpha_s \gg 1$. As a matter of fact, the significance of the potential (12) consists in its ability to demonstrate the desired dynamical symmetry breaking rather than in its practical usefulness as a good effective potential. Thus it may be worthwhile for us to take a more phenomenological point of view and to repeat the calculation using the following familiar

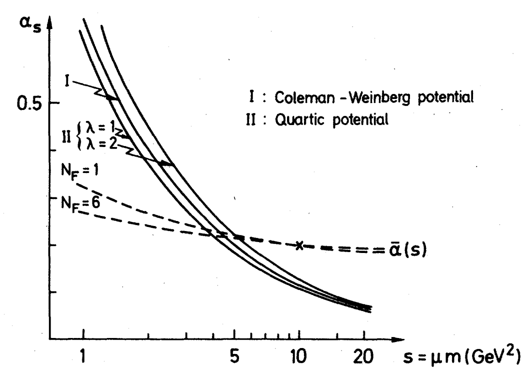

FIG. 2. An estimate of α_s. The solid curves I and II show α_s as functions of μm. The dashed curves represent the running coupling constant $\bar{\alpha}(s)$. The reference point is chosen as $\bar{\alpha}=0.2$ at $s=10$ GeV^2, and the number of the flavors is denoted by N_F.

effective potential:

$$V_{\rm eff}=\frac{48\pi^2}{g^4}\lambda(\phi^*\phi-\phi_0{}^2)^2\,. \qquad (21)$$

The advantage of the above potential is that we no longer have to satisfy the mass ratio (14) obtained by the one-loop approximation. From the phenomenological point of view this is important because *a priori* one does not know how good the strong-coupling assumption will be in reality. From the potential (21) one obtains

$$\kappa^2=\left(\frac{\mu}{m}\right)^2=\frac{3}{2\pi\alpha_s}\lambda\,. \qquad (22)$$

Notice that when $\lambda=1$ the above mass ratio reduces to (14). Now, the potential (21) yields the following equations of motion for the string:

$$\ddot{R}+\frac{1}{\xi}\dot{R}-\frac{3}{4\pi\alpha_s}\lambda(R^2-1)R=\frac{1}{2}\frac{\dot{E}^2}{R^3}\,,$$
$$\ddot{E}+\left(\frac{1}{\xi}-2\frac{\dot{R}}{R}\right)\dot{E}-R^2E=0\,. \qquad (23)$$

In this case γ is given by

$$\gamma=\int_0^\infty 2\pi\xi\left(\frac{1}{2}E^2+\dot{R}^2+\frac{1}{2}\frac{\dot{E}^2}{R^2}+\frac{3}{8\pi\alpha_s}\lambda(R^2-1)^2\right)d\xi$$
$$=\int_0^\infty 2\pi\xi\left(\frac{1}{2}E^2+\frac{3}{8\pi\alpha_s}\lambda(1-R^4)\right)d\xi\,. \qquad (24)$$

Then from (14) and (19) μ and m may be determined as functions of α_s. The numerical results[7] are listed in Table II.

If we make the same assumption as before that α_s is fixed by the running coupling constant $\overline{\alpha}(s)$ at $s=\mu m$, we find again $\alpha_s\simeq 0.22$ as shown in Fig. 2(II). From this we obtain $\mu\simeq 2.4$ GeV, $m\simeq 1.6$ GeV when $\lambda=1$, and $\mu\simeq 3.2$ GeV, $m\simeq 1.6$ GeV when $\lambda=2$. At this point one may have noticed that while the two potentials (12) and (21) should have yielded the same masses by construction when $\lambda=1$, this does not appear to be exactly the case in our numerical results. Indeed in principle one should

TABLE II. The masses of the magnetic glueballs as functions of α_s. The result is obtained with the quartic potential (21).

$\kappa^2 (=3\lambda/2\pi\alpha_s)$	α_s/λ	γ	μ(GeV), $\lambda=1$	m (GeV), $\lambda=1$
10	0.048	10.20	9.530	3.013
4	0.119	8.30	4.205	2.102
2	0.239	7.20	2.273	1.607
1	0.477	6.28	1.218	1.218
0.5	0.955	5.40	0.656	0.929
0.25	1.910	4.60	0.355	0.711
0.1	4.775	3.35	0.167	0.527

have predicted different values of α', using the same values of the masses and α_s as the experimental inputs. In practice, however, this is not feasible, so we have used the same input α' to predict the masses. This explains the small numerical discrepancy. Now that we have completed our estimate of the masses of the magnetic glueballs, in Sec. IV we will discuss how one may obtain the hadron spectrum of RCD.

