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Radiative corrections to Higgs-boson decays in the Weinberg-Salam model

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One-loop corrections to the Higgs-boson decays $H\tau^+\tau^-$, HW^+W^-, and HZZ are calculated up to Higgs-boson masses of about 1 TeV. The corrections are of the order of 10% for 200 GeV $\lesssim m_H \lesssim$ 1 TeV within the renormalization scheme adopted. Renormalization problems are discussed in detail. A complete set of one-loop counterterms in the 't Hooft gauge is presented.

I. INTRODUCTION

One of the most important general features of the Weinberg-Salam (WS) model[1] is its renormalizability, which makes systematic calculations of the radiative corrections possible for the electroweak interactions. The structurally important Higgs sector necessary for the construction of a renormalizable broken local $SU(2)_L \times U(1)_Y$ gauge theory remains experimentally completely unverified. It is commonly expected, however, that at least in some effective sense the WS model with its particular minimal Higgs structure could provide a correct description of electroweak interactions in nature up to energies as large as 1 TeV.

Although we are not immediately expecting the experimental results to confirm the detailed features of the Higgs sector, it is, nevertheless, interesting to extend the previous studies on this subject.[2-4] An excellent discussion on the relevance of the Higgs-boson effects has been given by Veltman.[5] In this paper we analyze the radiative corrections to the Higgs-boson decays. These corrections are expected to be particularly important for a heavy Higgs particle above the vector-boson thresholds. In this case, the radiative corrections to the predominant decays into the vector bosons turn out to be larger than the contributions from all the other decay channels, if we assume that no fermions heavier than about 50 GeV exist.

Another aim of the present investigation is to get some more insight into the practical aspects of the renormalization of the WS model. Our renormalization procedure differs in several respects from the procedures used by other authors. Therefore the renormalization problems are discussed in some detail. In particular we present a complete set of one-loop counterterms in the 't Hooft gauge.

Our paper is organized as follows. First, a review of some general properties of the Higgs sector is given in Sec. II. Section III is devoted to a description of the renormalization scheme. In Sec. IV we discuss the one-loop radiative corrections to some form factors. The Higgs-boson decays and the related bremsstrahlung are considered in Secs. V and VI, respectively. We finally discuss our numerical results in Sec. VII.

II. THE HIGGS SECTOR

For later use we will briefly summarize some well-known facts about the Higgs sector of the WS model. We essentially follow the notation of Abers and Lee.[6] The Higgs Lagrangian including the couplings of the Higgs field Φ_b to the vector bosons reads

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi_b)^+(D^\mu\Phi_b) - \lambda(\Phi_b^+\Phi_b)^2 + \mu^2(\Phi_b^+\Phi_b)\,, \quad (2.1)$$

where $D_\mu = \partial_\mu - i(g'/2)B_\mu - ig(\tau_a/2)W_{\mu a}$ is the covariant derivative and B_μ and $W_{\mu a}$ are, respectively, the $U(1)_Y$ and $SU(2)_L$ gauge-boson fields. Φ_b is the complex Higgs isodoublet and Φ_t is its Y-charge conjugate

$$\Phi_b = \begin{pmatrix} \varphi^+ \\ \varphi_0 \end{pmatrix}, \quad \Phi_t = \begin{pmatrix} \varphi_0^* \\ -\varphi^- \end{pmatrix}.$$

The couplings of the Higgs field to the fermions are then described by

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{fd} [G_b(\overline{\psi}_L \Phi_b \psi_{bR} + \text{H.c.}) + G_t(\overline{\psi}_L \Phi_t \psi_{tR} + \text{H.c.})]\,. \quad (2.2)$$

This sum extends over the three generations of the left-handed lepton and quark isodoublets

$$\psi_L = \begin{bmatrix} \psi_t \\ \psi_b \end{bmatrix}_L$$

and ψ_{tR} and ψ_{bR} are the associated right-handed isosinglets. We often suppress flavor and color indices. The Higgs-boson mass term $\mu^2(\Phi_b^+\Phi_b)$ is the only super-renormalizable term in the symmetric WS Lagrangian. Since $\mu^2 > 0$, by the Higgs mechanism, Φ_b develops a nonvanishing vacuum expectation value, which may be taken to have the form

$$\langle 0|\Phi_b|0\rangle = \frac{v}{\sqrt{2}}\chi\,; \quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad v>0\,. \tag{2.3}$$

By the shift

$$H_s = \frac{\varphi_0 + \varphi_0^*}{\sqrt{2}} = H + v \tag{2.4}$$

the Lagrangian is represented in terms of the physical Higgs field H. We denote by Y and X the field monomials multiplying H_s and H_s^2 in the original Lagrangian. Thus the shift yields

$$\begin{aligned} YH_s &= YH + vY\,, \\ \tfrac{1}{2}XH_s^2 &= \tfrac{1}{2}XH^2 + vXH + \tfrac{1}{2}v^2X\,, \\ \tfrac{1}{4}\lambda H_s^4 - \tfrac{1}{2}\mu^2 H_s^2 &= \tfrac{1}{4}\lambda H^4 + \lambda v H^3 + \tfrac{1}{2}(3\lambda v^2 - \mu^2)H^2 \\ &\quad + (\lambda v^3 - \mu^2 v)H\,, \end{aligned} \tag{2.5}$$

so it generates extra superrenormalizable terms.

The $SU(2)_L \times U(1)_Y$ symmetry is broken down into $U(1)_{em}$. In particular, the following tadpole term is induced:

$$-v(\lambda v^2 - \mu^2)H = -tH \tag{2.6}$$

plus the mass terms of the physical fields:

$$\begin{aligned} H &: m_H^2 = (3\lambda v^2 - \mu^2)\,, \\ Z &: M_Z^2 = \frac{v^2}{4}(g'^2 + g^2)\,, \\ W^\pm &: M_W^2 = \frac{v^2}{4}g^2\,, \\ \psi_f &: m_f = \frac{v}{\sqrt{2}}G_f\,. \end{aligned} \tag{2.7}$$

The proper ground state is characterized by $\mu^2 = \lambda v^2$ such that $t=0$ and $m_H^2 = 2\lambda v^2$. The mass matrix has been diagonalized by the orthogonal transformation

$$\begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{1}{(g'^2+g^2)^{1/2}} \begin{pmatrix} g & g' \\ g' & -g \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}. \tag{2.8}$$

The photon field A_μ associated with $U(1)_{em}$ remains massless. In our notation the charged-vector-boson field is $W_\mu^\pm = (W_{1\mu} \mp i W_{2\mu})/\sqrt{2}$.

With the mass parameters fixed we have the Lagrangian, in particular its free part $\mathcal{L}_0$, in a form suitable for the perturbation expansion. When we are given v and the masses from (2.7) the couplings are determined by the important relations

$$\begin{aligned} \lambda &= \frac{m_H^2}{2v^2}\,, \\ (g'^2+g^2)^{1/2} &= \frac{2M_Z}{v}\,, \\ g &= \frac{2M_W}{v}\,, \\ G_f &= \frac{\sqrt{2}m_f}{v}\,. \end{aligned} \tag{2.9}$$

It will often be convenient to use the electromagnetic charge e and $\sin^2\theta_W$ (θ_W the weak mixing angle) as parameters. They are given by

$$\begin{aligned} e &= 2\frac{M_W}{M_Z}(M_Z^2 - M_W^2)^{1/2}v^{-1}\,, \\ \sin^2\theta_W &= 1 - \frac{M_W^2}{M_Z^2}\,. \end{aligned} \tag{2.10}$$

Since we will not be considering quark and hadron amplitudes we shall ignore Cabbibo mixing for simplicity. The WS Lagrangian then gives rise in a natural way to a perturbation expansion. The propagators are given with physical masses m_H, M_Z, M_W, and m_f, and v^{-1} is the loop expansion parameter. This follows from the fact that quadrilinear couplings are of the form M^2/v^2 whereas trilinear couplings are of the form m/v or M^2/v. Thus with n_3, the number of trilinear vertices, and n_4, the number of quadrilinear vertices, a connected amplitude with N external legs at L-loop order obeys

$$2n_4 + n_3 = 2L + N - 2$$

and hence

$$A^{(N)} = v^{-(N-2)}(1 + a_1 v^{-2} + \cdots + a_L v^{-2L} + \cdots)\,.$$

This expansion is expected to make sense if M^2/v^2 and m/v are small enough for all renormalizable vertices of the model.

Except for the Higgs-boson mass, all parameters are fixed by low-energy phenomenology testing

$$\mathcal{L}_{\text{eff,int}} = \frac{4}{\sqrt{2}}G(J_\mu^+ J^{\mu -} + J_{\mu Z} J_Z^\mu) + e j_{em}^\mu A_\mu\,, \tag{2.11}$$

with $J_Z^\mu = -J_3^\mu + \sin^2\theta_W j_{em}^\mu$. From the Fermi coupling $G = 1.164 \times 10^{-5}\ \text{GeV}^{-2}$, the fine structure constant $\alpha = e^2/4\pi = (137)^{-1}$, and the weak mixing angle $\sin^2\theta_W = 0.23$ we have

$$\begin{aligned} v &= (\sqrt{2}G)^{-1/2} \simeq 246.5\ \text{GeV}\,, \\ M_W &= v\frac{e}{2\sin\theta_W} \simeq 77.8\ \text{GeV}\,, \\ M_Z &= \frac{M_W}{\cos\theta_W} \simeq 88.7\ \text{GeV}\,. \end{aligned} \tag{2.12}$$

A lower bound on the Higgs-boson mass follows from the fact that the couplings of H to the gauge bosons destabilize the ground state and the Higgs-boson self-couplings and Higgs-boson–fermion couplings must compensate this effect.[7] Owing to the fact that the known fermions only have small masses $m_f \ll M_W$, with the assumption that there are no unknown heavy fermions, the Higgs-boson mass must be large enough to keep the ground state stable. An interesting possibility is provided by the observation that the Higgs-boson mass can be generated by radiative corrections in a purely massless theory so that $\mu^2=0$ in the original WS Lagrangian.[7] In this case the Higgs-boson mass is fixed to one-loop order at $m_H \simeq 10$ GeV. In any case m_H is not expected to be much less than this value, if the theory is to be sound.

For large m_H the Higgs sector gets strongly coupled. Thus an increase of the radiative corrections is to be expected. Owing to the breakdown of the perturbation expansion for

$$m_H > v\ ,$$

it might, however, not be easy to get an upper bound on m_H from the observed smallness of radiative corrections. Actually, Veltman[5] has found that there is an important screening of Higgs-boson effects, due to the fact that the nonrenormalizability of the pure massive-vector-boson sector only shows up at the two-loop level; in the fermion sector it is suppressed by a factor $(m_f/M_W)^2$ at the one-loop level.

Nevertheless, an investigation of the Higgs sector is needed because the fundamental property of renormalizability of the WS model relies on it. The aim of the present investigation is to extend existing results on this subject.[2-4] As a first step, we present an extensive study of the one-loop corrections to Higgs-boson decays, particularly in the region of large Higgs-boson mass.

III. RENORMALIZATION

According to the preceding discussion, the perturbation expansion is parametrized in a natural way by the physical propagator masses m_H, M_W, M_Z, m_f and the loop expansion parameter v^{-1}. The bare quantities are defined by means of the dimensional regularization.[8] We assume γ_5 to anticommute with all the other γ matrices, thus avoiding spurious anomalies.[9] The amplitudes are considered in the 't Hooft gauge with free gauge parameter α, so that the vector-boson propagators used are of the form

$$-\left(g^{\mu\nu}-\frac{(1-\alpha)k^\mu k^\nu}{k^2-\alpha M^2+i\epsilon}\right)\frac{1}{k^2-M^2+i\epsilon}\ . \tag{3.1}$$

The gauge invariance is tested by the α independence of the physical on-shell amplitudes. The advantage of this procedure is that only physical amplitudes need to be calculated. When working in a fixed gauge, the gauge invariance can be checked only by the explicit use of the Slavnov-Taylor (ST) identities. This makes the evaluation of ghost amplitudes necessary, so that a much larger number of diagrams must be considered. Therefore, the advantage taken from the possible simple choice of the gauge, e.g., $\alpha=1$, which leads to the much simpler form of the vector-boson propagator

$$-\frac{g^{\mu\nu}}{k^2-M^2+i\epsilon}\ ,$$

is partial only.

In our choice of the renormalization procedure, we make use of the fact that apart from the gauge-dependent wave-function renormalizations, all other counterterms may be chosen to be gauge invariant. Starting from the bare Lagrangian we generate counterterms by the shifts

$$v_0=v+\delta v_t\ ,\quad M_0^{\,2}=M^2+\delta M^2\ ,\ \text{and}\ m_0=m+\delta m \tag{3.2}$$

of the parameters. Considering the *bare* amplitudes these counterterms are determined by the following procedures:

(i) The additive renormalization of v. For the proper value of v the tadpole condition $\langle H\rangle=0$ must hold. This condition thus fixes δv_t. In Appendix A we prove that, by taking into account the proper value of v, it amounts to an inclusion of the appropriate tadpole terms in the amplitudes.

(ii) The additive renormalization of the masses. The γ-Z mixing mass counterterm must be chosen such that the "mixing propagator" $\langle TA^\mu Z^\nu\rangle$ has no pole at $p^2=0$. For details we refer to Appendix B. As usual, δM^2 and δm are then determined so that M^2 and m are the real parts of the locations of the poles of the propagators of the physical fields. By definition δM^2 and δm are the differences between the bare mass and the renormalized mass, which are both gauge invariant. We notice that, if the mass counterterms were defined to involve wave-function renormalization factors, then, for example, $\delta M^2=ZM_0^{\,2}-M^2$ could *not* be gauge invariant.

We then perform the following procedures.

(iii) Multiplicative renormalizations of the physical fields. We write

$$A_0^\mu=\sqrt{Z_\gamma}A^\mu\ ,\quad W_0^\mu=\sqrt{Z}W^\mu\ ,\quad Z_0^\mu=\sqrt{Z_0}Z^\mu\ ,$$
$$H_0=\sqrt{Z_H}H\ \text{and}\ \psi_{0f}=\sqrt{Z_f}\psi_f\ . \tag{3.3}$$

The Z factors are determined to yield unity for the residues of the real parts of the corresponding propagator poles. As the fermion singlets and the doublets are renormalized independently, Z_f becomes a matrix of the form

$$Z_f=Z_R\,\frac{1+\gamma_5}{2}+Z_L\,\frac{1-\gamma_5}{2}=(1+z_a)+z_b\gamma_5\ . \tag{3.4}$$

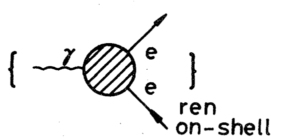
At this stage there is only *one* additional renormalization left.

(iv) Vertex renormalization. This corresponds to a multiplicative renormalization of the loop expansion parameter v^{-1}. The physically most suit-

able choice to fix this renormalization is charge renormalization

$$e_0 = e + \delta e\,, \tag{3.5}$$

with δe determined by the electron form factor



$$= -i\gamma^{\mu} e\,. \tag{3.6}$$

By using the relation

$$a = v^{-1} = \frac{e}{2}\,\frac{M_Z}{M_W}\,\frac{1}{(M_Z^{\,2} - M_W^{\,2})^{1/2}}\,, \tag{3.7}$$

we obtain to leading order

$$\frac{\delta a}{a} = \frac{\delta e}{e} - \frac{1}{2}\,\frac{1}{M_Z^{\,2} - M_W^{\,2}} \times \left[(M_Z^{\,2} - 2M_W^{\,2})\frac{\delta M_W^{\,2}}{M_W^{\,2}} + M_W^{\,2}\,\frac{\delta M_Z^{\,2}}{M_Z^{\,2}}\right] \tag{3.8}$$

for the renormalization of $a = v^{-1}$.

By these renormalizations of parameters and fields the WS model is finite and properly normalized. As a consequence all other vertices are nontrivially renormalized. The radiative corrections are finite calculable functions of the masses, and v^{-1} or e.

One advantage of this renormalization scheme is that all counterterms are determined by two-point functions. Actually by the Ward-Takahashi (WT) identity the electron vertex counterterm δe is given by a tadpole.

