Gauge invariance, semiminimal coupling, and propagating torsion

V. De Sabbata and M. Gasperini
Istituto di Fisica dell'Università, Bologna, Italy
and Istituto di Fisica dell'Università, Ferrara, Italy
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A dynamical theory of torsion is proposed in which the electromagnetic potential is dispensed from minimal coupling, and a "semiminimal" photon-torsion coupling is introduced and physically justified. Local gauge invariance of the total Lagrangian density is preserved in its usual form, provided that torsion is determined by the gradient of a propagating potential. From the field equations we deduce that a polarized macroscopic body is a source of a dipolar torsion field, and that a test body may feel torsion only if polarized. No disagreement is found with the Eötvös-Dicke-Braginskii tests of the equivalence principle.

I. INTRODUCTION

Recently Hojman, Rosenbaum, Ryan, and Shepley[1] (HRRS) have proposed a theory in which torsion and electromagnetism interact. In order that gauge invariance be compatible with the minimal coupling of the electromagnetic field to space-time geometry, they propose to modify the form of local gauge transformations in the presence of torsion, obtaining a dynamical theory which allows propagation of torsion in a vacuum. However, as shown by Ni[2] through an explicit computation, the field equations of this theory imply that test bodies of different electromagnetic content accelerate differently in the solar and terrestrial gravitational field, in disagreement with the experiments performed by Eötvös, Dicke, and Braginskii.

In the present paper we propose a theory in which photons, like all the other spinning particles, both generate and react to torsion; introducing a photon-torsion coupling (which we call semiminimal) different from the minimal one of the HRRS theory, we are able to retain the usual form of local gauge invariance, without modifications in the coupling of the electromagnetic field to matter fields. The result is again a propagating torsion theory, but our field equations do not seem to disagree with experiments.

In Sec. II of this paper, after a short review of the HRRS theory, we present the Lagrangian density for our theory, providing with a physical justification the photon-torsion coupling term. In Sec. III we give the field equations and in Sec. IV we calculate the torsionic field produced by a macroscopic body. In Sec. V we compare the implications of our theory with experiments, and in Sec. VI we point out the main differences between the HRRS theory and ours. For easy reference, we adopt the notation and conventions of the papers of Refs. 1 and 2.

II. GAUGE INVARIANCE AND THE PHOTON-TORSION INTERACTION

We start, as in the HRRS paper, with the Lagrangian density for a charged massless scalar field ψ in a space with curvature and torsion; the total Lagrangian density is

$$\mathfrak{L}=\frac{\sqrt{-g}}{16\pi}R'-\frac{\sqrt{-g}}{4\pi}\psi^*_{,\mu}\psi^{,\mu}\,, \tag{1}$$

where R' is the scalar curvature derived from the nonsymmetric connection $\Gamma^\alpha{}_{\mu\nu}$,

$$\Gamma^\alpha{}_{\mu\nu}=\left\{{\alpha \atop \mu\nu}\right\}-\tfrac{1}{2}(T^\alpha{}_{\mu\nu}-T_\mu{}^\alpha{}_\nu-T_\nu{}^\alpha{}_\mu)\,. \tag{2}$$

$\left\{{\alpha \atop \mu\nu}\right\}$ is the Christoffel symbol and $T^\alpha{}_{\mu\nu}=\Gamma^\alpha{}_{\nu\mu}-\Gamma^\alpha{}_{\mu\nu}$ is the torsion tensor. We consider the following local gauge transformation:

$$\psi\to\psi'=e^{iq\Lambda(x)}\psi\,, \tag{3}$$

where q is the electromagnetic coupling constant. As is well known, the Lagrangian density (1) can be made invariant under this transformation, provided that a coupling between the matter field ψ and the electromagnetic potential A_μ is introduced according to the prescription

$$\psi_{,\mu}\to D_\mu\psi=\psi_{,\mu}-iqA_\mu\psi \tag{4}$$

and provided that A_μ, under the gauge transformation (3), transforms as

$$A_\mu\to A'_\mu=A_\mu+\Lambda_{,\mu}\,. \tag{5}$$

The gauge-invariant Lagrangian density becomes then

$$\mathfrak{L}=\frac{\sqrt{-g}}{16\pi}R'-\frac{\sqrt{-g}}{4\pi}D_\mu\psi^*D^\mu\psi-\frac{\sqrt{-g}}{16\pi}F_{\mu\nu}F^{\mu\nu}\,, \tag{6}$$

where the last term is the free electromagnetic Lagrangian, and

$$F_{\mu\nu}=A_{\nu|\mu}-A_{\mu|\nu}=A_{\nu,\mu}-A_{\mu,\nu} \tag{7}$$

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(the bar symbol denotes a covariant derivative using only the Christoffel symbols of the metric).