IV. COLOR REFLECTION INVARIANCE AND HADRON SPECTRUM

In this section we will show how one can obtain the hadron spectrum of RCD. For this purpose first notice that the dual dynamics actually does *not* guarantee us color confinement, since it merely tells us that any physical state has to be color neutral. To clarify the situation, let us consider color-neutral $q\overline{q}$ mesons and qq baryons. There are four such states, with the following color quantum numbers C and C_3 in the magnetic gauge:

$$|C,C_3\rangle=|0,0\rangle=\frac{|r\overline{r}\rangle+|b\overline{b}\rangle}{\sqrt{2}}\,,$$
$$|C,C_3\rangle=|0,0\rangle=\frac{|rb\rangle-|br\rangle}{\sqrt{2}}\,, \qquad (25)$$

and

$$|C,C_3\rangle=|1,0\rangle=\frac{|r\overline{b}\rangle-|b\overline{r}\rangle}{\sqrt{2}}\,,$$
$$|C,C_3\rangle=|1,0\rangle=\frac{|rb\rangle+|br\rangle}{\sqrt{2}}\,, \qquad (26)$$

among which only the first two are color singlets. Clearly the issue then is how, if possible, one can get rid of the other two color-neutral states from the physical spectrum. To show how, it is crucial to observe that *even after one has chosen one's own magnetic gauge one can uniquely assign the color electric charge of a source only within a certain reflection degrees of freedom* which we specify in the following. The reflection invariance originates from the fact that within the framework of a non-Abelian gauge theory there is intrinsically no way to tell the absolute signature of the non-Abelian magnetic charge. This can easily be seen from the fact that if $\hat{m}$ is a magnetic symmetry which satisfies the condition (1), so must be also $-\hat{m}$. This, of course, is a restatement of a well-known fact that the non-Abelian magnetic charge is well defined only up to the Weyl reflection.[8] As a consequence any physical state has to be invariant under the reflection

$$\hat{m}\to-\hat{m}\,. \qquad (27)$$

Now under the reflection (28) the restricted potential (3) transforms as

$$\vec{B}_\mu = A_\mu \hat{m} - \frac{1}{g}\hat{m}\times\partial_\mu\hat{m}$$
$$\to -A_\mu\hat{m} - \frac{1}{g}\hat{m}\times\partial_\mu\hat{m}\,. \tag{28}$$

This means that the theory has to be invariant under the color reflection

$$A_\mu \to -A_\mu\,. \tag{29}$$

The effect of the reflection (29) on the quark doublet can easily be figured out. Since the reflection has to be a symmetry within the gauge group SU(2), the theory must be invariant under the following four-element reflection group made of matrices:

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\ \begin{pmatrix}0&1\\-1&0\end{pmatrix},$$
$$\begin{pmatrix}-1&0\\0&-1\end{pmatrix},\ \begin{pmatrix}0&-1\\1&0\end{pmatrix}. \tag{30}$$

Notice that the reflection group is neither the center group Z_2 nor the Weyl reflection group of SU(2). It is a generalization of the Weyl reflection. We will call this symmetry *the color reflection invariance*. With the reflection invariance one can now determine the hadron spectrum uniquely. Among the $q\bar{q}$ mesons and the qq baryons one may easily find that only the color-singlet states (25) satisfy the reflection invariance. In conclusion, the color reflection invariance of the magnetic condensation excludes any colored states from the physical spectrum of the theory.

The color reflection invariance is important enough that it is worthwhile to rephrase it in a different way. Applied to the monopole it means that the theory must be invariant under the Weyl reflection. Physically speaking this means that there is no way for us to tell whether the magnetic condensation of the vacuum is made of monopoles or antimonopoles in our theory. Consequently the magnetic current that confines the color electric flux has to be made of *the symmetric combination* of the two oppositely charged monopoles. For the string configuration this means that *the electric flux of the string does not carry any absolute sense of helicity*. Accordingly, only the symmetric combination of two opposite helical modes of the string must be acceptable as physical. Translated in terms of the quarks, this is of course precisely the reflection invariance that we have required for the physical hadron states in the above.

