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K-matrix analysis of the $J^P = 3^-$ and 2^+ dibaryon systems

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A multichannel K-matrix formalism which accounts for the effect of inelastic thresholds is used to study the $J^P = 3^-$ and 2^+ dibaryon systems. An analysis, carried out previously for the $J^P = 2^+$ system, is presented for the $J^P = 3^-$ system. The relationship between phase-shift behavior and background contributions to the scattering matrix is then probed by studying both J^P systems using explicit K-matrix poles. Possible effects of the πd channel are considered and a three-channel analysis is performed for the $J^P = 2^+$ system. The analyses are consistent with the existence of a $J^P = 3^-$ and a $J^P = 2^+$ dibaryon resonance.

I. INTRODUCTION

Structure which is suggestive of dibaryon resonances has been observed in polarized proton-proton scattering cross sections[1] and in the $pp(^1D_2)$ and $pp(^3F_3)$ partial-wave solutions of recent phase-shift analyses.[2,3] The interpretation of this structure is complicated by the proximity of a strong threshold in $pp \to N\Delta$ scattering. In a recent paper[4] a K-matrix analysis incorporating this threshold behavior[5] was applied to the $pp(^1D_2)$ and $n\Delta^{++}(^5S_2)$ channels which couple to the $J^P = (2^+)$ system. The analysis indicated the existence of a pole in the scattering matrix on an unphysical sheet whose features resembled those of the $J^P = 2^+$ 2.17-GeV dibaryon found in Hoshizaki's[2] single-channel study.

In this paper, the analysis of dibaryon systems with the K-matrix formalism is continued. In Sec. II, the $J^P = 3^-$ system is studied using recent $pp(^3F_3)$ phase-shift solutions.[2,3] In Sec. III, an alternate analysis is given for both the $J^P = 2^+$ and $J^P = 3^-$ systems which is intended to provide some insight into the behavior of the phase shift in the $n\Delta^{++}$ channel. In Sec. IV, the effect of the πd channel is discussed; in particular, an analysis of the $pp(^1D_2)$, $n\Delta^{++}(^5S_2)$, and $\pi^+ d(^3P_2)$ system is performed.

II. THE $J^P = 3^-$ SYSTEM

A. Choice of channels

The $pp(^3F_3)$ channel couples to many $J^P = 3^-$ inelastic channels of which only the $N\Delta^{++}(^5P_3)$ channel is treated here. The possible exclusive single-π channels include both the πd and $NN\pi$ channels. The πd channel is much less important than the $NN\pi$ channels[6] for the energy region of interest; its possible effects are discussed in Sec. IV. The $NN\pi$ three-body state is assumed to be dominated by the quasi-two-body $N\Delta$ state.[7] This, with isospin conservation, gives $pp \to np\pi^+$ production 5 times larger than $pp \to pp\pi^0$ production, in reasonable agreement with experiment.[6] The $(N\Delta)_{I=1}$ state is predominately $n\Delta^{++}$ and is referred to as such throughout; the analysis may, however, be regarded as a treatment of the full $(N\Delta)_{I=1}$ state.[8] The dominant partial wave of the $n\Delta^{++}$ state is the $^5P_3$ wave, the other partial waves ($^5F_3$, $^5H_3$, $^3F_3$) being negligible near threshold. Two π exclusive channels, though open for $s \gtrsim 4.6$ GeV^2, do not become significant until the (2Δ) threshold, $s \gtrsim 5.9$ GeV^2; these channels are also not included in this analysis.

This choice of channels is supported in part by measured inelastic cross sections. At $s \sim 5$ GeV^2, where the $J^P = 3^-$ resonancelike structure appears, the total inelastic pp cross section is approximately 21 mb of which about 16.1 mb is $pn\pi^+$,[9] 3.5 mb is $pp\pi^0$,[9] and 1.4 mb is $\pi^+ d$.[10] The total triplet inelastic cross section is about 1.4 mb.[11] This is consistent with a $^3F_3$ inelastic cross section of 6 mb ($\eta \sim 0.68$). If the $pp(^3F_3)$ inelastic cross sections occur in roughly the same proportions as the total inelastic cross sections, the $pp \to N\Delta \to NN\pi$ process accounts for more than 5 mb of the $pp(^3F_3)$ inelastic cross section, with $pp \to \pi d$ accounting for most of the remainder. These estimates should be compared with the results of Hoshizaki's[2] single-channel analysis which suggests that the $J^P = 3^-$ dibaryon gives an inelastic contribution of 3 mb. While threshold effects in $pp \to N\Delta$ or $pp \to \pi d$ might produce such a large resonancelike contribution, it is unlikely that any further channel could.

B The K-matrix formalism

The notation and conventions used here are those of Ref. 4. The unitary $\mathcal{S}$ matrix with proper threshold behavior in all channels is

$$\mathcal{S}(s) = 1 + 2i\mathcal{T}(s) = 1 + 2i\rho^{1/2}(s)T(s)\rho^{1/2}(s) , \quad (2.1)$$

where

$$T(s) = K(s)[1 - C(s)K(s)]^{-1} . \quad (2.2)$$

The K-matrix elements are meromorphic functions of s and the Chew-Mandelstam functions $C_i(s)$ [with $\rho_i(s) = \mathrm{Im}C_i(s)$] describe the kinematics in channel i.

In the present problem, channel 1 is $pp(^3F_3)$ and its Chew-Mandelstam function describes two stable particles of equal mass in an $L = 3$ wave. Channel 2 is $n\Delta^{++}(^5P_3)$ and its chew-Mandelstam function is that appropriate to two particles of unequal mass, one of which is unstable, in an $L = 1$ wave. These functions are given in the Appendix .

