Quark-matter diagnostics

G. Domokos and Joseph I. Goldman
Department of Physics, Johns Hopkins University, Baltimore, Maryland 21218
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We suggest that a phase transition between ordinary hadronic matter and quark matter is detectable in heavy-ion collisions by observing lepton-pair production spectra. Pair production rates are estimated on the basis of a "fireball model" of nuclear collisions. The presence of quark matter in the initial fireball should be revealed by an enhancement of the lepton-pair production rate at invariant masses between 200 and 600 MeV. Observation of lepton pairs within this mass interval is expected to serve as a sensitive diagnostic tool for probing the properties of bulk hadronic matter at high temperatures and compressions.

I. INTRODUCTION AND SOME QUALITATIVE CONSIDERATIONS

It is expected that hadronic matter undergoes a phase transition: At high temperatures and densities it appears as a gas composed of quarks, antiquarks, and gluons, whereas under "normal" circumstances (moderate temperatures and densities) it can be viewed as a fluid composed mainly of baryons and pions.[1] Both the existence and the nature of such a phase transition are a matter of conjecture at present. Under normal circumstances, the quark phase must be a highly unstable one; hence it is not expected to be seen on time scales much larger than the characteristic time of hadronic reactions, $t_0 \approx m_\pi^{-1} \approx 3\times 10^{-24}$ sec. Although the quark phase of hadronic matter may become stable in "exotic" environments (such as in the interior of heavy stars), a direct observation of the existence of this phase under terrestrial circumstances would be very desirable. As emphasized by Chin,[2] Olive,[3] and other authors, the quark phase of hadronic matter is likely to be present (for times of the order of 10^{-23} sec) when heavy nuclei undergo central collisions at sufficiently high energies.

The observation of the quark phase is made difficult by the fact that the dynamics of the collisions tends to mask its presence. In particular, physical hadrons emerge from the collisions as a result of complicated, multiple interactions. Even though the quark phase may be present at the initial stages of the collision, the final hadrons are unlikely to retain a memory of that stage.[4] It is physically plausible that a signature of the quark phase is carried by those reaction products which emerge from the initial "fireball" essentially without rescattering. This suggests that one should try to detect a phase transition from hadronic to quark matter by observing the production of photons, leptons, etc., in central collisions of heavy nuclei.

In this work we investigate lepton-pair production from such collisions, since, on intuitive grounds, heavy virtual photons are the most likely candidates for transmitting information about the initial stages of a nuclear collision. It is immediately obvious that—due to the extremely complicated dynamics of the collisions—some drastic simplifying assumptions have to be made in order to make the problem tractable. To this end, we imagine that the collision process between heavy nuclei can be broken up into essentially three stages; also, for the sake of simplicity, we treat only collisions between two equal nuclei of atomic numbers $A/2$. (The last assumption is inessential and it could be easily discarded within the framework of our three-stage scenario; however, no essentially new physics would be learned thereby.) The stages of the collision process are the following.

Stage 1. The colliding nuclei penetrate each other and the nucleons begin to exchange energy and momentum.

Stage 2. The colliding nuclei completely lose their identities and they merge into a hot initial fireball of baryon number A, which is at rest in the center-of-mass system (c.m.s.).

Stage 3. The fireball expands and cools, eventually ending up in nucleons, pions, etc., emerging from the collision.

We assume that at stage 2 the initial fireball is essentially in thermal equilibrium at a temperature which is determined—via the equation of state—by the available energy in the c.m.s. Its initial volume can be estimated by assuming that (after merging into a fireball) the hot hadronic matter is contained in the volume appropriate for $A/2$ nucleons in their ground state. The expansion process following this stage can be approximately described by means of classical relativistic hydrodynamics: this corresponds to stage 3 above. We thus estimate the initial volume of the fireball by the standard formula

$$V=\frac{4\pi}{3}\frac{r_0^3 A}{\gamma\kappa_0}, \tag{1.1}$$

where γ is the Lorentz factor in the c.m.s., viz.,

$$\gamma=\left(1+\frac{K}{2m_N}\right)^{1/2}, \tag{1.2}$$

K being the kinetic energy per nucleon of the projectile measured in the laboratory system, m_N is the mass of nucleons, while r_0 is the length determined by the equilibrium density ($n_0 = 0.17\ \mathrm{fm}^{-3}$) of normal nuclear matter: $r_0 = 1.12$ fm. As indicated before, we expect $\kappa_0 \approx 2$; however, in order to allow for the fact that nuclear matter may be excited during the penetration stage, we leave κ_0 as a free parameter. We remark in passing that $\kappa_0\gamma$ is the compression ratio of the hadronic matter enclosed in the initial (Lorentz-contracted) volume, i.e., $n/n_0 = \kappa_0\gamma$; thus κ_0 would be the compression ratio corresponding to two nuclei squeezed together *at rest*. Within this framework, we expect that timelike virtual photons (manifesting themselves in lepton pairs) are predominantly radiated during stages 1 and 2 of the collision process. Indeed, if phase transition into quark matter is taking place as expected, the dominant source of lepton pairs should be the annihilation of quark pairs into virtual photons during stage 2. The penetration stage (stage 1) is expected to contribute a background of lepton pairs which is unavoidably present, but it is irrelevant from our present point of view, whereas stage 3 is not expected to contribute substantially to the creation of lepton pairs.

It is obvious that without a detailed knowledge of the dynamics of heavy-ion collisions, it would be an entirely futile effort to strive for an accurate quantitative description of lepton-pair production in those collisions. Rather, we concentrate on what we believe to be the qualitatively important aspects of the problem at hand. Accordingly, in Sec. II we briefly review the statistical mechanics of hadron matter at high temperatures and densities from an elementary point of view. The production rate of lepton pairs from quark pairs annihilating in thermal equilibrium is estimated in Sec. III, while, for purposes of comparison, the same production rate is estimated in Sec. IV under the assumption that a phase transition to quark matter is not taking place. Section V is denoted to an elementary discussion of the expansion of the initial fireball; as a result, we are able to integrate our thermodynamical formulas over the history of the expanding fireball. Moreover, we acquire a refined and somewhat more reliable picture of the expansion; hence, the qualitative picture sketched above receives some support from actual calculations. The results are put together and discussed in Sec. VI.