If we had defined, following the minimal-coupling procedure, the electromagnetic field tensor using covariant differentiation (denoted by a semicolon) involving the full connection coefficients $\Gamma^{\alpha}{}_{\mu\nu}$, i.e.,

$$F_{\mu\nu}=A_{\nu;\mu}-A_{\mu;\nu}\,, \tag{8}$$

then we should not have obtained gauge invariance of the Lagrangian (6) under the local transformation (5). The usual procedure (proposed by Hehl *et al.*[3]) to secure gauge invariance in the presence of torsion is then to dispense A_μ from minimal coupling, and to define $F_{\mu\nu}$ according to (7). This prescription finds a very natural justification in the framework of the Poincaré gauge field theories of gravitation,[4] where the gauge fields (as the potential A_μ) are treated as Poincaré scalars. This means, however, that photons are decoupled from torsion, although they are spinning particles.

As an alternative to the choice of Hehl *et al.*, HRRS proposed to keep the minimal coupling between gravitation and electromagnetism in the form (8), introducing, however, a nonminimal prescription for the coupling of the matter field

$$\psi_{,\mu}\rightarrow\psi_{.\mu}=\psi_{,\mu}-iqe^{-\phi}A_\mu\psi \tag{9}$$

and a modified form of local gauge transformation

$$A_\mu\rightarrow A'_\mu=A_\mu+e^{\phi}\Lambda_{,\mu} \tag{10}$$

(ϕ is a scalar function). They showed that the minimal coupling (8) and gauge invariance under the transformations (10) are compatible, provided that torsion be determined by the gradient of the potential ϕ according to (see also Refs. 11 and 12)

$$T^{\alpha}{}_{\mu\nu}=\delta^{\alpha}{}_{\nu}\phi_{,\mu}-\delta^{\alpha}{}_{\mu}\phi_{,\nu}\,. \tag{11}$$

They obtained the field equations

$$F^{\mu\nu}{}_{|\nu}=e^{-\phi}J^{\mu}_{\psi}-F^{\mu\nu}\phi_{,\nu}$$
$$F_{[\mu\nu|\alpha]}=F_{[\mu\nu}\phi_{,\alpha]} \tag{12}$$

for the electromagnetic field, and

$$\phi^{|\alpha}{}_{\alpha}=\tfrac{1}{6}F_{\mu\nu}F^{\mu\nu} \tag{13}$$

for the torsion potential, where J^{μ}_{ψ} is the usual matter current density, and $F_{\mu\nu}$ is defined, according to (8), as

$$F_{\mu\nu}=A_{\nu,\mu}-A_{\mu,\nu}+A_\mu\phi_{,\nu}-A_\nu\phi_{,\mu}\,. \tag{14}$$

According to these equations, the electromagnetic field acts as a source for torsion, and torsion is allowed to propagate, as its potential obeys a wave equation. Moreover, torsion affects the electromagnetic field, generating an electric current, $J^{\mu}_{\phi}=-F^{\mu\nu}\phi_{,\nu}$, and a magnetic current density, $m^{\mu}_{\phi}=\frac{1}{2}\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}\phi_{,\nu}$, where $\eta_{\mu\nu\alpha\beta}=(-g)^{1/2}\epsilon_{\mu\nu\alpha\beta}$ is the completely antisymmetric symbol defined by $\eta^{0123}=(-g)^{-1/2}$.

The field equation (13) and the minimal coupling (14) disagree with the experimental results on the accelerations of different test bodies in the solar and in the earth's gravitational field.[2] Therefore, we propose a photon-torsion interaction dropping the assumption that the coupling is to be introduced according to the formal prescription (8). Starting from the Lagrangian density (6), we define $F_{\mu\nu}$ as in (7), and we introduce a nonminimal photon-torsion interaction Lagrangian, on the grounds of the following physical motivations: Even if torsion is not minimally coupled to electromagnetism, there is, however, a minimal coupling between torsion and the virtual pairs of massive fermions produced by photons according to the vacuum polarization effect. By virtue of this effect, we may expect an "indirect" coupling, which we may call "semiminimal," between torsion and the electromagnetic field itself.[5] Following the result of an explicit perturbative computation of the polarization tensor in the presence of torsion,[5] we add to the Lagrangian (6) the "phenomenological" interaction Lagrangian density

$$\mathcal{L}_I=\frac{\sqrt{-g}}{16\pi}\alpha\eta^{\mu\nu\alpha\beta}A_\mu F_{\nu\alpha}T_\beta\,, \tag{15}$$