The K-matrix elements are taken to be simple polynomials in s:

$$K_{ij}(s) = a_{ij} + b_{ij}s + c_{ij}s^2. \tag{2.3}$$

The effect of using poles in the K matrix is discussed in the next section.

At present $\mathcal{S}_{11}$ is known approximately for $s < 5.4$ GeV^2 through the phase and inelasticity parameters of the pp phase-shift analyses,[2,3]

$$\mathcal{S}_{11} = \eta e^{2i\delta_1} \ . \tag{2.4}$$

No constraints exist on the $n\Delta^{++}$ phase δ_2. A variety of K matrices are expected to reproduce the given δ_1 and η values.

The $\mathcal{S}$ matrix is, by construction, analytic with a sheet structure inherited from the known Chew-Mandelstam functions. Once the K matrix has been chosen, it is straightforward to examine $\mathcal{S}$ for poles at complex s. A pole occurring on an appropriate sheet is interpreted as a resonance.[4] Note that with a polynomial parametrization of the K matrix, the $\mathcal{S}$ matrix need not necessarily exhibit poles near the physical region—i.e., a dibaryon resonance is not assumed *a priori* to exist.

C. Results

Four solutions are presented here which are typical of many more which have been found. Solutions 1 and 2 are illustrated in Fig. 1, solutions 3 and 4 in Fig. 2. The K-matrix parameters of these solutions are given in Table I.

Solutions 1–3 reproduce the $pp(^3F_3)$ phases of Arndt's analyses[3] reasonably well, while solution 4 has $pp(^3F_3)$ phases more closely resembling those of Hoshizaki's analysis.[2] Solutions 1 and 3 have an inelasticity which varies quite slowly, reaching a minimum at $\eta \sim 0.68$. Solutions 2 and 4 show sharper η behavior with minima at $\eta = 0.66$ and $\eta = 0.55$, respectively. A variety of $n^{++}(^3P_2)$ phase behaviors are seen, from the undramatic ones of solutions 1 and 4 to the very dramatic one

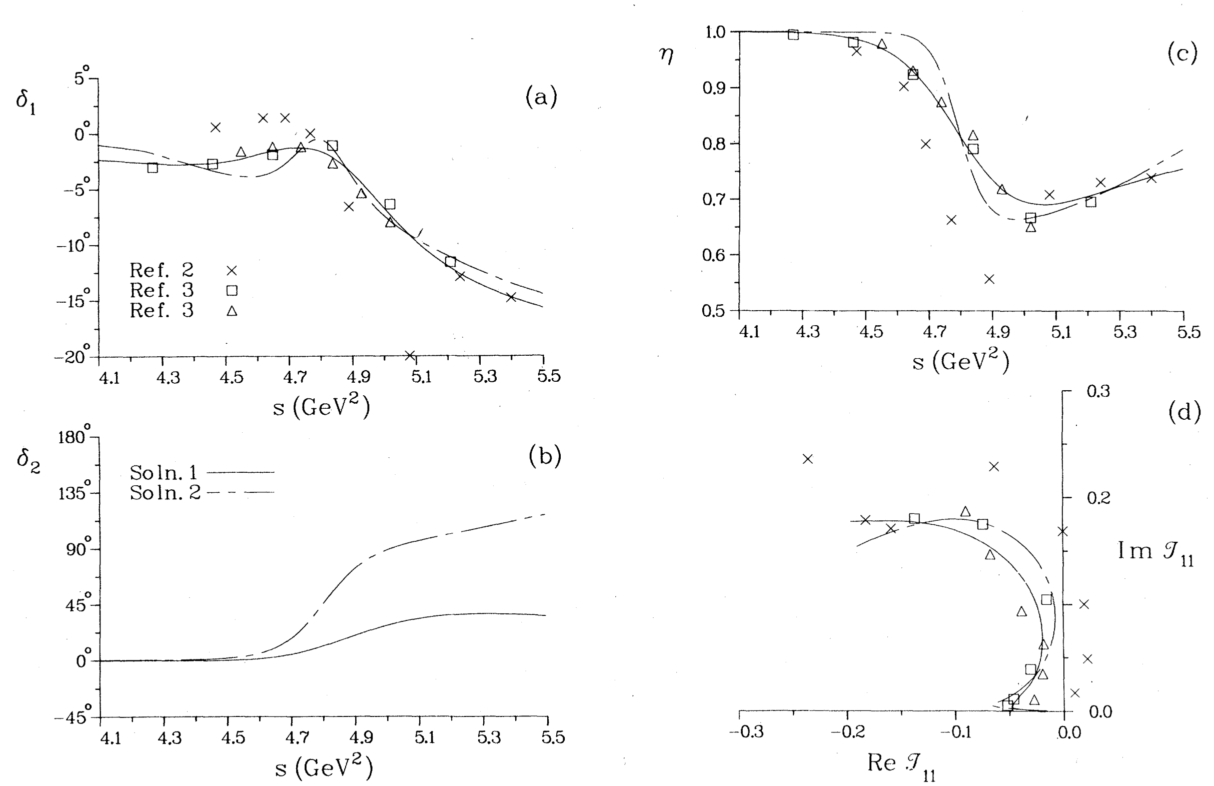

FIG. 1. Solutions 1 and 2 for the $J^P = 3^-$ system. The phase-shift-analysis results are those of Hoshizaki (Ref. 2) (crosses) and Arndt (Ref. 3) (energy independent—triangles; energy dependent—squares).