II. STATISTICAL MECHANICS OF HADRON MATTER: AN ELEMENTARY REVIEW

There is no truly satisfactory theory of the phase transition between normal hadronic matter and quark matter available. However, normal hadronic matter can be satisfactorily described by Walecka's effective field theory[5] supplemented by the explicit contribution of pions to the free energy.[2] Alternatively, one may use an effective chiral field theory, together with a neutral vector field.[6] (The latter is necessary in order to prevent hadronic matter from collapsing at high densities.) On the quark side, some truncated version of quantum chromodynamics (QCD) is used[2,6,7]; the equilibrium between both phases is then determined according to Gibbs' rules. (An exception to this practice is the calculation by Dicus *et al.*[8] These authors discover a phase transition in quark matter by using a self-consistent calculation based on truncated QCD alone. However, no detailed properties of ordinary hadronic matter are available from this calculation.)

The heating curves of hadronic matter as a function of the incident laboratory energy, K and the compression ratio, n/n_0 were calculated by Chin.[2] We recalculated these heating curves by taking into account the fact that the compression ratio itself is a function of the incident energy (see Sec. I). Thus, the heating curves are conveniently parametrized by K and the compression ratio *at rest*, κ_0. Our results are shown in Fig. 1, for κ_0 near its conjectured value ($\kappa_0 \approx 2$). We observe that the limiting temperature ($T_{\mathrm{lim}} \approx 190$ MeV) is reached somewhat slower than according to Chin's calculations. This is due to the fact that the compression ratio increases with the incident energy. At moderate values of n/n_0, the effective attraction dominates over the short-range repulsion; this tends to decrease the temperature of hadronic matter.

Critical baryon densities as calculated in Refs. 2 and 6 are plotted in Fig. 2. We also plot there the relationship between the density and temperature for $\kappa_0 = 2$, as computed on the basis of Eqs. (1.1) and (1.2) and the heating curves shown in Fig. 1. (The latter curve is parametrized by the incident energy K.) The results of Chin[2] and Kuti *et al.*[6] differ substantially. We believe that this is due to the fact that Chin takes a phenomenological approach to the long-range part of the gluon-mediated interaction between quarks (and he fits some low-lying hadron masses), whereas Kuti *et al.* calculate the properties of quark matter from

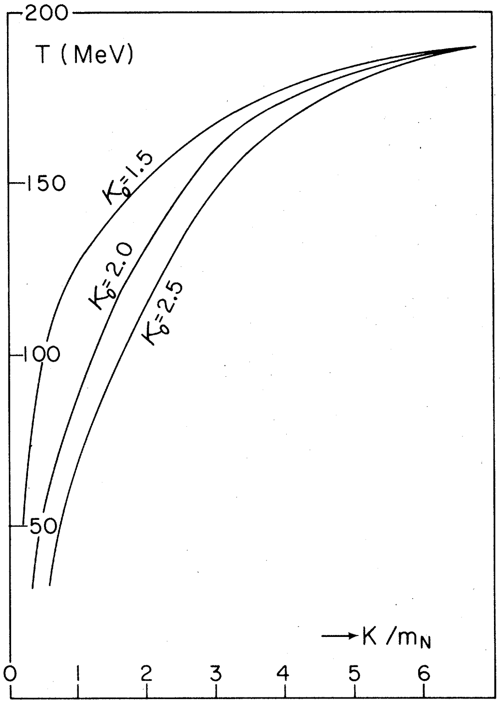

FIG. 1. Heating curves of hadron matter in heavy-nucleus collisions based on Ref. 2. Chin's heating curves are recalculated by taking the Lorentz contraction of the volume into account.

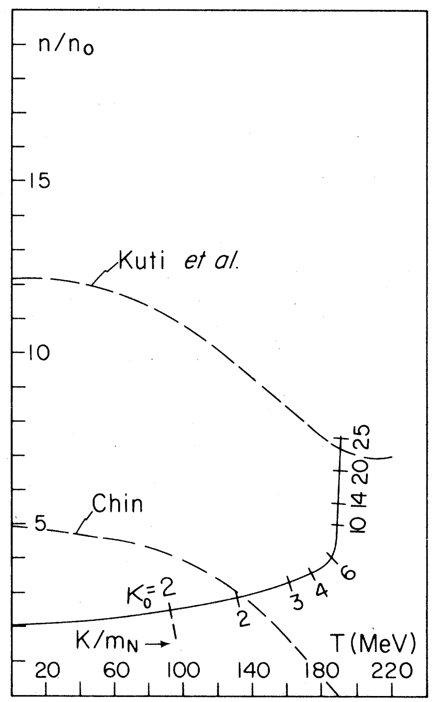

FIG. 2. Critical baryon densities according to Refs. 2 and 6 (dashed curves). Continuous line: density-temperature relationship of hadronic matter, parametrized by the incident laboratory energy per nucleon.

first principles, by taking the normalization mass Λ (and the value of the strong coupling constant at Λ) from high-energy data. At present it is hard to estimate the accuracy of either calculation; however, the difference between the two critical curves leads to dramatic consequences. By looking at the intersection of the heating curve with that of the critical density, we discover that according to Chin's calculation, the critical density is reached at an incident energy of about 2.4 GeV/nucleon. By contrast, using the critical curve of Kuti *et al.*, we would predict that the critical density is not reached until an incident energy of about 24 GeV/nucleon. Chin's semiphenomenological theory is parametrized so as to fit low-energy data; therefore, in what follows, we continue to use his results, despite the fact that the calculations reported in Ref. 6 are somewhat more satisfying from a theoretical point of view. (The calculations reported in Refs. 2 and 6, respectively, represent two extreme cases. Results obtained by other authors[7] tend to lie between these two.)

