Composite quarks and nonleptonic weak interactions

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Nonleptonic weak interactions are studied on the bases of some composite-quark models proposed currently and within the framework of the conventional minimal $SU(2)_L \times U(1)$ gauge theory. One of those models provides a further SU(4)-$\underline{20}$ enhancement by a factor of 2 as compared with the conventional Glashow-Iliopoulos-Maiani model.

Recently many authors[1-9] have discussed subquark models of leptons and quarks for the purpose of a unified description of leptons and quarks, but few authors have directed their attention to nonleptonic weak interactions.

In the conventional Glashow-Iliopoulos-Maiani (GIM) scheme,[10] the effective Hamiltonian for nonleptonic weak interactions is of the form

$$H_{\rm NL}^{\rm GIM}=\frac{G_F}{\sqrt{2}}\{\sin\theta_C\cos\theta_C[\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{u}\gamma^\mu(1-\gamma_5)s] + \cos^2\theta_C[\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{c}\gamma^\mu(1-\gamma_5)s]+\cdots\} + \text{H.c.}, \quad (1)$$

in the absence of strong interactions. It is well known that the factor $\sin\theta_C\cos\theta_C$ in the $\Delta C=0$, $\Delta S=1$ piece of the interaction (1) is too small to explain the observed $\Delta I=\frac{1}{2}$ amplitude in kaon decays, while it is somewhat large to account for the observed deviation from the $\Delta I=\frac{1}{2}$ rule. Moreover, the interaction (1) provides no satisfactory description for charm decays.

A major school of thought argues that the conventional weak Hamiltonian (1) is correct, but such characteristic features in nonleptonic weak decays of hadrons are provided by the strong interactions, and the problem is how to estimate such effects reliably.

Indeed, for example, the gluon corrections at short distance provide an enhancement of the SU(4)-$\underline{20}$ piece and a suppression of the SU(4)-$\underline{84}$ piece in the interaction (1):[11]

$$H_{\rm NL}^{\rm eff} = \frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C(\tfrac{1}{2}c_+^{LL}O_+^{LL}+\tfrac{1}{2}c_-^{LL}O_-^{LL}) + \text{H.c.}+\cdots, \quad (2)$$

where c_-^{LL} is the enhancement factor for the $\underline{20}$ piece and c_+^{LL} is the suppression factor for the $\underline{84}$ piece, and

$$O_\pm^{LL}=[\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{u}\gamma^\mu(1-\gamma_5)s] \pm[\bar{u}\gamma_\mu(1-\gamma_5)u][\bar{d}\gamma^\mu(1-\gamma_5)s]. \quad (3)$$

(Here and hereafter we drop small components[12] induced by "penguin" diagrams.) However, the interaction (2) is still unsatisfactory to explain the observed features in K and D decays.

On the other hand, a minor school of thought argues that the interaction (1) is incomplete and there are some additional terms which originate in a subquark structure of quarks.[13]

In this paper, we investigate possible features of nonleptonic weak interactions on the bases of some composite-quark models proposed currently but within the framework of the conventional minimal $SU(2)_L\times U(1)$ gauge theory.[14] Of course, we consider that the strong-interaction effects are inevitably present and they cannot be disregarded.

Since we confine ourselves to studying the weak interactions, we assume that the quark is composed of a fermion (ν, l), whose left-handed component is a doublet of the weak $SU(2)_L$, and the others Θ, although Θ may be composed of further fundamental subquarks. According to which constituent carries flavor and/or color quantum numbers, we classify the models as follows.

Model A. The fermion (ν, l) has flavor index f ($f=e, \mu, \tau$, and so on) and the boson Θ has color index i ($i=1,2,3,$):

$$u'=\nu^e\Theta_i,\ c'=\nu^\mu\Theta_i,\ \ldots, \quad d'=l^e\Theta_i,\ \ s'=l^\mu\Theta_i,\ \ldots. \quad (4)$$

Model B. The boson Θ has both flavor and color indices:

$$u'=\nu\Theta_i^e,\ \ c'=\nu\Theta_i^\mu,\ \ldots, \quad d'=l\Theta_i^e,\ \ s'=l^\mu\Theta_i,\ \ldots. \quad (5)$$

