Magnetic moment of massive neutrinos and the cosmic helium abundances

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We point out that if neutrinos have mass they will also develop a magnetic dipole moment. We calculate that moment in the simplest extension of the Weinberg-Salam theory. We then propose that spin precession in the magnetic fields of the early universe may be an important production mechanism for right-handed neutrinos from left-handed neutrinos. We consider two models of the primordial magnetic field—one based on the equipartition of energy and the other a "flux-freezing" model—and show that the angle of spin precession can be large. Since we would expect very different cosmic abundances of ^{4}He at present if right-handed neutrinos were to come into thermal equilibrium before the onset of ^{4}He production, we are thus able to put a constraint on the amount of spin precession of neutrinos in the early universe. This, in turn, allows us to set severe constraints on either the mass of *each* light neutrino in the $SU(2)_L \times U(1)$ model, on the magnetic moment in an arbitrary model, or on the magnitude of any intergalactic magnetic field which is a remnant of the early universe.

With the recent reports of the so-called "neutrino oscillations"[1] comes renewed interest in the masses of the known (and soon to be discovered) neutrinos. Nonzero masses could have profound cosmological implications and would force us to modify the "standard" theory of electromagnetic and weak interactions.

It is the purpose of this paper to resurrect the idea that a nonzero neutrino mass would mean that the neutrino would also develop a magnetic dipole moment.[2] We calculate that moment within the context of the simplest extension of the Weinberg-Salam $SU(2)_L \times U(1)$ model of weak and electromagnetic interactions in which the right-handed neutrinos do not interact with the gauge fields but do have Yukawa couplings to the Higgs fields, whence they get their mass.

We then consider the precession of the neutrino spin due to this magnetic moment in an external magnetic field as a production mechanism for right-handed neutrinos from initially left-handed neutrinos. We consider three models of this classical magnetic field—the last two within the context of the standard or "big bang" cosomology of an expanding universe—and point out that the amount of spin precession before the onset of ^{4}He production can be quite large. Since the cosmic abundances of ^{4}He are thought to place a bound on the total number of neutrino species in thermal equilibrium in the first few minutes of the early universe, we can put a bound on the amount of spin precession and, subsequently, on either the neutrino mass in the $SU(2)_L \times U(1)$ model, on the magnetic moment in an arbitrary model, or on the magnitude of any intergalactic field which is a remnant of the early universe.

The Feynman diagrams contributing to order g^3 in the electroweak coupling constant g to the magnetic moment of the jth neutrino are shown in Fig. 1. In 't Hooft's $\xi = 1$ gauge we must consider the contributions of the transverse gauge fields (W^+, W^-, W^0, A) and their longitudinal components (ϕ^+, ϕ^-, ϕ^0, 0) separately. Further, because of the severe restrictions imposed on the W^+ magnetic moment by the symmetry of the theory, the neutrino magnetic moment will be finite in the $SU(2)_L \times U(1)$ model.

The lepton-Higgs-coupling Lagrangian is written down in Ref. 3 and gives the neutrinos their masses. Using this and the Feynman rules of Ref. 4, we calculate the contributions to the magnetic moment from the Feynman diagrams in a straightforward way. They give the magnetic moment to order gG_F for the jth type of neutrino,

$$\mu_{\nu_j} = 3m_{\nu_j} m_e G_F \mu_B / (4\pi^2 \sqrt{2}) \tag{1}$$

$$\simeq 3.1 \times 10^{-19} \mu_B m_{\nu_j} / \text{eV}, \tag{2}$$

with G_F the Fermi constant, μ_B the Bohr magneton, and m_{ν_j} and m_e the masses of the jth neutrino and electron, respectively. This result is in agreement with special cases of the results of Bég *et al.* and Lee and Shrock.[5] Because the Kobayashi-Maskawa matrix[3] is unitary, the mag-

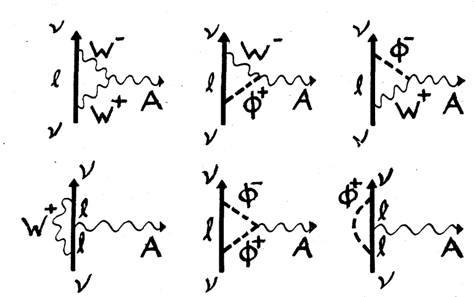

FIG. 1. Feynman diagrams contributing to neutrino magnetic moment to order g^3.

netic moment to this order does not depend on any of the mixing angles which presumably account for the neutrino oscillations.

