Restoration of symmetry by radiative corrections

R. Rajaraman
Centre for Theoretical Studies, Indian Institute of Science, Bangalore 560 012, India

M. Raj Lakshmi
Department of Physics, Indian Institute of Science, Bangalore 560 012, India
(Received 24 April 1980)

It is shown using an explicit model that radiative corrections can restore the symmetry of a system which may appear to be broken at the classical level. This is the reverse of the phenomenon demonstrated by Coleman and Weinberg. Our model is different from theirs, but the techniques are the same. The calculations are done up to the two-loop level and it is shown that the two-loop contribution is much smaller than the one-loop contribution, indicating good convergence of the loop expansion.

I. INTRODUCTION

In a very interesting article written some years ago, Coleman and Weinberg[1] gave explicit examples demonstrating spontaneous symmetry breaking through radiative corrections in relativistic quantum field theories. That is to say, the classical potential in the Lagrangians of these examples had an absolute minimum in field space which was symmetric under the action of some invariance group of the Lagrangian. When the effect of quantum corrections was included up to leading order in a loop expansion, the vacuum shifted to one of a set of asymmetric points, giving rise to spontaneous breaking of the symmetry. In the process, Coleman and Weinberg also developed the technical machinery for systematically studying the effect of radiative (i.e., quantum) corrections on the location of the vacuum. These techniques have been further discussed in other papers.[2,3]

In the examples studied in the Coleman-Weinberg work, the effect of the corrections was towards the direction of breaking a symmetry. But the basic lesson we learn from that work is that quantum corrections can, in suitably chosen models, significantly alter the location of the vacuum as compared to what the classical potential would indicate. Roughly speaking, the differences in zero-point energy can more than offset the differences in classical potentials. Given this underlying physics, there is no reason why the phenomenon cannot go in the reverse direction. That is, one may be able to find other examples, where the classical potential indicates an asymmetric vacuum, whereas when quantum corrections are added on, the vacuum shifts to a symmetric point. If such examples were found, they would correspond to "restoration of symmetry" due to radiative corrections, rather than breaking of symmetry. This paper is devoted to explicitly demonstrating this possibility with a simple example.

Our model consists of a real, scalar field $\tilde{\phi}(\tilde{x},\tilde{t})$ in $(1+1)$ dimensions, described by the Lagrangian density

$$\tilde{\mathcal{L}}(\tilde{x},\tilde{t})=\tfrac{1}{2}(\tilde{\partial}_\mu\tilde{\phi})^2-U(\tilde{\phi})\,, \quad (1)$$

where

$$U(\tilde{\phi})=(\tilde{\phi}^2+b^2)\left(\tilde{\phi}-\frac{1}{c^2}\right)^2-\frac{b^2}{c^4} \quad (2)$$

and

$$\tilde{\partial}_\mu\equiv\frac{\partial}{\partial\tilde{x}_\mu}\,.$$

By changing variables to $\phi=c\tilde{\phi}$, $x_\mu=(1/c^2)\tilde{x}_\mu$, one can rewrite this in terms of $\phi(x,t)$ as

$$\mathcal{L}(x,t)=\frac{1}{\lambda}\left[\tfrac{1}{2}(\partial_\mu\phi)^2-V_0(\phi)\right]\,, \quad (3)$$

where

$$V_0(\phi)=(\phi^2+a^2)(\phi^2-1)^2-a^2 \quad (4)$$

and

$$\lambda\equiv c^6\,,\quad a^2\equiv b^2c^2\,.$$

We will work with the simplified form (3) of the Lagrangian, where a "major coupling constant" λ has been factored out. We will also restrict ourselves to the range $0<a^2<\frac{1}{2}$. In this range, the classical potential $V_0(\phi)$ has three minima (Fig. 1). Two of them at $\phi=\pm1$ are absolute minima, while the third, at $\phi=0$, is a higher local minimum. In the absence of radiative corrections, we would expect the vacuum to be near $\phi=+1$ or $\phi=-1$. The Lagrangian enjoys $\phi\longleftrightarrow-\phi$ symmetry, so that a vacuum around $\phi=\pm1$ would correspond to spontaneous symmetry breaking. We proceed to study the effect of quantum fluctuations on this result by the standard procedure[1] of evaluating the so-called "effective potential," first to the one-loop level and then to the two-loop level. The evaluation up to

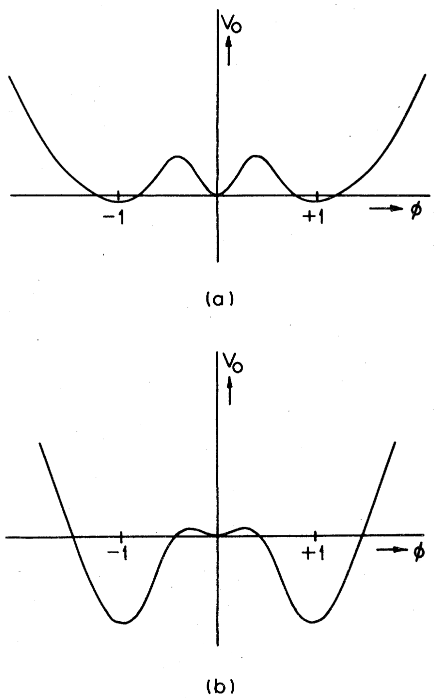

FIG. 1. Schematic plot of the classical potential V_0 vs ϕ for $0<a^2<\frac{1}{2}$ (a) for a^2 slightly greater than 0, (b) for a^2 slightly less than $\frac{1}{2}$.