Model C. The fermion (ν, l) has color index and the boson Θ has flavor index:

$$u'=\nu_i\Theta^e,\ \ c'=\nu_i\Theta^\mu,\ \ldots, \quad d'=l_i\Theta^e,\ \ s'=l_i\Theta^\mu,\ \ldots. \quad (6)$$

Here u', c', d', and s' are flavor-mixing states, $u'=u\cos\alpha-c\sin\alpha$, $c'=u\sin\alpha+c\cos\alpha$, $d'=d\cos\beta-s\sin\beta$, and $s'=d\sin\beta+s\cos\beta$, respectively, and the Cabibbo angle θ_C is given by $\theta_C=\alpha-\beta$. The model A corresponds to the so-called "new Nagoya model"[2] proposed in 1962 if we consider that "B matter" has colors, and to models proposed later by Pati and Salam,[3] Matumoto,[4] and Greenberg.[5] The model B corresponds to models proposed by Akama, Terazawa and Chikashige,[6] Yasuè,[7] Tanikawa and Saito,[8] and Ne'eman,[9] so far as the structure of the quark is concerned (although their models have much wealth of contents compared with the present treatment of the subquarks).

We assume that the weak-boson mass is so large in comparison to hadronic mass scale that we can regard the effective weak interactions of (ν, l) as the four-fermion interaction

$$H_w = \frac{G_F}{\sqrt{2}} \left(J_\mu J^{\dagger\mu} + \tfrac{1}{2} J_\mu^{(0)} J^{(0)\mu}\right), \tag{7}$$

where

$$J_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)l\,, \tag{8}$$

$$J_\mu^{(0)} = \bar{\nu}\gamma_\mu(1-4Q\sin^2\theta_W - \gamma_5)\nu - \bar{l}\gamma_\mu[1-4(1-Q)\sin^2\theta_W - \gamma_5]l\,, \tag{9}$$

and Q is the charge of ν. Then, there exist rearrangement diagrams as shown in Figs. 1(c) and 1(d), and those contribute to the nonleptonic weak interactions of quarks by the same weight as the usual one.

Under these assumptions, we can get the effective nonleptonic weak interactions of quarks as follows.

Model A. Flavor-changing interactions come only from the diagrams (a) and (c) (Fig. 1) but not from the Z-exchange diagrams. Since the W-exchange interaction is $(V-A)\times(V-A)$, the sum of the contributions from the diagrams (a) and (c) leads to the $\underline{84}$-dominance Hamiltonian

$$H_{\rm NL}^{\rm A} = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C\, c_+^{LL} O_+^{LL} + \text{H.c.} + \cdots\,. \tag{10}$$

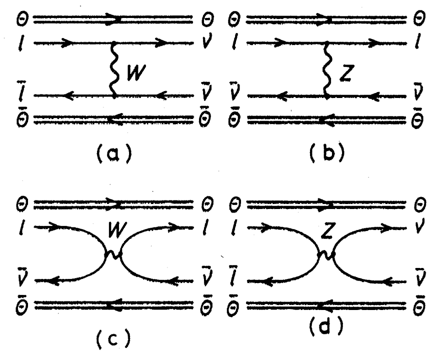

FIG. 1. Subquark diagrams for nonleptonic weak interactions of quarks.

Here and hereafter we represent only the $\Delta C=0$, $\Delta S=1$ term, since the other terms have identical structure with the $\Delta C=0$, $\Delta S=1$ term except for the Cabibbo-angle factors.

Obviously, the interaction (10) is unfavorable to describe the observed nonleptonic decays. However, note that if we suppose (although it is unlikely) that the spinor (ν, l) obeys Bose statistics, whereas Θ obeys Fermi statistics, the model can provide the $\underline{20}$-dominance Hamiltonian. The case is equivalent to the Bose-quark model proposed by Ishida.[15]

Model B. Additional flavor-changing interactions come from the diagram (d) but not from (c). Note that the color structure is of the form

$$[(\bar{d}'u') + (\bar{s}'c')][(\bar{u}'d') + (\bar{c}'s')],$$

while the Lorentz structure is not of the form $(V-A)\times(V-A)$, unlike the usual W-exchange interaction (1). Taking care for the Lorentz structure of the neutral current (9), we obtain the effective Hamiltonian

$$H_{\rm NL}^{\rm B} = \frac{G_F}{\sqrt{2}} \sin\theta_C\cos\theta_C \sin^2\theta_W \{[1-2Q(1-Q)\sin^2\theta_W](c_+^{LL}O_+^{LL} + c_-^{LL}O_-^{LL}) - 2Q(1-Q)\sin^2\theta_W(c_+^{RR}O_+^{RR} + c_-^{RR}O_-^{RR}) - 3(1-Q)(1-2Q\sin^2\theta_W)c_+^{LR}O_+^{LR} - 3Q[1-2(1-Q)\sin^2\theta_W]c_+^{RL}O_+^{RL}\} + \text{H.c.} + \cdots\,. \tag{11}$$