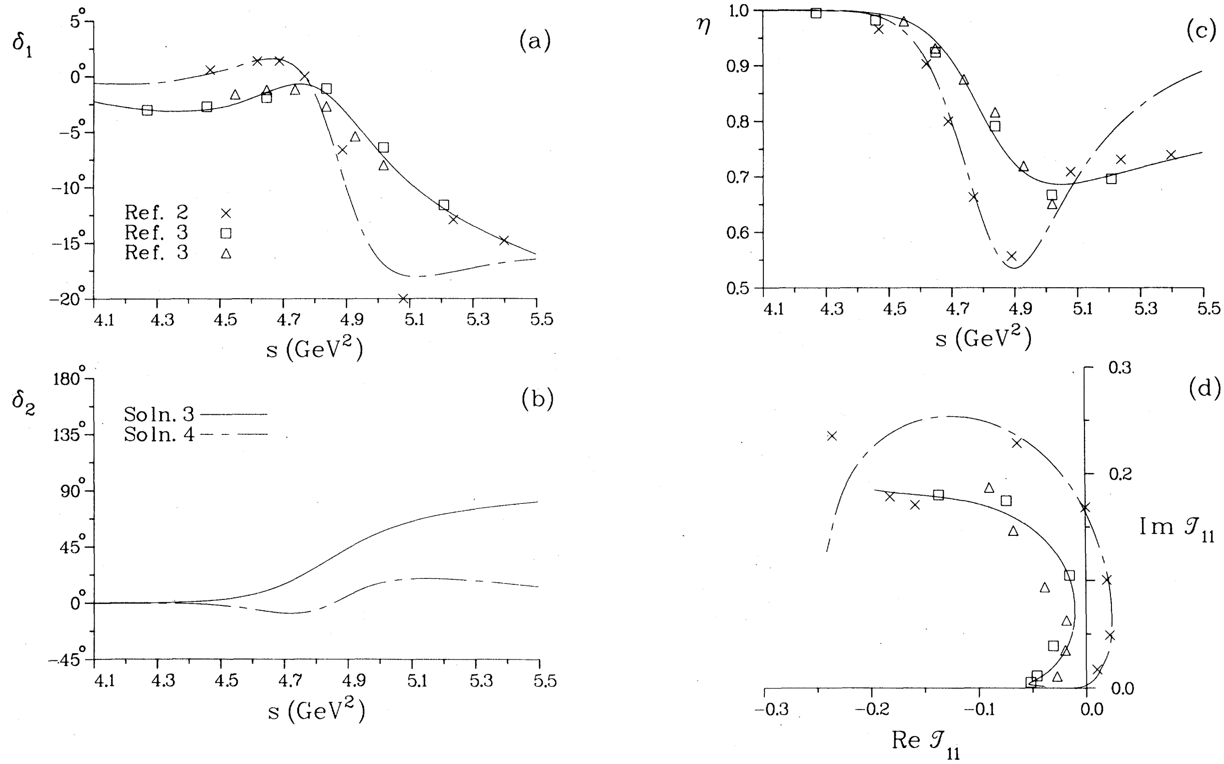

FIG. 2. Solutions 3 and 4 for the $J^P=3^-$ system. The phase-shift-analysis results are those of Hoshizaki (Ref. 2) (crosses) and Arndt (Ref. 3) (energy independent—triangles; energy dependent—squares).

of solution 2.

All solutions have a zero of $\det(1-CK)$, i.e., a pole of $\mathcal{S}$, on the second sheet of the pp and $np\pi^+$ cuts—see Fig. 3. This pole is interpreted as a resonance at an energy similar to but slightly lower than that found in Hoshizaki's single-channel analysis. Based on all the solutions obtained, the resonance parameters lie in the following ranges:

$$\begin{aligned} &M_R \sim 2.18\text{–}2.20 \text{ GeV},\\ &\Gamma_R \sim 50\text{–}160 \text{ MeV},\\ &\Gamma_1 \sim 5\text{–}95 \text{ MeV},\\ &\Gamma_2 \sim 40\text{–}95 \text{ MeV}. \end{aligned} \tag{2.5}$$

All of the solutions have another $\mathcal{S}$-matrix pole which is farther from the physical region. This pole is on the second sheet of all the cuts and is found by circling the $n\Delta^{++}$ branch point—see Fig. 3. Both poles have reflections on more distant sheets, as explained in Ref. 4.

TABLE I. Parameters of the $J^P=3^-$ system solutions.

	Sol. 1	Sol. 2	Sol. 3	Sol. 4
a_{11}	2.6393	650.4707	46.2203	59.0794
b_{11} (GeV^{-2})	1.2496	−276.0000	−16.0550	−22.3113
c_{11} (GeV^{-4})	−0.0399	30.0000	1.7709	2.3692
a_{12}	5.6014	191.8662	−0.9172	46.9766
b_{12} (GeV^{-2})	−2.8843	−87.8094	−0.6340	−19.9774
c_{12} (GeV^{-4})	0.3927	10.0700	0.2208	2.1437
a_{22}	40.3312	148.7824	51.7900	27.0349
b_{22} (GeV^{-2})	−9.5747	−58.4767	−14.8676	−3.4855
c_{22} (GeV^{-4})	0.6405	6.1373	1.2500	−0.0622
M_R (GeV)	2.19	2.19	2.18	2.20
Γ_R (MeV)	158	62	122	131
Γ_1 (MeV)	63	7	25	88
Γ_2 (MeV)	95	55	97	43

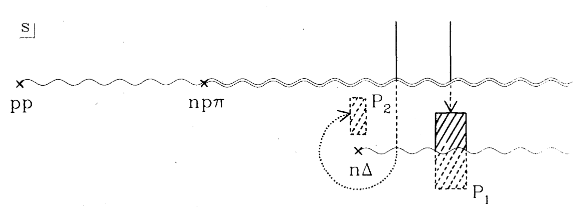

FIG. 3. The complex s plane indicating the pp, $np\pi$, and $n\Delta$ unitarity cuts. The approximate positions of the $\mathcal{S}$ matrix poles are indicated (P_1 and P_2).