these two orders is done exactly. We find that for a certain range of the parameters a^2 and λ, the classical result is reversed by the radiative corrections, which restore the symmetry. The effective potential develops an absolute minimum at $\phi=0$. This phenomenon already happens at the one-loop level (Sec. II) for λ greater than some λ_c, for a given a^2 in this range. Recall[4] that since $1/\lambda$ factors out of the whole Lagrangian in (3), the loop expansion will be a power series in λ. Thus, it is reasonable that the one-loop term may significantly alter the result from the zero-loop (classical) term only for λ larger than some minimal λ_c. This, however, raises an important question, anticipated and discussed by Coleman and Weinberg in the context of their models. If the one-loop contribution is comparable to or larger than the zero-loop term, will the two-loop and higher effects be even larger, rendering the loop expansion divergent, or, at the very least, a poor approximation scheme? To overcome this problem, these authors consider a model with two coupling constants. They show that by suitably choosing the relative strengths of the two couplings, it may be possible to have a significant effect from one-loop terms, without ruining the rapid convergence of the loop expansion. Notice that our Lagrangian also has two couplings λ and a^2. We calculate in Sec. III the exact and full two-loop contribution to the effective potential. We show that, for a range of λ and a^2, (i) the two-loop contribution preserves and enhances the one-loop effect of restoring symmetry while (ii) at the same time it is much smaller than the one-loop term indicating that the loop expansion may well be a good approximation. This still leaves the conclusion far from being rigorous. We do not know how to evaluate or even estimate arbitrary n-loop contributions, let alone obtain a closed expression for the effective potential free of loop expansions. But we feel that verification up to the two-loop order is a good indication, given the present state of the art.

A word about what motivated us to choose the model (3) to illustrate this phenomenon. This is not a pathological model—just a familiar ϕ^6 field theory. It is the simplest example which has (i) a symmetry, in this case under $\phi \leftrightarrow -\phi$; (ii) absolute minima at asymmetrical points, in this case at $\phi=\pm 1$; and (iii) a higher local minimum at the symmetric point $\phi=0$. Classically, the energy at $\phi=\pm 1$ is lower than at $\phi=0$. But note that the curvature of the potential $V_0'' \equiv d^2V_0/d\phi^2$ is higher at $\phi=\pm 1$ than at $\phi=0$. Thus, the quantum correction to the energy (zero-point energy), which typically increases with the curvature, will be higher for a trial vacuum state centered at $\phi=\pm 1$ than at $\phi=0$. This may, for a range of a^2 and λ, more than offset the difference in classical energy and thus restore the minimum to $\phi=0$. Of course, strictly speaking, in a relativistic quantum field theory requiring divergent counterterms, the notion of the classical ("bare") potential is not very meaningful. For a given renormalized theory, the bare parameters vary with the choice of the renormalization prescription. Therefore, the argument given above is intended at best to be a qualitative motivation for studying the model (3). As it turns out, careful calculations given below with divergences canceled through a proper renormalization prescription do indeed reveal the anticipated result for a range of a^2 and λ.

The nature of the symmetry restoration illustrated by this model is quite different from a more familiar class of symmetry restoration due to quantum (or statistical) fluctuations. The latter class comes under the well-known category of "absence of spontaneous symmetry breaking in low dimensions." Consider, for instance, the prototype case of a complex field $\phi=|\phi|e^{i\theta}$ with a Lagrangian

$$\mathcal{L}=\tfrac{1}{2}|\partial_\mu\phi|^2-(|\phi|^2-1)^2. \tag{5}$$

This model enjoys U(1) symmetry. At the zero-loop level, one would expect the vacuum to be at $\phi=1$ and at some phase angle θ chosen spontaneously. However, if the theory were in 1 space dimension, there are theorems[5,6] which prohibit spontaneous breaking of a continuous symmetry such as U(1). In physical terms, what will happen is that[7] quantum fluctuations will spread the ground state to all values of θ, so that there will be no preferred direction θ any longer and U(1) symmetry will be restored. However, this spread is still in the vicinity of $|\phi|=1$, although all phase angles may be equally sampled by the vacuum. The vacuum will not shift to the vicinity of the symmetric point $\phi=0$. In other words, in this example, although $\langle\Phi\rangle_{vac}=0$, $\langle|\Phi|^2\rangle_{vac}\sim 1$, where Φ is the field operator and $\langle\Phi^2\rangle$ is understood to be properly regularized. By contrast, in the type of symmetry restoration we will illustrate with the model in Eq. (3), the vacuum will shift from the asymmetrical point $\phi=+1$ or $\phi=-1$ to the symmetric point $\phi=0$. Not only will $\langle\Phi\rangle_{vac}=0$, but also $\langle\Phi^2\rangle_{vac}\sim 0$.

Also, whereas the restoration of symmetry in models such as (5) relies strongly on the smallness of space dimensionality, our phenomenon does not. There is no *a priori* reason why the type of symmetry restoration we will discuss cannot happen in, say, (3+1) dimensions. True, our model [Eq. (1)] was stated to be in (1+1) dimensions. But that was only for ensuring that, unlike in (3+1) dimensions, the ϕ^6 theory remains renormalizable. Renormalizability is important, to ensure that the model is sensible. But it is of only secondary importance to our physical question. Whereas renormalizability is an ultraviolet problem, the existence or absence of symmetry breaking is an infrared phenomenon, related to the infiniteness of volume. Therefore, if models such as (1) were extended to (3+1) dimensions, by putting them on a lattice to avoid ultraviolet problems, the type of symmetry restoration we are discussing may well happen in suitable cases. We do not present a lattice calculation here and this statement is only a conjecture based on the detailed nature of the calculation below.

It is well known that upon continuation to Euclidean metric, the effective potential is analogous to the free energy in statistical mechanics. Our phenomenon, with quantum fluctuations replaced by statistical fluctuations, should have its analog in phase transitions as well. In particular, first-order transitions are typically described by ϕ^6 theories as in (1). The analog of our phenomenon should alter the critical temperature as compared to its mean-field value.