The neutrino magnetic moment calculated in the $SU(2)_L \times U(1)$ model is so small that it appears difficult to detect. However, even such a small moment might affect the properties of the "blackbody" neutrinos thought to have been created in the early universe and to pervade all space. We propose that spin precession in the presence of external magnetic fields could be an important production mechanism for right-handed neutrinos from left-handed neutrinos which were produced in weak interactions in the early universe. This, in turn, could affect the abundances of ^{4}He.

First, consider an initially left-handed neutrino in a homogenous, isotropic, and static universe. We imagine that there is some overall uniform intergalactic magnetic field $\vec{B}_0$ which is perpendicular to the neutrino's direction of motion. Under the influence of this time-independent field, the neutrino spin will precess with frequency

$$\omega = \mu_\nu \hbar^{-1} |\vec{B}_0| \tag{3}$$

so that at a time τ later the neutrino will be right-handed:

$$\tau \simeq 7.3 \times 10^9 \, (50 \text{ eV}/m_\nu)(10^{-7} \text{ G}/|\vec{B}_0|)\text{yr} . \tag{4}$$

The magnitude of the intergalactic magnetic field is not known but it might not be much less than 10^{-7} G.[6] In that case (if we take $m_\nu = 50$ eV) we would expect a large fraction of present-day blackbody neutrinos to be right-handed. However, if we arbitrarily take the period of ^{4}He production to be the first few minutes in this model universe, there is no time for appreciable spin precession and ^{4}He production is unaffected. This model is, to say the least, unrealistic, and we drop it from further consideration.

Now we turn to a simplified version of the standard cosmological model of an expanding universe.[7] Some details of this model are outlined in the Appendix. We imagine that this too has an overall magnetic field $\vec{B}(t)$ which is time dependent but uniform in space.

Our first model of this magnetic field is based on the assumption that the energy density stored in the magnetic field is proportional to the kinetic energy density of electrons. From equipartition of energy we have

$$\vec{B}^2 \sim \eta_e T_e \tag{5}$$

with η_e the number density of electrons and T_e the electron temperature. If we use the results of the Appendix, this means that the magnetic field scales with the cosmic scale factor $R(t)$ during the radiation and matter-dominated periods as

$$\vec{B}_{\text{equipartition}} = \begin{cases} \vec{B}_0 R_0^{5/2} R_{\text{mat}}^{-1/2} R^{-2}, & \text{radiation} \quad (6) \\ \vec{B}_0 (R_0/R)^{5/2}, & \text{matter} \quad (7) \end{cases}$$

with $\vec{B}_0$ being the present intergalactic magnetic field and R_0, R_{mat}, and R_{rad} the cosmic scale factors at present and at the onset of the matter and radiation-dominated periods, respectively.

The angle of spin precession for a neutrino moving *perpendicular* to this field in a vacuum during the radiation-dominated period is

$$\Delta\theta^{\text{rad}}_{\text{equipartition}} = \hbar^{-1} \int_{t_{\text{rad}}}^{t} \mu_\nu |\vec{B}(t)| dt \tag{8}$$

$$= \hbar^{-1} \int_{R_{\text{rad}}}^{R} \mu_\nu |\vec{B}(R)| (dt/dR) dR \tag{9}$$

$$= 2\hbar^{-1} \mu_\nu |\vec{B}_0| t_0 (R_0/R_{\text{mat}}) \ln (R/R_{\text{rad}}) \tag{10}$$

with $t_0 \simeq \pi \times 10^{17}$ sec the present age of the universe. If we take R_{He} to be the cosmic scale factor at the onset of ^{4}He production ($t_{\text{He}} \simeq 180$ sec) and the values of R_{rad}, R_{He}, R_{mat}, and R_0 given in the Appendix as well as the value of μ_ν from (2), we have the total spin-precession angle before the onset of ^{4}He formation:

$$\Delta\theta^{\text{rad, He}}_{\text{equipartition}} \simeq 4.6 \times 10^{10} (m_\nu/\text{eV})(|\vec{B}_0|/\text{G})\text{rad}. \tag{11}$$