We have computed a complete set of one-loop counterterms (except for the quark propagators) in the 't Hooft gauge. They are listed in Appendix C. Appendix D contains a list of counterterms for the remaining physical vertices. These yield the vertices finite and on-shell gauge invariant. The given expressions for the counterterms thus provide important consistency tests of the renormalization. Our parametrization is not considered to be a very appropriate one to describe the low-energy data. In particular, the use of unstable particle masses might not be very useful. It is not difficult, however, to introduce more convenient parameters as, e.g., e, G, and $\sin^2\theta_W$ as defined by (2.11), replacing v, M_W, and M_Z by means of (2.12). After fixing δe, δG, and $\delta \sin^2\theta_W$ by appropriate four-fermion processes we could calculate δv^{-1}, $\delta M_W^{\,2}$, and $\delta M_Z^{\,2}$ from

$$\frac{\delta v^{-1}}{v^{-1}} = \frac{1}{2}\,\frac{\delta G}{G}\,,$$

$$\frac{\delta M_W^{\,2}}{M_W^{\,2}} = 2\,\frac{\delta e}{e} - \frac{\delta G}{G} - \frac{\delta \sin^2\theta_W}{\sin^2\theta_W}\,, \tag{3.9}$$

$$\frac{\delta M_Z^{\,2}}{M_Z^{\,2}} = 2\frac{\delta e}{e} - \frac{\delta G}{G} - \frac{1 - 2\sin^2\theta_W}{1 - \sin^2\theta_W}\,\frac{\delta \sin^2\theta_W}{\sin^2\theta_W}\,.$$

The masses M_W and M_Z would get extra contributions from radiative corrections.

Some remarks concerning the *unstable particles* are necessary. Only the low-mass fermions and the photon are stable within the WS model. Thus the S-matrix elements must be defined with some care. Unitarity requires the counterterms to be real so that they must be determined from the real parts of the bare amplitudes. We have listed the counterterms in Appendix C in analytic (complex) form. Only the real parts of these expressions are to be considered as the counterterms.

Owing to threshold effects, analyticity in the masses is lost for the renormalized amplitudes, which are given by

$$G^{(N)}_{\text{ren}}(p; M^2, m, v^{-1}, \alpha) = \frac{1}{\operatorname{Re}\sqrt{Z_1}\cdots \operatorname{Re}\sqrt{Z_N}} \times G^{(N)}_{\text{bare}}(p; M^2 + \operatorname{Re}\delta M^2, m + \operatorname{Re}\delta m, v^{-1} + \operatorname{Re}\delta v^{-1}, \alpha) \tag{3.10}$$

The analyticity in the momenta is not affected, of course, since the renormalization terms are polynomials in the momenta.

Problems with the gauge invariance show up if the imaginary parts of the "complex counterterms" are gauge dependent. This happens if the massive ghosts, having the masses $\sqrt{\alpha}M_Z$ or $\sqrt{\alpha}M_W$, are not heavy enough, so that physical particles can decay into them.

From our choice of the renormalization scheme only the Z factors are gauge dependent. Since the gauge-invariant integrals $B_0(m_1, m_2; m_3^{\,2})$ have gauge-invariant coefficients, only the integrals $B_0(\sqrt{\alpha}M_1, M; m^2)$ and $B_0(\sqrt{\alpha}M_1, \sqrt{\alpha}M_2; m^2)$ can develop gauge-dependent imaginary parts. M_1 and M_2 are gauge-boson masses. Both integrals are real provided

$$\alpha > \max\left[\frac{(m - M)^2}{M_1^{\,2}}\,,\ \frac{m^2}{(M_1 + M_2)^2}\right]. \tag{3.11}$$

The right-hand side is smaller than $m_{\max}^{\,2}/M_W^{\,2}$ with $m_{\max}$ the heaviest physical particle mass of the model. Supposing $m_f < M_Z$ we have to choose

$$\alpha > \frac{M_Z^2}{M_W^2} = \frac{1}{\cos\theta_W^2} \simeq 1.3 \tag{3.12}$$

in the case of a light Higgs particle $m_H < M_Z$ or

$$\alpha > \frac{m_H^2}{M_W^2} \tag{3.13}$$

for a heavy Higgs particle.

We conclude that for a proper evaluation of the imaginary parts of the amplitudes which are to be renormalized, the gauge parameter α must be chosen large enough so that $G^{(2)}_{\rm phys}$ is gauge invariant near the mass shell. In particular, the Feyman-'t Hooft gauge is not suitable for this purpose in general. At the one-loop level, however, there is no problem as the imaginary parts do not contribute to the width and the cross sections to this order if there is a Born term.

IV. RADIATIVE CORRECTIONS TO FORM FACTORS

We have computed the radiative corrections to the form factors γl^+l^-, Zl^+l^-, $H\gamma\gamma$, $HZ\gamma$, HZZ, HW^+W^-, and HHH containing about 200 diagrams through the use of the computer program SCHOONSCHIP.[10] The gauge invariance and the finiteness of the resulting on-shell amplitudes have been checked analytically. For the analytic investigations a reduction of all one-loop integrals of the form

$$\frac{1}{(2\pi)^d}\int d^dk \frac{k^{\mu_1}\cdots k^{\mu_n}}{(k^2-m_1^2+i0)[(k+p_1)^2-m_2^2+i0]\cdots[(k+p_l)^2-m_{l+1}^2+i0]} \tag{4.1}$$

to linear combinations of standard integrals is necessary. In our case $n=0,1,2,3,4$ and $l=0,1,2$. After making a covariant decomposition of (4.1), contracting with $g_{\mu_1\mu_2}$ and the $p_{i\mu_k}$'s, and inserting

$$k^2 = (k^2 - m_1^2) + m_1^2$$

and

$$2p_i \cdot k = [(k+p_i)^2 - m_{i+1}^2] - (k^2 - m_1^2) - p_i^2 + m_{i+1}^2 - m_1^2$$

into (4.1), one gets a representation of (4.1) in terms of the scalar one-loop integrals ($n=0$, $l=0,1,2,\ldots$). A systematic treatment with the explicit reduction formulas has been given by Passarino and Veltman.[11]

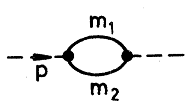
In $d = 4-\epsilon$ dimensions ($\epsilon \to +0$) the standard integrals we need are the following:

(i) The one-point function

$$= \frac{1}{(2\pi)^d}\int d^dk \frac{1}{k^2-m^2+i0} = \frac{-i}{16\pi^2}A_0(m),$$

$$A_0(m) = -m^2(\operatorname{Re} g + 1 - \ln m^2). \tag{4.2}$$

(ii) The two-point function



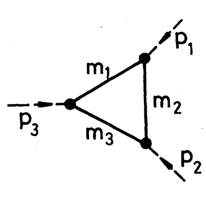
$$= \frac{1}{(2\pi)^d}\int d^dk \frac{1}{(k^2-m_1^2+i0)[(k+p)^2-m_2^2+i0]} = \frac{i}{16\pi^2}B_0(m_1,m_2;p^2),$$

$$B_0(m_1,m_2;s) = \operatorname{Re} g - \int_0^1 dz \ln[-sz(1-z) + m_1^2(1-z) + m_2^2 z - i0]. \tag{4.3}$$

The derivative of B_0 is denoted by

$$AB_0(m_1,m_2;s) = \frac{\partial}{\partial s}B_0(m_1,m_2;s).$$

(iii) The three-point function



$$= \frac{1}{(2\pi)^d}\int d^dk \frac{1}{(k^2-m_1^2+i0)[(k+p_1)^2-m_2^2+i0][(k+p_1+p_2)^2-m_3^2+i0]}$$

$$= \frac{-i}{16\pi^2}C_0(m_1,m_2,m_3;p_1^2,p_2^2,p_3^2),$$

$$C_0(m_1,m_2,m_3;s_1,s_2,s_3) = \int_0^1 dx \int_0^x dy\, (ax^2+by^2+cxy+dx+ey+f)^{-1}, \tag{4.4}$$

with

$$a=s_2,\quad b=s_1,\quad c=s_3-s_1-s_2,\quad d=m_2^2-m_3^2-s_2,\quad e=m_1^2-m_2^2+s_2-s_3,\ \text{and}\ f=m_3^2-i0.$$

The ϵ pole is included in

$$\mathrm{Reg}=\frac{2}{\epsilon}+\ln\mu^2+\gamma+\ln 4\pi\,, \tag{4.5}$$

where γ is Euler's constant and μ is the arbitrary renormalization mass scale. We have chosen the definitions of A_0, B_0, and C_0 to agree with the definitions given in[11] up to a sign in the p_i^2 in our notation and the replacement

$$\Delta=\frac{2}{\epsilon}+\gamma-\ln\pi\rightarrow \mathrm{Reg}\,.$$

Thus we have

$$X_0(\{m_i\};\{p_k^2\},\mathrm{Reg})=X_0(\{m_i\};\{-p_k^2\},\ \Delta+\ln\mu^2+\ln 4\pi^2)_{\mathrm{Velt}}\,. \tag{4.6}$$

Obviously the shift $\mathrm{Reg}\rightarrow\Delta$ does not affect renormalized (finite) quantities. The only relevant property of the pole term is

$$g^\mu_\mu\ \mathrm{Reg}=d\ \mathrm{Reg}=4\ \mathrm{Reg}-2\,.$$

Thus by

$$dA_0(m)=4A_0(m)+2m^2$$

and (4.7)

$$dB_0(m_1,m_2;s)=4\,B_0(m_1,m_2;s)-2\,,$$

constant terms appear in the standard decomposition of amplitudes.

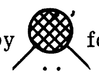
The scalar one-loop integrals have been calculated analytically for arbitrary masses and momenta in terms of Spence functions and logarithms by 't Hooft and Veltman.[12] The integrals B_0 are symmetric in the masses. The symmetry relations for the C_0's follow easily from the associated diagram.

The different functions $X_0(\{m_i\};\{p_k^2\})$, in general, form an independent set of integrals. Hence in a gauge-invariant amplitude the coefficients of gauge-dependent integrals must vanish and the coefficients of gauge-invariant integrals must be gauge invariant.

In special cases, e.g., due to the validity of a WT identity, the standard integrals are not independent. The relations between them, given in Appendix E, will then be needed to make obvious such general properties as gauge invariance.

For the necessary infrared regularization we consider the photon and the neutrinos to be massive. The zero-mass limits are performed whenever they exist.

In the following we briefly discuss the form factors which we have considered. A number of them have been investigated previously, mostly in the Feynman–'t Hooft gauge and neglecting terms proportional to fermion masses.

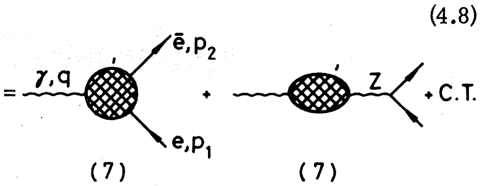
Since there is a large number of diagrams to be considered, we shall not give them explicitly. A collection of diagrams will be denoted by  for connected diagrams, by  for one-particle irreducible (1PI) diagrams, and by  for proper (nontrivial) 1PI diagrams. The number of diagrams is given in brackets. Crossed diagrams obtained by an interchange of identical particle lines are not counted.

A. Electron-photon vertex[11]

The irreducible bare vertex is given by

$$\Gamma^\mu_{\gamma ee}=i[-(e+\delta e)\gamma^\mu+\Pi^\mu_{\gamma ee}]\,,$$

where (4.8)

$i\Pi^\mu_{\gamma ee}=$ 

The covariant decomposition of Π^μ, for $p_2^2=p_1^2$, is

$$\Pi^\mu_{\gamma ee}=\gamma^\mu F_1+\frac{p^\mu}{2m_e}F_2+\gamma^\mu\gamma_5F_3+\frac{q^\mu}{2m_e}\gamma_5F_4-i\sigma^{\mu\alpha}\frac{q_\alpha}{2m_e}F_5\,, \tag{4.9}$$

with $\sigma^{\mu\nu}=i/2[\gamma^\mu,\gamma^\nu]$, $p^\mu=(p_2+p_1)^\mu$, and $q^\mu=(p_2-p_1)^\mu$. The WT identity reads

$$q_\mu\Pi^\mu_{\gamma ee}(p_2,p_1)=-[\Sigma_e(p_2)-\Sigma_e(p_1)]\,, \tag{4.10}$$

with Σ_e the irreducible self-energy of the electron. On the mass shell, using the Gordon decomposition, the amplitude may be written in the form

$$\Pi^\mu_{\gamma ee,\,\mathrm{os}}=[\gamma^\mu(F_1+F_2)-i\sigma^{\mu\alpha}\frac{q_\alpha}{2m_e}(F_2+F_5)+\gamma^\mu\gamma_5F_3]_{\mathrm{os}}\,.$$

By $(F)_{\mathrm{os}}$ we denote the on-shell limit of an amplitude F. The renormalized vertex is

$$\Gamma^\mu_{\gamma ee,\,\mathrm{ren}}=\sqrt{Z}_\gamma\gamma_0\sqrt{Z}_e\gamma_0\Gamma^\mu_{\gamma ee}\sqrt{Z}_e=i\left\{-e\gamma^\mu\left[1+\frac{\delta e}{e}+\frac{1}{2}(Z_\gamma-1)+(Z_e-1)\right]+\Pi^\mu_{\gamma ee}\right\}. \tag{4.11}$$

According to (3.4) we can write $(Z_e-1)=z_a+z_b\gamma_5$.

The electron charge form factor

$$F_V=-e+F_1+F_2-e[\tfrac{1}{2}(Z_\gamma-1)+z_a]-\delta e \qquad (4.12)$$

is used for the definition of e. $F_V(p_1^2, p_2^2; q^2=0)$ is infrared finite as $m_\gamma \to 0$. Hence e may be defined by the condition

$$\lim_{p_1^2=p_2^2\to m_e^2} F_V(p_1^2,p_2^2;q^2=0)=-e\,, \qquad (4.13)$$

by which

$$\delta e=\{F_1+F_2-e[\tfrac{1}{2}(Z_\gamma-1)+z_a]\}_{\text{os}}\,. \qquad (4.14)$$

The WT identity then yields

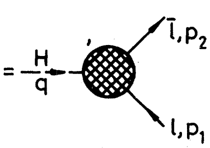
$$(F_3-ez_b)_{\text{os}}=0\,. \qquad (4.15)$$

Furthermore, due to the WT identity, δe takes an extremely simple form given in Appendix C. Important tests of the computations are (i) F_4 and F_2+F_5 must be finite and (ii) $(F_2+F_5)_{\text{os}}$ and δe must be gauge invariant.

B. Hl^+l^- vertex

The irreducible vertex

$$\Gamma_{Hll}=i\left(-\frac{m_l}{v}+\Pi_{Hll}\right),$$

$i\Pi_{Hll}=$ 

(4.16)

(11)

has the covariant decomposition

$$\Pi_{Hll}=F_1+\not{p}F_2+\not{p}\gamma_5F_3+\not{p}_1\not{p}_2F_4\,, \qquad (4.17)$$

for $p_2^2=p_1^2$. The on-shell amplitude is

$$\Pi_{Hll,\text{os}}=(F_1+2m_lF_2+m_l^2F_4)_{\text{os}}\,.$$

The renormalized vertex is given by

$$\Gamma_{Hll,\text{ren}}=i\left\{-\frac{m_l}{v}\left[1+\tfrac{1}{2}(Z_H-1)+z_a+\frac{\delta m_l}{m_l}+\frac{\delta a}{a}\right]+\Pi_{Hll}\right\}. \qquad (4.18)$$

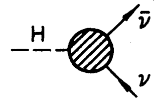
Keeping $m_\gamma>0$ the on-shell vertex exists. Thus,

$$\Gamma_{Hll,\text{ren,os}}=-i\frac{m_l}{v}(1+c_{1l}) \qquad (4.19)$$

is gauge invariant and finite for small m_γ. c_{1l} to one-loop order is of the form

$$c_{1l}=\frac{m^2}{16\pi^2v^2}f_{1l}\,,$$

with f_{1l} a dimensionless function of the masses and m one of the masses. An important property of the Hff vertex is that $\Gamma_{Hff,\text{os}}\propto m_f$. As a consequence the neutrino amplitude



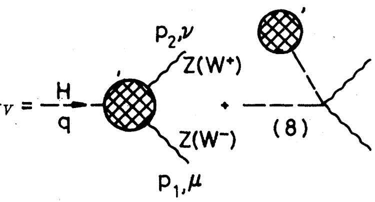
vanishes identically for massless on-shell neutrinos.

C. HZZ and HW^+W^- vertices

The vertex parts are

$$\Gamma^{\mu\nu}_{HVV}(p_1,p_2)=i\left[2\frac{M_V^2}{v}g^{\mu\nu}+\Pi^{\mu\nu}_{HVV}(p_1,p_2)\right],$$

with (4.20)

$i\Pi^{\mu\nu}_{HVV}=$ 

(19,30)

We can write, for $p_2^2=p_1^2$,

$$\Pi^{\mu\nu}_{HVV}=g^{\mu\nu}A_1+\frac{p_1^\nu p_2^\mu}{M_V^2}A_2+\frac{p_1^\mu p_2^\nu}{M_V^2}A_3\,, \qquad (4.21)$$

where the amplitudes A_2 and A_3 are finite. The renormalized vertex reads

$$\Gamma^{\mu\nu}_{HVV,\text{ren}}=i\left\{2\frac{M_V^2}{v}g^{\mu\nu}\left[1+\frac{1}{2}(Z_H-1)+(Z_V-1)+\frac{\delta M_V^2}{M_V^2}+\frac{\delta a}{a}\right]+\Pi^{\mu\nu}_{HVV}\right\} \qquad (4.22)$$

and thus

$$A_{1,\text{ren}}=A_1+2\frac{M_V^2}{v}\left[\frac{1}{2}(Z_H-1)+(Z_V-1)+\frac{\delta M_V^2}{M_V^2}+\frac{\delta a}{a}\right].$$

In the case of the charged vector boson the on-shell limit of $A_{1,\text{ren}}$ is infrared singular when $m_\gamma=0$. We therefore keep $m_\gamma>0$ but asymptotically small. The mass-shell restrictions $(A_{1,\text{ren}})_{\text{os}}$ and $(A_2)_{\text{os}}$ then exist and are gauge invariant.