Finally, the phenomenon we will demonstrate will be the reverse of what is named by Coleman and Callan[8] as the "fate of the false vacuum." They describe in detail how a false vacuum, i.e., one constructed about a higher minimum of the classical potential V_0, will decay into the correct vacuum around the absolute minimum. Our work shows that in models such as (3), for suitable choices of the parameters, the opposite happens. The so-called false vacuum, as for instance constructed around $\phi=0$ in our model, is in fact the real one even though $\phi=0$ is not the absolute minimum of V_0.

II. THE EFFECTIVE POTENTIAL TO ONE-LOOP ORDER

We now proceed to evaluate the effect of quantum corrections on the location of the vacuum for the model in Eq. (3) by using the Coleman-Weinberg method.[1] Let us recall the salient features of their method. It involves calculating the so-called effective potential $V_{eff}(\phi)$, which may be defined by its Taylor expansion

$$V_{eff}(\phi)=\sum_n \frac{\phi^n}{n!}G^n(0,0,\ldots,0), \qquad (6)$$

where $G^n(0,0,\ldots,0)$ is the sum of all amputated, one-particle irreducible (1PI) graphs with n external legs, all at zero energy-momentum. It has been shown[1,4] that the absolute minimum of the $V_{eff}(\phi)$ so defined will give $\langle\phi\rangle$, the vacuum expectation value of the field operator. Each $G^n(0,0,\ldots,0)$ may be expanded in a loop expansion, in powers of the number of loops in the corresponding 1PI graphs. This will also be a power series in the coupling λ which factors out of the Lagrangian (3), as per the counting argument in Ref. 1. Thus $V_{eff}(\phi)$ can also be written as a loop-expansion series:

$$V_{eff}(\phi)=\lambda^{-1}V_0(\phi)+\lambda^0V_1(\phi)+\lambda V_2(\phi)+\lambda^2V_3(\phi)+\cdots . \qquad (7)$$

In this section we will evaluate $V_{eff}(\phi)$ up to one-loop order, i.e., the first two terms of (7).

The leading term in (7) is obtained by inserting just the zero-loop (tree) graphs into the Taylor series (6). This will give, as is well known, just the classical potential in the Lagrangian, viz.,

$$\lambda^{-1}V_0(\phi)=\frac{1}{\lambda}[(\phi^2+a^2)(\phi^2-1)^2-a^2]. \qquad (8)$$

The V_0 in (7) is therefore the same as the V_0 in (4). The next step is to evaluate the one-loop term $V_1(\phi)$ by inserting all one-loop graphs into (6). There are infinitely many one-loop graphs which can, however, be compactly depicted as in Fig. 2. There the solid circle stands for a sum of vertices with zero, two, and four external lines arising from terms in V_0 of second, fourth, and sixth order in ϕ, respectively. Each external line carries

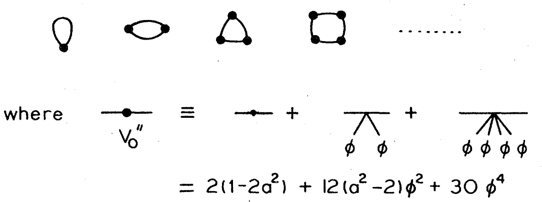

FIG. 2. The one-loop approximation for the effective potential.

zero energy-momentum and a factor of ϕ. The weightage of each of these terms is such that a vertex in Fig. 2 can be represented by $V_0''(\phi)$, where $V_0''\equiv d^2V_0/d\phi^2$.

The contribution from the one-loop graphs to $V_1(\phi)$ is

$$i\int\frac{d^2k}{(2\pi)^2}\sum_{n=1}^{\infty}\frac{1}{2n}\left[\frac{V_0''(\phi)}{k^2+i\epsilon}\right]^n. \tag{9}$$

$1/2n$ is a combinatoric factor. A cyclic permutation or a reversal of the order of the n vertices in the one-loop graphs does not lead to a new contraction in the Wick expansion, therefore, the $1/n!$ in Dyson's formula is not completely canceled.[1]

Rotating the integral (9) to Euclidean space and dropping the $i\epsilon$ gives

$$\int_0^{\Lambda^2}\frac{\pi}{2}\frac{d(k^2)}{(2\pi)^2}\ln\left(1+\frac{V_0''}{k^2}\right).$$

We have used a cutoff at $k^2=\Lambda^2$, since the integral is otherwise ultraviolet divergent. The value of the integral is

$$\frac{1}{8\pi}V_0''\left[\ln\left(\frac{\Lambda^2}{V_0''}\right)+1\right], \tag{10}$$

where terms that vanish as $\Lambda^2\to\infty$ have not been kept.

The contribution (10) diverges and this divergence has to be canceled by adding counterterms to the Lagrangian (3). This is a trivial example of renormalization in the sense that the divergence in (10) arises, for this example of a scalar field in (1+1) dimensions, solely from the lack of normal ordering of the Lagrangian (3). For a ϕ^6 theory, normal-ordering counterterms will have the form

$$V_{\text{c.t.}}(\phi)=A+\frac{B}{2}\phi^2+\frac{C}{4!}\phi^4, \tag{11}$$

where one can again expand in powers of λ,

$$A\equiv A_1+\lambda A_2+\lambda^2A_3+\cdots,$$

and similarly for B and C. The constants A, B, and C are so designed as to cancel, order by order, the divergences produced by the loop integration and leave behind a finite part which is fixed by specific renormalization conditions. The conditions we will use are

$$V_{\text{eff}}(\phi)\big|_{\phi=0}=0=V_0(\phi)\big|_{\phi=0}, \tag{12a}$$

$$\frac{d^2V_{\text{eff}}(\phi)}{d\phi^2}\bigg|_{\phi=0}=\frac{2}{\lambda}(1-2a^2)=\frac{d^2V_0}{d\phi^2}\bigg|_{\phi=0}, \tag{12b}$$

$$\frac{d^4V_{\text{eff}}(\phi)}{d\phi^4}\bigg|_{\phi=0}=\frac{4!}{\lambda}(a^2-2)=\frac{d^4V_0}{d\phi^4}\bigg|_{\phi=0}. \tag{12c}$$

