Off-resonance production of heavy vector quarkonium states in e^+e^- annihilation

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The inclusive production of the heavy quarkonium states V ($=\psi, \Upsilon$....) in e^+e^- annihilation is studied in the framework of quantum-chromodynamic perturbative theory. The rate and spectrum allow the study of the jet structure produced via two intermediate gluons. The similar process of the Z^0 decay into VX is found to be extremely rare.

The study of heavy vector quarkonium states ($\psi, \Upsilon, \cdots$) provides tests[1] of the quantum chromodynamics (QCD) and its related models. Most information[2] about ψ and Υ has been obtained from their resonances in $e^+ e^-$ annihilation and their decay channels. In this paper, we consider ψ and Υ produced off resonance and study their production mechanisms. Such a process in the lowest order of QCD can be visualized in the parton picture as $e^+e^- \to \psi gg$ (Υgg) with two gluon jets recoiling against the produced ψ (Υ). (See Fig. 1.) Hereafter, we use V as a generic symbol for all heavy vector quarkonia including the presumed states made of top-quark–antiquark pair ($t\bar{t}$). The The V vertex is determined by the wave function at the origin $\Phi(0)$ in the nonrelativistic limit. Our perturbative calculation therefore provides a test of QCD. It has been argued[3] that other diagrams such as Figs. 2(a) and 2(b) may contribute significantly. In Fig. 2(a), the heavy-quark pair $Q\bar{Q}$ produced along with one *hard* gluon bremsstrahlung eventually turns into the V meson in the final state. There are uncertainties about the effects of the soft gluons released from the color-octet $Q\bar{Q}$ in order to form an observable color-singlet V. We expect that such color rearrangement has been correctly described in the process of Fig. 1. In Fig. 2(b), the nonperturbative nature of the $VQ\bar{Q}$ vertex introduces uncertainties in the evaluation of this process and its overall strength.

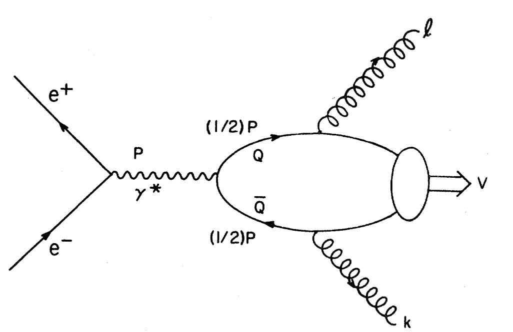

FIG. 1. Lowest-order QCD diagrams for $e^+e^- \to Vgg$. The five crossed diagrams are not shown.

Our perturbative calculation is therefore the minimal expected contribution to the V inclusive production in e^+e^- annihilation. Future high-statistics e^+e^- experiments could help to settle this important issue regarding possible nonperturbative contributions.

We define the scaling variables for the process $e^+e^- \to \gamma^* \to V + g(l) + g(k)$:

$$x_i = 2E_i/\sqrt{s}\,, \quad i = V, l, k,$$
$$x_V + x_l + x_k = 2, \tag{1}$$
$$\lambda = M^2/s.$$

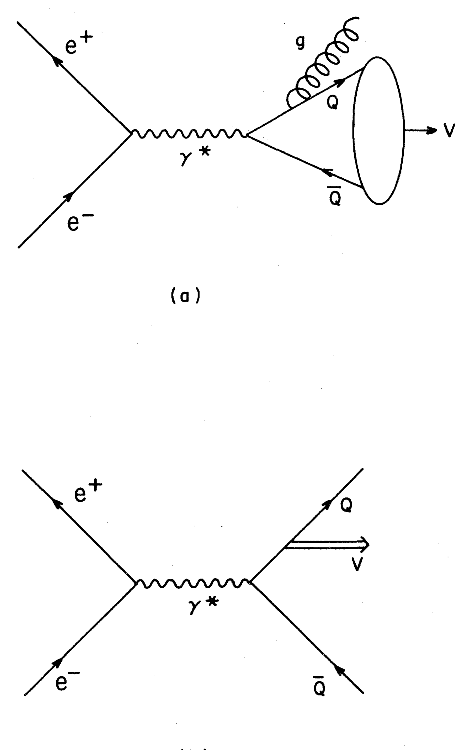

FIG. 2. Diagrams for inclusive V production in e^+e^- annihilation (a) through one *hard* gluon bremsstrahlung and (b) through nonperturbative $VQ\bar{Q}$ vertex. Crossed diagrams are now shown.

Here E_i is the energy measured in the e^+e^- c.m. frame and M is the mass of the V meson. The differential cross section after integrating over the angles between the plane of Vgg and the direction of e^+e^- beam is related to the "decay width" $\Gamma(\gamma^* \to Vgg)$ of the virtual photon γ^* as follows:

$$d\sigma/dx_V dx_l = 4\pi\alpha s^{-3/2}\, d\Gamma(\gamma^* \to Vgg)/dx_V dx_l \qquad (2)$$

with

$$\frac{d\Gamma(\gamma^*\to Vgg)}{dx_V dx_l} = \tfrac{128}{9} e_Q^2 \alpha \alpha_s^2 s^{-3/2} M |\Phi(0)|^2 \times \left\{ \frac{(2+x_k)x_k}{(2-x_V)^2(1-x_l-\lambda)^2} + \frac{(2+x_l)x_l}{(2-x_V)^2(1-x_k-\lambda)^2} + \frac{(x_V-\lambda)^2-1}{(1-x_k-\lambda)^2(1-x_l-\lambda)^2} + \frac{1}{(2-x_V)^2}\left[\frac{6(1+\lambda-x_V)^2}{(1-x_k-\lambda)^2(1-x_l-\lambda)^2} + \frac{2(1-x_V)(1-\lambda)}{(1-x_k-\lambda)(1-x_l-\lambda)\lambda} + \frac{1}{\lambda}\right]\right\}. \qquad (3)$$