In our second model of the intergalactic magnetic field, we take the view that the universe acts as a perfect conductor and the magnetic field obeys a "flux-freezing" law:

$$\vec{B}_{\text{flux-freezing}} = \vec{B}_0 (R_0/R)^2 \quad . \tag{12}$$

In this case, the spin precession up until the onset of ^{4}He formation is about two orders of magnitude smaller:

$$\Delta\theta^{\text{rad, He}}_{\text{flux-freezing}} = 2\hbar^{-1} \mu_\nu |\vec{B}_0| t_0 (R_0/R_{\text{mat}})^{1/2} \ln(R_{\text{He}}/R_{\text{rad}}) \tag{13}$$

$$\simeq 7.5 \times 10^8 (m_\nu/\text{eV})(|\vec{B}_0|/\text{G})\text{rad}. \tag{14}$$

Similar calculations yield values of the total spin precession during the *entire* radiation- and matter-dominated periods for each of the two models of the magnetic field. All these results are displayed in Table I.

The connection between the number of two-component neutrino species and the ^{4}He abundance has been explored carefully by Yang *et al.*[8] assuming that only left-handed neutrinos interact weakly. They conclude that the number of *two-component* neutrino species in thermal equilibrium during the early universe before the onset of ^{4}He formation, N_L, is bounded:

$$N_L \leq 3 \; . \tag{15}$$

TABLE I. Spin-precession angle in standard cosmology for two models of the intergalactic magnetic field $\vec{B}$ in the Weinberg-Salam theory $[\Delta\theta/(m_\nu|\vec{B}_0|)$ in rad/(eV G)].

	To time of ^{4}He formation	Entire radiation-dominated period	Entire matter-dominated period
Equipartition model $\vec{B}^2 \sim \eta_e T_e$	4.6×10^{10}	1.2×10^{11}	$3\mu_\nu t_0 R_0/(2\hbar m_\nu R_{\text{mat}})$ $\simeq 4.7\times10^9$
Flux-freezing model $\lvert\vec{B}\rvert R^2 \sim \text{const.}$	7.5×10^8	2.0×10^9	$3\mu_\nu t_0 (R_0/R_{\text{mat}})^{1/2}/(m_\nu \hbar)$ $\simeq 1.5\times10^8$

We assume ν_τ to be light, so this leaves room only for the left-handed ν_e, ν_μ, and ν_τ (and their antiparticles) and no new species.

If the spin precession of massive left-handed neutrinos produced in weak interactions before the time of ^{4}He production into right-handed neutrinos is large, these too will come into thermal equilibrium by that time, the extra energy density will help drive the expansion of the universe and they will act just like an extra two-component species for the purposes of the calculation of the ^{4}He abundances. If we accept that calculation and assume ν_τ to be light, we must then conclude that there can be no appreciable spin precession of left-handed neutrinos before the time of ^{4}He formation. We can, therefore, set the limits

$$\Delta\theta^{\text{rad, He}}_{\text{equipartition}} < \pi, \tag{16}$$

$$\Delta\theta^{\text{rad,He}}_{\text{flux-freezing}} < \pi, \tag{17}$$

which give rise, via the above calculations, to the bounds

$$m_\nu|\vec{B}_0| < 7\times10^{-11}\ \text{eV G}, \quad \text{equipartition}, \tag{18}$$

$$m_\nu|\vec{B}_0| < 4\times10^{-9}\ \text{eV G}, \quad \text{flux-freezing} \tag{19}$$

for *each* type of neutrino.