III. PHASE BEHAVIOR IN THE SECOND CHANNEL

A. Eigenphase behavior

All of the solutions found for the $J^P = 2^+$ and 3^- systems display an $\mathcal{S}$-matrix pole corresponding to a dibaryon resonance. However, not all of these solutions demonstrate dramatic phase-shift behavior. Because phase behavior may be used in conjunction with a generalized Levinson's theorem[4,12] to characterize solutions as elementary or dynamical resonances, it is important to understand the cause of the instances of untraditional resonance phase behavior.[13]

The distorting effects of a large inelasticity may be removed by examining the eigenphases of $\mathcal{T}$:

$$\phi_1 = \frac{1}{2}\left\{\delta_1 + \delta_2 + \arctan\frac{[1-\eta^2\cos^2(\delta_1-\delta_2)]^{1/2}}{\eta\cos(\delta_1-\delta_1)}\right\},$$
$$\phi_2 = \frac{1}{2}\left\{\delta_1 + \delta_2 - \arctan\frac{[1-\eta^2\cos^2(\delta_1-\delta_2)]^{1/2}}{\eta\cos(\delta_1-\delta_2)}\right\}. \quad (3.1)$$

The phase shifts and eigenphases for two $J^P = 2^+$ solutions are shown in Fig. 4 and for two $J^P = 3^-$ solutions in Fig. 5. In all instances, the eigenphase behavior proves only as dramatic as that of the corresponding phase shifts.

Possible effects of threshold kinematics may be removed by examining the eigenphases of T.[14] For the $J^P = 2^+$ and $J^P = 3^-$ dibaryon systems, these eigenphases differ from those of $\mathcal{T}$ by less than $10°$ with the largest difference occurring near the resonance position.

The remaining standard explanation[13] for the undramatic phase behavior is the presence in these systems of a large nonresonant inelastic background. In fact, Hoshizaki[2] had to assume such a background in his single-channel analysis.

B. *K*-matrix poles

One way to separate background and resonance contributions is to use explicit poles in the K matrix.[5] The analyses of the $J^P = 2^+$ and 3^- systems performed earlier are now repeated, allowing for K-matrix elements with a smooth background and an explicit pole.

The K-matrix elements are taken to be

$$K_{ij}(s) = a_{ij} + b_{ij}s + c_{ij}s^2 + \frac{g_i g_i}{s_R - s}. \quad (3.2)$$

Note that since K_{ij} is assumed to be real, s_R is real. The corresponding $\mathcal{S}$-matrix pole will occur at a complex value of s whose real part is approximately s_R. Since a K-matrix separation of background and resonance does not correspond directly to a $\mathcal{T}$-matrix separation, the "coupling

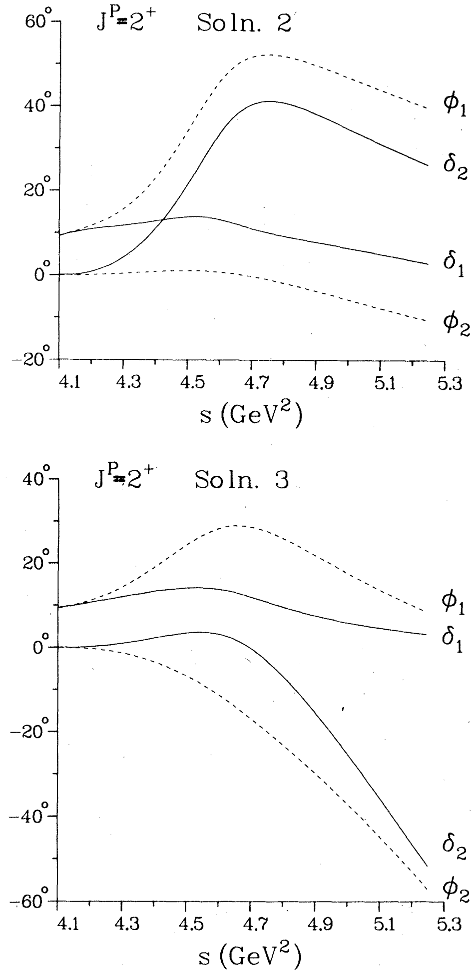

FIG. 4. The phase shifts (δ) and eigenphases (ϕ) for two $J^P = 2^+$ solutions (Ref. 4).

strengths" g_i of (3.2) are not immediately connected to the resonance partial widths. In fact, the ratio g_1/g_2 is easily extracted from elastic channel phase-shift analyses.[15]