For the renormalized on-shell vertex we write

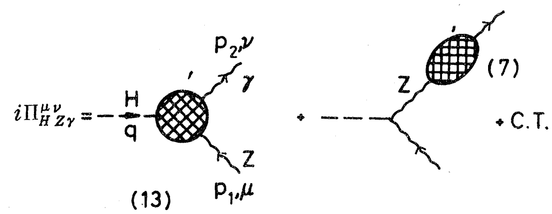
$$\Gamma^{\mu\nu}_{HVV,\text{ren,os}}=i2\frac{M_V^2}{v}\left[1+c_{1V}+\frac{p_1^\nu p_2^\mu}{2M_V^2}c_{2V}\right]. \qquad (4.23)$$

To the one-loop order the amplitudes c_{iV} are of the form

$$c_{iV}=\frac{m^2}{16\pi^2v^2}f_{iV}(m_H,M_W,M_Z,m_f)\,,$$

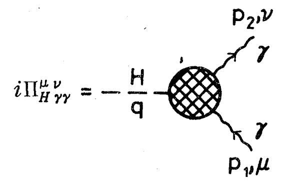
where the f_{iV} are dimensionless functions of the mass ratios, m is one of the masses.

Related to the vertex HZZ are the vertices $HZ\gamma$ (Ref. 13) and $H\gamma\gamma$ (Ref. 3), which vanish in the Born approximation. The amplitudes



(4.24)

and



are due to loop corrections from charged massive particles (γ couples to charge, H to mass). Writing, for $p_2{}^2 \to 0$,

$$\Pi^{\mu\nu} = g^{\mu\nu} A_1 + p_1^\nu p_2^\mu A_2 + p_1^\mu p_2^\nu A_3 - i\epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma} A_4 , \tag{4.25}$$

the WT identity

$$p_{2\nu}\Pi^{\mu\nu} = 0$$

implies

$$A_2 = -(p_1 p_2)^{-1} A_1 . \tag{4.26}$$

By parity invariance of $\Pi_{H\gamma\gamma}$ obviously $A_{4H\gamma\gamma} = 0$. $\Pi^{\mu\nu}$ is finite and on-shell gauge invariant. The on-shell values are

$$\Pi^{\mu\nu}_{HZ\gamma,\,\mathrm{os}} = \left(g^{\mu\nu} + \frac{2}{m_H{}^2 - M_Z{}^2} p_1{}^\nu p_2{}^\mu\right) a_{1Z} - i\,\frac{2}{m_H{}^2 - M_Z{}^2}\,\epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma} a_{2Z} \tag{4.27}$$

and

$$\Pi^{\mu\nu}_{H\gamma\gamma,\,\mathrm{os}} = \left(g^{\mu\nu} + \frac{2}{m_H{}^2} p_1^\nu p_2^\mu\right) a_{1\gamma} ,$$

where $a_1 = (A_1)_{\mathrm{os}}$ and $a_2 = (A_4)_{\mathrm{os}} \frac{1}{2}(m_H{}^2 - M_Z{}^2)$. To the one-loop order the amplitudes a_i have the form

$$a_{iZ} = \frac{eM_Z}{16\pi^2 v^2} m^2 f_{iZ} , \quad a_{1\gamma} = \frac{e^2}{16\pi^2 v} m^2 f_{1\gamma} .$$

Again the f_i's are dimensionless functions of mass ratios only. m is one of the masses.

The above amplitudes have been computed analytically in the 't Hooft gauge. Such general properties as gauge invariance, WT identities, and finiteness of the renormalized vertices have been verified. The results for the physical amplitudes, represented in terms of the standard integrals, are lengthy and will not be given here. As a simple application we use them for the calculation of the radiative corrections to Higgs-boson decay.

V. HIGGS-BOSON DECAY

The Higgs particle couples directly to all massive particles. The coupling is proportional to the mass of the particle to which it couples. The Higgs boson only couples to the photon through radiative corrections via charged massive particles. It does not couple to massless neutrinos. Accordingly H decays mainly into the heaviest particles which are energetically accessible.

Up to one-loop order the decay widths are given by

$$\Gamma_{Hf\bar{f}} = \frac{1}{8\pi m_H} \frac{m_f{}^2 m_H{}^2}{v^2} \left(1 - \frac{4m_f{}^2}{m_H{}^2}\right)^{3/2} (1 + 2\,\mathrm{Re}\,c_{1f} + c_{\mathrm{br}f}) , \tag{5.1}$$

$$\Gamma_{H\gamma\gamma} = \frac{1}{16\pi m_H} |a_{1\gamma}|^2 , \tag{5.2}$$

$$\Gamma_{HZ\gamma} = \frac{1}{8\pi m_H} \left(1 - \frac{M_Z{}^2}{m_H{}^2}\right) (|a_{1Z}|^2 + |a_{2Z}|^2) , \tag{5.3}$$

$$\Gamma_{HVV} = \frac{1}{2\sigma_V \pi m_H} \frac{M_V{}^4}{v^2} \left(1 - \frac{4M_V{}^2}{m_H{}^2}\right)^{1/2} \times \left\{ \left[1 + \frac{1}{2}\left(\frac{m_H{}^2}{2M_V{}^2} - 1\right)^2\right] (1 + 2\,\mathrm{Re}\,c_{1V} + c_{\mathrm{br}V}) + \frac{1}{2}\left(\frac{m_H{}^2}{2M_V{}^2} - 1\right)\left[\left(\frac{m_H{}^2}{2M_V{}^2} - 1\right)^2 - 1\right] \mathrm{Re}\,c_{2V} \right\} ,$$

$$\sigma_W = 1 , \quad \sigma_Z = 2 . \tag{5.4}$$

The amplitudes a_i and c_i have been defined in the preceding section. In charged decays, the soft-photon bremsstrahlung c_{br} has to be taken into account in order to obtain finite widths as $m_\gamma \to 0$. The detailed treatment of bremsstrahlung will be considered in the following section.

In general, the predictive power of the WS model is limited by considerable ambiguities in our knowledge of the strong interactions of the quarks. In the low-energy region strong interactions may be taken into account using current algebra methods.[14] At high energies (relative to strong-interaction scales) the quark-parton picture applies by asymptotic freedom of quantum chromodynamics (QCD). Here we can use the phenomenological insight from e^+e^- annihilation and inclusive lepton-hadron scattering.[15] The latter scheme is expected to apply to Higgs-boson decay, since the Higgs-boson mass is expected to be rather large. It may be assumed that about a few GeV above the quark-antiquark threshold, which is formally defined in terms of the current quark masses,[16] the total width for the decay into all hadrons involving a certain flavor is essentially given by the corre-

TABLE I. Physical parameters in GeV (Refs. 15 and 17).

$v=246.5$		$M_W=77.82$	$M_Z=88.68$
Leptons ($q_2=-1$)	: $m_e=0.511\times10^{-3}$	$m_\mu=105.66\times10^{-3}$	$m_\tau=1.807$
Top quarks ($q_1=\frac{2}{3}$)	: $m_u=4.2\times10^{-3}$	$m_c=1.2$	$m_t=35(20)$
Bottom quarks ($q_2=-\frac{1}{3}$)	: $m_d=7.5\times10^{-3}$	$m_s=150\times10^{-3}$	$m_b=4.4$

sponding quark-antiquark decay width. This has been discussed extensively by Ellis, Gaillard, and Nanopoulos.[3]

Accordingly, an estimate for the total width for decay into leptons and hadrons is

$$\Gamma_{Hf\bar{f},\,\mathrm{tot}}=\sum_l \Gamma_{Hl\bar{l}}+3\sum_q \Gamma_{Hq\bar{q}}\,, \qquad (5.5)$$

where $\Gamma_{Hf\bar{f}}$ is given by (5.1). The sums extend over the flavors excited up to a given m_H. In the hadronic resonance regions we may consider (5.5) to be an "estimate in the mean." Away from the thresholds (5.5) yields in the Born approximation

$$\Gamma_{Hf\bar{f},\,\mathrm{tot}}\simeq\frac{m_H}{8\pi v^2}\left(\sum_l m_l^{\,2}+3\sum_q m_q^{\,2}\right). \qquad (5.6)$$

For definiteness we assume a t-quark at 35 GeV in addition to the known fermions. We have evaluated the H width particularly in the heavy-Higgs-boson region above 70 GeV. Lower m_H have been considered in Ref. 3. In Table I we compile the parameters we have used[15,17] and in Table II we give some values for the Higgs-boson widths. Figure 1 shows the branching ratios. A detailed discussion of our results will be given in Sec. VII. At this point a few comments are in order.

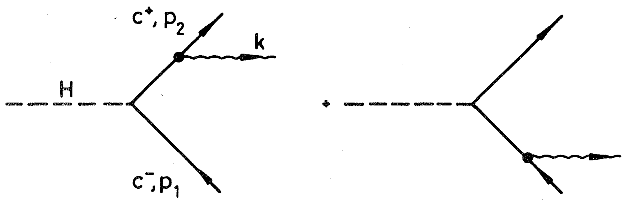
Provided no further unknown heavy fermions exist, say above 50 GeV, the WS model makes pretty definite predictions in the case of a heavy Higgs particle. For $m_H\geq 300$ GeV the hadronic decays get negligible and radiative corrections become important. However, this also is the region where a breakdown of the perturbation expansion is to be expected. At the one-loop level radiative corrections become large only for $m_H\sim 1$ TeV. We cannot conclude from this fact that our results are sensible for $m_H\gtrsim 300$ GeV, since screening of the one-loop corrections is expected.[7] In any case, the Higgs-boson lifetime $\tau=\Gamma_{H\,\mathrm{tot}}^{-1}$ decreases extremely fast with increasing m_H such that in the 1-TeV region $\Gamma_{H\,\mathrm{tot}}\sim m_H$. Thus a very heavy-Higgs-boson particle is so unstable that it will never be detected in the particle spectrum. It is amusing to observe that in this case, although the Higgs boson hardly could be considered an existing particle, it would, nevertheless, do what it was aimed to do, that is to render the WS model renormalizable. Although in this case there would be no Higgs particle, the Higgs field would show up in the structure of the weak processes of fermions and vector bosons. Effects related to a heavy Higgs particle in such processes have recently been analyzed.[18]

VI. BREMSSTRAHLUNG

The contributions from soft-photon bremsstrahlung to Higgs-boson decay into charged particles c

TABLE II. Some Higgs-boson decay widths Γ_{Hxy} in GeV ($m_t=35$ GeV). Quark contributions to Γ_{tot} are estimated using Eq. (5.6).

m_H (GeV)	10	20	50	100	200	500	1000
$\Gamma_{H\gamma\gamma}$	2.8×10^{-9}	2.7×10^{-8}	3.9×10^{-7}	3.7×10^{-6}	8.0×10^{-5}	1.6×10^{-4}	4.0×10^{-4}
$\Gamma_{H\tau\tau}$	1.8×10^{-5}	4.2×10^{-5}	1.1×10^{-4}	2.2×10^{-4}	4.1×10^{-4}	9.6×10^{-4}	2.1×10^{-3}
$\Gamma_{HZ\gamma}$					2.0×10^{-4}	6.7×10^{-4}	7.0×10^{-4}
Γ_{HWW}					1.4	$4.0\times10^{+1}$	$3.9\times10^{+2}$
Γ_{HZZ}					4.7×10^{-1}	$1.8\times10^{+1}$	$1.7\times10^{+2}$
Γ_{tot}	4.3×10^{-4}	8.6×10^{-4}	2.2×10^{-3}	9.8×10^{-2}	2.0	$5.4\times10^{+1}$	$4.7\times10^{+2}$



has the standard form

$$d\Gamma_{Hc\bar{c},\mathrm{br}} \simeq d\Gamma_{Hc\bar{c},0} \frac{e^2}{(2\pi)^3} \int_{|\vec{k}|<\omega} \frac{d^3k}{2\omega_k} \left[\frac{2(p_1\cdot p_2)}{(k\cdot p_1)(k\cdot p_2)} - \frac{m_c^2}{(k\cdot p_1)^2} - \frac{m_c^2}{(k\cdot p_2)^2}\right], \tag{6.1}$$

$d\Gamma_0$ is the Born-term differential width, ω is the photon cutoff energy, and $\omega_k = (\vec{k}^2 + m_\gamma^2)^{1/2}$ with $m_\gamma^2 \ll \omega^2$. The condition for the validity of the soft-photon approximation (6.1) is that the photon four-momentum is negligible in four-momentum conservation. This must be taken care of by restricting ω appropriately. For "elastic events" $k=0$ we have $E_1 = E_2 = \frac{1}{2}m_H$, $\vec{p}_1 = -\vec{p}_2$, and $|\vec{p}_1| = \frac{1}{2}m_H(1-y)^{1/2}$, where $y = 4m_c^2/m_H^2$. For the given energy $E_1 = (\vec{p}_1^2 + m_c^2)^{1/2}$ the emitted γ is soft when it has a minimum energy of

$$|\vec{k}|_{\min} = \frac{m_H}{2}\left(\frac{m_H - 2E_1}{m_H - E_1 + |\vec{p}_1|}\right).$$

In this case $\vec{k}$ is parallel to $\vec{p}_1$. If x is the fraction of the "elastic" energy going into soft γ's, such that $E_1 = (1-x)\frac{1}{2}m_H$, we obtain

$$|\vec{k}|_{\min} = m_H\left\{\frac{x}{1+x+[(1-x)^2-y]^{1/2}}\right\}.$$

x is restricted by the threshold condition

$$1 - \frac{2m_c}{m_H} > x > 0.$$

Hence, by choosing for x the energy resolution x_r of the charged-particle counter, we set[19]

$$\omega = m_H\left\{\frac{x_r}{1+x_r+[(1-x_r)^2-y]^{1/2}}\right\}. \tag{6.2}$$

For small enough x_r the factorization (6.1) is then justified.

The bremsstrahlung integral is given by

$$c_{\mathrm{br}} = \frac{e^2}{\pi^2}\left[\left(1-\frac{y}{2}\right)I_1^\omega(y) - \frac{y}{2}I_0^\omega(y)\right], \tag{6.3}$$

where

$$I_0^\omega(y) = y^{-1}\left(\ln\frac{2\omega}{m_\gamma} - F_1(y)\right),$$
$$I_1^\omega(y) = F_1(y)\ln\frac{2\omega}{m_\gamma} - F_2(y) \tag{6.4}$$

and

$$F_1(y) = \frac{1}{2(1-y)^{1/2}}\ln\frac{1+(1-y)^{1/2}}{1-(1-y)^{1/2}},$$

$$F_2(y) = \frac{1}{2(1-y)^{1/2}}\left(\mathrm{Sp}[(1-y)^{1/2}] - \mathrm{Sp}[-(1-y)^{1/2}] + \tfrac{1}{2}\mathrm{Sp}\left[\frac{1-(1-y)^{1/2}}{2}\right] - \tfrac{1}{2}\mathrm{Sp}\left[\frac{1+(1-y)^{1/2}}{2}\right]\right.$$
$$\left. + \tfrac{1}{4}\{\ln[1-(1-y)^{1/2}]\}^2 - \tfrac{1}{4}\{\ln[1+(1-y)^{1/2}]\}^2 + \tfrac{1}{2}\ln 2\ln\frac{1+(1-y)^{1/2}}{1-(1-y)^{1/2}}\right). \tag{6.5}$$

The Spence function is defined by $\mathrm{Sp}(x) = -\int_0^1 dt/t \times \ln(1-xt)$. The infrared singular parts must cancel with corresponding terms in the renormalized amplitudes c_{1c}. Actually,

$$(c_{1c})_{\mathrm{IR}} = \frac{1}{2}\frac{e^2}{16\pi^2}\left[4m_H^2\left(1-\frac{y}{2}\right)C_0(m_\gamma, m_c, m_c; m_c^2, m_H^2, m_c^2) + 8m_c^2 AB_0(m_\gamma, m_c; m_c^2)\right]$$

exhibits the infrared-singular terms as

$$AB_0(m_\gamma, m_c; m_c^2) = m_c^{-2}\left(\ln\frac{m_c}{m_\gamma} - 1\right)$$

and

$$C_0(m_\gamma, m_c, m_c; m_c^2, m_H^2, m_c^2) = \frac{4}{m_H^2}F_3(y)\ln\frac{m_\gamma}{m_H} + F_4.$$

F_4 is regular[12] and

$$F_3(y) = \frac{1}{2(1-y)^{1/2}}\ln\frac{1+(1-y)^{1/2}}{-1+(1-y)^{1/2}}, \quad \mathrm{Re}F_3 = F_1.$$

Up to the one-loop order only $\mathrm{Re}\,c_{1c}$ contributes. Thus we have that

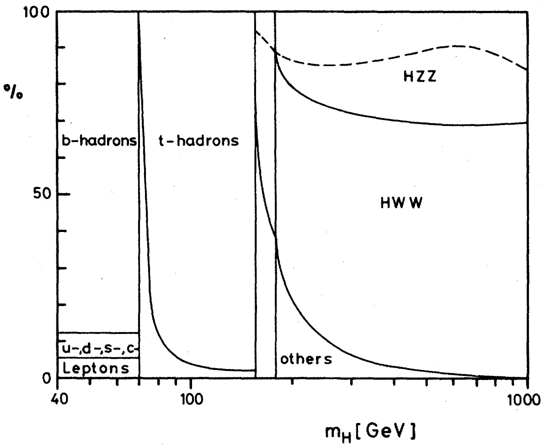


FIG. 1. Relative contributions to the total H decay width in per cent. The dashed line gives the Born-term contribution above the WW threshold.

$$(2\,\mathrm{Re}\,c_{1c}+c_{\mathrm{brc}})_{\mathrm{IR}}=\frac{e^2}{2\pi^2}\left[(2-y)F_1(y)\ln\frac{2\omega}{m_H}-\ln\frac{2\omega}{m_c}\right] \tag{6.6}$$

is infrared finite.