We take into account quarks of two flavors[9] and of three colors; gluons are assumed to be massless (hence transverse), forming a color octet; hence their degeneracy factor is $G = 16$. Neglecting the short-range part of the gluon-mediated interactions entirely, the thermodynamic potential of the quark-gluon system becomes

$$p = -\frac{\Omega}{V} = -B + \frac{1}{6\pi^2} \times \int_0^\infty dq\, q^3 \left[\frac{g}{e^{\beta(q-\mu)}+1} + \frac{g}{e^{\beta(q+\mu)}+1} + \frac{G}{e^{\beta q}-1}\right]. \tag{2.1}$$

In this equation, $\beta = T^{-1}$. The first term on the right-hand side is proportional to the density of condensation energy needed to form hadrons out of quarks and gluons; we treat this term as a phenomenological parameter, independent of the temperature and chemical potential. (This amounts to assuming a constant "bag pressure" in the bag model of hadrons. Presumably, such a term accounts for the long-range part of gluon-mediated interactions in a mean-field approximation.) The three terms of the integrand in (2.1) correspond to the contributions of quarks, antiquarks and gluons to Ω/V, respectively, μ being the chemical potential corresponding to baryon charge. No chemical potentials are introduced for flavor and color quantum numbers. The contribution of gluons to the pressure is easily evaluated; we find $p(\text{glue}) = 8\pi^2 T^4/45$. Turning now to the contribution of quarks and antiquarks, we put $\beta q = x$, $\beta\mu = y$, and $g = 12$; this gives

$$p(q+\bar{q}) = \frac{8T^4}{3\pi^2}\int_0^\infty dx\, x^3\left(\frac{1}{e^{x+y}+1}+\frac{1}{e^{x-y}+1}\right)$$
$$\equiv \frac{8T^4}{3\pi^2}[F_+(y)+F_-(y)], \qquad (2.2)$$

where

$$F_\pm(y) = \int_0^\infty \frac{dx\, x^3}{e^{x\pm y}+1}.$$

We substitute new variables by $t = x \pm y$, respectively, which gives

$$F_\pm(y) = \int_0^\infty \frac{dt(t\mp y)^3}{e^t+1} - \int_0^y \frac{dt(t-y)^3}{e^{\pm t}+1}.$$

The combination occurring in (2.2) can be evaluated in closed form because the sum of the integrals over the interval $(0,y)$ reduces to the integral of a polynomial. We find

$$p(q+\bar{q}) = T^4\left(\frac{7}{30}\pi^2 + \frac{\mu^2}{T^2} + \frac{1}{2\pi^2}\frac{\mu^4}{T^4}\right). \qquad (2.3)$$

Thus, the expression of the total pressure becomes

$$p = T^4\left(\frac{37}{90} + \frac{\mu^2}{T^2} + \frac{1}{2\pi^2}\frac{\mu^4}{T^4}\right) - B. \qquad (2.4)$$

The temperature and the chemical potential can be determined by means of standard thermodynamic arguments from the conservation of the net baryon charge and energy. We have

$$\frac{3A}{V} = \frac{\partial p}{\partial\mu}, \quad \epsilon \equiv \frac{E}{V} = -p - \beta\frac{\partial p}{\partial\beta} + \mu\frac{\partial p}{\partial\mu}, \qquad (2.5)$$

where $E = m_N\gamma A$ is the total energy available in the c.m.s. This leads to the equations

$$\frac{9\gamma\kappa_0}{4\pi r_0^3} = 2\mu T^2\left(1+\frac{1}{\pi^2}\frac{\mu^2}{T^2}\right),$$
$$\frac{m_N\gamma^2\kappa_0}{4\pi r_0^3} = \frac{37\pi^2}{90}T^4\left(1+\frac{90}{37\pi^2}\frac{\mu^2}{T^2}+\frac{45}{37\pi^4}\frac{\mu^4}{T^4}\right)+\frac{B}{3}. \qquad (2.6)$$

Obviously, these equations make sense physically only at sufficiently large temperatures and/or compressions; otherwise matter exists in its hadronic phase. On keeping κ_0 fixed and letting $\gamma \to \infty$, we discover that μ tends to a finite value, given by

$$\mu_\infty = \frac{9}{8\pi}\left(\frac{74\pi^3}{45}\right)^{1/2}\frac{\kappa_0^{1/2}}{r_0(r_0 m_N)^{1/2}} \approx (196\text{ MeV})\kappa_0^{1/2}. \qquad (2.7)$$

In the high-temperature limit we have the useful estimate

$$T \sim \left(\frac{45}{74\pi^3}\frac{m_N\gamma^2\kappa_0}{r_0^3} - \frac{30}{37\pi^2}B\right)^{1/4}. \qquad (2.8)$$

This formula can be used for $90\mu_\infty^2/37\pi^2T^2 \lesssim 1$, i.e., for $T \gtrsim 130$ MeV. We determine the value of the parameter B by matching the temperatures of the quark and hadronic phases at the point where the heating curve intersects Chin's curve of critical baryon density.[2] In this way we obtain $B^{1/4} \approx 180$ MeV. This is to be compared with the value, $B^{1/4} \approx 190$ MeV, used in Ref. 2. The difference of about 10 MeV is a rough measure of the short-range contribution to the thermodynamic potential. It is completely neglected in our calculations, whereas it is included in Ref. 2 in its Akhiezer-Peletminskii form.[10] We see that the short-range contribution to Ω is quite small; this fact is easily understood in qualitative terms. At high temperatures ($T \gtrsim 130$ MeV), the Fermi sea is not completely depleted [see Eq. (2.7)]; this is due to the fact that with rising temperatures the quark gas is also increasingly compressed. The low-momentum components of the single-gluon-exchange (Akhiezer-Peletminskii) contribution to Ω are therefore suppressed due to the Pauli principle, whereas its high-momentum part contributes little due to the rapid decrease of the Fermi distribution for $q \gtrsim \mu_\infty$. In concluding this section, we notice that the equation of state reads