To one-loop order, we add counterterms $A_1+(B_1/2)\phi^2+(C_1/4!)\phi^4$ to the loop contribution (10) and set

$$V_1(\phi)=\frac{V_0''(\phi)}{8\pi}\left\{\ln\left[\frac{\Lambda^2}{V_0''(\phi)}\right]+1\right\}+A_1+\frac{B_1}{2}\phi^2+\frac{C_1}{4!}\phi^4. \tag{13}$$

The constants A_1, B_1, and C_1 are so chosen that the renormalization conditions (12) are satisfied. Notice that these conditions are so prescribed that they are already satisfied by the leading term $(1/\lambda)V_0(\phi)$. Hence the radiative corrections must contribute zero to each equation in (12). To one-loop order,

$$0=V_1(\phi)\big|_0=V_1''(\phi)\big|_0=V_1''''(\phi)\big|_0. \tag{14}$$

These three conditions fix A_1, B_1, and C_1. Some trivial algebra yields

$$A_1=\frac{1-2a^2}{4\pi}[\ln(2-4a^2)-\ln\Lambda^2-1], \tag{15a}$$

$$B_1=\frac{3}{\pi}(2-a^2)[\ln\Lambda^2-\ln(2-4a^2)], \tag{15b}$$

$$C_1=\frac{108}{\pi}\frac{(2-a^2)^2}{1-2a^2}+\frac{90}{\pi}[\ln(2-4a^2)-\ln\Lambda^2]. \tag{15c}$$

On inserting this into (13), one gets a finite expression for $V_1(\phi)$ satisfying (14). This expression along with the zero-loop term (8) gives the following result for $V_{\text{eff}}(\phi)$ up to one-loop order:

$$V_{\text{eff}}(\phi)=\frac{1}{\lambda}\phi^6+\phi^4\left(\frac{a^2-2}{\lambda}+\frac{9}{2\pi}\frac{(a^2-a)^2}{1-2a^2}+\frac{15}{4\pi}[1+\ln(2-4a^2)]\right)$$
$$+\phi^2\left(\frac{1-2a^2}{\lambda}-\frac{3}{2\pi}(2-a^2)[1+\ln(2-4a^2)]\right)-\frac{1}{8\pi}V_0''(\phi)\ln V_0''(\phi)+\frac{1-2a^2}{4\pi}\ln(2-4a^2)+O(\lambda). \tag{16}$$

Thus we have a closed analytic expression for the effective potential to $O(\lambda^0)$. Notice that it becomes complex in the region where V_0'' is negative. Such problems did not arise in Ref. 1, because the classical potential in their model was nowhere concave. When V_0 is not convex everywhere, this problem is to be expected. Its existence may be physically interpreted as follows. In physical terms $V_{\text{eff}}(\phi)$ is essentially the energy of the lowest state subject to the constraint $\langle\Phi\rangle = \phi$. If we try to construct a ground-state wave functional in a region where the potential is concave, it will be unstable. The imaginary part of the value of $V_{\text{eff}}(\phi)$ reflects this instability. Also, it is well known that the leading quantum correction to the energy is related to the curvature of the potential. Now, it is possible that $V_{\text{eff}}(\phi)$, when calculated exactly to all orders, may become real everywhere and convex.[9] As we have remarked earlier $V_{\text{eff}}(\phi)$, when continued to Euclidean metric, is formally akin to the Gibbs free energy used in statistical mechanics, and there are theorems which require the latter to be everywhere real and convex.[10] Hence, this unpleasant feature of $V_{\text{eff}}(\phi)$ in Eq. (16) may be an artifact of the loop expansion. In this work, however, we are basically interested in whether the vacuum lies near $\phi = \pm 1$, or near $\phi = 0$. We will therefore confine ourselves to comparing the values of $V_{\text{eff}}(\phi)$ in the vicinity of these points, where it is certainly real. Up to any given order in the loop expansion, which is the only available technique for studying this question, one can do no better.

Even though (16) is an analytic expression, it is quite complicated, and the functional dependence of V_{eff} on ϕ, a^2, and λ is not transparent. Our conclusions which follow are based on a numerical evaluation of this function for a grid of points in ϕ, a^2, and λ space. The reader can independently verify these statements starting from the result (16). Consider, for example, the case $a^2 = 0.05$ and $\lambda = 0.007$. The sum $[(1/\lambda)V_0 + V_1]$ is plotted in Fig. 3, curve II. We see that there are two potential wells, near $\phi = 0$ and $\phi = 1$, in V_{eff}. The effect of V_1 is to raise the minimum near $\phi = 1$ as compared to what existed in $(1/\lambda)V_0$. Thus the leading quantum correction tends to restore symmetry. The precise location of this minimum is also shifted by V_1. Nevertheless, the minimum near $\phi = 1$ is still lower than the one at $\phi = 0$, and symmetry breaking will persist to this order for the values $a^2 = 0.05$, $\lambda = 0.007$. This is because this value of λ is too small for V_1 to significantly alter the classical term V_0/λ. However, as λ is increased, keeping a^2 at 0.05, there comes a critical value λ_c beyond which the minimum near $\phi = 1$ is raised higher than that at $\phi = 0$. Figure 3, curve III, gives $(V_0/\lambda + V_1)$ for the cases $a^2 = 0.05$ and $\lambda = 0.01$. We see that $\phi = 0$ is the absolute minimum of $V_{\text{eff}}(\phi)$ in the regions where the latter is real. From this we would conclude that up to one-loop order, and for this pair of values of a^2 and λ, the vacuum lies at the symmetric point $\phi = 0$, and that the symmetry is therefore restored by the