Although the above calculations are very crude and should not be construed as better than order-of-magnitude estimates, let us take them seriously for a moment. If we take $|\vec{B}_0| \simeq 10^{-10}$ G,[6] we have limits of $m_\nu < 0.7$ eV from the equipartition and $m_\nu < 40$ eV from the flux-freezing models of the intergalactic magnetic field for *each* of the neutrinos ν_e, ν_μ, and ν_τ. Alternately, if we require the energy density of the matter-dominated phase to come primarily (90%) from the masses of these three light neutrinos, then at least one of the neutrinos ν_e, ν_μ, or ν_τ must have $m_\nu > 5$ eV (Ref. 9) and we may set limits on the intergalactic magnetic field of $|\vec{B}_0| < 1.4\times10^{-11}$ G for the equipartition model and $|\vec{B}_0| < 9\times10^{-10}$ G for the flux-freezing model.

We must hesitate, however, before accepting the above scenario at face value. If neutrinos have appreciable mass, we would have to modify the description of the matter-dominated phase and this could affect our estimate of the age of the universe[9]: $t_0 \simeq \pi\times10^{17}$ sec. Further, if neutrinos have mass, the universe and in particular binary galaxies and small groups and large clusters of galaxies may not be dominated by nucleons. One is then forced to use a significantly lower limit on the baryon density (perhaps $\Omega \geq 0.006$) rather than $\Omega \geq 0.04$ which was used to derive the limit $N_\nu \leq 3$. In this case there may be no limits on the number of neutrino types and the above results would be invalid. A more serious problem lies in the assumption of an overall intergalactic magnetic field. Not only is this field not liable to be uniform in space but there is a conceptual difficulty in the assumption that it points in some given direction throughout space. In order for that to be true, causally unconnected regions of the very early universe would have to conspire to choose that direction.[10] A more reasonable assumption is that there are domains of net magnetic polarization in space but that these domains are uncorrelated. Thus, the neutrino spin would undergo a "random walk" as it moved from domain to domain. We have implicitly assumed above that the magnetic field at the time of ^{4}He formation was correlated over distance scales $d \simeq 200$ light-seconds (about $R_0 d/R_{\text{He}} \simeq 2400$ light-years now) and that the magnitude of the magnetic field was the same in all domains. If we instead assume that the intergalactic magnetic field which is a remnant of the early universe is correlated over distances of d_0 now, we must multiply $\Delta\theta^{\text{rad,He}}$ by a random-walk factor

$$(R_{\text{He}} d_0/(R_0 d))^{1/2}. \tag{20}$$

If we arbitrarily choose $d_0 \simeq 24$ light-years, we *loosen* the constraints (18) and (19) by an order of magnitude.

Another possibility is that the fundamental gauge theory is not $SU(2)_L \times U(1)$ but some other model such as the left-right symmetric model of Bég *et al.*[5] which could give a much larger magnetic moment for the neutrino and, therefore, a much smaller limit on the magnetic field. If we want to consider an arbitrary gauge theory, then we should properly use the above reasoning to set limits on the neutrino magnetic moment rather than the mass. If we forget about the random walk and use $|\vec{B}_0| \simeq 10^{-10}$ G, the constraints (16) and (17) are readily translated into constraints on the magnetic moment which are independent of any gauge theory:

$$|\mu_\nu| < 2 \times 10^{-19} \mu_B\,, \quad \text{equipartition,} \tag{21}$$

$$|\mu_\nu| < 1 \times 10^{-17} \mu_B\,, \quad \text{flux-freezing} \tag{22}$$

for each light neutrino. There is also the possibility that $m_\nu = \mu_\nu = 0$ exactly as would occur if the Weinberg-Salam theory were properly imbedded in a grand unified theory (GUT) based on SU(5). Other GUT's allow only "Majorana" masses for neutrinos and, in these cases, the above reasoning is inapplicable.