$$\epsilon = 3p + 4B. \qquad (2.9)$$

III. PAIR PRODUCTION FROM THE QUARK PHASE

We compute the rate of lepton-pair production arising from the annihilation of quark pairs in the fireball. We use a semiclassical approximation, assuming that the fireball can be subdivided into portions which are locally in thermal equilibrium. This implies that the phases of the quark wave functions are random, hence, the production is predominantly incoherent. Under these assumptions, the number of lepton pairs in a space-time volume element d^4x and of invariant mass M is given by the elementary formula

$$\frac{dN}{d^4x\, dM^2} = \sigma\sum_k\int d^4q\, d^4\bar{q}\, \delta(q^2)\delta(\bar{q}^2)q_0\bar{q}_0\theta(q_0)\theta(\bar{q}_0)$$
$$\times Q_k^2 v n_k(q_0)\bar{n}_k(\bar{q}_0)\delta(M^2-(q+\bar{q})^2). \qquad (3.1)$$

In this equation, σ is the elementary annihilation cross section into leptons of mass m_l:

$$\sigma = \frac{4\pi\alpha^2}{M^2}\left(1+\frac{2m_l^2}{M^2}\right)\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}. \qquad (3.2)$$

n_k ($\bar{n}_k$) stand for the distribution of quarks (antiquarks) of flavor, color, and helicity k and of charge Q_k, whereas v stands for the relative velocity of the annihilating quark pair. It is given by

$$v=\frac{q\cdot\bar{q}}{q_0\bar{q}_0}\,. \tag{3.3}$$

We take into account two flavors (u,d); heavy flavors are suppressed at the temperatures of interest.

With this, Eq. (3.1) is reduced to the following expression:

$$\frac{dN}{d^4x\,dM^2}=\frac{30}{9}\,\frac{M^2\sigma}{2(2\pi)^4}\int_0^\infty dq_0\,d\bar{q}_0 ds\, s\,\delta(M^2-s)\times\theta(4q_0\bar{q}_0-s)\,n(q_0)\,\bar{n}(\bar{q}_0)$$
$$=\frac{5}{3}\,\frac{M^2\sigma}{(2\pi)^4}\int_0^\infty\frac{dq_0 d\bar{q}_0\theta(4q_0\bar{q}_0-M^2)}{(e^{\beta(q_0-\mu)}+1)(e^{\beta(\bar{q}_0+\mu)}+1)}\,. \tag{3.4}$$

We introduce dimensionless variables by the substitution $\beta q_0=x$, $\beta\bar{q}_0=y$, $\beta\mu=z$, and $\beta M=u$. We have then

$$\frac{dN}{d^4x\,dM^2}=\frac{5}{3}\,\frac{\sigma T^2M^2}{(2\pi)^4}F(u,z)\,, \tag{3.5}$$

where

$$F(u,z)=\int_0^\infty\frac{dx\,dy\;\theta(xy-u^2/4)}{(e^{x-z}+1)(e^{y+z}+1)}\,. \tag{3.6}$$

The function $F(u,z)$ cannot be evaluated in terms of known transcendental functions; however, it can be approximated by a simple and convenient interpolation formula. To this end, we notice that $F(u,z)$ can be explicitly computed in the limits $u\to 0$ and $u\to\infty$. We have by elementary integration

$$F(u,z)\underset{(u\to 0)}{\sim}\left[\left(\ln 2\cosh\frac{z}{2}\right)^2-\frac{z^2}{4}\right]\equiv F_0(z)\,. \tag{3.7}$$

In order to evaluate $F(u,z)$ in the limit as $u\to\infty$, we first integrate over y. This results in the following expression:

$$F(u,z)=\int_0^\infty\frac{dx}{e^{x-z}+1}\ln(1+e^{-u^2/4x-z})$$
$$\underset{(u\to\infty)}{\sim}e^{-z}\int_0^\infty\frac{dx}{e^{x-z}+1}\,e^{-u^2/4x}\,.$$

In the last integral, contributions from small values of x are exponentially suppressed. Therefore, we may replace the first factor of the integrand by its Boltzmann approximation. This gives finally

$$F(u,z)\sim\int_0^\infty dx\,e^{-x-u^2/4x}=uK_1(u)\equiv F_\infty(u)\,. \tag{3.8}$$

There are many formulas which interpolate between Eqs. (3.7) and (3.8). A reasonably accurate and convenient interpolation is provided by diagonal Padé approximants to F_0/F_∞; on choosing the (1,1) approximant for the sake of simplicity, we get the following approximate expression for $F(u,z)$:

$$F(u,z)\approx\frac{u+(\ln 2\cosh z/2)^2-z^2/4}{u+1}uK_1(u)\,. \tag{3.9}$$

We can now put our results together; the production rate of lepton pairs from the quark phase of the fireball is given by the approximate formula

$$\left.\frac{dN}{d^4x\,dM^2}\right|_{q\bar{q}}=\frac{5}{3}\,\frac{M^2\sigma T^2}{(2\pi)^4}\,\frac{u+(\ln 2\cosh z/2)^2-z^2/4}{u+1}uK_1(u)\,. \tag{3.10}$$

It is worth noting that the (1,1) Padé approximant is found to be quite accurate in the high-temperature region, where $z\to 0$. More accurate results can be obtained by numerical evaluation of the integrals; however, the present approximation is simple and adequate.