At this point it is perhaps instructive to compare our confinement mechanism with that of an ordinary superconductor. Unlike our case, in ordinary superconductors the magnetic flux line is accompanied by the surrounding supercurrent which is exclusively made of the electron pairs alone. There is no supercurrent made of the positron pairs in an ordinary superconductor. Accordingly, the string has one well-defined helical mode. On the other hand, in our case the "supercurrent" is made of the symmetric combination of two oppositely charged monopoles. In this respect we notice that the two confinement mechanisms are not exactly dual to each other.

Before we close this section let us discuss the J^{PC} property of the magnetic glueballs. First the scalar mode may naturally be identified as a 0^{++} object. As for the vector mode, one would ordinarily identify it as 1^{+-} object. However, we will now argue that the vector mode could be identified as a 1^{++} object. To show this one has to determine how the magnetic potential C_μ should transform under the charge-conjugation operation. Now, since the potential C_μ has been extracted from $\hat{m}$, one might first look for the charge-conjugation property of $\hat{m}$ before one determines that of C_μ. Clearly $\hat{m}$ must transform to $-\hat{m}$ under the charge conjugation. Under the reflection, however, the magnetic part of the potential (3) should obviously remain invariant. This suggests that the vector mode must carry a positive charge-conjugation quantum number. This is not to say that our Lagrangian (8) should violate the charge-conjugation invariance. Under the reflection $\hat{m}$ to $-\hat{m}$ the magnetic charge $4\pi/g$ of the monopole automatically changes its signature. This peculiar feature comes from the fact that the magnetic charge is topological in its origin. Consequently, the Lagrangian (8) remains invariant under charge conjugation.

V. EXTENDED GAUGE THEORY

The restricted chromodynamics has been obtained by imposing the magnetic symmetry (1) which restricts the potential to the form (3). To construct the unrestricted theory one obviously has to remove the magnetic symmetry and reactivate the suppressed dynamical degrees of freedom. For this purpose let us write the unrestricted potential as

$$\vec{B}_\mu = A_\mu\hat{m} - \frac{1}{g}\hat{m}\times\partial_\mu\hat{m} + \vec{X}_\mu\,, \tag{31}$$

where, as before, $\hat{m}$ is an arbitrary scalar triplet with unit length which forms an adjoint representation of the group, A_μ is the Abelian component of the potential which is parallel to $\hat{m}$

$$A_\mu = \hat{m}\cdot \vec{B}_\mu ,$$

and we have expressed the reactivated part by $\vec{X}_\mu$. Now let us clarify the physical motivation behind the decomposition, and the meaning of each term in the potential. Clearly the first two terms are just the components needed for the restricted theory. As has been emphasized in Sec. II, the introduction of the magnetic multiplet $\hat{m}$ into the potential allows us to include the topological degrees of freedom into the dynamics and to deal with both the local and the global structures of the gauge symmetry on the same footing at the level of the dynamics. In this respect notice that the potential (31) is *not* the same as the one that one has in conventional gauge theory which contains only the local gauge degrees of freedom. To understand this, notice that in the magnetic gauge the potential (31) may be written as

$$\vec{B}_\mu = X^1_\mu \hat{\xi}_1 + X^2_\mu \hat{\xi}_2 + (A_\mu + C^*_\mu)\hat{\xi}_3 . \tag{32}$$

Obviously had we taken into account only the local degrees of freedom as one normally does in conventional gauge theory, we would have ended up with only X^1_μ, X^2_μ, and A_μ, but not with the magnetic potential C^*_μ in (32). Clearly the magnetic degrees of freedom represent the nontrivial topological degrees of freedom that the global structure of the underlying gauge symmetry can exhibit, and contain the information of *the gauge symmetry in the large*. For this reason we will call the potential (31) *the extended potential* and the corresponding dynamics *the extended QCD*, or *QCD in the large*.