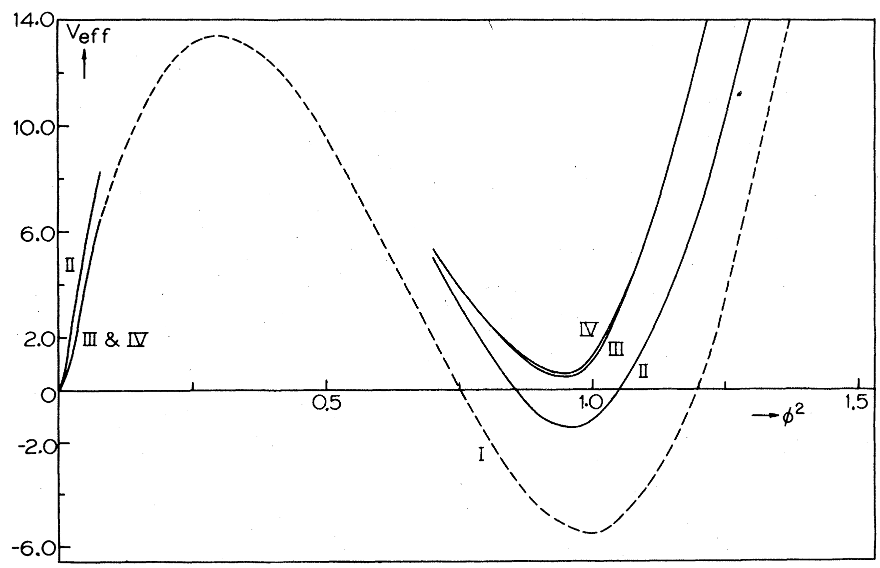

FIG. 3. Plots of V_{eff} vs ϕ^2 for $a^2 = 0.05$. Curves I, III, and IV show the profile of the effective potential calculated to the zero-, one-, and two-loop orders, respectively, all for $\lambda = 0.01$ (which is greater than $\lambda_c = 0.009$). Curve II shows the effective potential to the one-loop order for $\lambda = 0.007$ ($<\lambda_c$). Curves II, III, and IV are drawn only on regions where V_{eff} is real. For small values of ϕ^2 (i.e., before V_{eff} turns complex), the effective potentials to the zero-, one-, and two loop orders are almost the same; curves I, III, and IV are therefore shown by a single line for small values of ϕ^2.

radiative term V_1. Our numerical estimate is that for $a^2 = 0.05$, the critical value of λ is around 0.009. It is evident that $V_{\text{eff}}(\phi)$ is a smooth function of a^2 and λ, and therefore this phenomenon will happen for a dense range of values of a^2 and λ. In Fig. 4, we give another example where $a^2 = 0.20$ and $\lambda = 0.025$ and 0.037. For the smaller λ, symmetry is still broken, but for $\lambda = 0.037$ the minimum of the outer well in V_{eff} is higher than the minimum at $\phi = 0$, indicating restoration of symmetry. Table I gives, for a set of values of a^2, the approximate minimal values λ_c beyond which symmetry restoration happens. We have restricted the discussion to the positive ϕ region since $V_{\text{eff}}(-\phi) = V_{\text{eff}}(\phi)$.

Two qualifying remarks should be made about this conclusion. The first is the existence of the embarrassing region in between the two potential wells in V_{eff}, where V_{eff} is complex. This difficulty is unavoidable in any given order of the loop expansion. We can only hope, guided by the theorem quoted, that in the exact result to all orders the potential will become real and convex in this region as well, along a line connecting the two minima. In that case, the lowest minimum, in this case $\phi = 0$, will support the vacuum.

The second remark regards the convergence of the loop expansion even within the range where V_{eff} is real. Note that symmetry is restored only for $\lambda >$ some $\lambda_c(a^2)$. One can ask whether, for such λ, the higher-loop contributions will be small enough for the loop expansion to converge rapidly.

TABLE I. Variation of λ_c with a^2.

a^2	λ_c
0.05	0.009
0.1	0.018
0.2	0.033
0.25	0.038
0.3	0.044

Otherwise, our conclusions based on one-loop order will be meaningless. We can partially answer this question by computing the two-loop contribution to V_{eff}, explicitly for our model. This we do in Sec. III.

III. THE EFFECTIVE POTENTIAL TO TWO-LOOP ORDER

In this section, we calculate the $O(\lambda)$ contribution to V_{eff}. These come from three sources: (i) the insertion of the earlier counterterms $(B_1/2)\phi^2 + (C_1/4!)\phi^4$ into the one-loop diagrams, (ii) genuine two-loop graphs, and (iii) new counterterms $A_2 + (B_2/2)\phi^2 + (C_2/4!)\phi^4$ needed for renormalizing the net result. The first contribution is easy to obtain. We merely replace $V_0''(\phi)$ by $V_0''(\phi) + \lambda[(B_1\phi^2)/2) + (C_1/4!)\phi^4]''$ in Eq. (10). This gives a contribution to V_2 of the form