IV. LEPTON-PAIR PRODUCTION WITHOUT PHASE TRANSITION

Lepton-pair production from the normal hadronic phase can be computed along the same lines as outlined in the previous section. There are several processes which can give rise to lepton pairs from the hadronic phase, for instance,

$$\pi^+\pi^-\to l\bar{l}+X\,, \tag{4.1a}$$
$$\pi N\to l\bar{l}+X\,, \tag{4.1b}$$
$$N\bar{N}\to l\bar{l}+X\,. \tag{4.1c}$$

Given the fact that the hadronic phase quickly reaches a limiting temperature $T_{\rm lim}\approx 190$ MeV and thus the average energy per particle is of the same order of magnitude, the reactions (4.1) can be safely replaced by their exclusive counterparts for kinematic reasons. The contribution of (4.1c) is found to be negligibly small, essentially because there are hardly any antibaryons present at those temperatures. We also estimated the contribution of lepton pairs from (4.1b) in its exclusive forms, $\pi N\to l\bar{l}+N$, $\pi N\to l\bar{l}+\Delta$, taking N and Δ intermediate states into account. Transition form factors were approximated as in Ref. 11. We found that the contribution from (4.1b) is a small fraction of the pion annihilation contribution, at least for invariant masses $M\gtrsim 100$ MeV. This result can be understood on intuitive grounds. The density of pairs of hadrons (e.g., πN) as a function of their invariant c.m.s. energy $\sqrt{s}$ is given by expressions of the same type as discussed in the previous section, cf. Eq. (3.4). Such densities decrease exponentially for $(\sqrt{s}-\sum M_i)>T$, where M_i stand for the rest masses of the initial hadrons. At small c.m.s.

energies the process $\pi N \to l\bar{l} + N$ is suppressed by the large energy denominators in the matrix element. Near the Δ – mass, where the cross section would be enhanced by the resonance, the probability of having a sufficient invariant energy is already in its rapidly decreasing tail.

With these considerations in mind, we estimate lepton-pair production in the hadron phase by computing the rate from the reaction $\pi^+\pi^- \to l\bar{l}$. (As discussed before, this is a *slight* underestimate.) The pair production rate is given by an expression very similar to (3.3) with two important differences: (i) the weight of a $\pi^+\pi^-$ pair annihilating into a lepton pair is 1 instead of $\frac{30}{9}$ ($Q=1$, but no spin, color, and flavor degeneracy); (ii) the elementary cross section (3.2) is modified due to the presence of a pion form factor $F_\pi(M^2)$. We estimate the number of annihilating pion pairs of invariant mass M by the same procedure as in the previous section; however, now the rest mass cannot be completely neglected. After performing the trivial integrations, we find the following expression for the lepton-pair production rate from the hadron phase:

$$\frac{dN}{d^4x\,dM^2} = \frac{1}{2}\frac{\sigma|F_\pi(M^2)|^2}{(2\pi)^4} M(M^2-4m^2)^{1/2}\int_m^\infty dE\,d\bar{E}\,\nu(E)\nu(\bar{E})\,\theta\left(\frac{M^2}{2} - m^2 - E\bar{E} + [(E^2-m^2)(\bar{E}^2-m^2)]^{1/2}\right)$$
$$\times\,\theta\left(m^2 + E\bar{E} + [(E^2-m^2)(\bar{E}^2-m^2)]^{1/2} - \frac{M^2}{2}\right), \tag{4.2}$$

where m is the pion mass and $\nu(E) = (-1+\exp\beta E)^{-1}$ is the equilibrium distribution of pions. Again, we introduce dimensionless variables by the substitution $\beta E = x$, $\beta\bar{E} = y$, $v^2 = \beta^2(M^2-4m^2)$. We evaluate the resulting integral in the high-temperature limit by neglecting terms of the order of $\beta^2 m^2$; this approximation simplifies the calculations considerably and its accuracy is adequate for our present purposes. In this approximation we have

$$\frac{dN}{d^4x\,dM^2} = \frac{T^2\sigma|F_\pi(M^2)|^2}{2(2\pi)^4} M(M^2-4m^2)^{1/2}\theta(M^2-4m^2)G\,, \tag{4.3}$$

where

$$G(v,\beta m) = \int_{\beta m}^\infty dx\,dy\,\theta(4xy - v^2)\nu(x)\nu(y)\,. \tag{4.4}$$

Elementary integration leads to the following expression:

$$G(v,\beta m) = (\ln\beta m)^2\theta(2\beta m - v) + \theta(v - 2\beta m)\left[\ln\beta m\,\ln(1 - e^{-v^2/4\beta m}) - \int_{\beta m}^{v^2/4\beta m}\frac{dx}{e^x-1}\ln(1-e^{-v^2/4x})\right] + O(\beta^2m^2)\,. \tag{4.5}$$

The last integral can be approximated by expanding the integrand as in the case of quark-pair annihilation. A simple and reasonably good interpolation formula is

$$\int_{\beta m}^{v^2/4\beta m}\frac{dx}{e^x-1}\ln(1-e^{-v^2/4x}) \approx -(v-2\beta m)K_1(v)\,.$$

Thus, we have finally

$$G(v,\beta m) \approx (\ln\beta m)^2\theta(2\beta m - v) + \theta(v-2\beta m)[\ln\beta m\,\ln(1-e^{-v^2/4\beta m}) + (v-2\beta m)K_1(v)]\,. \tag{4.6}$$

(The derivative of this approximate expression has a small discontinuity at $v = 2\beta m$; however, this is not noticeable for the temperatures of interest to us.)