The matrix element above is related to the corresponding matrix element[4] of the crossed-channel reaction $V \to \gamma^* gg$ with appropriate variable replacements. The allowed region of the phase space is

$$2\sqrt{\lambda} < x_V < 1+\lambda\ ,$$
$$x_l \lesseqgtr \tfrac{1}{2}[2 - x_V \pm (x_V^2 - 4\lambda)^{1/2}]\,. \qquad (4)$$

For ψ and Υ, the wave functions at the origin $\Phi(0)$ were measured through the leptonic decay widths Γ_{ee},

$$|\Phi(0)|^2 = M^2\Gamma_{ee}/16\pi\alpha^2 e_Q^2\ . \qquad (5)$$

The values of $|\Phi(0)|^2$ for ψ and Υ are therefore 0.04 and 0.4 GeV^3, respectively.[2] We also set the strong coupling constant $\alpha_s = 0.3$ in our calculations. For the $t\bar{t}\ \zeta$ production, we assume $M_\zeta = 40$ GeV and approximate the wave-function value $|\Phi(0)|^2$ from the calculation based on a Coulomb-type binding potential

$$|\Phi_\zeta(0)|^2 = \alpha_s^3 M^3/27\pi\ . \qquad (6)$$

Our results are shown in Fig. 3. The cross sections for the inclusive V production are small but not unmeasurably so.

The process $e^+e^- \to VX$ can in principle allow a study of the final state from two gluon jets over a full range of kinematics $M_X < (\sqrt{s} - M)$. The invariant mass squared of the hadronic recoil system is $M_X^2 = s(1 - x_V + \lambda)$. Figure 4 shows the spectrum of M_X in $e^+e^- \to VX$.

It is natural to consider the Z^0 resonance in e^+e^- annihilation as a source for Vgg production. The differential decay rate is given by Eq. (3) with the following substitutions:

$$\alpha \to \sqrt{2}\, G_F M_Z^2 \pi\ ,$$
$$e_Q \to \tfrac{1}{4} - |e_Q| \sin^2\theta_W\ , \qquad (7)$$
$$\sqrt{s} \to M_{Z^0}\ .$$

Here we use the standard electroweak model[5] and set $\sin^2\theta_W = 0.23$. There is no contribution from the axial-vector coupling as a result of charge

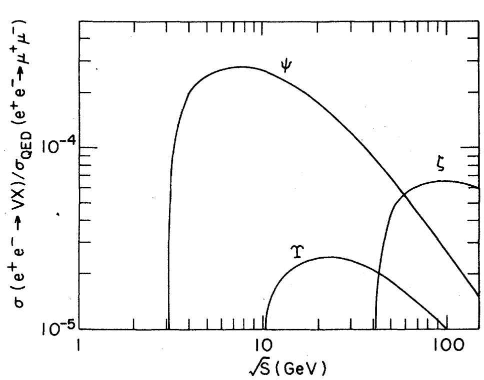

FIG. 3. Ratio of total $e^+e^- \to VX$ cross section to the pure QED $e^+e^- \to \mu^+\mu^-$ cross section versus $\sqrt{s}$ for the cases V being ψ, Υ, and ζ.

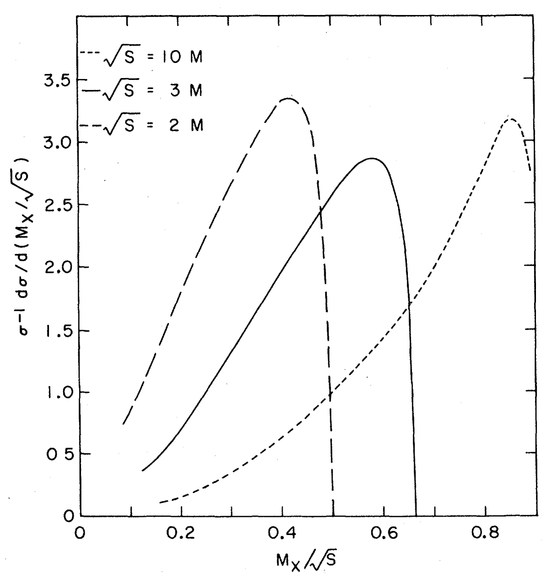

FIG. 4. Hadronic mass spectrum $\sigma^{-1} d\sigma/d(M_x/\sqrt{s})$ predicted for the process $e^+e^- \to VX$. The cases shown have $\sqrt{s} = 2M$, $3M$, and $10M$.

conjugation. The decay width $\Gamma(Z^0 \to Vgg)$ is very small because of the smallness of the vector coupling in the standard electroweak model. Results are

$$\Gamma(Z^0 \to Vgg)/\Gamma(Z^0 \to \mu\overline{\mu}) = \begin{cases} 1.1 \times 10^{-5}, & V=\psi \\ 4.7 \times 10^{-5}, & V=\Upsilon \\ 2.2 \times 10^{-5}, & V=\zeta\,. \end{cases} \quad (8)$$

In conclusion, we study the process $e^+e^- \to VX$ with two gluon jets as the recoiling system X. Rates and spectra are predicted. The decay mode $Z^0 \to VX$ in the Z^0 resonance is extremely rare.

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