In fact, the infrared problem leads to a breakdown of naive perturbation theory. The charged one-particle states do not exist and ω cannot be made arbitrarily small. Physically, the charged one-particle states are "dressed" by many soft photons. This is taken into account properly by exponentiation of the one-photon IR-sensitive terms.[20] Therefore, we have

$$1+(2\,\mathrm{Re}\,c_{1c}+c_{\mathrm{brc}})_{\mathrm{IR}}+\cdots=\exp[(2\,\mathrm{Re}\,c_{1c}+c_{\mathrm{brc}})_{\mathrm{IR}}]$$

$$=\exp\left[-\frac{e^2}{2\pi^2}\left(\ln\frac{2\omega}{m_c}-(2-y)F_1(y)\ln\frac{2\omega}{m_H}\right)\right]. \tag{6.7}$$

In the Hcc decay width accordingly we set

$$1+c\simeq e^{c_{\mathrm{IR}}}+(c-c_{\mathrm{IR}})\,, \tag{6.8}$$

with $c=2\,\mathrm{Re}\,c_{1c}+c_{\mathrm{brc}}$. The exponential represents the dressed Born term and the remainder, the radiative corrections. This interpretation is physically reasonable. For $x_r\to 0$, $\omega\to x_r\{m_H/[1+(1-y)^{1/2}]\}$ and

$$e^{c_{\mathrm{IR}}}\simeq (x_r)^{e^2/2\pi^2[(2-y)F_1(y)-1]}\to 0$$

since $(2-y)F_1(y)-1>0$ for $y<1$, i.e., the probability of finding a highly resolved charged particle tends to zero. Furthermore, at the threshold

$$e^{c_{\mathrm{IR}}}=\exp\left(-\frac{e^2}{2\pi^2}\ln 2\right)\simeq 1$$

is independent of ω. Since $c_{\mathrm{IR}}<0$, there is a screening effect on the naive Born term due to soft photons.

VII. RESULTS

We have evaluated numerically the amplitudes Hl^+l^-, HW^+W^-, HZZ, $HZ\gamma$, and $H\gamma\gamma$, and the corresponding Higgs-boson decay widths as functions of the Higgs-boson mass m_H in the range from 2–1500 GeV. As input for the numerical computer program, we use the reduced analytic forms of the one-loop amplitudes in the 't Hooft gauge, which have been discussed in the preceding sections. The standard integrals A_0, B_0, AB_0, and C_0 are computed using Veltman's computer program FORMF.[21] The parameters we use for the extended WS model,[22] including three generations of leptons and quarks, are given in Table I. For the unknown mass m_t of the, as yet unobserved, t quark we have considered the values $m_t=20$ and 35 GeV.

Numerical results are reproduced in Figs. 1–6.

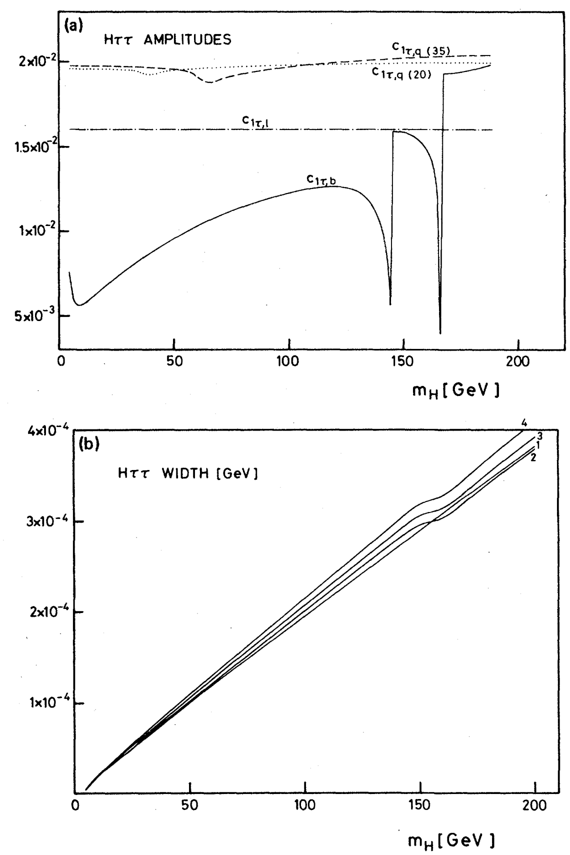


FIG. 2. (a) $H\tau\tau$ decay amplitudes $c_{i\tau,x}$ ($i=1,2$) in relation to the Born term. $x=b$, l, and q stands for the virtual bosons, leptons, and quarks ($m_t=20$ and 35 GeV) in the intermediate states, respectively. (b) $H\tau\tau$ width up to $m_H=200$ GeV. Curve 1 is the dressed Born-term width, curve 2 includes the corrections from vector bosons, in curve 3 the lepton contributions are added, and in the curve 4 the quark contributions are added (m_t =35 GeV).

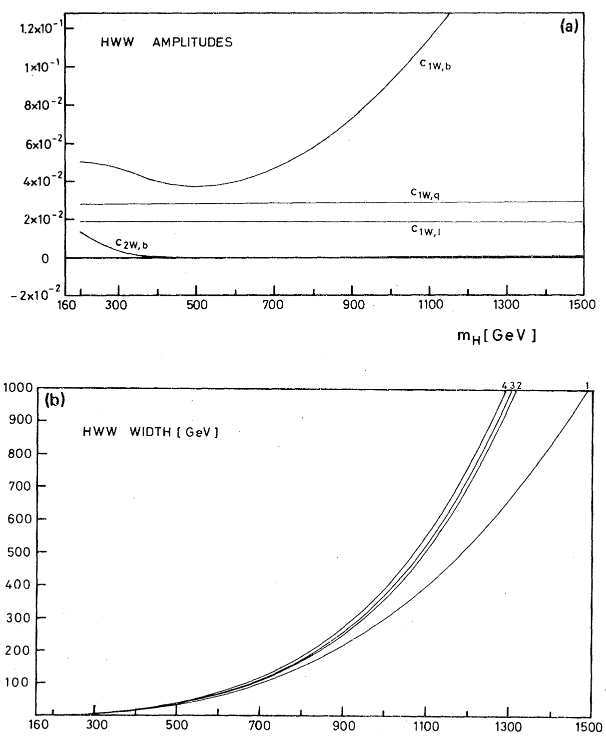


FIG. 3. (a) HWW decay amplitudes $c_{iW,x}$ ($i=1,2$) in relation to the Born term, labeled as in Fig. 2(a). (b) HWW width up to $m_H=1500$ GeV, labeled as in Fig. 2(b).

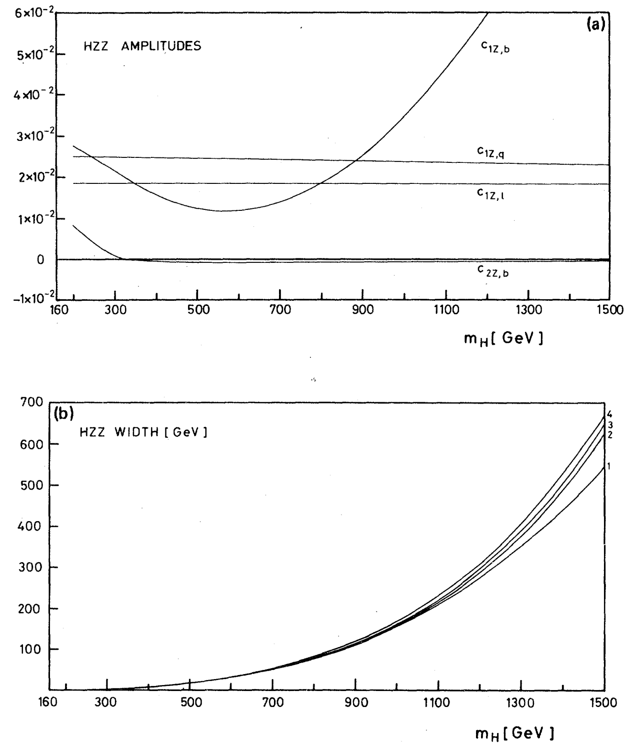


FIG. 4. (a) HZZ decay amplitudes $c_{iZ,x}$ ($i=1,2$) in relation to the Born term, labeled as in Fig. 2(a). (b) HZZ width up to $m_H=1500$ GeV, labeled as in Fig. 2(b).

The amplitudes c_{1l}, c_{1W}, and c_{1Z}, defined by (4.19) and (4.23), are normalized relative to the Born terms. We have split the amplitudes into the loop contributions from the bosons $c_{ix,b}$, the leptons $c_{ix,l}$, and the quarks $c_{ix,q}$. The quark contributions are drawn for $m_t=35$ GeV. For the cases with a significant m_t dependence we have included $m_t=20$ GeV. Instead of the infrared singular amplitudes $c_{1l,b}$ and $c_{1W,b}$, we have plotted $c^*_{1c,b}=\frac{1}{2}(c-c_{\rm IR})$ with $c=2\,{\rm Re}c_{1c,b}+c_{\rm brc}$, i.e., the infrared regular part of $c_{1c,b}$ including the infrared regular part of the associated soft-photon bremsstrahlung, which has been defined in (6.8).

For the widths we have added up successively the contributions from the Born term, the bosons, the leptons, and the quarks. For the decays into the charged particles, we have depicted the Born-term width using the dressed Born term $\exp(c_{\rm IR})$, defined in (6.8), with $x_r=0.1$. This leads to a physically reasonable separation of the radiative corrections from the singular soft-photon effects.

From the figures we can read off the following features. The $H\tau\tau$ amplitudes clearly exhibit the threshold effects, which are more pronounced the heavier the excited virtual particles relative to the external particles are. We observe that the fermion contributions are roughly constant and that the quark threshold effects are relatively small. According to our discussion in Sec. V, when we are a few hadronic binding energies away from the

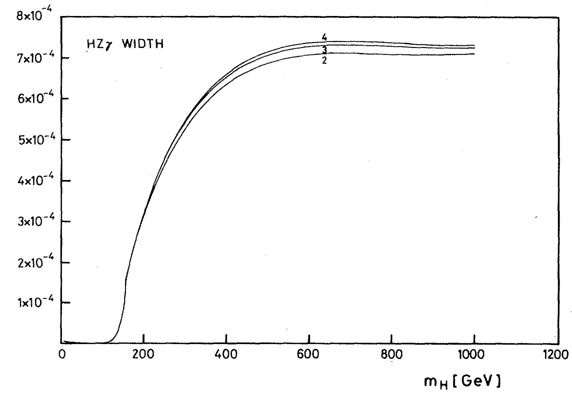


FIG. 5. $HZ\gamma$ width up to $m_H=1000$ GeV, labeled as in Fig. 2(b). The cusp occurs at the WW threshold. Below the $Z\gamma$ threshold the indicated width is the $ZH\gamma$ decay width ($m_t=35$ GeV).

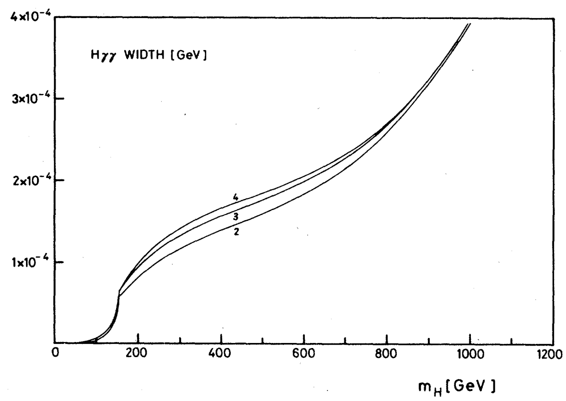


FIG. 6. $H\gamma\gamma$ width up to $m_H=1000$ GeV. The cusp at the WW threshold is very much pronounced ($m_t=35$ GeV).

quark thresholds, we can assume strong-interaction effects to be negligible. Then, since the quark contributions are relatively structureless and by the relatively weak dependence on m_t, we conclude that the WS model makes rather unambiguous predictions for the radiative corrections to the processes involving the heavier nonhadronic particles. This is the case particularly for the HW^+W^- and HZZ amplitudes due to the heavy external particles involved, and also because we are far enough above all the fermion thresholds.

In the case of the HW^+W^- and HZZ amplitudes, the dominant bosonic contributions $c_{1W,b}$ and $c_{1Z,b}$ show a characteristic minimum at $m_H \simeq 500$ GeV and $m_H \simeq 600$ GeV, respectively. This implies that the radiative corrections decrease at first with increasing m_H in the region where we would expect them to grow. Of course, this is due to the fact that we are not yet far enough from the vector-boson thresholds. Only above $m_H \simeq 700$ GeV the corrections grow fast. The amplitudes $c_{2W,b}$ and $c_{2Z,b}$ tend to small, nonvanishing values. According to (5.4) the c_{2V} contribution to the width is enhanced kinematically by a factor $m_H{}^2/M_V{}^2$ relative to the c_{1V} contribution.

We have to point out that the curves drawn up to about 1 TeV do not show up the asymptotic behavior of the amplitudes for $m_H \to \infty$. The asymptotic behavior actually sets in only at much larger values of m_H of about 10 TeV. This will be further discussed at the end of this section.

The corrections to the widths reflect the features of the amplitudes. The $H\tau\tau$ width clearly exhibits the excitation of vector bosons yielding a destructive contribution above the thresholds. The decrease of the radiative corrections in the HVV widths, which we mentioned above, can be seen clearly in Fig. 4(b). For the $HZ\gamma$ and the $H\gamma\gamma$ widths, which have no Born-term contribution, the threshold effects are particularly pronounced.

Figure 1 shows the branching ratios. The fermionic contributions for $m_f \ll m_H$ have been estimated using (5.6). Instead of the t quark we have taken into account a 35-GeV lepton supplied with a color factor 3 in order to estimate the magnitude and the qualitative behavior of the t-hadron contributions. The radiative corrections to the HVV decays are of the order of 10% in the range up to 1 TeV (see also Fig. 1).

The stability and the accuracy of the numerical computations have been tested by varying the small photon mass, the renormalization ϵ-pole term Reg, and the gauge parameter α. In any case, changes in the numerical results occur far beyond a 0.1% accuracy and are in agreement with simple estimates.

The dependence of the charged decay widths on the charged-particle counter resolution x_r follows from (6.7). The x_r dependence is small near the thresholds and increases monotonically with $m_H/M_c \to \infty$ according to

$$\exp\{c_{\rm IR}\} \propto (x_r)^{-(e^2/2\pi^2)(1+2\ln 2)} \left(\frac{m_H}{m_c}\right)^{(-e^2/2\pi^2)(1-2\ln x_r)} . \tag{7.1}$$

Changing x_r from 0.1 to 0.01 lowers the HW^+W^- width by about 0.5% at 200 GeV and by 4% at 1000 GeV. For the lighter leptons the dependence increases for given m_H as follows from (7.1).

