

Quark-gluon separation in three-jet events

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We discuss the necessity of a separation of quarks and gluons in $q\bar{q}g$ three-jet events in e^+e^- annihilation in order to obtain quantitative tests of quantum chromodynamics. The possibility of such a separation is investigated with special emphasis on the case that one or two of the jets are identified as quark jets by a semileptonic decay of a c or b quark. It is shown in detail how gluon energy spectra and gluon-jet properties can be deduced from these measurements.

I. INTRODUCTION

Three-jet events¹ in e^+e^- annihilation have proven to be useful as a (qualitative) test of perturbative quantum chromodynamics (QCD),² although up to now they have not shed light on the properties of the gluon and the jets originated by it.^{3,4} A unique reaction to study these properties would be the decay of a heavy quarkonium state⁵ ($M > 30$ GeV) into three gluons, but such a state (if existent at all) seems to lie outside the presently available energy range. With the increasing statistics in $e^+e^- \rightarrow q\bar{q}g$ three-jet events one could try to extract the properties of the gluon from this reaction. It is the purpose of this paper to investigate the associated questions: (i) What is the energy spectrum of the gluon? (ii) How can one learn about properties of gluon jets such as multiplicity and transverse-momentum distributions? We will show that in the reaction $e^+e^- \rightarrow c\bar{c}g$ (c denotes the charmed quark) in which at least one of the jets is identified through a semileptonic decay to be a quark (or antiquark) jet these questions could be answered. The paper is organized as follows. In Sec. II we discuss the kinematics and first-order QCD distributions for three-jet events with regard to special properties of the gluon spectra. Section III exploits the information one can obtain in the case that the charmed quark decays semileptonically and in Sec. IV we discuss some of the questions that may be encountered in measuring the proposed quantities. Section V summarizes our results.

II. QUARK AND GLUON SPECTRA IN THE REACTION $e^+e^- \rightarrow q\bar{q}g$

We consider three-jet events where the quark (q), antiquark (\bar{q}), and gluon (g) jets have energy fractions x_1 , x_2 , and x_3 ($x_i = 2E_i/\sqrt{s}$, $0 \leq x_i \leq 1$, $x_1 + x_2 + x_3 = 2$) and where the energy fraction of the most energetic jet is less than $1 - \epsilon$, i.e., $\max(x_1, x_2, x_3) \leq 1 - \epsilon$.⁶ A first-order QCD calcula-

tion⁷ leads to the following spectra ($m_q = 0$):

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_2 d \cos \theta_1} = \frac{\alpha_s}{4\pi} \left[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} (1 + \cos^2 \theta_1) + \frac{2(x_1 + x_2 - 1)}{x_2^2} (1 - 3 \cos^2 \theta_1) \right], \quad (1)$$

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_2 d \cos \theta_2} = (1 - 2), \quad (2)$$

$$\frac{1}{\sigma_0} \frac{d\sigma}{dx_1 dx_3 d \cos \theta_3} = \frac{\alpha_s}{4\pi} \left[\frac{x_1^2 + (2 - x_1 - x_2)^2}{(1-x_1)(x_1 + x_3 - 1)} (1 + \cos^2 \theta_3) + \frac{4(1-x_2)}{x_3^2} (1 - 3 \cos^2 \theta_3) \right], \quad (3)$$

where σ_0 is the lowest-order $e^+e^- \rightarrow q\bar{q}$ cross section and θ_i is the angle between the jet of energy x_i and the e^- beam direction. The spectra in (1)–(3) are not separately measurable as long as q , \bar{q} , and g jets are not identified. Let us first see what could be done in the absence of such an identification. We order the three jets according to their energy $T = x_i \geq U = x_j \geq V = x_k$ so that $T = \max(x_1, x_2, x_3)$ and $V = \min(x_1, x_2, x_3)$, obeying the kinematic constraints $\frac{2}{3} \leq T \leq 1 - \epsilon$, $(1 + \epsilon)/2 \leq U \leq 1 - \epsilon$, $2\epsilon \leq V \leq \frac{2}{3}$. The T , U , and V spectra are given in Fig. 1(a). These distributions (perhaps with suitable fragmentation corrections^{3,8}) could be measured but are unfortunately not very specific predictions of QCD, reflecting mainly three-body phase space restrictions.⁴ In order to illustrate this fact we computed the same quantities in a model with scalar gluons. The T , U , and V distributions are similar in both cases [compare Fig. 2(a)] whereas the quark and gluon energy spectra show a large difference [Fig. 2(b)]. Probabilities for quarks and gluons to be the T , U , or V jets are given in Table I and the corresponding distributions are displayed in Figs. 1(b) and 1(c). First-order QCD predicts that the gluon jets are predominantly soft, giving 65% of the jets in the jet sample of lowest energy (V jet). This enhance-

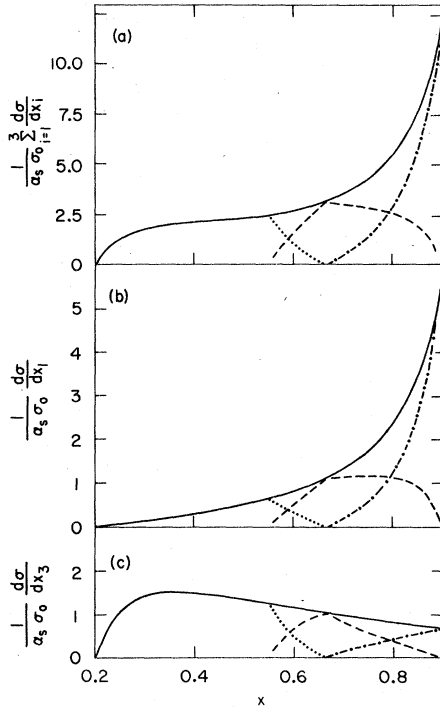


FIG. 1. Jet energy spectra with a cut $T \leq 0.9$. The dotted (dashed, dashed-dotted) lines show the corresponding V (U , T) spectra.

ment of the number of gluon jets could be used to extract gluon-jet properties from those of the V jet,^{9,10} but it is necessary to show experimentally whether the enhancement is true. We therefore propose in the next section a possibility to prove that. It will in addition enable us to select kinematic regions where the probability of a jet being a gluon jet is even more strongly enhanced.