where $\alpha=e^2/4\pi\hbar c$ is the fine-structure constant, T_μ is the totally antisymmetric part of the torsion tensor

$$T_\mu=\frac{1}{3!}\eta_{\mu\nu\alpha\beta}T^{\nu\alpha\beta}\,, \tag{16}$$

and $F_{\mu\nu}$ is defined by (7). The coupling constant α, justified by the explicit computation of Ref. 5, is due to the fact that vacuum polarization is a second-order electromagnetic effect; notice also that only the totally antisymmetric part of the torsion takes part in the interaction, because of the axial character of the spin tensor of the virtual Dirac particles, as shown in Ref. 5. The total Lagrangian density, $\mathcal{L}+\mathcal{L}_I$, is gauge invariant under the usual local transformation (5), provided that $T_{[\mu,\nu]}=0$; therefore gauge invariance is satisfied if we require that torsion be generated by a potential φ, i.e., $T_\mu=\varphi_{,\mu}$.

In the absence of other torsionic sources besides the electromagnetic field, we can set

$$T^{\mu}{}_{\alpha\beta}=\eta^{\mu}{}_{\alpha\beta\nu}\varphi^{,\nu} \tag{17}$$

and the gravitational Lagrangian density R', separating the torsion contributions, becomes

$$\frac{\sqrt{-g}}{16\pi}R'=\frac{\sqrt{-g}}{16\pi}(R+\tfrac{3}{2}\varphi_{,\alpha}\varphi^{,\alpha})\,, \tag{18}$$

where R is the scalar curvature derived from the Christoffel connection.

III. FIELD EQUATIONS

The total Lagrangian density is ($c=1,\ G=1$)

$$\mathcal{L}=\frac{\sqrt{-g}}{16\pi}(R+\tfrac{3}{2}\varphi_{,\alpha}\varphi^{,\alpha}-F_{\mu\nu}F^{\mu\nu}-4D_\mu\psi^*D^\mu\psi + \alpha\eta^{\mu\nu\alpha\beta}A_\mu F_{\nu\alpha}\varphi_{,\beta}), \quad (19)$$

where $F_{\mu\nu}=A_{\nu,\mu}-A_{\mu,\nu}$ and $D_\mu\psi=\psi_{,\mu}-iqA_\mu\psi$. It is invariant under the usual local gauge transformations

$$\psi\to\psi'=e^{iq\Lambda(x)}\psi,\quad A_\mu\to A'_\mu=A_\mu+\Lambda_{,\mu} \quad (20)$$

and the electromagnetic field is coupled in a minimal way to the matter field ψ, and in a semiminimal way to the torsion potential φ. From independent variation of ψ^*, ψ, A_μ, and φ, we obtain the following field equations:

$$(D^\mu\psi)_{|\mu}=iqA_\mu D^\mu\psi,\quad (D^\mu\psi^*)_{|\mu}=-iqA_\mu D^\mu\psi^*, \quad (21)$$

$$F^{\mu\nu}{}_{|\nu}=4\pi J^\mu_\psi+\tfrac{1}{2}\alpha\eta^{\mu\nu\alpha\beta}F_{\nu\alpha}\varphi_{,\beta}, \quad (22)$$

$$\varphi^{|\alpha}{}_{\alpha}=\tfrac{1}{6}\alpha\eta^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \quad (23)$$

where

$$J^\mu_\psi=\frac{i}{4\pi}q(\psi D^\mu\psi^*-\psi^*D^\mu\psi) \quad (24)$$

is the current density of the charged matter field. Unlike the HRRS theory, notice that ψ is not directly coupled to torsion, in agreement with the expectation that a spinless particle should be unaffected by torsion.

According to Eq. (23), torsion may propagate, as in the HRRS paper; in our theory, however, the density of "torsionic charge" is a pseudoscalar, such as the torsionic potential φ. Equation (23) shows that a free radiation field, satisfying $\vec{E}\cdot\vec{B}=0$, does not serve as a source for torsion: This agrees with the fact that there is no linear vacuum polarization effect for traveling electromagnetic waves in empty space[6] and with the physical interpretation given to the semiminimal coupling term.