Other mechanisms for converting massive left-handed into right-handed neutrinos are possible, such as gravitational scattering. There might even be primordial right-handed neutrinos left over from the very early universe due to spin precession in the huge magnetic fields of that era.[11]

Although the connection between the neutrino magnetic moment and the known ^{4}He abundance given above is *very* crude and should be considered only an order-of-magnitude estimate, it does pose one severe constraint on cosmological phenomena from the very basis of the fundamental theory of electroweak interactions—the higher-order effects which were the motivation for the invention of gauge theories in the first place. These estimates are encouraging because the magnitudes of the magnetic fields quoted above are not out of the question for *local* intergalactic magnetic fields. A *correct* calculation would involve dispensing with the classical magnetic field altogether and looking at the early universe from an elementary-particle-physics point of view. Note, however, that the spatial coherence of the classical magnetic field was crucial for the effect to be large in the first place. This field coherence over distances large compared to the mean free path of neutrinos may be mimicked by the effects of turbulence in the radiation-dominated era. Coherent processes of elementary particles over relatively large "turbulence scales" could generate relatively intense magnetic fields which are coherent over these scales.[12] We then imagine left-handed neutrinos colliding with photons, leptons, and nucleons and ask for the total production rate for right-handed neutrinos in the extreme conditions (high-flux densities, etc.) of that epoch for a given gauge theory of electroweak interactions [possibly imbedded in a GUT such as that based on SO(10)]. By requiring that right-handed neutrinos *not* come into thermal equilibrium before the onset of ^{4}He production, we might then be able to put constraints on m_ν or some other parameter of the theory or the cosmological model. The above connection between the magnetic dipole moment of neutrinos and the cosmic ^{4}He abundance therefore deserves careful examination and refinement so that any inconsistencies may be resolved. We will then have a more tightly constrained theoretical framework in which to do fundamental physics.

Note added: After this work was finished, it was found that Shapiro and Wasserman have independently reached similar conclusions for the flux-freezing model of an intergalactic magnetic field.[13]

I would like to thank and give credit to G. Feinberg for his original proposal of the above ideas. I also should like to thank M. A. B. Bég, J. Bernstein, and M. Ruderman for helpful conversations and S. A. Teukolsky, S. L. Shapiro, and I. Wasserman for their useful correspondence. This research was supported in part by the United States Department of Energy.

APPENDIX

The time development of the universe in the standard cosmological model as outlined in Ref. 7 is divided into three main periods. The very early universe (temperatures $T > 10^{12}$ °K) contains a great many particle species, including strongly interacting particles, in thermal equilibrium. For temperatures 10^3–$10^4 < T < 10^{12}$ °K the dynamics of the universe is dominated by relativistic electrons, positrons, neutrinos, antineutrinos, and photons as well as a few nucleons. ^{4}He is produced in the first few minutes ($t_{\text{He}} \simeq 180$ sec) of this radiation-dominated period and it will be the main focus of this paper. The onset of the matter-dominated period occurs when the universe has cooled to $T < 10^3$–10^4 °K and from then on the dynamics is dominated by nonrelativistic particles. We take the cosmic scale factor $R(t)$ at the onset of the radiation-dominated period, the period of ^{4}He formation, and the matter-dominated period to be, respectively,

$$R_{\text{rad}} \simeq 1.9 \times 10^{-12} R_0\,, \tag{A1}$$

$$R_{\text{He}} \simeq 2.6 \times 10^{-9} R_0\,, \tag{A2}$$

$$R_{\text{mat}} \simeq 2.7 \times 10^{-4} R_0\,, \tag{A3}$$

with R_0 the cosmic scale factor at the present time

$$t_0 \simeq \pi \times 10^{17} \text{ sec} \tag{A4}$$

even though we expect the dynamics of the matter-dominated period to be very different from that outlined by Weinberg if neutrinos have appreciable mass.

The cosmic scale factor $R(t)$ is given by Einstein's equations with a Robertson-Walker metric. If we neglect spatial curvature (not necessarily a good idea) we have

$$(dR/dt)^2 = 8\pi G\rho R^2/3 \qquad \text{(A5)}$$

with the energy density ρ and the pressure P related by energy conservation,

$$d(\rho R^3)/dR = -3PR^2\,, \qquad \text{(A6)}$$

and G, Newton's gravitational constant.