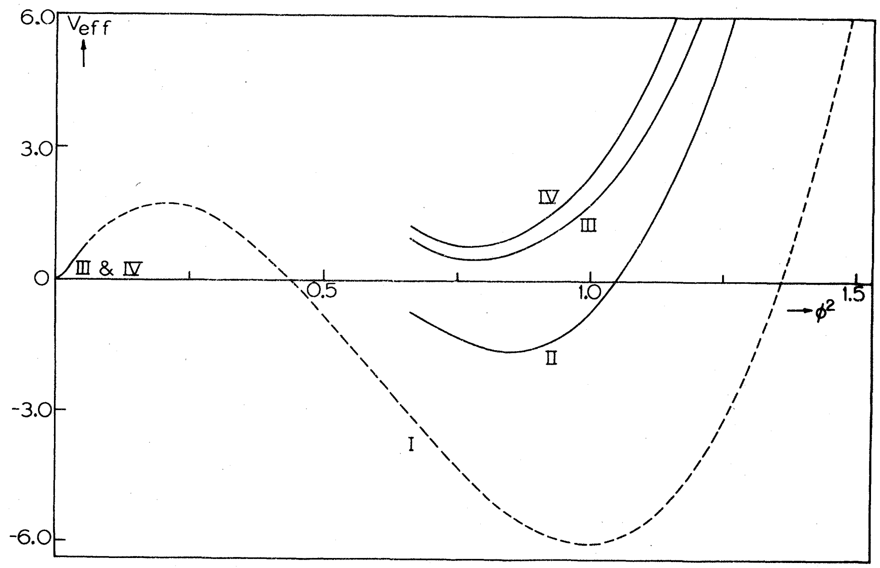

FIG. 4. Plots of V_{eff} vs ϕ^2 for $a^2 = 0.2$. Curves I, III, and IV show the profile of the effective potential calculated to the zero-, one-, and two-loop orders, respectively, for $\lambda = 0.037$ (which is greater than $\lambda_c = 0.033$). Curve II shows the effective potential to the one-loop order for $\lambda = 0.025$ ($<\lambda_c$). Curves II, III, and IV are drawn only in regions where V_{eff} is real. For small values of ϕ^2 (i.e., before V_{eff} turns complex), the effective potentials to the zero-, one-, and two-loop orders are almost the same; curves I, III, and IV are therefore shown by a single line for small values of ϕ^2.

$$\lambda V_2^{(1)}(\phi)=\frac{1}{8\pi}\left(V_0''+\lambda B_1+\frac{\lambda C_1}{2}\phi^2\right)\times\left[\ln\Lambda^2+1-\ln\left(V_0''+\lambda B_1+\frac{\lambda C_1}{2}\phi^2\right)\right]-\frac{1}{8\pi}V_0''(\ln\Lambda^2-\ln V_0''+1)\,. \qquad (17)$$

Retaining only order λ terms consistent with two-loop order, this reduces to

$$\lambda V_2^{(1)}(\phi)=\frac{\lambda}{8\pi}\left(B_1+\frac{C_1}{2}\phi^2\right)(\ln\Lambda^2-\ln V_0'')+O(\lambda^2)\,. \qquad (18)$$

On substituting the expressions (15) for B_1 and C_1 which themselves contain $\ln\Lambda^2$ terms, we see that there are two cutoff-dependent terms that are nonpolynomial in ϕ. These are of the form $\alpha\ln\Lambda^2\ln V_0''(\phi)$ and $\beta\phi^2\ln\Lambda^2\ln V_0''(\phi)$, where α and β are functions of a^2. Unlike the other cutoff-dependent terms that are either of second or fourth order in ϕ, these two terms cannot be canceled by the renormalization counterterms. However, as we shall soon see, this need not cause worry because these terms will be annulled by similar ones arising from the two-loop graphs.

The genuine two-loop graphs, in a compact notation, are shown in Fig. 5. Each of the two graphs stands for an infinite number of graphs obtained by inserting an arbitrary number of vertices that are represented by V_0'' (such vertices have been discussed in Sec. II), into all the internal lines. The sum of all such insertions just leads to an independent geometric series for each internal line. The thick internal lines in Fig. 5 represent these sums. Computationally, the effect of these insertions can be taken into account by making the substitution $i/(k^2+i\epsilon)\to i/(k^2-V_0''+i\epsilon)$ for every internal propagator. Furthermore, the vertex in Fig. 5(a) marked by a solid square stands for the sum of two vertices, both having four internal lines, but differing in the number of external lines; one has two and the other no external lines. It can be shown by explicitly working out the combinatorics to obtain the weights of the two vertices that these can be collectively represented by $V_0''''(\phi)\equiv d^4V_0/d\phi^4$. All this holds true for the graph in Fig. 5(b) except that here the vertex marked by a solid triangle represents a sum of vertices each with three internal lines. This can be represented by $V_0'''(\phi)\equiv d^3V_0/d\phi^3$ and has been so done in the figure.

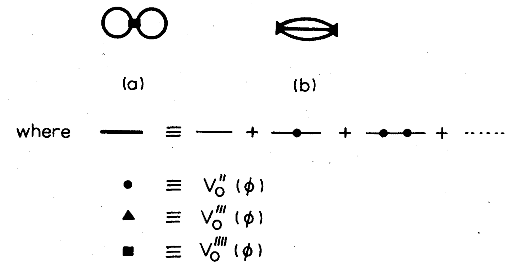

FIG. 5. The two-loop approximation for the effective potential.