In our theory, the Maxwell equations are modified as follows[7]:

$$F^{\mu\nu}{}_{|\nu}=4\pi J^\mu+\tfrac{1}{2}\alpha\eta^{\mu\nu\alpha\beta}F_{\nu\alpha}\varphi_{,\beta},\qquad F_{[\mu\nu|\alpha]}=0. \quad (25)$$

Torsion does not generate a magnetic current, unlike the HRRS theory, but only an electric current $J^\mu_\varphi=\frac{1}{2}\alpha\eta^{\mu\nu\alpha\beta}F_{\nu\alpha}\varphi_{,\beta}$, which is conserved by itself, i.e., $J^\mu_{\varphi|\mu}=0$. The total electric charge Q associated with J^μ_φ vanishes, however. In fact,

$$Q=\int\sqrt{-g}\,d^3x\,J^0_\varphi=\frac{\alpha}{2}\int d^3x(\epsilon^{0ijk}F_{ij}\varphi)_{,k} \quad (26)$$

(Latin indices range from 1 to 3) and this vanishes in a simply connected space if the fields fall off to zero sufficiently fast at spatial infinity. Notice that also this fact has a simple explanation: If torsion is due to the virtual particle-antiparticle pairs associated with a "physical" photon, then the net total charge must be zero.

IV. THE TORSION FIELD OF A MACROSCOPIC BODY

In this section we will compare the torsionic potential φ produced by a celestial macroscopic body such as the Sun or the Earth, with its Newtonian gravitational potential.

In first approximation, setting $E_k=F_{k0}$ and $B_k=\epsilon_{kij}F^{ij}/2$, Eq. (23) reduces to

$$\nabla^2\varphi=-\tfrac{4}{3}\alpha E_kB^k \quad (27)$$

and the solution is given by

$$\varphi=\frac{\alpha}{3\pi}\int d^3x\,\frac{E_kB^k}{r}, \quad (28)$$

where $r=(x_kx^k)^{1/2}$. Since stars and planets are neutral, the outer space, where $E=0$, gives no contribution to the integral; for the same reason also the atoms, inside the body, do not contribute. The only contribution may come from the single components, nuclei and electrons.

Suppose that a body is composed of particles with charge e, mass m_0, magnetic moment μ^k, and radius $r_0=h/m_0c$. First of all we shall evaluate the torsion field produced by one of these particles. As we are concerned with the field far from the body, Eq. (28) may be expanded as follows:

$$\varphi(x)=\frac{\alpha}{3\pi}\left[\frac{1}{r}\int d^3x'E_k(x')B^k(x') + \frac{x^i}{r^3}\int d^3x'E_k(x')B^k(x')x'_i+\cdots\right]. \quad (29)$$

In the hypothesis of a Coulombian electric field, $E^k=ex^k/r^3$, and of a dipolar magnetic field, $B^k=(3x^k\mu_ix^i-\mu^kr^2)/r^5$, the first integration in Eq. (29) vanishes, and the second integral, which may be called a "torsionic dipole" d^i, gives

$$d_i=\frac{\alpha}{3\pi}\int d^3x\,E_kB^kx_i=\tfrac{8}{9}\alpha\frac{e}{r_0}\mu_i. \quad (30)$$

Therefore, a particle with a Coulombian electric field and a dipolar magnetic field has also a torsionic dipolar moment proportional to its magnetic moment, and it generates a torsion field described by the following potential:

$$\varphi = \frac{x^i d_i}{r^3} = \tfrac{8}{9}\alpha \frac{e}{r_0}\frac{x^i \mu_i}{r^3}. \tag{31}$$

Inside a macroscopic body, to each charged spinning particle is associated then a torsionic dipole: In the absence of spin alignment, however, these dipoles are oriented chaotically, the total averaged torsionic moment vanishes, and there is no torsion field outside the body.

In the case of spin orientation,[8] on the contrary, the total magnetic and torsionic moments are nonzero, and a dipolar torsion field is produced, proportional to the magnetic field. Supposing that the magnetic moment $\mathfrak{M}_k$ of a celestial body of mass M is entirely due to spin alignment, we must expect then a torsion field associated with the magnetic field, and we may compare the dipolar torsionic potential φ with the gravitational potential $U = M/r$ at a distance r. For the Sun we obtain, from Eq. (31),

$$\varphi \sim \frac{8}{9}\frac{e^3 m_0}{4\pi\hbar^2 c^2}\frac{\mathfrak{M}}{M}\frac{U}{r} \sim 0.6\times 10^{-19}\frac{U}{r}, \tag{32}$$

where m_0 is the proton mass and we have used a solar magnetic moment per unit mass $\mathfrak{M}/M = 4.5$. At the Earth's surface then $\varphi \sim 0.4\times 10^{-32} U$ according to our theory, while, in the HRRS theory, $\varphi \sim 0.6\times 10^{-4} U$. It must be stressed that if the solar magnetic field is not entirely due to the alignment of the nuclear spins in its interior, then $\varphi < 10^{-19} U/r$; in particular, if there is no spin alignment and the magnetic field has a different origin, then $\varphi = 0$.