We can complete our discussion by considering the asymptotic behavior of the HZZ amplitudes c_{iZ}, defined by (4.23), for $m_f \ll M_W$, M_Z, m_H and for large m_H.

The asymptotic expansion for $m_f \ll M_W, M_Z \ll m_H$ yields for the fermionic contributions

$$\begin{aligned}
\mathrm{Re}c_{1Z,f} = \frac{1}{16\pi^2 v^2} \sum_{f_d} \Bigg\{ & \tfrac{4}{3}\Bigg[(3M_Z{}^2 - 10M_W{}^2 + 8M_Z{}^{-2}M_W{}^4) + 8q_1q_2(M_Z{}^2 - 3M_W{}^2 + 2M_Z{}^{-2}M_W{}^4) \\
& + (\tfrac{1}{2}M_Z{}^2\cot^2\theta_W - M_W{}^2)\ln\frac{M_Z{}^2}{M_W{}^2} + 2M_W{}^2\sin^2\theta_W\left(q_1{}^2\ln\frac{M_Z{}^2}{m_1{}^2} + q_2{}^2\ln\frac{M_Z{}^2}{m_2{}^2}\right)\Bigg] \\
& - 4\sin^2\theta_W(m_1{}^2q_1a_1 - m_2{}^2q_2a_2)\left(\ln^2\frac{4M_Z{}^2}{m_H{}^2} + \frac{\pi^2}{3}\right) + (m_1{}^2 + m_2{}^2)\left(\ln\frac{m_H{}^2}{M_W{}^2} - \cot^2\theta_W\ln\frac{M_Z{}^2}{M_W{}^2}\right) \\
& - [m_1{}^2(1 - 8q_1a_1\cos2\theta_W) + m_2{}^2(1 + 8q_2a_2\cos2\theta_W)] + O\!\left(\frac{m_f{}^4}{M_W{}^2}, m_f{}^2\frac{M_W{}^2}{m_H{}^2}\right)\Bigg\},
\end{aligned} \tag{7.2}$$

and

$$\mathrm{Re}c_{2Z,f}=\frac{1}{16\pi^2v^2}\sum_{f_d}8\frac{M_Z^2}{m_H^2}\left\{-2\sin^2\theta_W(m_1^2q_1a_1-m_2^2q_2a_2)\left(\ln^2\frac{4M_Z^2}{m_H^2}+\frac{\pi^2}{3}\right)+(m_1^2+m_2^2)\ln\frac{m_H^2}{M_Z^2}\right.$$
$$\left.-[m_1^2(1-4q_1a_1\sin^2\theta_W)+m_2^2(1+4q_2a_2\sin^2\theta_W)]+O\left(\frac{m_f^4}{M_W^2},m_f^2\frac{M_W^2}{m_H^2}\right)\right\}, \qquad (7.3)$$

where $a_1=1-2q_1\sin^2\theta_W$ and $a_2=1+2q_2\sin^2\theta_W$. In the region $m_f \ll M_W, m_H \sim M_W$ the dominant contribution is given by the terms without a factor m_f^2. Thus we get

$$\mathrm{Re}c_{1Z,l}\simeq 3.871\times10^{-4}\sum_l\left(\ln\frac{M_Z^2}{m_l^2}+0.789\right)$$

for the lepton contribution. The main contribution is due to the electron. The value we get from this term agrees with the result obtained by the numerical analysis in the range 2 GeV $\leq m_H \leq$ 1 TeV within 0.05% accuracy. The $m_H\to\infty$ asymptotic terms proportional to $\ln^2 4M_Z^2/m_H^2$ and $\ln m_H^2/M_W^2$ are suppressed by a factor m_f^2/M_W^2.

For the quark contribution we have

$$\mathrm{Re}c_{1Z,q}\simeq 3.871\times10^{-4}\times\tfrac{1}{3}\times\sum_{q_d}\left(4\ln\frac{M_Z^2}{m_1^2}+\ln\frac{M_Z^2}{m_2^2}+12.711\right)$$

and the light quarks contribute predominantly. This explains the relatively weak m_t dependence mentioned before. The values obtained agree with the values from the numerical analysis in the range 2 GeV $\leq m_H \lesssim$ 800 GeV within 5%. Since m_t is quite large, the corrections proportional to m_t^2 are important now, in particular for the larger values of m_H. In the region $m_H \gg M_Z$ we obtain from (7.2) the corresponding correction

$$\Delta\,\mathrm{Re}c_{1Z,q}\simeq-\frac{m_t^2}{16\pi^2v^2}\times\left(1.276\ln^2\frac{4M_Z^2}{m_H^2}-\ln\frac{m_H^2}{M_W^2}+7.269\right).$$

This term of course dominates for asymptotically large m_H.

The asymptotic behavior of the bosonic contributions for $2M_Z \ll m_H$ is given by

$$\mathrm{Re}c_{1Z,b}=\frac{1}{16\pi^2v^2}\left[\left(15+2\sqrt{3}\pi-2\pi^2-16\frac{M_W^2}{M_Z^2}\sin^2\theta_W\right)\frac{m_H^2}{4}+O\left(M_Z^2\ln\frac{m_H^2}{M_W^2}\right)\right]$$
$$\simeq 6.781\times10^{-4}\frac{m_H^2}{M_Z^2} \qquad (7.4)$$

and

$$\mathrm{Re}c_{2Z,b}=\frac{1}{16\pi^2v^2}\left\{[(8\sqrt{3}\pi-\tfrac{11}{3}\pi^2-4)M_Z^2-16M_W^2\sin^2\theta_W]+O\left(\frac{M_Z^4}{m_H^2}\ln\frac{m_H^2}{M_W^2}\right)\right\}$$
$$\simeq 4.170\times10^{-4}. \qquad (7.5)$$

Obviously the numerical results in the 1-TeV region do not yet show this behavior. It turns out that only above 10 TeV the asymptotic tails clearly show up.

The conclusions following from the above considerations may be summarized as follows.

(i) At the one-loop level the radiative corrections to the HVV form factors asymptotically grow as m_H^2/M_V^2 due to the bosonic contributions. The amplitudes c_{1V} and c_{2V} yield comparable contributions to the widths, since c_{2V} is enhanced kinematically by m_H^2/M_V^2.

(ii) The contributions from fermion loops are asymptotically negligible since they grow logarithmically only.

(iii) Owing to the logarithmic approach to the asymptotes, the leading asymptotic regime sets in only at about 10 TeV.

(iv) Although the region of the numerical analysis is far from being asymptotic, the radiative corrections are large for 200 GeV $\lesssim m_H$ and grow fast above 700 GeV.

A final remark may be necessary concerning the significance of the radiative corrections to on-shell form factors. Obviously the *splitting* of the value of an on-shell form factor into a Born term and the radiative corrections depends upon the particular renormalization scheme (i.e., the choice and the definitions of the physical parameters) used. As an example, in our case, we could redefine the masses M_V such that

$$\frac{2M_V^2}{v}(1+\mathrm{Re}c_{1V})=\frac{2\bar{M}_V^2}{v},$$

and there would be no radiative corrections due to $\mathrm{Re}c_{1V}$ in the new parametrization. Obviously, by such a parameter change we cannot, however, renormalize away $\mathrm{Im}c_{1V}$ and c_{2V}. The reparametrization does not change the physics, since the widths are not altered. The radiative corrections would show up at different places as in our example in the locations of the propagator poles (see also Sec. III).

ACKNOWLEDGMENTS

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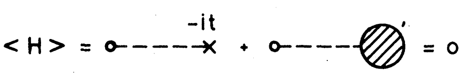
APPENDIX A: TADPOLES

In this appendix we consider the problems related to the shift (2.4) of the Higgs field. The quantities considered in this appendix are the bare ones if not indicated otherwise. If the shift parameter v has the correct value, the physical Higgs field satisfies the gauge-invariant condition



(A1)

when the trivial tadpole

$$t = vm_0^2 = v(\lambda v^2 - \mu^2) \tag{A2}$$

and the Higgs-boson mass

$$m_H^2 = 3\lambda v^2 - \mu^2 \tag{A3}$$

are given the ground-state values

$$m_0^2 = 0 \text{ and } m_H^2 = 2\lambda v^2 . \tag{A4}$$

The proper value of v, however, is not known *a priori* and it must be determined order by order in the perturbation expansion. We denote by v_0 and v the proper values of v to the nth and the $(n+1)$th order, respectively. Thus we write

$$v = v_0 + \delta v_t . \tag{A5}$$

Inserting (A5) into (A2), (A3) and the relations (2.5) and (2.7) we generate terms proportional to δv_t. Considering these terms as counterterms, fixed by the condition (A1), and identifying v_0 with the proper value v, we achieve the proper treatment of the v shift in perturbation theory.

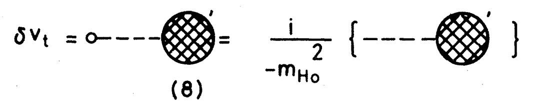
To lowest order (A1) yields

$$t_0 = 0 , \quad \mu^2 = \lambda v_0^2 , \text{ and } m_{H0}^2 = 2\lambda v_0^2 . \tag{A6}$$

At the one-loop order from (A2), (A5), and (A6) we have

$$t = \delta t = 2\lambda v_0^2 \delta v_t = m_{H0}^2 \delta v_t . \tag{A7}$$

This result together with (A1) yields



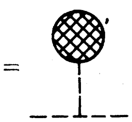
(A8)

For the Higgs-boson mass we get

$$m_H^2 = 3\lambda v_0^2 - \mu^2 + 6\lambda v_0 \delta v_t = m_{H0}^2 + \Delta m_H^2 . \tag{A9}$$

Graphically Δm_H^2 is given by

$$-i\Delta m_H^2 = -i6\lambda v_0 \delta v_t =$$


. (A10)

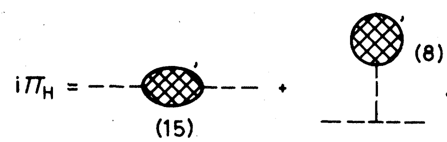
Thus the bare negative inverse Higgs-boson propagator may be represented by

$$\Gamma_H^{(2)} = i[p^2 - m_{H\text{ren}}^2 - \delta m_H^2 + \Pi_H(p^2)] , \tag{A11}$$

with

$$\delta m_H^2 = m_{H0}^2 - m_{H\text{ren}}^2$$

and

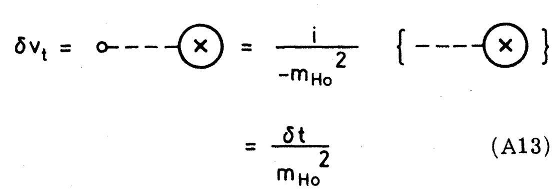


$m_{H\text{ren}}$ is the renormalized mass and hence

$$\delta m_H^2 = \Pi_H(m_{H\text{ren}}^2) . \tag{A12}$$

m_{H0} has to be identified with the proper bare Higgs-boson mass (A3). By this definition, which takes into account the proper v, the mass counterterm δm_H^2 is *gauge invariant*.

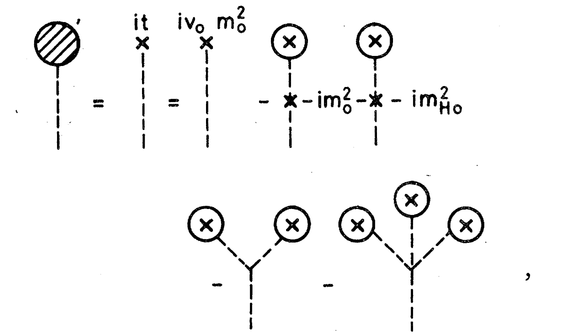
Similarly the proper value of v is taken into account in the remaining mass terms and vertices appearing in (2.5). Before we can give the corresponding relations, we give the proper definition of



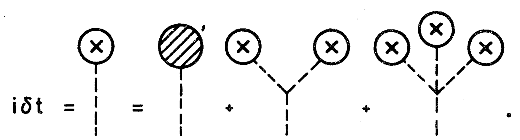
(A13)

valid now to all orders in the loop expansion.

By amputation of the external H leg in (A1) we obtain



where $m_0{}^2=\lambda v_0{}^2-\mu^2=0$ and $m_{H0}{}^2=2\lambda v_0{}^2$. Thus δv_t is defined by the nonlinear relation[23]



(A14)

In these equations, the couplings and combinatorical factors are given by the Feynman rules with $-\delta tH$ as a Lagrangian tadpole counterterm. Equation (A14) can be solved recursively in perturbation theory. The solution to the one-loop order is given by (A8).

Now by inserting (A5) into (2.5) and using the Feynman rules we get the relations depicted in Fig. 7. With t fixed by (A1) we identify v_0 with the proper v. By this procedure we achieve that the correct value of v is taken into account in the fundamental relations (2.7) and (2.9) between the bare parameters.

The crucial point of a proper inclusion of the tadpoles is that mass and coupling counterterms can be defined in a gauge-invariant manner.

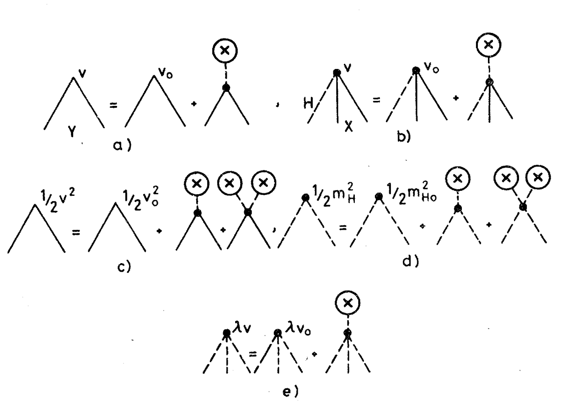


FIG. 7. Tadpole terms induced by the relation $v=v_0+\delta v_t$ [Eq. (A5)].

APPENDIX B: γ-Z mixing

We briefly discuss some problems in perturbation theory related to the mixing of the photon and the neutral vector boson. If not indicated otherwise all quantities considered are the bare ones.

1. Mass mixing

The bare free γ-Z Lagrangian

$$\mathcal{L}_{0\gamma Z}=-\tfrac{1}{4}(B_{\mu\nu}B^{\mu\nu}+W_{3\mu\nu}W_3^{\mu\nu})+\tfrac{1}{8}v^2(g'B-gW_3)^2 \tag{B1}$$

is diagonalized by the orthogonal transformation (2.8) yielding

$$\mathcal{L}_{0\gamma Z}=-\tfrac{1}{2}(A_{\mu\nu}A^{\mu\nu}+Z_{\mu\nu}Z^{\mu\nu})+\tfrac{1}{2}M_Z{}^2Z^2\,. \tag{B2}$$

The orthogonality implies the form invariance of the kinetic term.

In the perturbation expansion we denote by g_0' and g_0 the couplings and by (A_0,Z_0) the fields which diagonalize $\mathcal{L}_{0\gamma Z}$ to nth order. To $(n+1)$th order with the couplings $g'=g_0'+\delta g'$ and $g=g_0+\delta g$ we obtain

$$\begin{aligned}g'B-gW_3&=(g'^2+g^2)^{1/2}Z\\&=(g_0{}'^2+g_0{}^2)^{1/2}[(1+c)Z_0+bA_0]\,,\end{aligned} \tag{B3}$$

with

$$c=\frac{g_0'\delta g'+g_0\delta g}{g_0{}'^2+g_0{}^2}$$

and (B4)

$$b=\frac{g_0\delta g'-g_0'\delta g}{g_0{}'^2+g_0{}^2}\,.$$

It follows that the fields which diagonalize $\mathcal{L}_{0\gamma Z}$ are related to (A_0,Z_0) by the orthogonal transformation

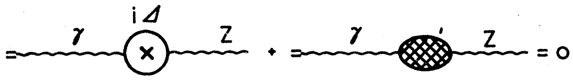
$$\begin{aligned}\begin{pmatrix}A\\Z\end{pmatrix}&=\frac{1}{[(1+c)^2+b^2]^{1/2}}\begin{pmatrix}1+c&-b\\b&1+c\end{pmatrix}\begin{pmatrix}A_0\\Z_0\end{pmatrix}\\&=\begin{pmatrix}1&-b\\b&1\end{pmatrix}\begin{pmatrix}A_0\\Z_0\end{pmatrix}+O(\delta^2)\,.\end{aligned} \tag{B5}$$

According to (B3) the mass term has the form

$$\mathcal{L}_{\text{mass},\gamma Z}=\tfrac{1}{2}[(M_{0Z}{}^2+\delta M_Z{}^2)Z_0{}^2+2\Delta A_0Z_0+O(\delta^2)]\,. \tag{B6}$$

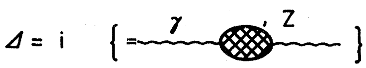
We notice that the bare photon mass remains zero and, apart from the usual bare-Z-mass shift $\delta M_Z{}^2=2cM_{0Z}{}^2$, there appears an induced γ-Z mixing mass term $\Delta=bM_{0Z}{}^2$. The renormalization of v entering as an overall factor does not affect the diagonalization.