Here we define the multiplicatively renormalizable operators $O_\pm^{LR}$ as

$$O_+^{LR} = -\tfrac{2}{3}[\bar{d}(1-\gamma_5)u][\bar{u}(1+\gamma_5)s]\,, \tag{12}$$

$$O_-^{LR} = -\sum_{i=1}^{8} [\bar{d}\lambda_i(1-\gamma_5)u][\bar{u}\lambda_i(1+\gamma_5)s]\,. \tag{13}$$

The operators $O_\pm^{RR}$ and $O_\pm^{RL}$ are defined by substituting $(1\pm\gamma_5)$ for $(1\mp\gamma_5)$ in the operators $O_\pm^{LL}$ and $O_\pm^{LR}$, respectively.

We assume that the gluon corrections for the composite quarks at short distance can be estimated by a similar way to those for the usual structureless quarks: $c_+^{LR} = (c_-^{LL})^2 = (1/c_+^{LL})^4 = (1/c_-^{LR})^8 = (L \leftrightarrow R)$.

Model C. Similarly to model B, but differently from the model B in the color structure, we obtain the effective Hamiltonian

$$H_{\rm NL}^{\rm C}=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C\,\{\sin^2\theta_W[1-2Q(1-Q)\sin^2\theta_W]c_+^{LL}O_+^{LL}+[1-\sin^2\theta_W+2Q(1-Q)\sin^4\theta_W]c_-^{LL}O_-^{LL}$$
$$-2Q(1-Q)\sin^4\theta_W(c_+^{RR}O_+^{RR}-c_-^{RR}O_-^{RR})+2(1-Q)\sin^2\theta_W(1-2Q\sin^2\theta_W)(c_+^{LR}O_+^{LR}+c_-^{LR}O_-^{LR})$$
$$+2Q\sin^2\theta_W[1-2(1-Q)\sin^2\theta_W](c_+^{RL}O_+^{RL}+c_-^{RL}O_-^{RL})\}+\text{H.c.}+\cdots. \tag{14}$$

Note that the Hamiltonian (14) satisfies 20 dominance in the limit of $\sin^2\theta_W\to 0$. This is essentially due to the opposite sign of the effective coupling constant of the neutral-current interaction $(\bar{\nu}\nu)(\bar{l}l)$ relative to that of the charged-current interaction $(\bar{l}\nu)(\bar{\nu}l)$.

Up to here, we have not discussed rearrangement diagrams for Θ (or a further fundamental constituent of Θ) analogous to those for (ν,l) as shown in Fig. 1. We assume that the boson Θ, in general, interacts with Z so that neutral-current interaction of quarks is identical with that in the conventional minimal $SU(2)_L\times U(1)$ model[14] with no subquark structure. Therefore, if Θ has charge, that is, $Q\neq\frac{2}{3}$, there are rearrangement diagrams for Θ, and the contributions modify the parity-conserving terms in the results (11) and (14). However, for simplicity, we confine ourselves to discuss parity-violating terms in this paper, so that we take no account of such a rearrangement diagram for Θ.

In order to examine the role of the new operators $O_\pm^{LR}$ and $O_\pm^{RL}$, as an example, let us discuss $K\to 2\pi$ decay amplitudes under the $\Delta C=0$, $\Delta S=1$ nonleptonic weak Hamiltonian

$$H_{\rm NL}^{\rm eff}=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C[f_+^{LL}O_+^{LL}+f_-^{LL}O_-^{LL}+f_+^{LR}O_+^{LR}+f_-^{LR}O_-^{LR}+(L\leftrightarrow R)]+\text{H.c.} \tag{15}$$