The reactivated component $\vec{X}_\mu$ may be interpreted as a *gauge-covariant colored vector source* of the theory. To justify this interpretation notice that $\vec{X}_\mu$ can formally be written as

$$\vec{X}_\mu = \frac{1}{g}\hat{m}\times D_\mu \hat{m} , \tag{33}$$

which tells us that $\vec{X}_\mu$ is a gauge-covariant vector field. Indeed one can easily verify this fact directly from (31). To do this notice that under an arbitrary infinitesimal gauge transformation specified by $\vec{\theta}(x)$, one must have

$$\begin{aligned} \delta\vec{B}_\mu &= -\vec{\theta}\times\vec{B}_\mu - \frac{1}{g}\partial_\mu\vec{\theta} , \\ \delta\hat{m} &= -\vec{\theta}\times\hat{m} , \\ \delta A_\mu &= \delta\hat{m}\cdot\vec{B}_\mu + \hat{m}\cdot\delta\vec{B}_\mu . \end{aligned} \tag{34}$$

From this one can easily deduce that

$$\begin{aligned} \delta\vec{X}_\mu &= \delta\vec{B}_\mu - \delta\left(A_\mu\hat{m} - \frac{1}{g}\hat{m}\times\partial_\mu\hat{m}\right) , \\ &= -\vec{\theta}\times\vec{X}_\mu . \end{aligned} \tag{35}$$

This confirms the fact that $\vec{X}_\mu$ transforms covariantly under the gauge transformation. It is remarkable that one can single out the gauge-covariant piece of the potential so easily by the decomposition (31). Now it becomes clear that reactivitating the multiplet $\vec{X}_\mu$ from RCD will merely amount to putting in an additional colored covariant vector source in the theory. Then it comes as natural for us to call $\vec{X}_\mu$ the *valence gluons* and the other part of the potential the *binding gluons*. Thus in the magnetic gauge the complex vector field X_μ

$$X_\mu = \frac{X^1_\mu + iX^2_\mu}{\sqrt{2}} \tag{36}$$

should carry the unit color charge and form the valence gluons, whereas A_μ and C^*_μ make the binding gluons. In short, the *potential in the extended gauge theory is made of two parts, the binding gluons that will confine any colored source and the valence gluons which will behave as a gauge-covariant colored source of the binding gluons*.

The effective Lagrangian of the extended QCD can easily be obtained from the QCD Lagrangian (7) with the help of the potential (31). In the magnetic gauge we find

$$\mathcal{L} = \mathcal{L}_{(R)} + \mathcal{L}_{(X)} \tag{37}$$

where $\mathcal{L}_{(R)}$ is the RCD Lagrangian (8), and $\mathcal{L}_{(X)}$ is the reactivated piece,

$$\begin{aligned} \mathcal{L}_{(X)} = -\tfrac{1}{2}\,|\,&[\partial_\mu + ig(A_\mu + C^*_\mu)]X_\nu \\ &-[\partial_\nu + ig(A_\nu + C^*_\nu)]X_\mu|^2 \\ &+ (g/\sqrt{2})(\bar{b}\gamma^\mu r X^*_\mu + \bar{r}\gamma^\mu b X_\mu) \\ &- ig(F_{\mu\nu} + H_{\mu\nu})X_\mu X^*_\nu \\ &+ \tfrac{1}{2}g^2[X_\mu{}^2 X^{*2}_\nu - (X_\mu X_\mu{}^*)^2]. \end{aligned} \tag{38}$$

Thus, extended QCD may still be described by the dual dynamics of RCD, but with an additional colored vector source $\vec{X}_\mu$ in it. This means, first of all, that the mechanism of color confinement in RCD will remain valid in extended QCD. Furthermore, the physical particle spectrum of RCD, i.e., the meson and baryon trajectories made of quark pairs as well as the magnetic glueballs 0^{++} and 1^{++}, all will appear unchanged in the particle spectrum of extended QCD. However, the extended theory will obviously contain an additional set of particles made of valence gluons. Owing to the color-confinement mechanism the valence gluons will have to be confined in pairs to form color-singlet "electric glueballs" just as the quarks had to be confined in pairs to form the mesons. So one would expect a set of linearly rising glueball trajectories made of the valence-gluon pair.