V. TEST-BODY ACCELERATIONS

We will follow the method introduced in Refs. 2 and 9 to calculate the accelerations of a macroscopic test body in a space with curvature and torsion. The four-momentum vector of a test body is

$$P_\mu = \int d^3x\, \tau_\mu{}^0, \tag{33}$$

where $\tau_\mu{}^\nu$ is the total stress-energy tensor density, including also the electromagnetic contributions. Starting with the Lagrangian $\mathcal{L}'$,

$$\mathcal{L}' = -\frac{\sqrt{-g}}{16\pi}(F_{\mu\nu}F^{\mu\nu} - \alpha\eta^{\mu\nu\alpha\beta}A_\mu F_{\nu\alpha}\varphi_{,\beta}) + \mathcal{L}^{(P)}, \tag{34}$$

where $\mathcal{L}^{(P)}$ is the test-particle Lagrangian density, and using the matter-response equations $\tau_\mu{}^\nu{}_{,\nu} = \partial\mathcal{L}'/\partial x^\mu$, we get, in a "local inertial frame" such that the Christoffel symbols vanish at the location of the body,

$$\begin{aligned}\dot P_\mu &= \frac{\alpha}{16\pi}\int d^3x\,\eta^{\sigma\nu\alpha\beta}(A_\sigma F_{\nu\alpha}\varphi_{,\beta})_{,\mu}\\ &= \dot P^{(0)}_\mu + \dot P^{(1)}_\mu + \cdots,\end{aligned} \tag{35}$$

where

$$\dot P^{(0)}_\mu = \frac{\alpha}{32\pi}\varphi_{,\mu}\int d^3x\,\eta^{\sigma\nu\alpha\beta}F_{\sigma\nu}F_{\alpha\beta}, \tag{36}$$

$$\dot P^{(1)}_\mu = \frac{\alpha}{32\pi}\varphi_{,\mu,\gamma}\int d^3x\,\eta^{\sigma\nu\alpha\beta}F_{\sigma\nu}F_{\alpha\beta}x^\gamma \tag{37}$$

(we neglect higher-order derivatives of φ). If the test body is unpolarized (i.e., no spin alignment in its interior) the two integrals vanish identically: Therefore unpolarized matter of different composition fall with the same acceleration in the gravitational field of the Sun and of the Earth, even assuming that a celestial body generates a torsion field (i.e., $\varphi_{,\mu}\neq 0$). Our theory, unlike the HRRS theory, is then in full agreement with the Eötvös-Dicke-Braginskii[10] experiments, performed with unpolarized aluminum and platinum matter.

According to our theory, test bodies of different polarization accelerate differently in the gravitational field produced by a polarized body. Defining the center of mass of the test particle as[2]

$$X^\mu = \frac{1}{P^0}\int d^3x\, x^\mu \tau_0{}^0 \tag{38}$$

in the local inertial frame we get, in first approximation, $P^0\ddot X_\mu = m\ddot X_\mu = \dot P_\mu$, where m is the mass of the test body. In this frame, a polarized test body is then subject to an acceleration $\ddot X_k$ given by Eq. (37),

$$\ddot X_k = -\frac{3}{4m}(T_k)_{,i}D^i, \tag{39}$$

where $T_k = \varphi_{,k}$ is the torsion field and D^i is the torsionic dipolar moment of the test body, related to its total magnetic moment according to Eq. (30).

VI. SUMMARY AND CONCLUSION

In this paper we have shown that it is possible to propose a theory in which torsion and electromagnetism interact, without modifying the form of local gauge invariance, provided that a semiminimal photon-torsion coupling is chosen on the grounds of physical reasonableness. The Maxwell equations are modified, but no magnetic current is generated and the total electric charge induced by torsion is zero. Moreover, torsion is not directly coupled to matter scalar fields, even if they are charged. As in the HRRS theory, torsion is generated by a propagating potential; in our case, however, the torsionic charge density $\rho(x)$ is a pseudoscalar,

$$\rho(x) = -\frac{\alpha}{24\pi}\eta^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}, \tag{40}$$

because only the axial-vector part of the torsion tensor is coupled to the electromagnetic field. A macroscopic body, if polarized, generates a dipolar torsion field whose potential is

$$\varphi(x) = \frac{x^k}{r^3}\int d^3x'\rho(x')x'_k, \tag{41}$$

where $\rho(x)$ is given by Eq. (40). The theory proposed in this paper agrees with all the present tests of the equivalence principle, and deviations from geodesic motion at a macroscopic level are predicted only in the case of gravitational interactions between two polarized bodies.

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