Earlier treatment[4] of the $pp(^1D_2)$ and $n\Delta^{++}(^5S_2)$ system indicated an $\mathcal{S}$-matrix pole at $s \sim (4.55 \to 4.70) - i(0.23 \to 0.45)$ GeV^2 on the second sheet of the pp and $np\pi^+$ cuts and on the first sheet of the $n\Delta^{++}$ cut. Several K-matrix-pole solutions have been found for $s_R \sim 4.625$ GeV^2 ($g_1/g_2 \sim 1.7$), each differing in its choice of background and g_1. In all instances, a rather large background K matrix is necessary. The behavior of δ_2 resembles solutions 2 and 4 of Ref. 4—rising to about $30°$ near $s = 4.7$ GeV^2 and leveling off or falling slowly thereafter. The pole in the $\mathcal{S}$ matrix is located near $s \sim 4.6 - i\,0.2$ GeV^2, not unlike in the analysis with no explicit K-matrix poles. Furthermore, the partial widths are consistent

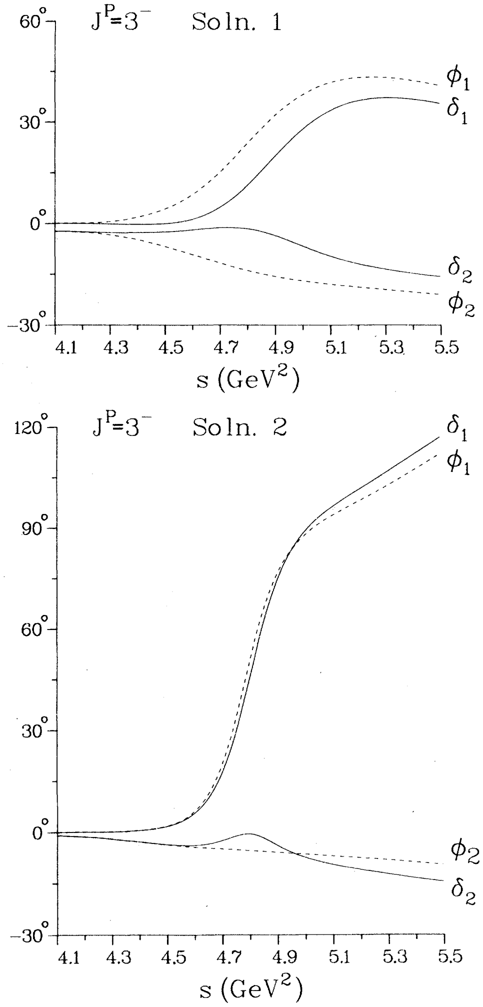

FIG. 5. The phase shifts (δ) and eigenphases (ϕ) for two $J^P=3^-$ solutions.

with those obtained earlier.

In Sec. II, the $pp(^3F_3)$ and $n\Delta^{++}(^5P_2)$ analysis demonstrated an $\mathcal{S}$-matrix pole at $s\sim(4.7\rightarrow 4.9)-i\,(0.13\rightarrow 0.35)$ GeV^2. Solutions with K-matrix poles have been found using $s_R\sim 4.725$ GeV^2 ($g_1/g_2\sim 8.3$) and a large background. These solutions resemble solution 2 presented in Sec. II—δ_2 rises quickly through 90° leveling off about 150°. As before, two $\mathcal{S}$-matrix poles appear. One is on the second sheet of the pp and $np\pi^+$ cuts and on the first sheet of the $n\Delta^{++}$ cut near $s\sim 4.8-i\,0.1$ GeV^2, quite close to the physical region. The second is on the second sheet of all the cuts, not far from the $n\Delta^{++}$ branch point, $s\sim 4.6-i\,0.1$ GeV^2.

An explicit pole in the K matrix is quite analogous to a Castillejo-Dalitz-Dyson pole in the D matrix of an N/D calculation. The *ad hoc* nature of both these structures leads naturally to their interpretation as elementary. The positive, though not necessarily dramatic, phase behavior found in the K-matrix-pole solutions for the $J^P=2^+$ and 3^- systems adds credence to the Levinson's-theorem mnemonics of Ref. 4—solutions with $\delta_1+\delta_2$ increasing have elementary dibaryons while solutions with $\delta_1+\delta_2$ level or decreasing have dynamical dibaryons.

IV. EFFECTS OF THE πd CHANNEL

A. Cross sections and phase-shift analyses for $pp\rightarrow\pi d$

The total $pp\rightarrow\pi d$ cross section is well measured[10] over the energy region of concern in this analysis—it rises steadily from threshold ($s\sim 4.1$ GeV^2) to about 3.2 mb at $s\sim 4.6$ GeV^2 and is down to 2.5 mb at $s\sim 4.8$ GeV^2. A sizable fraction of this cross section is expected to arise from the coupling of πd to the $pp(^1D_2)$ and $pp(^3F_3)$ channels and it is important to understand the effects of this coupling on the results and interpretation of the resonance analysis.

Several analyzing-power coefficients for $pp\rightarrow\pi d$ have been measured[16] for $4.1<s<5.0$ GeV^2; these indicate that more than one pp partial wave contributes to the $pp\rightarrow\pi d$ scattering. These analyzing-power coefficients may be used to place constraints on the partial-wave amplitudes[17] a_i. It is clear that $a_2(pp(^1D_2)\rightarrow\pi d(^3P_2))$ and $a_1(pp(^3P_1)\rightarrow\pi d(^3S_1))$ are the most important amplitudes but that $a_4(pp(^3P_2)\rightarrow\pi d(^3D_2))$, $a_6(pp(^3F_3)\rightarrow\pi d(^3D_3))$ (Ref. 6), and $a_5(pp(^3F_2)\rightarrow\pi d(^3D_2))$ (Ref. 18) cannot be neglected. The polarization information is consistent with

$$\sigma(pp(^1D_2)\rightarrow\pi d(^3P_2))\sim\tfrac{2}{3}\sigma^{\rm tot}(pp\rightarrow\pi d) \tag{4.1}$$

for $s\lesssim 5$ GeV^2.