In conclusion, in extended QCD we obtain, in addition to mesons and baryons, two types of glueballs, i.e., electric and magnetic glueballs. These two types of glueballs are, however, completely different in their character and origin. *The magnetic glueballs appear as collective modes of the condensed vacuum and do not belong to any trajectory, whereas the electric glueballs appear as bound states of valence gluons and form a family of linear trajectories of their own.*

The remaining question now is, of course, how one can find the full spectrum of the electric glueballs. Since the spectrum must form linearly rising trajectories the general character of the full spectrum may be described by the number of the leading trajectories, their slopes, and the masses of their ground states. In the following section we will show how to obtain these characteristics as far as possible.

VI. ELECTRIC GLUEBALLS

To obtain the glueball spectrum remember that the meson spectrum made of the spin-$\frac{1}{2}$ quark pair has 2×2 leading trajectories which may be specified by their low-lying states $^1S_0$, $^3S_1$, $^3P_0$, and $^3P_1$, or π, ρ, δ, and A_1, respectively. Now, since the electric glueballs are made of the spin-1 valence-gluon pair, one would expect 3×3 leading trajectories[10] for the glueballs, which may be listed by their characteristic low-lying states $^5S_2$, $^1S_0$, $^3P_2$, $^3P_1$, $^3P_0$, $^5D_3$, $^5D_2$, $^5D_1$, and $^5D_0$. Of course, each leading trajectory will have its own daughter trajectories.

The confinement mechanism of the theory unambiguously provides us with the string picture of the glueballs for higher excited states when one of the valence gluons is hit with a large momentum, whereas it will provide the bag model picture for the low-lying states of the masses comparable to those of the magnetic glueballs. So one expects that the glueball trajectories will rise linearly with a slope fixed by the string tension k_X of the valence-gluon pair. Now one may easily estimate the slope parameter by calculating the string tension of the gluon pair. If we assume the effective potential (12), the string equations of the gluon pair will become identical to (16) with the same boundary condition (17), but with a different color flux quantization condition

$$\frac{1}{\sqrt{2}}\int_0^\infty \zeta E(\zeta)d\zeta = 2\,. \tag{39}$$

The difference, of course, is due to the fact that the valence gluons carry a unit color charge while the quarks carry a half-unit charge. The string tension k_X and the corresponding slope parameter α'_X may now be written as

$$k_X = \frac{1}{2\pi\alpha'_X} = \gamma_X\phi_0^2 = \frac{\alpha_s}{8\pi}\gamma_X m^2\,, \tag{40}$$

where γ_X may be evaluated with the help of (20), but now with the new solutions satisfying the quantization condition (39). The ratio R of the slopes of the meson and the glueball trajectories

$$R = \frac{\alpha'_X}{\alpha'} = \frac{\gamma}{\gamma_X} \tag{41}$$

may then be calculated as a function of α_s. A computer calculation[7] of the ratio R is shown in Fig. 3(I). As before, if we use the value $\alpha_s \simeq 0.22$, which is fixed by the running coupling constant $\overline{\alpha}(s)$ at $s=\mu m$, we obtain

$$\alpha'_X \simeq 0.5\alpha' \simeq 0.45\ \text{GeV}^{-2}\,. \tag{42}$$

One may carry out a similar calculation using the potential (21) instead of (12) to fix the value of the slope parameter α'_X. In Fig. 3(II) we show the ratio R obtained from the potential (21). Again with the assumption that α_s is given by $\overline{\alpha}(s)$ at $s=\mu m$, we obtain

$$\alpha'_X \simeq \begin{cases} 0.47\alpha' \simeq 0.42\ \text{GeV}^{-2}, & \lambda=1 \\ 0.45\alpha' \simeq 0.41\ \text{GeV}^{-2}, & \lambda=2\,. \end{cases} \tag{43}$$

With the precise numerical results set aside, however, notice that in spite of the difference in their exact forms both of the potentials (12) and (21) predict

$$R \simeq 0.5 \tag{44}$$

over a wide range of α_s. Thus the slope of the new trajectories would, in any reasonable circumstance, be roughly $\frac{1}{2}$ smaller than the Regge slope α' of the meson trajectories.