Instead of transforming the fields, the diagonalization of the mass matrix may also be achieved by adjusting the mixing mass term as a counterterm through the condition



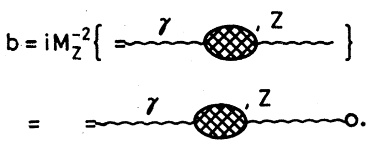
(B7)

and identifying couplings and fields with g', g and (A,Z), respectively. The equals sign attached to a line means that this line is considered on the mass shell. From (B7) it follows



(B7a)

and



(B7b)

2. Vertex mixing

The γ-Z mixing of vertices follows from the transformation (B5) of the fields. Accordingly the bare vertices are represented by

$$G_Z X_Z^\mu Z_\mu = G_Z X_Z^\mu Z_{0\mu} + bG_Z X_Z^\mu A_{0\mu} + O(\delta^2)$$

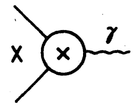
and (B8)

$$G_A X_A^\mu A_\mu = G_A X_A^\mu A_{0\mu} - bG_A X_A^\mu Z_{0\mu} + O(\delta^2)\,,$$

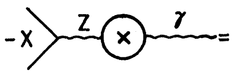
where X^μ denotes some field monomial and G the corresponding coupling. Considering the induced mixing vertices as counterterms fixed by the condition (B7) and identifying couplings and fields with g', g and (A,Z), respectively, the γ-Z mixing is properly accounted for.

Graphically the $(n+1)$th-order contribution to an amputated amplitude is represented as shown in Fig. 8.

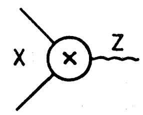
According to (B8) and (B7) the mixing vertices are given up to a sign by the on-shell values of the mixing-mass-term contribution



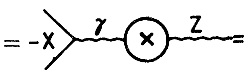
$$= iG_{ZX}b = -iG_{ZX}\frac{-i}{p^2-M_Z{}^2+i\epsilon}\, i\Delta\,\Big|_{p^2=0}$$

$= -$ 

and (B9)



$$= -iG_{AX}b = -iG_{AX}\frac{-i}{p^2+i\epsilon}\, i\Delta\,\Big|_{p^2=M_Z{}^2}$$

$= -$  .

As a result for *on-shell* A and Z lines, the mixing counterterms cancel. Thus the proper on-shell amplitudes are given by including the nontrivial mixing diagrams only as depicted in Fig. 9.

3. The γ-Z propagator

Owing to the mixing the γ-Z propagator is to be considered as a symmetric 2×2 matrix $\hat{G}$. In the 't Hooft gauge, the gauge function is chosen such that, to the lowest order, the free propagator $\hat{G}_0$ is diagonal. Thus the negative inverse propagator $\hat{\Gamma}$ has the form

$$\hat{\Gamma} = -\hat{G}^{-1} = -\hat{G}_0{}^{-1} - i\hat{\Pi} = \begin{pmatrix} -G_{0\gamma}{}^{-1} - i\Pi_{\gamma\gamma} & -i\Pi_{\gamma Z} \\ -i\Pi_{\gamma Z} & -G_{0Z}{}^{-1} - i\Pi_{ZZ} \end{pmatrix}. \tag{B10}$$

The irreducible self-energies are given diagramatically by

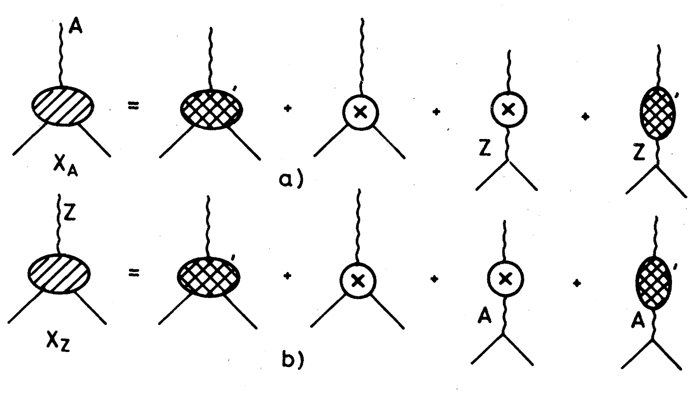


FIG. 8. Off-shell vertex contributions induced by γ-Z mixing.

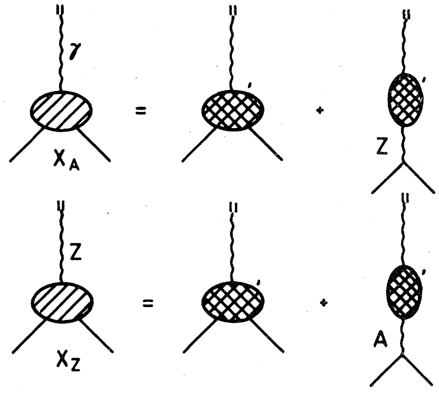
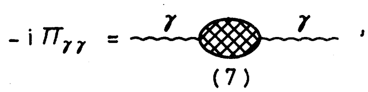
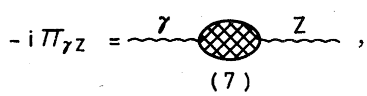
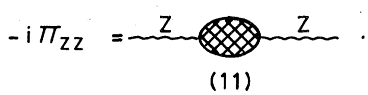


FIG. 9. On-shell vertex contributions, induced by γ-Z mixing, containing only the nontrivial mixing diagrams.





and



$\hat{G}$ is then formally given by the geometrical series generated by

$$\hat{G}=\hat{G}_0\frac{1}{1+i\hat{\Pi}\hat{G}_0}\,. \tag{B11}$$

For simplicity we shall restrict our discussion to the $g^{\mu\nu}$ parts $\hat{\Gamma}_1$ and $\hat{G}_1$ of $\hat{\Gamma}$ and $\hat{G}$, respectively. The $p^\mu p^\nu$ parts $\hat{\Gamma}_2$ and $\hat{G}_2$ are determined, given the $g^{\mu\nu}$ parts by the WT or ST identities. $\hat{G}_1$, which exhibits the pole structure, is given by

$$\hat{G}_1=\frac{1}{\Gamma_{1\gamma\gamma}\Gamma_{1ZZ}-(\Gamma_{1\gamma Z})^2}\begin{pmatrix}-\Gamma_{1ZZ} & \Gamma_{1\gamma Z}\\ \Gamma_{1\gamma Z} & -\Gamma_{1\gamma\gamma}\end{pmatrix}\,. \tag{B12}$$

Obviously the matrix $\hat{\Gamma}$ can only be diagonalized for *one* particular value of p^2 by fixing the γ-Z mixing counterterm introduced above.

The relevant condition is that the photon is stable and massless. Although we have from the electromagnetic WT identity

$$\Pi^{\mu\nu}_{\gamma\gamma}=(p^\mu p^\nu-p^2g^{\mu\nu})\Pi_{2\gamma\gamma}(p^2,\alpha)$$

and (B13)

$$\Pi^{\mu\nu}_{\gamma Z}=(p^\mu p^\nu-p^2g^{\mu\nu})\Pi_{2\gamma Z}(p^2,\alpha)\,,$$

the masslessness of the photon no longer follows from the existence of $\Pi_{2\gamma\gamma}(0,\alpha)$ alone. Actually $\Pi_{2\gamma Z}(p^2,\alpha)$ is singular at $p^2=0$ and

$$\Pi_{1\gamma Z}(0,\alpha)=-\lim_{p^2\to 0}p^2\Pi_{2\gamma Z}(p^2,\alpha)\neq 0\,. \tag{B14}$$

Hence the γ-Z counterterm must be fixed by the condition (B7), that is

$$\lim_{p^2\to 0}\Gamma_{1\gamma Z}=i[bM_Z{}^2-\Pi'_{1\gamma Z}(0)]=0\,. \tag{B15}$$

It follows that

$$G_{1\gamma\gamma}\simeq-\Gamma_{1\gamma\gamma}{}^{-1}\,,\quad p^2\simeq 0\,, \tag{B16}$$

so that the pole of the γ propagator is given by the zero of $\Gamma_{1\gamma\gamma}$ as usual.

On the other hand the pole of the Z propagator

$$G_{1ZZ}=\frac{1}{-\Gamma_{1ZZ}+\Gamma_{1\gamma\gamma}{}^{-1}(\Gamma_{1\gamma Z})^2} \tag{B17}$$

appears shifted, relative to the zero of Γ_{1ZZ}. Furthermore, we notice that the condition (B15) also guarantees that an additional pole at $p^2=0$ is absent in G_{1ZZ}.

According to (B17) the Z-mass counterterm $\delta M_Z{}^2$ is determined from the condition

$$\lim_{p^2\to M_Z{}^2}[\Gamma_{1ZZ}-\Gamma_{1\gamma\gamma}{}^{-1}(\Gamma_{1\gamma Z})^2]=0\,, \tag{B18}$$

where

$$\begin{aligned}\Gamma_{1\gamma\gamma}&=i(-p^2-\Pi_{1\gamma\gamma})=-ip^2(1-\Pi_{2\gamma\gamma})\,,\\ \Gamma_{1\gamma Z}&=i(bM_Z{}^2-\Pi_{1\gamma Z})\,,\\ \Gamma_{1ZZ}&=i(-p^2+M_{Z\,\mathrm{ren}}{}^2+\delta M_Z{}^2-\Pi_{1ZZ})\,.\end{aligned} \tag{B19}$$

To the one-loop order the mixing term does not contribute and we have as usual

$$\delta M_Z{}^2=\mathrm{Re}\Pi_{1ZZ}(M_{Z\,\mathrm{ren}}{}^2)\,. \tag{B20}$$

The "mixing propagator" $G_{1\gamma Z}$ is given by

$$G_{1\gamma Z}=-G_{1ZZ}\Gamma_{1\gamma\gamma}{}^{-1}\Gamma_{1\gamma Z} \tag{B21}$$

and by virtue of (B15) has no pole at $p^2=0$. However, the Z-propagator pole is obviously present.

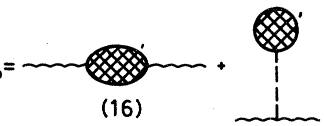
APPENDIX C: COUNTERTERMS

Below we give a list of the one-loop renormalization counterterms in the 't Hooft gauge. Apart from the one vertex counterterm, which we have considered in Secs. III and IV [Eqs. (3.8) and (4.14)], we need the mass and wave-function renormalizations of the physical fields. In this appendix m_H, M_W, M_Z, m_f, and v^{-1} are the renormalized parameters; bare quantities are indicated by an index b. The renormalized propagators are defined by $G^{(2)}_{\mathrm{ren}}=Z^{-1}G^{(2)}_b$ with Z the wave-function renormalization factor. By $\Gamma^{(2)}=-G^{(2)-1}$ we denote the negative inverse propagators. The tadpoles and the mixing terms are taken into account according to our discussion in the Appendices A and B. In particular the irreducible self-energies always include the nontrivial tadpoles. For our purpose we need to consider only the $g^{\mu\nu}$ parts.

The bare H, γ, and Z propagators have been given above in (A11) and (B19). For the W propagator we have

$$\Gamma_{1WWb}=i(-p^2+M_W{}^2+\delta M_W{}^2-\Pi_{1WWb})\,,$$

with

$-i\,\Pi_{1WWb}=$ 

(C1)

The gauge-invariant mass counterterms of the massive boson fields X are then given by

$$\delta m_X{}^2=\Pi_{Xb}(m_X{}^2) \tag{C2}$$

and the wave-function renormalizations by

$$Z_X=\left[1+\frac{\partial\Pi_{Xb}}{\partial p^2}(m_X{}^2)\right]^{-1}\simeq 1-\frac{\partial\Pi_{Xb}}{\partial p^2}(m_X{}^2)\,. \tag{C3}$$

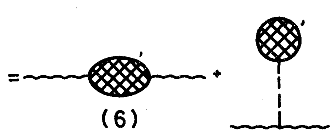
From our discussion in Appendix B the expressions (C2) and (C3) have to be modified beyond the one-loop approximation. For the massless photon we have

$$Z_\gamma=[1-\Pi_{2\gamma\gamma b}(0,\alpha)]^{-1}\simeq 1+\Pi_{2\gamma\gamma b}(0,\alpha)\,. \tag{C4}$$

We further have to consider the lepton propagators. For the inverse l propagator we write

$$\Gamma_{lb}=-S_{lb}{}^{-1}(\not p)=i[\not p-m_l-\delta m_l-\Sigma_{lb}(\not p)]\,, \tag{C5}$$

with the irreducible self-energy

$-i\Sigma_{lb}=$ 

.

We use the anticommuting γ_5 so that the covariant decomposition of Σ_{lb} is given by

$$\Sigma_{lb}=\not p(A+B\gamma_5)+m_l(C+D\gamma_5)\,. \tag{C6}$$

By hermiticity $\gamma_0\Sigma^\dagger\gamma_0=\Sigma$ we must have $D\equiv 0$. By virtue of the relation

$$\bar u(p,r')\gamma_5 u(p,r)=0$$

and the mass-renormalization condition

$$\bar u(p,r')[\Sigma_{lb}(\not p)+\delta m_l]u(p,r)=0$$

the gauge-invariant mass counterterm is determined from

$$\delta m_l=-m_l(A+C)(m_l{}^2)\,. \tag{C7}$$

Furthermore, the renormalized propagator

$$S_{l\,\mathrm{ren}}=\frac{1}{\sqrt{Z_l}}S_{lb}\frac{1}{\gamma_0\sqrt{Z_l}\gamma_0}\,,\quad Z_l-1=z_a+z_b\gamma_5 \tag{C8}$$

must satisfy

$$\lim_{\not p\to m_l}-iS_{l\,\mathrm{ren}}(\not p-m_l)u(p,r)=u(p,r)\,,$$

which implies

$$z_a=\{A+2m_l{}^2[\partial/\partial p^2(A+C)]\}(m_l{}^2)$$

and (C9)

$$z_b=B(m_l{}^2)\,.$$

The above relations are valid as well for the massless neutrinos in the limit $m_l\to 0$, where $\delta m_\nu=0$ and $z_a=-z_b$, such that the neutrino remains purely left handed after renormalization.

The resulting counterterms are explicitly given below in analytic complex form. Only the real parts of these expressions serve as renormalization counterterms as indicated in (3.10).

For the definitions of the standard integrals we refer to Sec. IV. The sums $\sum_{fs}$ and $\sum_{fd}$ extend, respectively, over the single fermions and the fermion doublets. a_f and b_f are, respectively, the vector and the axial-vector coefficients of the neutral currents. They are given by

$$(a_f,b_f)_t=(q_f\sin^2\Theta_W-\tfrac14,\tfrac14)$$

and (C10)

$$(a_f,b_f)_b=(q_f\sin^2\Theta_W+\tfrac14,-\tfrac14)\,,$$

respectively, for the flavors in the top and the bottom components of the weak isodoublets. α is the gauge parameter.