We assume the factorization of the matrix elements. Neglecting the mass difference between $\pi^\pm$ and π^0, we obtain

$$\sqrt{2}\,A(K^-\to\pi^-\pi^0)=(1/\sqrt{2})A(K_s\to\pi^+\pi^-)-A(K_S\to\pi^0\pi^0)=[\tfrac{8}{3}g_+^{LL}+\tfrac{1}{9}(g_+^{LR}+8g_-^{LR})]A-\tfrac{2}{3}g_+^{LR}B, \tag{16}$$

$$A(K_S\to\pi^0\pi^0)=[\tfrac{2}{3}(g_-^{LL}-2g_+^{LL})-\tfrac{1}{9}(g_+^{LR}+8g_-^{LR})]A, \tag{17}$$

where

$$A=\langle\pi|(\bar{d}\gamma_\mu\gamma_5 u)|0\rangle\langle\pi|(\bar{u}\gamma^\mu s)|K\rangle=f_\pi(m_K^2-m_\pi^2)f_s(m_\pi^2)$$

[$f_s(q^2)$ is the scalar form factor of the $K\to\pi$ current],

$$B=\langle\pi|(\bar{d}\gamma_5 u)|0\rangle\langle\pi|(\bar{u}s)|K\rangle,$$

$g_\pm^{LL}=f_\pm^{LL}-f_\pm^{RR}$, and $g_\pm^{LR}=f_\pm^{LR}-f_\pm^{RL}$.

Model B leads to $g_\pm^{LL}=c_\pm^{LL}\sin^2\theta_W$, $g_+^{LR}=-3c_+^{LR}\times(1-2Q)\sin^2\theta_W$, and $g_-^{LR}=0$. The suppression of the $\Delta I=\frac{3}{2}$ amplitude highly depends on the estimate of the reduced matrix element B. It is required that $B/A\simeq\frac{1}{6}$. Then the enhancement of the $\Delta I=\frac{1}{2}$ amplitude is ascribed to the presence of the large enhancement factor c_+^{LR} ($\gg c_-^{LL}\gg 1$).

On the other hand, model C leads to $g_-^{LL}=c_-^{LL}(1-\sin^2\theta_W)$, $g_+^{LL}=c_+^{LL}\sin^2\theta_W$, and $g_\pm^{LR}=2c_\pm^{LR}(1-2Q)\sin^2\theta_W$. The enhancement of the $\Delta I=\frac{1}{2}$ amplitude is provided by both the terms g_-^{LL} and $(g_+^{LR}+8g_-^{LR})/9$. In order that both the terms constructively contribute to the $\Delta I=\frac{1}{2}$ amplitude, the charge of ν is $Q>\frac{1}{2}$. We suppose $Q=\frac{2}{3}$ or $Q=1$. If $Q=\frac{2}{3}$, the term $(g_+^{LR}+8g_-^{LR})/9$ does not so dominantly contribute to the $\Delta I=\frac{1}{2}$ amplitude in spite of $c_+^{LR}\gg c_-^{LL}\gg 1$.

The suppression of the $\Delta I=\frac{3}{2}$ amplitude in the model C depends on the estimate of B/A as well as in model B, but it is not so sensitive as in model B. If we apply the free field equations to the quark densities $(\bar{u}s)$ and $(\bar{d}\gamma_5 s)$,[12] we get $B/A=m_\pi^2/(m_u+m_d)(m_s-m_u)$. For example, "current" quark masses ($m_u=m_d=5.4$ MeV and $m_s=150$ MeV) (Ref. 12) and "constituent" quark masses ($m_u=m_d=340$ MeV and $m_s=540$ MeV)(Ref. 16) lead to $B/A=12.5$ and 0.29, respectively. Obviously the current quark masses provide a large violation of the $\Delta I=\frac{1}{2}$ rule.

Similarly, we can discuss $D\to K\pi$ decay amplitudes. A further 20 enhancement by a factor of 2 in model C is enough to account for $\tau(D^+)\gg\tau(D^0)$,[17] although it is still somewhat insufficient to explain the observed $\Delta I=\frac{1}{2}$ amplitude in K decays.

In conclusion, we have studied nonleptonic weak interactions of quarks on the bases of some subquark models proposed currently. Model C provides a further 20 enhancement by a factor of 2 as compared with the conventional model (2) and the relative enhancement of 20 to 84 is of a factor of $c_-^{LL}/c_+^{LL}\sin^2\theta_W$. Model C is promising for a unified understanding of weak-interaction phenomena, but it is not conclusive since we have no reliable estimate of the reduced matrix element B.

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