For relativistic particles $P=\rho/3$ and for blackbody radiation $\rho \sim T^4$, so that during the radiation-dominated period

$$\rho \sim R^{-4}, \qquad \text{(A7)}$$

$$T \sim R^{-1}\,, \qquad \text{(A8)}$$

$$t = t_0 R^2 R_{\rm mat}{}^{-1/2} R_0{}^{-3/2}\,. \qquad \text{(A9)}$$

On the other hand, during the matter-dominated period $P \ll \rho$, so we have

$$\rho \sim R^{-3}\,, \qquad \text{(A10)}$$

$$T_{\rm mat} \sim R^{-3(\gamma-1)}\,, \qquad \text{(A11)}$$

$$t = t_0 (R/R_0)^{3/2}\,. \qquad \text{(A12)}$$

Here $T_{\rm mat}$ is the matter temperature (different from the radiation temperature after the time of formation of atomic hydrogen) while γ is the specific heat of the matter gas; for free electrons $\gamma = \frac{5}{3}$. Note that the coefficients of (A9) and (A12) have been chosen to match at $R = R_{\rm mat}$. We assume conservation of the total number of electrons during both the matter and radiation-dominated periods so that the electron number density scales as

$$\eta_e \sim R^{-3}\,. \qquad \text{(A13)}$$

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[3]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1972); W. J. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977). We take the lepton-Higgs coupling Lagrangian in analogy with the Kobayashi-Maskawa mass terms for quarks:

$$L_{\rm mass} = -F^{-1} \sum_{i,j=1}^{N} [M^{\nu}_{ij} \bar{\Psi}_i K (1-\gamma_5)\nu_j/2 + M^{l}_{ij} \bar{\Psi}_i \tilde{K}(1-\gamma_5) l_j/2 + \text{H. c.}]$$

Here, N is the total number of lepton doublets, F is the Higgs vacuum expectation value, and the fields are defined as

$$K = 2^{-1/2}\begin{pmatrix} 2^{1/2}F + Z + i\phi^0 \\ -\phi_2 + i\phi_1 \end{pmatrix},$$

$$\Psi_i = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu_i \\ l_i \end{pmatrix}$$

the ith left-handed doublet, with l_i and ν_i the ith charged lepton and neutrino, respectively. M^{ν} and M^{l} are Hermitian matrices of c-numbers while Z is the real massive Higgs field with $\tilde{K} = -i\sigma_2 K^*$. After diagonalization of the lepton mass matrix (and redefinition of the fields), the charged currents depend on Kobayashi-Maskawa mixing angles while the neutral currents do not.

[4]All notation and convention as well as the Feynman rules used in the calculation of the neutrino magnetic moment are as in G. 't Hooft and M. Veltman, in *Particle Interactions at Very High Energies*, edited by D. Speiser, F. Halzen, and J. Weyers (Plenum, New York, 1974), Part B, p. 177; and G. Passarino and M. Veltman, Nucl. Phys. B160, 151 (1979).

[5]M. A. B. Bég, W. J. Marciano, and M. Ruderman, Phys. Rev. D 17, 1395 (1978). If we take their equation (4.9) for the neutrino magnetic moment in $SU(2)_L \times SU(2)_R \times U(1)$ and set $M_{W_2} = \infty$ and $\zeta = 0$ our result follows. It is also possible to get the magnetic moment from B. W. Lee and R. E. Shrock, Phys. Rev. D 16, 1444 (1977) with a little work.

[6]S. L. Shapiro, S. A. Teukolsky, and I. Wasserman report in a private communication that various upper limits on any intergalactic magnetic field range from $10^{-6} - 10^{-10}$ G depending on what assumptions are made. See also Ref. 13.

[7]We use here the standard cosmological model details of which can be found in Chap. 15 of S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

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[9]J. Bernstein and G. Feinberg, Phys. Lett. (to be published).

[10]We are indebted to M. Ruderman for this remark.

[11]It is at the present time thought that SU(2) symmetry is restored above temperatures $T > 5 \times 10^{15}$ °K and, therefore, that $m_\nu = \mu_\nu = 0$ in the very early universe. This may not be the case, however, if the Weinberg-Salam theory were imbedded in a GUT with two distinct energy scales of 10^2 and 10^{14} GeV. This possibility will be the subject of a future paper.

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