H propagator:

$$\begin{aligned}\delta m_H{}^2=\frac{1}{16\pi^2v^2}\Big\{&A_0(m_H)3m_H{}^2+A_0(M_Z)(m_H{}^2+6M_Z{}^2)+A_0(M_W)(2m_H{}^2+12M_W{}^2)\\&+B_0(m_H,m_H;m_H{}^2)\tfrac92 m_H{}^4+B_0(M_Z,M_Z;m_H{}^2)(\tfrac12 m_H{}^4-2m_H{}^2M_Z{}^2+6M_Z{}^4)\\&+B_0(M_W,M_W;m_H{}^2)(m_H{}^4-4m_H{}^2M_W{}^2+12M_W{}^4)\\&+\sum_{fs}[A_0(m_f)(-8m_f{}^2)+B_0(m_f,m_f;m_H{}^2)(2m_H{}^2m_f{}^2-8m_f{}^4)]\Big\}\,,\end{aligned}$$

$$Z_H - 1 = \frac{1}{16\pi^2 v^2}\Big\{-A_0(M_Z) - A_0(M_W)2 + B_0(M_Z, M_Z; m_H^2)(-m_H^2 + 2M_Z^2) + B_0(M_W, M_W; m_H^2)(-2m_H^2 + 4M_W^2)$$

$$+ AB_0(m_H, m_H; m_H^2)(-\tfrac{9}{2}m_H^4) + AB_0(M_Z, M_Z; m_H^2)(-\tfrac{1}{2}m_H^4 + 2m_H^2 M_Z^2 - 6M_Z^4)$$

$$+ AB_0(M_W, M_W; m_H^2)(-m_H^4 + 4m_H^2 M_W^2 - 12M_W^4) + A_0(\sqrt{\alpha}M_Z) + A_0(\sqrt{\alpha}M_W)2$$

$$+ B_0(\sqrt{\alpha}M_Z, \sqrt{\alpha}M_Z; m_H^2)m_H^2 + B_0(\sqrt{\alpha}M_W, \sqrt{\alpha}M_W; m_H^2)2m_H^2$$

$$+\sum_{f_s}[B_0(m_f, m_f; m_H^2)(-2m_f^2) + AB_0(m_f, m_f; m_H^2)(-2m_H^2 m_f^2 + 8m_f^4)]\Big\}.$$

γ propagator:

$$Z_\gamma = \frac{1}{16\pi^2 v^2}(ev)^2\Big[-\tfrac{19}{3} + A_0(M_W)(-\tfrac{20}{3}M_W^{-2}) + \tfrac{2}{3}\alpha + A_0(\sqrt{\alpha}M_W)M_W^{-2} + B_0(M_W, \sqrt{\alpha}M_W; m_\gamma^2)(-\tfrac{8}{3})$$

$$+\sum_{f_s} q_f^2(\tfrac{16}{9} + A_0(m_f)\tfrac{4}{3}m_f^{-2})\Big].$$

Z propagator:

$$\delta M_Z^2 = \frac{1}{16\pi^2 v^2}\Big((-\tfrac{2}{3}m_H^2 M_Z^2 + 4m_H^{-2}M_Z^6 + 8m_H^{-2}M_Z^2 M_W^4 - \tfrac{2}{9}M_Z^4 - \tfrac{20}{9}M_Z^2 M_W^2 - 16M_Z^{-2}M_W^6 + \tfrac{16}{3}M_W^4)$$

$$+ A_0(m_H)(\tfrac{1}{3}m_H^2 + 2M_Z^2) + A_0(M_Z)(-\tfrac{1}{3}m_H^2 + 6m_H^{-2}M_Z^4 + \tfrac{2}{3}M_Z^2)$$

$$+ A_0(M_W)(12m_H^{-2}M_Z^2 M_W^2 + \tfrac{2}{3}M_Z^2 - 16M_Z^{-2}M_W^4 + \tfrac{16}{3}M_W^2) + B_0(m_H, M_Z; M_Z^2)(\tfrac{1}{3}m_H^4 - \tfrac{4}{3}m_H^2 M_Z^2 + 4M_Z^4)$$

$$+ B_0(M_W, M_W; M_Z^2)(\tfrac{1}{3}M_Z^4 + \tfrac{16}{3}M_Z^2 M_W^2 - 16M_Z^{-2}M_W^6 - \tfrac{68}{3}M_W^4)$$

$$+\sum_{f_s}\{(-\tfrac{16}{9}M_Z^4 + \tfrac{32}{3}M_Z^2 m_f^2)(a_f^2 + b_f^2) + A_0(m_f)[-8m_H^{-2}M_Z^2 m_f^2 + \tfrac{32}{3}M_Z^2(a_f^2 + b_f^2)]$$

$$+ B_0(m_f, m_f; M_Z^2)[\tfrac{16}{3}M_Z^4(a_f^2 + b_f^2) + \tfrac{32}{3}M_Z^2 m_f^2(a_f^2 - 2b_f^2)]\}\Big),$$

$$Z_0 - 1 = \frac{1}{16\pi^2 v^2}\Big((-\tfrac{4}{9}M_Z^2 - \tfrac{8}{3}M_Z^{-2}M_W^4 + \tfrac{8}{9}M_W^2) + A_0(m_H)(\tfrac{1}{3}m_H^2 M_Z^{-2} - \tfrac{1}{3}) + A_0(M_Z)(-\tfrac{1}{3}m_H^2 M_Z^{-2} + \tfrac{1}{3})$$

$$+ A_0(M_W)(-\tfrac{16}{3}M_Z^{-2}M_W^2 - \tfrac{4}{3}M_Z^{-4}M_W^4) + B_0(m_H, M_Z; M_Z^2)(\tfrac{1}{3}m_H^4 M_Z^{-2} - \tfrac{2}{3}m_H^2)$$

$$+ B_0(M_W, M_W; M_Z^2)(-M_Z^2 + \tfrac{68}{3}M_Z^{-2}M_W^4 - \tfrac{32}{3}M_W^2)$$

$$+ AB_0(m_H, M_Z; M_Z^2)(-\tfrac{1}{3}m_H^4 + \tfrac{4}{3}m_H^2 M_Z^2 - 4M_Z^4)$$

$$+ AB_0(M_W, M_W; M_Z^2)(-\tfrac{1}{3}M_Z^4 - \tfrac{16}{3}M_Z^2 M_W^2 + 16M_Z^{-2}M_W^6 + \tfrac{68}{3}M_W^4) + (\tfrac{8}{3}M_Z^{-2}M_W^4\alpha)$$

$$+ A_0(M_W)(-\tfrac{4}{3}M_Z^{-2}M_W^2\alpha + \tfrac{4}{3}M_Z^{-4}M_W^4\alpha) + A_0(\sqrt{\alpha}M_W)[\tfrac{16}{3}M_Z^{-2}M_W^2 + \tfrac{4}{3}M_Z^{-2}M_W^2\alpha + \tfrac{4}{3}M_Z^{-4}M_W^4(1-\alpha)]$$

$$+ B_0(M_W, \sqrt{\alpha}M_W; M_Z^2)[\tfrac{4}{3}M_Z^2 - 12M_Z^{-2}M_W^4 + \tfrac{4}{3}M_Z^{-2}M_W^4\alpha^2 - \tfrac{4}{3}M_Z^{-4}M_W^6(1-\alpha)^2 + 12M_W^2 - \tfrac{8}{3}M_W^2\alpha]$$

$$+ B_0(\sqrt{\alpha}M_W, \sqrt{\alpha}M_W; M_Z^2)(-\tfrac{2}{3}M_Z^2 + \tfrac{8}{3}M_W^2\alpha)$$

$$+\sum_{f_s}\{\tfrac{16}{9}M_Z^2(a_f^2 + b_f^2) + B_0(m_f, m_f; M_Z^2)[-\tfrac{16}{3}M_Z^2(a_f^2 + b_f^2)]$$

$$+ AB_0(m_f, m_f; M_Z^2)[-\tfrac{16}{3}M_Z^4(a_f^2 + b_f^2) - \tfrac{32}{3}M_Z^2 m_f^2(a_f^2 - 2b_f^2)]\}\Big).$$

$W^\pm$ propagator:

$$\delta M_W^2 = \frac{1}{16\pi^2 v^2}\Big((-\tfrac{2}{3}m_H^2 M_W^2 + 4m_H^{-2}M_Z^4 M_W^2 + 8m_H^{-2}M_W^6 - \tfrac{2}{3}M_Z^2 M_W^2 - \tfrac{112}{9}M_W^4) + A_0(m_H)(\tfrac{1}{3}m_H^2 + 2M_W^2)$$

$$+A_0(M_Z)(6m_H^{-2}M_Z^2M_W^2+\tfrac{1}{3}M_Z^2-8M_Z^{-2}M_W^4+\tfrac{8}{3}M_W^2)+A_0(M_W)(-\tfrac{1}{3}m_H^2+12m_H^{-2}M_W^4-\tfrac{1}{3}M_Z^2-4M_W^2)$$
$$+B_0(m_H,M_W;M_W^2)(\tfrac{1}{3}m_H^4-\tfrac{4}{3}m_H^2M_W^2+4M_W^4)$$
$$+B_0(M_Z,M_W;M_W^2)(\tfrac{1}{3}M_Z^4+\tfrac{16}{3}M_Z^2M_W^2-16M_Z^{-2}M_W^6-\tfrac{68}{3}M_W^4)+B_0(m_\gamma,M_W;M_W^2)[-4(ev)^2M_W^2]$$
$$+\sum_{fd}\{[-\tfrac{4}{9}M_W^4+\tfrac{4}{3}M_W^2(m_1^2+m_2^2)]+A_0(m_1)[-8m_H^{-2}M_W^2m_1^2+\tfrac{4}{3}M_W^2-\tfrac{2}{3}(m_1^2-m_2^2)]$$
$$+A_0(m_2)[-8m_H^{-2}M_W^2m_2^2+\tfrac{4}{3}M_W^2+\tfrac{2}{3}(m_1^2-m_2^2)]$$
$$+B_0(m_1,m_2;M_W^2)[\tfrac{4}{3}M_W^4-\tfrac{2}{3}M_W^2(m_1^2+m_2^2)-\tfrac{2}{3}(m_1^2-m_2^2)^2]\}\Bigg),$$

$$Z-1=\frac{1}{16\pi^2v^2}\Bigg((-\tfrac{20}{9}M_W^2)+A_0(m_H)(\tfrac{1}{3}m_H^2M_W^{-2}-\tfrac{1}{3})+A_0(M_Z)(\tfrac{1}{3}M_Z^2M_W^{-2}-\tfrac{8}{3}M_Z^{-2}M_W^2-\tfrac{10}{3}M_Z^{-4}M_W^4+\tfrac{7}{3})$$
$$+A_0(M_W)(-\tfrac{1}{3}m_H^2M_W^{-2}-\tfrac{1}{3}M_Z^2M_W^{-2}-\tfrac{8}{3})+B_0(m_H,M_W;M_W^2)(\tfrac{1}{3}m_H^4M_W^{-2}-\tfrac{2}{3}m_H^2)$$
$$+B_0(M_Z,M_W;M_W^2)(\tfrac{1}{3}M_Z^4M_W^{-2}+\tfrac{8}{3}M_Z^2+8M_Z^{-2}M_W^4)$$
$$+B_0(m_\gamma,M_W;M_W^2)[2(ev)^2]+AB_0(m_H,M_W;M_W^2)(-\tfrac{1}{3}m_H^4+\tfrac{4}{3}m_H^2M_W^2-4M_W^4)$$
$$+AB_0(M_Z,M_W;M_W^2)(-\tfrac{1}{3}M_Z^4-\tfrac{16}{3}M_Z^2M_W^2+16M_Z^{-2}M_W^6+\tfrac{68}{3}M_W^4)+AB_0(m_\gamma,M_W;M_W^2)[4(ev)^2M_W^2]$$
$$+(\tfrac{8}{3}M_W^2\alpha)+[A_0(\sqrt{\alpha}m_\gamma)-A_0(m_\gamma)]m_\gamma^{-2}[\tfrac{1}{6}(ev)^2(5+\alpha)]+A_0(M_Z)(\tfrac{2}{3}M_Z^{-2}M_W^2\alpha-\tfrac{2}{3}M_Z^{-4}M_W^4\alpha)$$
$$+A_0(\sqrt{\alpha}M_Z)[-\tfrac{2}{3}M_Z^{-2}M_W^2\alpha+\tfrac{2}{3}M_Z^{-4}M_W^4(5+\alpha)]+A_0(\sqrt{\alpha}M_W)(\tfrac{10}{3})$$
$$+B_0(M_Z,\sqrt{\alpha}M_W;M_W^2)[-\tfrac{2}{3}M_Z^2+6M_Z^{-2}M_W^4-\tfrac{2}{3}M_Z^{-2}M_W^4\alpha^2+\tfrac{2}{3}M_Z^{-4}M_W^6(1-\alpha)^2-6M_W^2+\tfrac{4}{3}M_W^2\alpha]$$
$$+B_0(\sqrt{\alpha}M_Z,\sqrt{\alpha}M_W;M_W^2)[\tfrac{4}{3}M_Z^{-2}M_W^4\alpha(1+\alpha)-\tfrac{2}{3}M_Z^{-4}M_W^6(1-\alpha)^2-\tfrac{2}{3}M_W^2\alpha^2]$$
$$+B_0(m_\gamma,\sqrt{\alpha}M_W;M_W^2)[\tfrac{1}{6}(ev)^2(9+2\alpha+\alpha^2)]$$
$$+\sum_{f_d}\{\tfrac{4}{9}M_W^2-[A_0(m_1)-A_0(m_2)][\tfrac{2}{3}M_W^{-2}(m_1^2-m_2^2)]+B_0(m_1,m_2;M_W^2)[-\tfrac{4}{3}M_W^2-\tfrac{2}{3}M_W^{-2}(m_1^2-m_2^2)^2]$$
$$+AB_0(m_1,m_2;M_W^2)[-\tfrac{4}{3}M_W^4+\tfrac{2}{3}M_W^2(m_1^2+m_2^2)+\tfrac{2}{3}(m_1^2-m_2^2)^2]\}\Bigg),$$

$l^\pm$ propagator:

$$\delta m_l=\frac{1}{16\pi^2v^2}m_l\Big[(2m_H^{-2}M_Z^4+4m_H^{-2}M_W^4+\tfrac{3}{2}M_Z^2-3M_W^2)+A_0(m_H)$$
$$+A_0(M_Z)(3m_H^{-2}M_Z^2-\tfrac{5}{2}M_Z^2m_l^{-2}+6M_W^2m_l^{-2}-4M_Z^{-2}M_W^4m_l^{-2})+A_0(M_W)(\tfrac{1}{2}+6m_H^{-2}M_W^2-M_W^2m_l^{-2})$$
$$+A_0(m_l)(1+\tfrac{5}{2}M_Z^2m_l^{-2}-2M_W^2m_l^{-2})+B_0(m_H,m_l;m_l^2)(-\tfrac{1}{2}m_H^2+2m_l^2)$$
$$+B_0(M_Z,m_l;m_l^2)(-\tfrac{5}{2}M_Z^4m_l^{-2}-\tfrac{7}{2}M_Z^2-4M_W^4m_l^{-2}+12M_W^2-8M_Z^{-2}M_W^4+6M_Z^2M_W^2m_l^{-2})$$
$$+B_0(M_W,m_\nu;m_l^2)(-M_W^4m_l^{-2}+\tfrac{1}{2}M_W^2+\tfrac{1}{2}m_l^2)+B_0(m_\gamma,m_l;m_l^2)[-2(ev)^2]+\sum_{f_S}A_0(m_f)(-4m_H^{-2}m_f^2)\Big],$$

$$Z_l-1=z_a+z_b\gamma_5,$$

$$z_a=-\frac{1}{16\pi^2v^2}[(-\tfrac{5}{2}M_Z^2+M_W^2)+A_0(m_H)\tfrac{1}{2}+A_0(M_Z)(\tfrac{5}{2}+\tfrac{5}{2}M_Z^2m_l^{-2}-6M_Z^{-2}M_W^2+4M_Z^{-2}M_W^4m_l^{-2}+4M_Z^{-4}M_W^4-6M_W^2m_l^{-2})$$
$$+A_0(M_W)(\tfrac{1}{2}+M_W^2m_l^{-2})+A_0(m_l)(-1-\tfrac{5}{2}M_Z^2m_l^{-2}+2M_W^2m_l^{-2})+B_0(m_H,m_l;m_l^2)(\tfrac{1}{2}m_H^2)$$
$$+B_0(M_Z,m_l;m_l^2)(\tfrac{5}{2}M_z^4m_l^{-2}-6M_Z^2M_W^2m_l^{-2}+4M_W^4m_l^{-2})+B_0(M_W,m_\nu;m_l^2)(M_W^4m_l^{-2}+\tfrac{1}{2}M_W^2+\tfrac{3}{2}m_l^2)$$
$$+AB_0(m_H,m_l;m_l^2)(-m_H^2m_l^2+4m_l^4)$$
$$+AB_0(M_Z,m_l;m_l^2)(-5M_Z^4+12M_Z^2M_W^2-7M_Z^2m_l^2-16M_Z^{-2}M_W^4m_l^2-8M_W^4+24M_W^2m_l^2)$$
$$+AB_0(M_W,m_\nu;m_l^2)(-2M_W^4+M_W^2m_l^2+m_l^4)+AB_0(m_\gamma,m_l;m_l^2)[-4(ev)^2m_l^2]$$
$$+[A_0(m_\gamma)-A_0(\sqrt{\alpha}m_\gamma)]m_\gamma^{-2}(ev)^2+A_0(\sqrt{\alpha}M_Z)(-2+6M_Z^{-2}M_W^2-4M_Z^{-4}M_W^4)+B_0(\sqrt{\alpha}M_Z,m_l;m_l^2)(\tfrac{1}{2}M_Z^2\alpha)$$
$$+B_0(\sqrt{\alpha}M_W,m_\nu;m_l^2)(M_W^2\alpha-m_l^2)]$$

$$z_b = \frac{1}{16\pi^2 v^2}[(-\tfrac{3}{2}M_z^{\,2}+3M_W^{\,2}) + A_0(M_Z)(-\tfrac{3}{2}M_Z^{\,2}m_l^{\,-2}+2M_W^{\,2}m_l^{\,-2}) + A_0(M_W)(-\tfrac{1}{2}+M_W^{\,2}m_l^{\,-2})$$
$$+A_0(m_l)(\tfrac{3}{2}M_Z^{\,2}m_l^{\,-2}-2M_W^{\,2}m_l^{\,-2}) + B_0(M_Z, m_l; m_l^{\,2})(-\tfrac{3}{2}M_Z^{\,4}m_l^{\,-2}+\tfrac{3}{2}M_Z^{\,2}+2M_Z^{\,2}M_W^{\,2}m_l^{\,-2}-2M_W^{\,2})$$
$$+B_0(M_W, m_\nu; m_l^{\,2})(-\tfrac{1}{2}M_W^{\,2}+M_W^{\,4}m_l^{\,-2}-\tfrac{1}{2}m_l^{\,2}) + B(\sqrt{\alpha}M_Z, m_l; m_l^{\,2})(\tfrac{3}{2}M_Z^{\,2}\alpha - 2M_W^{\,2}\alpha)$$
$$+B_0(\sqrt{\alpha}M_W, m_\nu; m_l^{\,2})(-M_W^{\,2}\alpha + m_l^{\,2})] .$$