The pion form factor can be well approximated by the Gounaris-Sakurai formula.[12] For our purposes, however, a simple pole approximation is adequate since pion annihilation near threshold is kinematically suppressed. Therefore, we choose

$$F_\pi(M^2) \approx \frac{m_\rho^2}{m_\rho^2 - M^2 - im_\rho\Gamma}\,, \tag{4.7}$$

with $m_\rho = 776$ MeV, $\Gamma = 155$ MeV. Putting our results together, we have the following estimate of pair production from the hadron phase:

$$\left.\frac{dN}{d^4x\,dM^2}\right|_{\pi\pi} \approx \frac{\sigma T^2 M(M^2-4m^2)^{1/2}}{2(2\pi)^4}\frac{m_\rho^4}{(m_\rho^2-M^2)^2 + m_\rho^2\Gamma^2}\theta(M^2-4m^2)$$
$$\times\{(\ln\beta m)^2\theta(8m^2-M^2) + \theta(M^2-8m^2)[\ln\beta m\,\ln(1-e^{-v^2/4\beta m}) + (v-2\beta m)K_1(v)]\}\,. \tag{4.8}$$

V. EXPANSION OF THE FIREBALL: HYDRODYNAMIC AVERAGES

The expansion of a relativistic fireball has been investigated by several authors.[13] Unfortunately, those works do not suit our purposes very well; owing to the fact that in Ref. 13 the integration of the equations of hydrodynamics was carried out numerically, it is not easy to use the results for averaging the pair production rates. We argue, however, that a very accurate knowledge of the entire history of the expansion is not necessary for our purposes. Indeed, we found in the previous sections that for not too low invariant masses the lepton production rate is essentially of the form

$$\frac{dN}{d^4x\,dM^2} \propto T^2 f(M/T)\,,$$

where f is some dimensionless function. On integrating over M^2, we get

$$\frac{dN}{d^4x} \propto T^2 \int dM^2 f(M/T) \propto T^4\,. \tag{5.1}$$

By a simple dimensional argument, the *average* temperature over the expanding fireball should drop with time roughly as t^{-1}, since at least the initial stages of the expansion are essentially isentropic. Hence, lepton-pair production is significant for times $t \lesssim T_0^{-1}$, where T_0 is the initial temperature of the fireball.

The hydrodynamics of a "young" fireball, however, is quite simple. This is due to the fact that the initial fireball is hot, hence a simplified equation of state, Eq. (2.9), can be used. Further, the expansion takes place from an initial state with essentially no hydrodynamic flow; hence, the flow velocities will remain small for some time and the equations of hydrodynamics can be simplified accordingly.

In an arbitrary coordinate system, the equations of relativistic hydrodynamics are

$$g^{\mu\nu}\partial_\nu p + \frac{1}{\sqrt{-g}}\partial_\nu(w\sqrt{-g}\,u^\mu u^\nu) + w\Gamma^\mu_{\nu\lambda}u^\nu u^\lambda = 0\,. \tag{5.2}$$

Here p is the pressure, $w = \epsilon + p$ is the heat function per unit proper volume, and u^μ are the components of the four-velocity with $g_{\mu\nu}u^\mu u^\nu = -1$; $\Gamma^\mu_{\nu\lambda}$ are connection coefficients. We assume that the initial fireball is spherically symmetric; at present energies this is a reasonable approximation, since typical c.m.s. Lorentz factors are of the order of 1.5. Hence, we choose spherical polar coordinates and retain only the radial component of the flow velocity, $u^\phi \approx 0, u^\theta \approx 0$. With this, (5.2) simplifies to the two equations

$$\begin{aligned}&\partial_0(wu^{02} - p) + \frac{1}{r^2}\partial_r(w\,r^2u^0u^r) = 0\,,\\ &\partial_0(w\,u^r u^0) + \partial_r p + \frac{1}{r^2}\partial_r(w\,r^2u^{r2}) = 0\,.\end{aligned} \tag{5.3}$$

All quantities are functions of r and $x^0 = t$ only. The pressure and the heat function are connected by Eq. (2.9). Corresponding to our previous argument, we make a small velocity approximation: We neglect quantities which are of the order of u^{r2}, in essence, the flow is nonrelativistic. This further simplifies the equations to the following:

$$\begin{aligned}&3\partial_0 p + \frac{1}{r^2}\partial_r(r^2uw) = 0\,,\\ &\partial_0(uw) + \partial_r p = 0\\ &(u^r \equiv u\,,\ u^0 \approx -1)\,.\end{aligned} \tag{5.4}$$

We introduce a velocity potential by the relationship $uw = \partial_r\phi$; on substituting into (5.4) we obtain the equations

$$\begin{aligned}&\frac{1}{r^2}\partial_r(r^2\partial_r\phi) - 3\partial_0{}^2\phi = 0\,,\\ &p = -\partial_0\phi\,,\\ &u = \frac{\partial_r\phi}{4(B - \partial_0\phi)}\,.\end{aligned} \tag{5.5}$$

We seek a solution with the initial conditions $u(r,0) = 0$, $p(r,0) = p_0\theta(R-r)$, $R = r_0(A/\kappa_0)^{1/3}$, corresponding to the initial fireball. This is a standard problem of hydrodynamics (see, e.g., Ref. 14). The potential is given by the expression

$$\begin{aligned}\phi = \frac{p_0\sqrt{3}}{4r}\Bigg[&\left(\frac{t}{\sqrt{3}} - R\right)^2\theta\left(R - \left|\frac{t}{\sqrt{3}} - r\right|\right)\\ &+ R^2\theta\left(\left|\frac{t}{\sqrt{3}} - r\right| - R\right) - R^2\theta\left(\frac{t}{\sqrt{3}} + R - r\right)\\ &- \left(\frac{t}{\sqrt{3}} + r\right)^2\theta\left(R - \frac{t}{\sqrt{3}} - r\right)\Bigg]\,,\end{aligned} \tag{5.6}$$

where p_0 is determined by Eq. (2.3). From here we obtain the pressure

$$p = \frac{p_0}{2r}\left[\left(r - \frac{t}{\sqrt{3}}\right)\theta\left(R - \left|\frac{t}{\sqrt{3}} - r\right|\right) + \left(r + \frac{t}{\sqrt{3}}\right)\theta\left(R - \frac{t}{\sqrt{3}} - r\right)\right]\,. \tag{5.7}$$

This solution describes an exploding fireball with two wave fronts starting from the surface $(r = R)$ at $t = 0$; one of the fronts is moving towards the center, the other outward. Both fronts move with the speed of sound, $c = (\partial p/\partial\epsilon)^{1/2} = 1/\sqrt{3}$. The pressure at the center is given by

$$p(0,t) = p_0\theta\left(R - \frac{t}{\sqrt{3}}\right)\,. \tag{5.8}$$

On analyzing the solution (5.7), we discover that it, formally, gives rise to zones of negative pressure. For instance, taking sufficiently large times, say $t > \nu R\sqrt{3}$ (with $\nu \geq 2$), we find that the pressure differs from zero in the shell $(\nu-1)R < r < (\nu+1)R$ and, hence, all the matter is completely outside the initial volume. The pressure varies between $-p_0(2\nu-2)^{-1}$ and $p_0(2\nu+2)^{-1}$ in this interval; it crosses zero at $r=\nu R$. Clearly, negative pressures are physically meaningless. Their appearance reflects the facts that our equation of state, Eq. (2.7), is not valid near the condensation point of quark matter and that the nonrelativistic approximation ($u^2 \approx 0$) is not valid everywhere.