A phase-shift analysis has recently been performed[19] for several $pp\rightarrow\pi d$ partial waves for $s\lesssim 4.6$ GeV^2. This analysis supports the suggestion that $pp(^1D_2)\rightarrow\pi d(^3P_2)$ is the dominant wave and is reasonably consistent with (4.1). Results for $pp(^3F_3)\rightarrow\pi d(^3D_3)$ vary widely giving $\sigma(pp(^3F_3)\rightarrow\pi d(^3D_3))$ as low as 0.2 mb or as high as 1.5 mb at $s=4.6$ GeV^2.

B. Three-channel calculation for the $J^P=2^+$ system

The inelastic cross section for $pp(^1D_2)$ is about 5 mb at the energy where the two-channel analysis indicates an $\mathcal{S}$-matrix pole. Of this, by (4.1), 2 mb is expected to arise from $pp(^1D_2)\rightarrow\pi d(^3P_2)$. The possibility exists that the $\mathcal{S}$-matrix pole found in the two-channel analyses is a manifestation of forcing all the inelasticity into a single inelastic channel, the $n\Delta^{++}(^5S_2)$, and that this pole would disappear if the $\pi d(^3P_2)$ channel were considered as well.[20] In this section, this possibility is investigated by carrying out the three-channel analysis.

A three-channel symmetric unitary $\mathcal{S}$ matrix may be parametrized by three phase shifts and three inelasticity parameters:

$$\mathcal{S}_{ii} = \eta_i e^{2i\delta_i} ,$$
$$\mathcal{S}_{ij} = i\left(\frac{1+\eta_k{}^2 - \eta_i{}^2 - \eta_j{}^2}{2}\right)^{1/2} e^{i(\delta_i + \delta_j + \phi_{ij})} \quad (i \neq j) , \tag{4.2}$$

where the extra phases ϕ_{ij} in the off-diagonal terms are known functions of the three η's. Equivalently, $\mathcal{S}$ may be parametrized by a 3×3 real K matrix according to (2.1) and (2.2). The Chew-Mandelstam functions for the $pp(^1D_2)$ and $n\Delta^{++}(^5S_2)$ channels are taken as before.[4] The $\pi d(^3P_2)$ channel consists of two stable particles of unequal masses in a relative $L=1$ state; this Chew-Mandelstam function is given in the Appendix. The discussion of Sec. III indicated that a polynomial parametrization of the K-matrix elements is not only simpler than including explicit poles in the K matrix, but it perhaps also admits a wider variety of solutions. Here, the six K-matrix elements are taken to be quadratic functions of s [compare (2.3)].

The pp phase-shift analyses[2,3] provide δ_1 and η_1. The estimate (4.1) of $\sigma(pp(^1D_2) \to \pi d(^3P_2))$ determines $|\mathcal{S}_{13}|$ and hence $(1+\eta_2{}^2 - \eta_1{}^2 - \eta_3{}^2)^{1/2}$. For s values near threshold, this estimate suggests that $pp(^1D_2) \to \pi d(^3P_2)$ saturates the $pp(^1D_2)$ inelasticity and so, near threshold, $\eta_1 \sim \eta_3$ and $\eta_1 < \eta_2 \sim 1$.

The phases and inelasticities of one three-channel solution are presented in Fig. 6. Notice that the pp inelasticity drops from 1.0 at 4.1 GeV^2, through 0.99 at 4.2 GeV^2 and 0.93 at 4.4 GeV^2 to 0.77 at 4.6 GeV^2, in agreement with the phase-shift analyses.[2,3] This is a marked improvement over the two-channel solutions of Ref. 4 for which η remained almost 1.0 until 4.4 GeV^2. This solution has a pole at $s \sim 4.63 - i\,0.24$ GeV^2 on the second sheet of the pp, $np\pi^+$, and πd cuts and on the first sheet of the $n\Delta^{++}$ cut. The resonance parameters are

$$\begin{aligned} &M_R \sim 2.15 \text{ GeV},\\ &\Gamma_{\text{tot}} \sim 112 \text{ MeV},\\ &\Gamma_1 \sim 33 \text{ MeV},\\ &\Gamma_2 \sim 46 \text{ MeV},\\ &\Gamma_3 \sim 33 \text{ MeV}. \end{aligned} \tag{4.3}$$

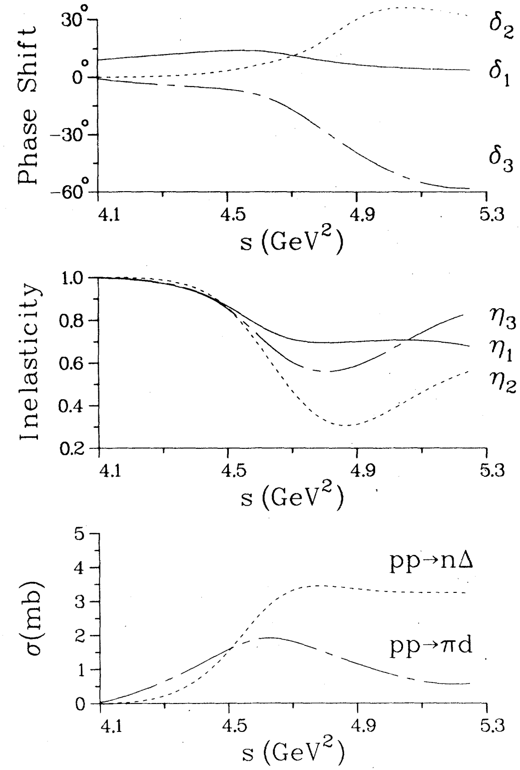

FIG. 6. A three-channel solution for the $J^P = 2^+$ system. The channels include $pp(^1D_2)$, $n\Delta^{++}(^5S_2)$, and $\pi d(^3P_2)$.