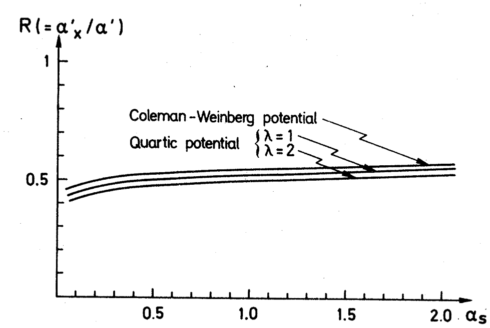

FIG. 3. The ratio R of the meson and the glueball slope parameters α' and α'_X as functions of α_s.

To fix the glueball spectrum completely one must be able to tell the mass spectrum of the low-lying states of the trajectories. This, of course, is a very difficult task which requires knowledge of the detailed dynamics of the confinement. However, if one is willing to make a most naive approximation, one could obtain a crude estimate of the masses of the low-lying states of the trajectories. So let us for the time being suppose that the low-lying states of both the gluon and the quark pairs may be described by a nonrelativistic Schrödinger equation with a binding potential of the type

$$V(r)=kr^{\nu} \quad (0<\nu<\infty)\,. \tag{45}$$

In the semiclassical WKB approximation the energy spectrum of such a system is given by[10]

$$E_{nl}=k^{2/(2+\nu)}(2\mu_0)^{-\nu/(2+\nu)}\left[A(\nu)\left(n+\frac{l}{2}+\frac{3}{4}\right)\right]^{2\nu/(2+\nu)} \times (n,l=0,1,2,\ldots)\,, \tag{46}$$

where

$$A(\nu)=\frac{2\nu\sqrt{\pi}\,\Gamma(1/\nu+\frac{3}{2})}{\Gamma(1/\nu)}\,,$$

μ_0 is the reduced mass, n and l are the principal and the orbital angular momentum quantum numbers of the two-body system. So if we denote the constants k and μ_0 of the quark pair and the gluon pair by k_q, μ_q and k_X, μ_X, respectively, the ratio of their energy spectrum may be written as[11]

$$\frac{E_{X,nl}}{E_{q,nl}}=\left(\frac{\mu_X}{\mu_q}\right)^{-\nu/(2+\nu)}\left(\frac{k_X}{k_q}\right)^{2/(2+\nu)} =\left(\frac{\langle r_q\rangle_n}{\langle r_X\rangle_n}\right)^2\frac{k_X}{k_q}\,, \tag{47}$$

where $\langle r_q\rangle_n$ and $\langle r_X\rangle_n$ are the mean radii of the two systems. The last equality in (47) comes from the fact that[10]

$$\frac{\langle r_q\rangle_n}{\langle r_X\rangle_n}=\left(\frac{\mu_X k_X}{\mu_q k_q}\right)^{1/(2+\nu)}.$$

Now if one assumes

$$\langle r_q\rangle_n \simeq \langle r_X\rangle_n \tag{48}$$

and use the value of the string potentials for k_q and k_X

$$\frac{k_X}{k_q}=\frac{\alpha'_X}{\alpha'}\simeq 2\,, \tag{49}$$

one would deduce that the typical mass scales of the low-lying glueballs are twice as heavy as the corresponding mass scales of the quark pair. So, for example, the typical mass scale of the S-state glueballs will be about twice as large ($\simeq$1.6 GeV) as the ρ mass, the typical mass scale of the S-state quark pairs. Now in reality, of course, the above nonrelativistic approximation should not be taken too seriously.[11] Nonetheless one may hope that the above analysis could give an acceptable (if crude) estimate of the typical mass scale of the low-lying glueballs. If so, the expected glueball spectrum, compared with the meson spectrum, may be characterized by the following two features: The low-lying states of the glueballs will be heavier than (about twice as heavy as) the corresponding low-lying states of the mesons. For the higher excited states the slope of the glueball trajectories will be smaller than (about half as small as) that of the meson trajectories.

VII. DISCUSSIONS

In this paper we have presented an extended gauge theory in which not only the local structure but also the global structure of a non-Abelian gauge symmetry is taken into account into its dynamics. The extended theory has many attractive features. First of all, the theory could successfully explain color confinement in QCD. The color reflection invariance of the confinement mechanism guarantees us the fact that the physical spectrum of the theory must contain only color singlets. Besides, the confinement mechanism unmistakably produces Nambu's string picture[12] of hadrons in the high-energy limit. Furthermore the theory predicts unambiguously the existence of two types of glueballs, the magnetic glueballs which are the massive collective modes of the condensed vacuum and the electric glueballs made of the valence-gluon pair which form a family of linear trajectories of their own, in its physical spectrum. Perhaps more importantly it can predict the slope of these glueball trajectories.