In a more realistic approach, McLerran integrated Eqs. (5.3) numerically for the initial stages of the expansion.[15] Initially, the numerical solution and our approximate analytic solution are in qualitative agreement with each other. Instead of physically meaningless negative pressure zones, however, the numerical solution indicates the development of sharp, perhaps infinite, gradients in the distributions of the physical quantities, beyond which the straightforward numerical integration cannot be continued. (We suspect that this is an indication for the onset of the well-known phenomenon of "breaking" of nonlinear waves.) Nevertheless, bearing in mind that lepton-pair creation is practically zero at low temperatures, our approximate solution can still be used for the purpose of integrating the thermodynamic formulas over the history of the fireball, at least, as an order-of-magnitude estimate.

To this end, we notice that the pressure, and hence the temperature, is much lower in the "mantle" of hadronic matter blown off in the course of the expansion than in the central part which has not been reached yet by the inward-moving wave front. Hence, the predominant majority of pairs is produced by this central core which stays at the initial temperature. Such a central core survives until $t \approx R\sqrt{3}$; at that moment the inward-moving front reaches the center. Consequently, to a good approximation, the lepton-pair spectrum integrated over the history of the fireball is given by the simple expression

$$\frac{dN}{dM^2} \approx \left(\frac{dN}{d^4x\, dM^2}\right)_{T=T_0} 4\pi \int_0^{R\sqrt{3}} dt \int_0^{R-t/\sqrt{3}} r^2 dr = \frac{\pi\sqrt{3}}{3} R^4 \left(\frac{dN}{d^4x\, dM^2}\right)_{T=T_0}, \tag{5.9}$$

where, as before, $R = r_0(A/\kappa_0)^{1/3}$. This expression is valid irrespective of the phase of hadron matter from which the pairs are produced.

VI. SUMMARY OF THE RESULTS AND DISCUSSION

We are now in the position to put together the results of the previous calculations. Our first prediction is that the pair production rate should vary with the atomic number as $A^{4/3}$, cf. (5.9). Equivalently, the inclusive cross section is given by

$$\frac{d\sigma}{dM^2} \approx P\sigma_t \frac{dN}{dM^2}, \tag{6.1}$$

where σ_t is the total cross section of the colliding nuclei and P is the probability of fireball formation. For heavy nuclei we estimate P by noticing that the geometrical cross section for collisions with impact parameters $b < b_0$ is $\sigma_0 \approx \pi b_0^2$. It is reasonable to assume that a fireball is formed whenever b_0 is less than some fraction of the nuclear radius, say $b_0 = fR$. $f \lesssim 1$. Consequently,

$$P \approx \frac{\pi b_0^2}{\sigma_t} \frac{b_0^2}{R^2} f^2; \tag{6.2}$$

hence, it is roughly independent of A. Therefore, with $\sigma_t \propto A^{2/3}$, we expect that *the inclusive cross section of pair production is proportional to* A^2. There is much to be gained by studying collisions between heavy nuclei.

Next, we observe that the lepton-production rates are relatively slowly varying functions of the incident energy: This is due to the fact that the temperature of the initial fireball varies essentially as $K^{1/4}$. However, the lepton-pair spectrum depends sensitively on the *initial phase* of the fireball. On inspecting Eqs. (3.10), (4.8), and (5.9), we can isolate a convenient *phase-sensitive function* of the invariant mass which is expected to carry information about the initial phase, but is relatively weakly dependent on irrelevant details of the collision. A convenient choice for such a function is the following:

$$P(M) = \left(\frac{r_0}{R}\right)^4 \frac{1}{\sigma} \frac{dN}{dM^2}. \tag{6.3}$$

The theoretical expressions we obtain for $P(M)$ are the following.

Quark phase:

$$P_Q(M) \approx \frac{\pi}{(2\pi)^4\sqrt{3}} \frac{5}{3} r_0^4 M^2 T^2 \frac{u + (\ln 2\cosh z/2)^2 - \frac{z^2}{4}}{u+1} uK_1(u). \tag{6.4a}$$

Hadron phase:

$$P_H(M) \approx \frac{\pi}{(2\pi)^4\sqrt{3}}\,\frac{1}{2} r_0{}^4 T^2 M (M^2-4m^2)^{1/2}\theta(M^2-4m^2)\frac{m_\rho{}^4}{(M^2-m_\rho{}^2)^2+m_\rho{}^2\Gamma^2}$$
$$\times\{(\ln\beta m)^2\theta(8m^2-M^2)+\theta(M^2-8m^2)[(\ln\beta m)\ln(1-e^{-v^2/4\beta m})+(v-2\beta m)K_1(v)]\}\,, \qquad (6.4b)$$

where the variables u, v, z have been defined previously, *viz.*,

$$u=\beta M\,,\quad v=\beta(M^2-4m^2)^{1/2}\,,\quad z=\beta\mu\,.$$

In order to assess the sensitivity of the function $P(M)$ to the initial phase of the fireball, we choose a temperature slightly above the temperature at which the quark phase is expected to become stable. On assuming that shock waves are absent, i.e., $\kappa_0 \approx 2$, inspection of Fig. 2 shows that at $T \approx 150$ MeV the quark phase should just become stabilized; this corresponds roughly[16] to $K \approx 3$ GeV/nucleon. The normalized lepton-pair yields $P(M)$, corresponding to the quark and hadronic phases are plotted in Fig. 3.