All other three-channel solutions demonstrate the same improved η_1 behavior and a pole in approximately the same position. In all instances, a δ_2 behavior similar to that obtained in a two-channel solution is found. A variety of δ_3 behaviors also emerges, none of which are particularly dramatic.

Inclusion of the $\pi d(^3P_2)$ channel yields solutions which better reproduce the phase-shift parameters for $pp(^1D_2)$ elastic scattering, but do not appear to affect the arguments concerning the existence of a $J^P = 2^+$ dibaryon resonance.

C. The $J^P = 3^-$ system

The $pp(^3F_3)$ inelastic cross section is 5–9 mb at $s \sim 4.8$ GeV^2, where the $\mathcal{S}$-matrix pole is observed. By (4.1), the $pp(^3F_3) \to \pi d(^3D_3)$ cross section is expected to be less than 1 mb. The results of the $J^P = 2^+$ system three-channel calculation, where the πd channel is relatively more important, suggest that the pole structure of the $J^P = 3^-$ system scattering matrix should be unaltered by the consideration of the $\pi d(^3D_3)$ channel.

Inclusion of this channel would, however, affect small details in the behavior of the phase-shift parameters. The solutions of Sec. II seem to complement those of the $pp \to \pi d$ phase-shift analysis[19] well—solutions 1, 3, and 4 are consistent with a small $pp(^3F_3) \to \pi d(^3D_3)$ cross section, while solution 2 clearly could be improved in a three-channel analysis by allowing a substantial $pp(^3F_3) \to \pi d(^3D_3)$ cross section. Until the $pp(^3F_3) \to \pi d(^3D_3)$ amplitude is better understood, a three-channel analysis of the $J^P = 3^-$ system would be premature.

V. SUMMARY OF RESULTS

The $J^P = 2^+$ dibaryon system has been studied with a two-channel and three-channel K-matrix approach. In both instances, the $\mathcal{S}$ matrix reconstructed from $pp(^1D_2)$ phase-shift information demonstrated a pole indicative of a dibaryon resonance. The calculation using K-matrix poles suggested that the undramatic phase behavior found for this system was a result of a large background. The inclusion of the πd channel assured that the dibaryon was not an artifact of forcing all of the pp inelasticity into the $n\Delta^{++}$ channel—the $J^P = 2^+$ dibaryon is a feature of each of the pp, $n\Delta^{++}$, and πd systems.

The $J^P = 3^-$ system has been examined using a two-channel K-matrix approach. A dibaryon resonance, similar in features to that found in Hoshizaki's single-channel analysis,[2] was found in analyses with and without K-matrix poles. Some solutions displayed standard resonancelike variation of the phase shift in the $n\Delta^{++}$ channel.

The second pole found in all of the $J^P = 3^-$ solutions, though rather far from the physical region, may prove an interesting subject of speculation. Its position near the $n\Delta$ branch point suggests that it may be a recursion of the $J^P = 2^+$ pole discussed above and, if so, these two poles are likely of dynamical origin.[21]

As emphasized before,[4,12] the questions of interpretation are not clear cut. The distinctions between resonances and virtual bound states and between dynamical and elementary origin may be impossible to make. The analysis just presented has, however, been able to address carefully the existence of $\mathcal{S}$-matrix poles—both the $J^P = 2^+$ and $J^P = 3^-$ systems do admit $\mathcal{S}$-matrix poles which correspond in a natural way to dibaryons.

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APPENDIX I: CHEW-MANDELSTAM FUNCTIONS

The Chew-Mandelstam function for two stable particles of mass m in a relative $L = 3$ state is obtained from

$$\rho(s) = \left(\frac{s - 4m^2}{s}\right)^{7/2} \tag{A1}$$

by the once-subtracted dispersion relation

$$C(s) = \frac{(s - s_0)}{\pi} \int_{\text{threshold}}^{\infty} ds' \, \frac{\rho(s')}{(s' - s_0)(s' - s)} \,. \tag{A2}$$

Performing the subtraction at $s = 0$,

$$C(s) = -\frac{2}{\pi}\left[\rho(s)\ln\frac{(4m^2 - s)^{1/2} + (-s)^{1/2}}{2m} + \left(\frac{4m^2 - s}{s}\right)^3 - \frac{1}{3}\left(\frac{4m^2 - s}{s}\right)^2 + \frac{1}{5}\left(\frac{4m^2 - s}{s}\right) - \frac{1}{7}\right] , \tag{A3}$$

C is real for $s < 4m^2$ and has the required discontinuity of $2i\rho$ for $s > 4m^2$.