Taking proper note of all the combinatorics, the two-loop graph in Fig. 5(a) gives a contribution

$$V_2^{(2)}(\phi)=\frac{V_0''''}{8}\int\frac{d^2k_1d^2k_2}{(2\pi)^4}\,\frac{1}{k_1^2-V_0''+i\epsilon}\,\frac{1}{k_2^2-V_0''+i\epsilon}=\frac{V_0''''}{8(2\pi)^4}\left(\int\frac{d^2k}{k^2-V_0''+i\epsilon}\right)^2\,. \qquad (19)$$

Figure 5(b) gives

$$V_2^{(3)}(\phi)=\frac{(V_0''')^2}{12}\int\frac{d^2k_1d^2k_2d^2k_3(-i)(2\pi)^2\delta^2(k_1+k_2+k_3)}{(2\pi)^6(k_1^2-V_0''+i\epsilon)(k_2^2-V_0''+i\epsilon)(k_3^2-V_0''+i\epsilon)}\equiv\frac{(V_0''')^2}{12}I\,. \qquad (20)$$

The integral in $V_2^{(2)}$ is ultraviolet divergent. So, we use a cutoff at $k^2=\Lambda^2$. Rotating the integral into Euclidean space and dropping the $i\epsilon$ gives

$$V_2^{(2)}=\frac{V_0''''}{8(2\pi)^4}\left[\int_0^{\Lambda^2}\frac{\pi d(k^2)}{k^2+V_0''}\right]^2=\frac{V_0''''}{128\pi^2}(\ln\Lambda^2-\ln V_0'')^2\,, \qquad (21)$$

where terms that vanish as $\Lambda\to\infty$ have been dropped.

The integral in I is not ultraviolet divergent as can be seen by a simple power counting of k. It can be simplified by making use of the δ function to integrate away one of the variables k_1, k_2, k_3. This gives, after rotation into Euclidean space,

$$I=\int \frac{1}{(2\pi)^4}\, d^2k_1 d^2k_2\, \frac{1}{(k_1{}^2+V_0'')(k_2{}^2+V_0'')[(k_1+k_2)^2+V_0'']}\ . \tag{22}$$

This can now be evaluated using Feynman parameters.[11] Introducing these, we get

$$I=\int \frac{1}{(2\pi)^4}\, d^2k_1 d^2k_2 \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\, \frac{\delta(\alpha_1+\alpha_2+\alpha_3-1)}{\{(k_1{}^2+V_0'')\alpha_1+(k_2{}^2+V_0'')\alpha_2+[(k_1+k_2)^2+V_0'']\alpha_3\}^3}\ . \tag{23}$$

The integrations over k_1 and k_2 can be done by using the known integral

$$\int \frac{d^n p}{(p^2+2pq+m^2)^\alpha}=\frac{i\pi^{n/2}}{(m^2-q^2)^{\alpha-n/2}}\,\frac{\Gamma(\alpha-n/2)}{\Gamma(\alpha)} \tag{24}$$

to give

$$I=\int_0^1 \frac{1}{(2\pi)^4}\, d\alpha_1 d\alpha_2 d\alpha_3\, \frac{\delta(\alpha_1+\alpha_2+\alpha_3-1)(-\pi^2)}{(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)}\,\frac{1}{V_0''}\ . \tag{25}$$

A simple transformation of the integration variables $\alpha_1=\rho x$, $\alpha_2=\rho-\rho x$, $\alpha_3=1-\rho$ gets rid of the δ function and reduces the integral to

$$I=\frac{-1}{16\pi^2 V_0''}\int_0^1 dx \int_0^1 \rho\, d\rho\, \frac{1}{\rho[1+p(x-x^2-1)]}\ .$$

The integral over ρ can be easily evaluated, and then the integration over x gives[12] for the double integral a value $K\approx 2.34$.

Therefore,

$$I=-\frac{K}{16\pi^2 V_0''} \tag{26}$$

and thus

$$V_2^{(3)}(\phi)=\frac{-K(V_0''')^2}{192\pi^2 V_0''}\ . \tag{27}$$

In addition to these contributions $V_2^{(1)}$, $V_2^{(2)}$, and $V_2^{(3)}$, we need to add fresh counterterms $\lambda[A_2+(B_2/2)\phi^2+(C_2/4!)\phi^4]$ to remove the divergences to this order. The net sum of all these terms is

$$V_2(\phi)=\frac{1}{8\pi}\left(B_1+\frac{C_1\phi^2}{2}\right)(\ln\Lambda^2-\ln V_0'')-\frac{K(V_0''')^2}{192\pi^2 V_0''}+\frac{V_0''''}{128\pi^2}(\ln\Lambda^2-\ln V_0'')^2+\frac{B_2}{2}\phi^2+\frac{C_2}{4!}\phi^4\ . \tag{28}$$

Notice that B_1 and C_1 contain $\ln\Lambda^2$, so that the first term in (28) contains terms of the nonpolynomial divergent form $\ln\Lambda^2 \ln V_0''$ referred to earlier. These are precisely canceled by the $\ln\Lambda^2 \ln V_0''$ pieces of the third term in (28). The remaining divergences are polynomial in ϕ and are canceled by a suitable choice of $\lambda[A_2+(B_2/2)\phi^2+(C_2/4!)\phi^4]$. To pick A_2, B_2, and C_2, recall the renormalization conditions, which require that

$$V_2(\phi=0)=V_2''(\phi=0)=V_2''''(\phi=0)=0\ . \tag{29}$$

Imposing (29) on (28) fixes A_2, B_2, and C_2. Omitting trivial algebra, the net finite result for $V_2(\phi)$ is

$$V_2(\phi)=-\frac{75K}{\pi^2}\frac{\phi^6}{V_0''}+\phi^4\left[\frac{30K}{\pi^2}(2-a^2)\frac{1}{V_0''}-\frac{15K}{\pi^2}\frac{2-a^2}{(1-2a^2)}+\frac{27}{8\pi^2}\frac{(2-a^2)^3}{(1-2a^2)^2}+\frac{9K}{\pi^2}\frac{(2-a^2)^3}{(1-2a^2)^2}\right]$$
$$+\phi^2\left[-\frac{3K}{\pi^2}(2-a^2)\frac{1}{V_0''}+\frac{45}{16\pi^2}\ln^2\left(\frac{V_0''}{2-4a^2}\right)-\frac{27}{4\pi^2}\frac{(2-a^2)^2}{(1-2a^2)}\ln\left(\frac{V_0''}{2-4a^2}\right)+\frac{3K}{2\pi^2}\frac{(2-a^2)^2}{(1-2a^2)}\right]. \tag{30}$$