Lepton pairs arising from processes not identifiable as quark- or pion-pair annihilation in the initial fireball constitute a background to the "interesting" production process. This background can come from several sources.

(i) Processes taking place in the initial fireball, but not accounted for by Eqs. (6.4a) and (6.4b). We found (cf. Sec. IV) that reactions competing with (4.1a) give a negligible contribution. Likewise, a standard estimate (based on lowest-order QCD diagrams) shows that the contribution of reactions like $g+q\to l\bar{l}+q$, $q+\bar{q}\to l\bar{l}+g$, etc., is negligibly small compared to the basic reaction $q\bar{q}\to l\bar{l}$; here g stands for an on-shell, massless, transverse gluon.

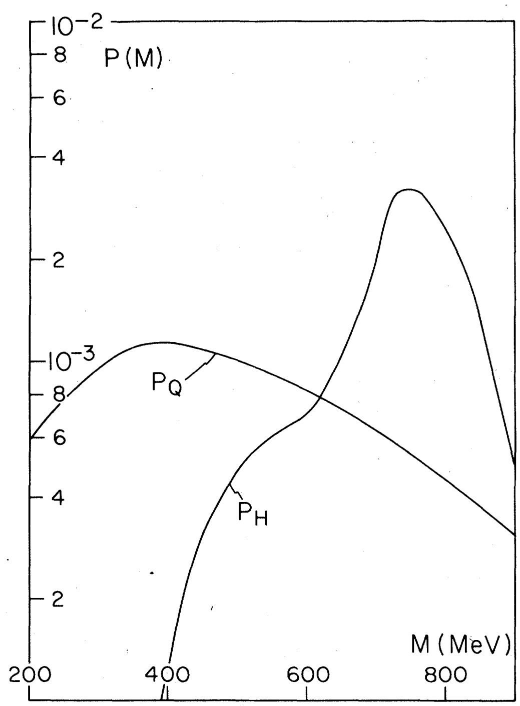

FIG. 3. Normalized lepton-pair production rates in the quark and hadron phases.

(ii) "Soft" nuclear processes: $A^*\to A+l\bar{l}$ (A^* being a highly excited state of the initial nuclear system with $E^*>1$ MeV), Dalitz pairs, $A/2+A/2\to\pi^0+X\to l\bar{l}+X$, etc. Most of these soft processes are, however, expected to produce lepton pairs of invariant masses below 100 MeV. By looking at the $\mu^+\mu^-$ inclusive channel, one can very effectively rid oneself of the soft background. The price to be paid, however, is that the threshold factor $(1-4m_l{}^2/M^2)^{1/2}$ suppresses muon-pair production considerably more than it does electron pair production. Clearly, an appropriate compromise has to be chosen depending on the facilities available.

(iii) Lepton-pair production via a Drell-Yan mechanism during the initial stage of the collision. This reaction contributes mostly hard lepton pairs as compared to those arising from the fireball. The reactions leading to Drell-Yan pairs at high and intermediate energies have been extensively studied. A representative sample of the relevant works is listed in Ref. 17; the papers listed there contain a practically complete list of references to previous works in this area. Although the data reported in Ref. 17 are not entirely free of ambiguities, one concludes from them that the Drell-Yan formula (supplemented with some QCD corrections) furnishes a basically adequate description of the experimental results. In particular, the A dependence of the inclusive cross section indicates that most of the lepton pairs arise from the interior of the target nucleus. There is an appreciable radiation from the surface; however, it contributes mostly softer pairs. As a consequence, we conclude that there exists a "window" of invariant masses, roughly defined by $100 \lesssim M \lesssim 1000$ MeV, through which we may get a glimpse to the state of hot hadronic matter under high compression. The sides of the window are delineated on the one hand by soft, nuclear, processes, and on the other hand, by the now well-known onset of the Bjorkên-scaling regime. For practical purposes, however, the difference between the quark and hadronic phases is best observed below the resonance region, so that the window through which a possible phase transition can be observed is narrowed to the interval say, $200 \lesssim M \lesssim 600$ MeV, cf. Fig. 3. In this interval of invariant masses the effects of a phase transition to quark matter should be quite spectacular: Lepton-pair production rates increase by almost an order of magnitude when (and if) phase transition to quark matter occurs. The high-energy tail of the pair spectrum is, of course, essentially exponential[18] until it merges into the Drell-Yan regime.

In view of these results, a "practical" scenario of observing a phase transition into quark matter would involve essentially two steps.

(a) First, one is to select a large sample of central collisions between heavy nuclei in the energy interval, say, $1.5 \lesssim K \lesssim 3$ GeV. The central collisions are, it is hoped, characterized by an almost isotropic and basically thermal distribution of pions.

(b) In this sample, one should plot invariant-mass distributions of the lepton pairs. Presumably, the normalized mass distributions at low energies should follow the typical hadronic curve $P_H(M)$, with minor corrections. As soon as the critical energy ($K \approx 2.3$ GeV/nucleon) is reached, however, *the normalized lepton-pair spectrum is expected to undergo a discontinuous change in shape;* in essence, it should look like the curve marked P_Q in Fig. 3.

No doubt, the calculations reported here leave much to be desired; in particular, the discontinuous change in the lepton spectrum as outlined above is an artifact due to the semiclassical treatment of the fireball. Nevertheless, it is not unreasonable to expect a rapid change in the shape of the pair spectrum if a phase transition to quark matter occurs in high-energy nuclear collisions. One may also speculate that similar changes could be observed in the distribution of transverse momenta in the production of *hard* pairs in high-energy collisions, cf. the model of Hwa and Lam.[19]

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