The Chew-Mandelstam function for two stable particles of masses m and M in a relative $L = 1$ state is obtained from

$$\rho(s, M, m) = \frac{\{[s - (M+m)^2][s - (M-m)^2]\}^{3/2}}{s^3} , \tag{A4}$$

$$C(s, M, m) = -\frac{2}{\pi}\left\{\rho(s, M, m)\ln\frac{[(M+m)^2 - s]^{1/2} + [(M-m)^2 - s]^{1/2}}{(2Mm)^{1/2}} + \tfrac{1}{2}\ln\frac{M}{m}\left[\frac{(M^2 - m^2)^3}{s^3} - \frac{3(M^2 - m^2)(M^2 + m^2)}{s^2} + \frac{3(M^4 + m^4)}{s(M^2 - m^2)} - \frac{M^6 - 3M^4m^2 - 3M^2m^4 + m^6}{(M^2 - m^2)^3}\right] - \frac{(M^2 - m^2)^2}{2s^2} + \frac{5(M^2 + m^2)}{2s} - \frac{11M^4 - 10M^2m^2 + 11m^4}{12(M^2 - m^2)^2}\right\} . \tag{A5}$$

If the particle of mass M is unstable, this function must be smeared[5] over a range of M values,

$$C(s, M^*, m)=\frac{1}{\pi}\int_{\text{threshold}} ds' W(s')C(s, \sqrt{s'}, m)\ . \tag{A6}$$

The weighting function W for Δ^{++} decay into $p\pi^+$ is described in detail in Ref. 4. The same smearing is performed here on the $n\Delta^{++}({}^5P_3)$ Chew-Mandelstam function as was performed there on the $n\Delta^{++}({}^5S_2)$ Chew-Mandelstam function.

The extension of the Chew-Mandelstam functions to complex s is analogous to the extensions performed in Ref. 4. The $L=3$ equivalent of Ref. 4 (A9) is

$$\int_{(m_p+m_\pi)^2}^{(\sqrt{s}-m_n)^2} ds' \frac{1}{s'-m_\Delta^{*2}}\rho^{L=3}(s, \sqrt{s'}, m_n)$$
$$=\frac{1}{s^3}\left\{-\frac{1}{3}(s_++s_0)^{3/2}(s_-+s_0)^{3/2}-\frac{(2s_0+s_++s_-)(s_++s_-)+8s_+s_-}{8}(s_++s_0)^{1/2}(s_-s_0)^{1/2}\right.$$
$$+\frac{(s_++s_-)(s_+{}^2-10s_+s_-+s_-{}^2)}{8}\ln\frac{(s_++s_0)^{1/2}+(s_-+s_0)^{1/2}}{(s_+-s_-)^{1/2}}$$
$$\left.+2s_+{}^{3/2}s_-{}^{3/2}\left[\ln\frac{[s_+(s_-+s_0)]^{1/2}+[s_-(s_++s_0)]^{1/2}}{[s_0(s_+-s_-)]^{1/2}}-i\epsilon\frac{\pi}{2}\right]\right\},$$

where

$$s_+=(\sqrt{s}+m_n)^2-m_\Delta^{*2},\quad s_-=(\sqrt{s}-m_n)^2-m_\Delta^{*2},\quad s_0=m_\Delta^{*2}-(m_p+m_\pi)^2\ , \tag{A7}$$

and

$\epsilon=1$ for $\mathrm{Im}(s_-)>0$, $\epsilon=-1$ for $\mathrm{Im}(s_-)<0$.

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[8]The full $(N\Delta)_{I=1}$ problem uses three independent $NN\pi$ states: $p(p\pi^0)_{I=3/2}$, $p(n\pi^+)_{I=3/2}$ $n(p\pi^+)_{I=3/2}$. By isospin invariance, the K-matrix elements for transitions involving these states are related by Clebsch-Gordon coefficients and, since the masses are almost the same, the Chew-Mandelstam functions for these states are essentially identical. A four-channel treatment of pp and these three $NN\pi$ states then reduces to the two-channel pp and $N\Delta$ problem—the T, K, and $1-KC$ matrices each block diagonalize, displaying 2×2 matrices for the $pp\to N\Delta$ problem and 2×2 matrices for channels with no scattering. The pp and $(N\Delta)_{I=1}$ problem is seen to be formally identical to the pp and $n\Delta^{++}$ problem, the only difference being a constant multiplication factor in the K-matrix elements.

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[12]As stressed in Ref. 4, the questions of interpretation of a particlelike structure are extremely hazy. In particular, the distinction between dynamical and elementary may be artificial since what appears elementary in a few-channel context may be dynamical under the view of more channels.

[13]An excellent review of the interconnections of phase shifts, eigenphases, inelasticity, background effects, and K matrices, with and without poles, is given in R. Levi Setti and T. Lasinski, *Strongly Interacting Particles* (University of Chicago Press, Chicago, 1973).

[14]The possibility that threshold effects might be distorting the phase behavior in the second channel and that, if so, the eigenphases of T (rather than $\mathcal{T}$) would be useful was suggested by D. V. Bugg (private communication).

[15]Elastic-channel scattering may be parametrized by a complex effective K matrix (Ref. 13): $\mathcal{S}_{11}=\eta e^{2i\delta_1}=(1-K^{\text{eff}}\bar{C}_1)/(1-K^{\text{eff}}C_1)$. The $pp({}^1D_2)$ and $pp({}^3F_3)$ effective K matrices reconstructed from phase-shift solutions[2,3] are smooth and structureless near s_R. Since $K^{\text{eff}}=K_{11}+K_{12}{}^2C_2/(1-K_{22}C_2)$, the pole parts of K_{ij} must cancel giving $(g_1/g_2)^2=|C_2|^2\mathrm{Im}K^{\text{eff}}/\rho_2|_{s=s_R}$.

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Phys. Lett. 81B, 156 (1979) and references therein.

[17]For notation, see G. Jones in *Nucleon-Nucleon Interactions—1977,* proceedings of the Second International Conference, Vancouver, edited by H. Fearing, D. Measday, and A. Strathdee (AIP, New York, 1978), p. 292.

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