This is an exact renormalized expression for the two-loop contribution to V_{eff}. We notice again that, similar to $V_1(\phi)$ and for the same reasons, $V_2(\phi)$ also becomes complex in the region where $V_0''(\phi)<0$. In fact, as we approach this region from either side, $V_2(\phi)$, while real, diverges at the boundary of this region, owing to $1/V_0''$ factors in Eq. (30). This is an infrared divergence and is to be expected. Recall that $V_0''(\phi)$ is something like a mass-squared term for states built around ϕ, and its vanishing can lead to infrared divergences. Such divergences did not exist in the Coleman-Weinberg calculation of V_{eff} even though V_0'' did vanish in their model at $\phi=0$. The reason was that their calculation was up to one loop in (3+1) dimensions. Our calculation is in (1+1) dimensions and goes up to two loops. These differences enhance infrared divergences and hence the $1/V_0''$ terms are only to be expected. Our attitude with respect to these problems will be the same as in the preceding section. Assuming that these pathologies will go away in the exact $V_{\text{eff}}(\phi)$, we confine ourselves to the neighborhood of the two minima at $\phi=0$ and $\phi=1$, where $V_2(\phi)$ is real and sensible.

IV. CONCLUSIONS

Our purpose in computing the two-loop contribution was to check if it is small compared to the one-loop term, for the range of a^2 and λ for which symmetry restoration was indicated at the one-loop order. The expression (30) leads to the conclusion that it is small, when ϕ is in the neighborhood of either minimum. Since (30) is a complicated expression, the simplest way to verify this is by numerically evaluating this function for sample values of a^2 and λ. For instance, for $a^2 = 0.05$ and $\lambda = 0.01$, we have plotted $V_{\text{eff}} = (1/\lambda)V_0 + V_1 + \lambda V_2$ in curve IV of Fig. 3. It can be seen that the additional contribution of $\lambda V_2(\phi)$ is quite small as compared to $V_1(\phi)$ (curve III minus curve I), in both potential wells near $\phi = 0$ and $\phi = 1$. For example, $\lambda V_2(\phi)/V_1(\phi)|_{\phi=1, a^2=0.05, \lambda=0.01}$ is about 0.01. Similarly, for $a^2 = 0.2$ and $\lambda = 0.037$, curve IV in Fig. 4, gives V_{eff} up to two-loop order. Once again, the contribution of $\lambda V_2(\phi)$ is small compared to $V_1(\phi)$. In both Figs. 3 and 4, the contribution of $V_2(\phi)$ is to raise the minimum near $\phi = 1$ even higher and further enhance symmetry restoration. This is just a fortuitous agreement of signs between $V_1(\phi)$ and $V_2(\phi)$. We do not draw much satisfaction from this since the signs of the higher-loop contributions may well have the opposite sign. The more important point is that the magnitude of $\lambda V_2(\phi)$ is much smaller than that of $V_1(\phi)$ in the regions of interest, encouraging us to hope that the higher-loop terms will be even smaller and that our conclusion will hold.

The examples discussed above are typical in the range $0 < a^2 < 0.3$, $\lambda > \lambda_c(a^2)$ (λ_c is the critical value of λ, discussed in Sec. II). When $\lambda < \lambda_c$, the radiative corrections will not be strong enough to overturn the symmetry breaking implied at the tree level. When $\lambda > \lambda_c$, $\phi = 0$ becomes the absolute minimum, and symmetry will be restored. Of course, if λ is very large, although this phenomenon will happen at the one- or two-loop level, the convergence of the loop expansion will be destroyed.

ACKNOWLEDGMENTS

We thank Dr. D. Dhar, Dr. E. Eichten, Dr. H. R. Krishnamurthy, and Dr. E. Weinberg for helpful discussions. One of us (M. R. L.) thanks the National Council for Educational Research and Training, India, for financial support.

[1]S. Coleman and E. J. Weinberg, Phys. Rev. D 7, 1888 (1973).

[2]B. L. Ioffe, V. A. Novikov, and M. A. Shifman, Phys. Lett. 53B, 467 (1975); A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. 23, 73 (1976) [JETP Lett. 23, 64 (1976)].

[3]R. Jackiw, Phys. Rev. D 9, 1686 (1974); L. Dolan and R. Jackiw, *ibid.* 9, 2904 (1974); S. Coleman, R. Jackiw, and H. D. Politzer, *ibid.* 10, 2491 (1974).

[4]S. Coleman, in *Laws of Hadronic Matter*, edited by A. Zichichi (Academic, New York, 1975).

[5]N. D. Mermin and H. Wagner, Phys. Rev. Lett. 17, 1133 (1966); P. C. Hohenberg, Phys. Rev. 158, 133 (1967).

[6]S. Coleman, Comm. Math. Phys. 31, 259 (1973).

[7]S-K Ma and R. Rajaraman, Phys. Rev. D 11, 1701 (1975).

[8]S. Coleman, Phys. Rev. D 15, 2929 (1977); C. G. Callan, Jr. and S. Coleman, *ibid.* 16, 1762 (1977).

[9]R. Fukuda, Prog. Theor. Phys. 56, 258 (1976).

[10]R. B. Griffiths, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, New York, 1972), Vol. I.

[11]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[12]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).