There are a few points to be discussed and clarified on the mass spectrum of the extended theory. First, we notice that with the color reflection invariance alone one cannot, in general, exclude possible exotic color-singlet states in one's physical spectrum. For example with the group SU(2) the above criterion may easily rule out any exotic meson or baryon state made of more than one quark pair from the physical spectrum. However, this criterion does not exclude possible exotic glueball states made of more than one pair of valence gluons. Although one might attempt to exclude these exotic states with a hitherto unknown *dynamical* selection rule, it seems unlikely that they can be successfully excluded from the physical spectrum. Another point is that although the structure of the physical spectrum, in particular the existence of the two types of glueballs and the linearly rising family of electric glueballs, seems unavoidable in our theory, the predicted mass spectrum of these glueball states presented in the above may not be taken at their face values for the following reasons. First, they are evaluated based on the group SU(2) in this paper. This defect may be corrected without much difficulty by considering the realistic SU(3)-symmetric extended QCD.[4] However, there are other *technical assumptions* involved in our predictions. First, all of our numerical predictions are based on the assumption that α_s of the theory could be fixed by the running coupling constant $\overline{\alpha}(s)$ of QCD at $s=\mu m$. Although this assumption seems reasonable, it remains an assumption which has yet to be checked by experiments. Besides, the masses of the magnetic glueballs are evaluated based on the mass ratio (14) obtained from the one-loop approximation, which may be reliable only in the strong-coupling limit. Also for the ground-state masses of the electric glueballs the naive nonrelativistic approximation adopted in the above is undoubtedly questionable. Accordingly these numerical predictions should be interpreted with due caution with these technical assumptions in mind.

The extended theory in its present form considered in this paper contains a few unsettled questions. For instance, although so far we were able to avoid dealing with the singular "dual potentials"[2,3] A_μ^* and C_μ^* in this paper, the precise mathematical meaning of the singular objects and their dual couplings has to be clarified before one could perform various higher-order calculations. Another novel feature of the theory is the fact that the magnetic condensation of the vacuum introduces a new parameter ϕ_0 into the theory. Thus the theory contains two arbitrary parameters, the coupling constant g and the vacuum expectation value ϕ_0. Consequently the Regge slope α' alone cannot fix the fine-structure constant α_s in our theory. In fact it was precisely due to this reason that we had to estimate α_s from the running coupling constant $\overline{\alpha}(s)$. This feature is in contrast with the contemporary belief that the coupling constant alone should fix the theory completely. At the moment we do not know whether this is an intrinsic feature of the extended gauge theory, or merely due to our inability to calculate the vacuum expectation value in terms of the coupling constant. On this issue it is perhaps illuminating to compare our theory again with the theory of superconductivity. Within the framework of the Ginsburg-Landau theory there is no way to calculate the mass gap in a superconductor. It is only in the microscopic BCS theory where one is able to calculate the mass gap. Likewise it is an open possibility that in a more elaborate and microscopic formulation of extended QCD α_s alone (and thus only one universal mass scale α' alone) could fix the theory completely.

Finally, from the theoretical point of view one of the most important questions to be settled is to what extent extended QCD is different from *conventional* QCD. The issue here is the precise nature of the duality[2,3] of the theory. Guided by physical intuition rather than a rigorous mathematical deduction, we have introduced the magnetic (i.e., the topological) degrees of freedom (the magnetic potential and the monopole field) as fundamental fields to take into account the topological structure of the underlying gauge symmetry explicitly. Accordingly the magnetic degrees appear independent of the electric (i.e., the local) degrees of freedom[3] in our formalism. On the other hand one could think of the possibility[2] that the magnetic degrees, or at least the magnetic potential, could be obtained from the electric potential through some kind of dual transformation. In this latter point of view the two degrees of freedom appear two opposite sides of one and the same physical degrees of freedom. Although phenomenologically both points of view would likely lead us to the same physical spectrum, from the theoretical point of view it would indeed by very interesting to see to what extent, if at all, the topological degrees could be derived from the local degrees of freedom.

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