ν propagator:

$$\delta m_\nu = 0, \quad Z_\nu - 1 = z_\nu(1-\gamma_5) ,$$

$$z_\nu = \frac{1}{16\pi^2\nu^2}[(\tfrac{1}{2}M_z^{\,2}+M_W^{\,2}) - A_0(m_l) + B_0(M_W, m_l; m_\nu^{\,2})(\tfrac{1}{2}m_l^{\,2}) + A_0(m_\nu)m_\nu^{\,-2}(-\tfrac{1}{2}M_Z^{\,2}) + A_0(\sqrt{\alpha}M_Z)\tfrac{1}{2}$$
$$+A_0(\sqrt{\alpha}M_W) + B_0(\sqrt{\alpha}M_W, m_l; m_\nu^{\,2})(-2m_l^{\,2})] .$$

Tadpoles:

$$\delta v_t = v\frac{1}{16\pi^2 v^2} m_H^{\,-2}\Big[-2M_Z^{\,4} - 4M_W^{\,4} + A_0(m_H)(-\tfrac{3}{2}m_H^{\,2}) + A_0(M_Z)(-3M_Z^{\,2}) + A_0(M_W)(-6M_W^{\,2})$$
$$+A_0(\sqrt{\alpha}M_Z)(-\tfrac{1}{2}m_H^{\,2}) + A_0(\sqrt{\alpha}M_W)(-m_H^{\,2}) + \sum_{f_s} A_0(m_f)4m_f^{\,2}\Big] .$$

Charge renormalization:

$$\delta e = e\frac{1}{16\pi^2 v^2}14\sin^2\Theta_W[M_W^{\,2}+A_0(M_W)] .$$

γZ mixing propagator:

$$\Pi'_{1\gamma Z}(s) = \frac{1}{16\pi^2 v^2}\tfrac{4}{3}(ev)M_Z^{\,-1}M_W^{\,4}\Big\{(\tfrac{1}{3}M_Z^{\,2}M_W^{\,-2}+2\alpha-2)s/M_W^{\,2} + [-M_Z^{\,2}M_W^{\,-2}(\alpha+1)+12] + A_0(M_W)M_W^{\,-2}(-2M_Z^{\,2}M_W^{\,-2}+12)$$
$$+[A_0(M_W) - A_0(\sqrt{\alpha}M_W)]M_W^{\,-2}[-\tfrac{1}{2}(\alpha+9)s/M_W^{\,2}+\tfrac{5}{2}M_Z^{\,2}M_W^{\,-2}+\alpha-1-\tfrac{1}{2}(\alpha-1)M_Z^{\,2}/s]$$
$$+B_0(M_W, M_W; s)(-\tfrac{1}{4}s^3/M_W^{\,6} - 4s^2/M_W^{\,4}+17s/M_W^{\,2}+12)$$
$$+B_0(\sqrt{\alpha}M_W, M_W; s)[\tfrac{1}{2}s^3/M_W^{\,6} - (\alpha-4)s^2/M_W^{\,2} + \tfrac{1}{2}(\alpha^2+2\alpha-19+M_Z^{\,2}M_W^{\,-2})s/M_W^{\,2}$$
$$-(\alpha-1)^2 - (\alpha-5)M_Z^{\,2}M_W^{\,-2} + \tfrac{1}{2}(\alpha-1)^2M_Z^{\,2}/s]$$
$$+B_0(\sqrt{\alpha}M_W, \sqrt{\alpha}M_W; s)(-\tfrac{1}{4}s^3/M_W^{\,6}+\alpha s^2/M_W^{\,4})$$
$$+4\sum_{f_s} a_f q_f M_Z^{\,2}M_W^{\,-2}[-\tfrac{1}{3}s/M_W^{\,2}+2m_f^{\,2}M_W^{\,-2}+A_0(m_f)2M_W^{\,-2}$$
$$+B_0(m_f, m_f; s)(s/M_W^{\,2}+2m_f^{\,2}M_W^{\,-2})]\Big\} ,$$

$$\Delta = bM_Z^{\,2} = \Pi'_{1\gamma Z}(0) = \frac{-1}{16\pi^2 v^2}\tfrac{2}{3}(ev)M_Z M_W^{\,2}[2\alpha+2+A_0(M_W)M_W^{\,-2}+3A_0(\sqrt{\alpha}M_W)M_W^{\,-2} - 8B_0(\sqrt{\alpha}M_W, M_W; 0)] .$$

APPENDIX D: COUNTERTERMS FOR THE PHYSICAL VERTICES

Except for the electron-photon vertex, which is used for the normalization of the electric charge e, all vertex renormalizations are determined, from the counterterms given in Appendix C, by virtue of the relations (2.9), (3.2), and (3.8). In the following, we list the resulting counterterms which render the vertices finite and gauge invariant on the mass shell. For the ghost vertices similar expressions can be derived. We shall include the Born terms for the purpose of normalization. For the kinematical tensors we shall use the abbreviations

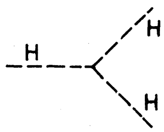
$$T^{\mu\nu,\rho\sigma} = (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$

and

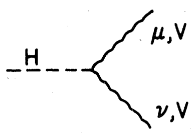
$$V^{\rho\sigma,\mu}(p)=g^{\rho\sigma}(p_2-p_1)^\mu+g^{\rho\mu}(p_1-p_3)^\sigma+g^{\sigma\mu}(p_3-p_2)^\rho .$$

Furthermore we denote by $\Pi_-=\frac{1}{2}(1-\gamma_5)$ the left-handed chiral projector and by $T_a=\frac{1}{2}\tau_a$ the SU(2) generators. q_i is the fermion charge in units of e and $a=v^{-1}$. e is given by (2.10) in our parametrization. We write δZ for $Z-1$. Our expressions below are valid up to $O(\delta^2)$ terms and hence are suitable in this form for one-loop calculations.

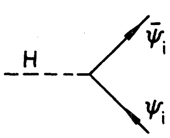
1. The trilinear vertices



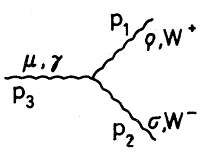
$$-3am_H^2\left(1+\frac{3}{2}\delta Z_H+\frac{\delta m_H^2}{m_H^2}+\frac{\delta a}{a}\right),$$



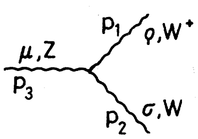
$$2aM_V^2 g^{\mu\nu}\left(1+\frac{1}{2}\delta Z_H+\delta Z_V+\frac{\delta M_V^2}{M_V^2}+\frac{\delta a}{a}\right),\quad V=Z\ ,W^\pm$$



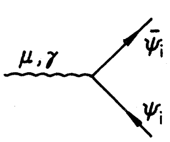
$$-am_i\left(1+\frac{1}{2}\delta Z_H+z_{ai}+\frac{\delta m_i}{m_i}+\frac{\delta a}{a}\right),$$



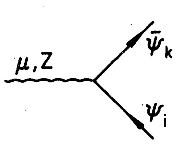
$$eV^{\rho\sigma,\mu}(p)\left(1+\frac{1}{2}\delta Z_H+\delta Z+\frac{\delta e}{e}\right),$$



$$-2a\frac{M_W^2}{M_Z}V^{\rho\sigma,\mu}(p)\left(1+\frac{1}{2}\delta Z_0+\delta Z+\frac{\delta M_W^2}{M_W^2}-\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{\delta a}{a}\right),$$

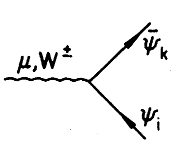


$$eq_i\gamma^\mu\left(1+\frac{1}{2}\delta Z_\gamma+\delta Z_i+\frac{\delta e}{e}\right),$$



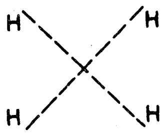
$$2aM_Z\gamma^\mu\left\{\left[1-\frac{M_W^2}{M_Z^2}\left(1+\frac{\delta M_W^2}{M_W^2}-\frac{\delta M_Z^2}{M_Z^2}\right)\right]\delta_{ik}q_k-\Pi_-T_{3ik}\right\}$$

$$\times\left(1+\frac{1}{2}\delta Z_0+\frac{1}{2}\delta Z_i+\frac{1}{2}\delta Z_k+\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{\delta a}{a}\right),$$

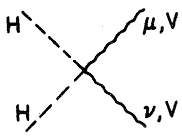


$$2aM_W\gamma^\mu\Pi_-T_{\pm ik}\left(1+\frac{1}{2}\delta Z+\frac{1}{2}\delta Z_i+\frac{1}{2}\delta Z_k+\frac{1}{2}\frac{\delta M_W^2}{M_W^2}+\frac{\delta a}{a}\right).$$

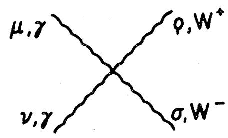
2. The quadrilinear vertices



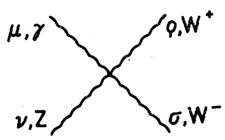
$$-3a^2 m_H^2\left(1+2\delta Z_H+\frac{\delta m_H^2}{m_H^2}+2\frac{\delta a}{a}\right),$$



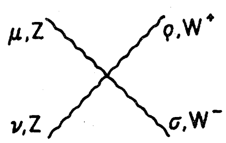
$$2a^2 M_V^2 g^{\mu\nu}\left(1+\delta Z_H+\delta Z_V+\frac{\delta M_V^2}{M_V^2}+2\frac{\delta a}{a}\right),\quad V=Z,W^{\pm}$$



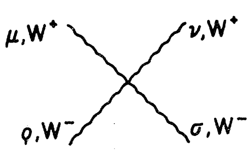
$$-e^2 T^{\mu\nu,\rho\sigma}\left(1+\delta Z_\gamma+\delta Z+2\frac{\delta e}{e}\right),$$



$$2ea\frac{M_W^2}{M_Z}T^{\mu\nu,\rho\sigma}\left(1+\frac{1}{2}\delta Z_\gamma+\frac{1}{2}\delta Z_0+\delta Z+\frac{\delta M_W^2}{M_W^2}-\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{\delta e}{e}+\frac{\delta a}{a}\right),$$



$$-4a^2\frac{M_W^4}{M_Z^2}T^{\mu\nu,\rho\sigma}\left(1+\delta Z_0+\delta Z+2\frac{\delta M_W^2}{M_W^2}-\frac{\delta M_Z^2}{M_Z^2}+2\frac{\delta a}{a}\right),$$



$$4a^2 M_W^2 T^{\mu\nu,\rho\sigma}\left(1+2\delta Z+\frac{\delta M_W^2}{M_W^2}+2\frac{\delta a}{a}\right).$$

APPENDIX E: RELATIONS BETWEEN INTEGRALS

The standard integrals A_0, B_0, and C_0 defined in Sec. IV are not independent in special kinematical situations. In particular there are relations for on-shell integrals related to the electromagnetic WT identity. Below we give a number of such relations which are needed for analytical checks of gauge invariance and other general features of on-shell amplitudes.

(i) Zero-mass integrals:

$$A_0(0)=0\ ,\quad B_0(0,0;s)=\mathrm{Re}g+2-\ln(-s+i0)\ ,\quad AB_0(0,0;s)=s^{-1}\ ,$$

$$C_0(0,0,0;s,s,m^2)=\frac{2}{m^2(1-y)^{1/2}}\left\{\mathrm{Sp}\left[\frac{(1-y)^{1/2}+1}{(1-y)^{1/2}-1}\right]-\mathrm{Sp}\left[\frac{(1-y)^{1/2}-1}{(1-y)^{1/2}+1}\right]\right\},\quad y=\frac{4s}{m^2}\ ,$$

$$B_0(0,m;s)=1-m^{-2}A_0(m)+\frac{m^2-s}{s}\ln\left(1-\frac{s}{m^2}\right),\quad AB_0(0,m;s)=-s^{-1}\left[1+\frac{m^2}{s}\ln\left(1-\frac{s}{m^2}\right)\right].$$

(ii) Two-point functions at zero momentum:

$$B_0(0,m;0)=-m^{-2}A_0(m)\ ,\quad AB_0(0,m;0)=\tfrac{1}{2}m^{-2}\ ,\quad B_0(m,m;0)=-[1+m^{-2}A_0(m)]\ ,\quad AB_0(m,m;0)=\tfrac{1}{6}m^{-2}\ ,$$

$$(m_1^2-m_2^2)B_0(m_1,m_2;0)=-[A_0(m_1)-A_0(m_2)]\ ,$$

$$2(m_1^2-m_2^2)^2 AB_0(m_1,m_2;0)=A_0(m_1)+A_0(m_2)+(m_1^2+m_2^2)[1+B_0(m_1,m_2;0)]\ .$$

(iii) Three-point functions at zero momentum transfer:

$$(m_2^2 - m_3^2)C_0(m_1,m_2,m_3;s,0,s,) = -[B_0(m_1,m_2;s) - B_0(m_1,m_3;s)]\,,$$

$$C_0(M,m,m;s,0,s,) = -\frac{\partial}{\partial m^2}B_0(M,m;s)\,,\quad C_0(0,M,M;s,0,s) = -\frac{\partial}{\partial M^2}B_0(0,M;s)\,,$$

$$AB_0(M,m;m^2) = m^{-2}[-1 - m^{-2}A_0(m) + M^{-2}A_0(M) - (M^2 - 3m^2)C_0(M,m,m;m^2,0,m^2)]\,,$$

$$AB_0(0,M;m^2) = m^{-2}[-1 + M^2C_0(0,M,M;m^2,0,m^2)]\,,$$

$$C_0(M,m,m;m^2,0,m^2) = (M^2 - 4m^2)^{-1}[B_0(M,m;m^2) - 1 - m^{-2}A_0(m) + 2M^{-2}A_0(M)]\,,$$

$$C_0(0,M,M;m^2,0,m^2) = -(M^2 - m^2)^{-1}[B_0(0,M;m^2) - 1 + M^{-2}A_0(M)]\,.$$

(iv) Infrared-singular integrals: The limits $m_\gamma^2 \to 0$ for $s = m^2$ and $\mu^2 = m^2 - s \to 0$ for $m_\gamma^2 = 0$ coincide upon identifying $\mu^2 = mm_\gamma$.

$$m_\gamma^{-2}A_0(m_\gamma) = -(\text{Reg} + 1 - \ln m_\gamma^2)\,,\quad B_0(m_\gamma,m;s) \simeq 1 - m^{-2}A_0(m)\ \ (\text{regular})\,,$$

$$AB_0(m_\gamma,m;s) \simeq m^{-2}\left(-1 + \ln\frac{m}{m_\gamma}\right),\quad C_0(m_\gamma,m,m;s,0,s) \simeq m^{-2}\ln\frac{m}{m_\gamma}\,,$$

$$C_0(m_\gamma,m,m;s,t,s) \simeq t^{-1}\left[F_1(y)\ln\frac{m_\gamma^2}{t} + F_2(y)\right],$$

$$F_1(y) = \frac{1}{(1-y)^{1/2}}\ln\frac{(1-y)^{1/2}+1}{(1-y)^{1/2}-1}\,,\quad y = 4m^2t^{-1}\,,$$

$$F_2(y) = \frac{1}{(1-y)^{1/2}}\left\{\ln\frac{(1-y)^{1/2}+1}{(1-y)^{1/2}-1}\{\ln[(1-y)^{1/2}+1] + \ln[(1-y)^{1/2}-1] - 2\ln(1-y)^{1/2}\}\right.$$
$$\left. + 2\,\mathrm{Sp}\left[\frac{(1-y)^{1/2}+1}{(1-y)^{1/2}}\right] - 2\,\mathrm{Sp}\left[\frac{(1-y)^{1/2}-1}{(1-y)^{1/2}}\right]\right\}.$$

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