III. LEPTON AS QUARK-JET IDENTIFICATION

The production of the leptons associated with a gluon jet is expected to be tiny (10^{-3}) as will be

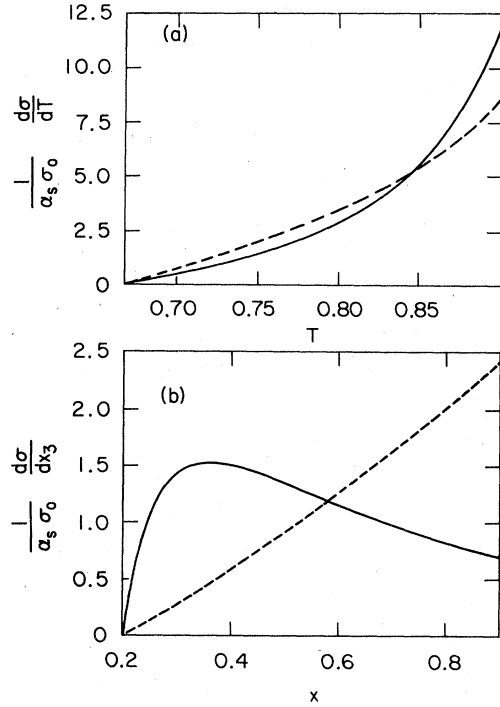


FIG. 2. T distributions (a) and gluon energy spectra (b) for QCD (solid line) and scalar gluons (dashed line).

discussed in Sec. IV, so that in three-jet events of the type $e^+e^- \rightarrow c\bar{c}g$ the semileptonic decay of the charmed quarks could be used to identify quark jets.¹¹ There will exist the rare case that both the q and \bar{q} decay semileptonically, allowing an isolation of the gluon jet on an event-by-event basis and a separate measurement of the quantities in (1)–(3) as well as the gluon-jet properties. Since this case requires rather large statistics we will in addition consider events where only one semileptonic decay occurs (q or \bar{q}). An identification of the gluon jet could still be possible in this case, using the fact that the decay products of a charmed particle contain a kaon most of the time. In con-

TABLE I. Global properties of the quark and gluon spectra for two different energy cuts ($T \leq 1 - \epsilon$, $V \geq \delta$). The $\langle x \rangle$ column gives the mean values of the energy fractions x_i and the T , U , and V columns contain the probabilities (in %) for quarks (gluons) to be the T , U , or V jet.

		QCD				Scalar gluons			
		$\langle x \rangle$	T	U	V	$\langle x \rangle$	T	U	V
$\epsilon = 0.1$,	q (\bar{q})	0.74	44.5	38	17.5	0.66	34.5	30.5	35
$\delta = 0.2$	g	0.52	11	24	65	0.68	31	39	30
$\epsilon = 0.1$,	q (\bar{q})	0.73	44	37	19	0.665	34.5	31	34.5
$\delta = 0.3$	g	0.54	12	26	62	0.67	31	38	31

junction with an estimate of kaon-pair production in gluon jets (which we will not do here) this case should be kept in mind as an interesting possibility.

We now turn to the case that all the information we have is just the fact that one of the jets is identified by a lepton to be a quark (or antiquark) jet. Let x_1 be the energy fraction of this jet (independent of whether it is a quark or an antiquark since the spectra of q and \bar{q} are identical). The energy spectrum $(1/\sigma_0)d\sigma/dx_1$ [Fig. 1(b)]¹³ can now be measured as well as the spectrum of the remaining two jets $(1/\sigma_0)(d\sigma/dx_2+d\sigma/dx_3)$ (Fig. 3). A subtraction of these two spectra gives the gluon-jet energy spectrum [Fig. 1(c)] and this information will allow a measurement of the predictions in Table I. We want to emphasize that this information is sensitive to specific properties of QCD (such as the spin of the gluon) and will be superior to presently available results. In addition it turns out to be useful even to consider angular distributions, parametrized as

$$\frac{1}{\sigma_0} \frac{d\sigma}{d \cos\theta_2} \propto 1 + A_i \cos^2\theta_i.$$

The corresponding A_i are given in Table II. They differ substantially for the various jets, which may allow experimental verification without enormous statistics.

Just the information that one jet is a quark jet will not allow an isolation of gluon jets on an event-by-event basis, but will enable us to select kinematic regions where gluon-jet probabilities will be enhanced. We define $x_H = \max(x_2, x_3)$ and $x_L = \min(x_2, x_3)$ (remember that x_1 is the energy fraction of the identified jet). The corresponding distributions are given in Fig. 4. First-order QCD predicts $\langle x_H \rangle = 0.79$, $\langle x_L \rangle = 0.47$ and a prob-

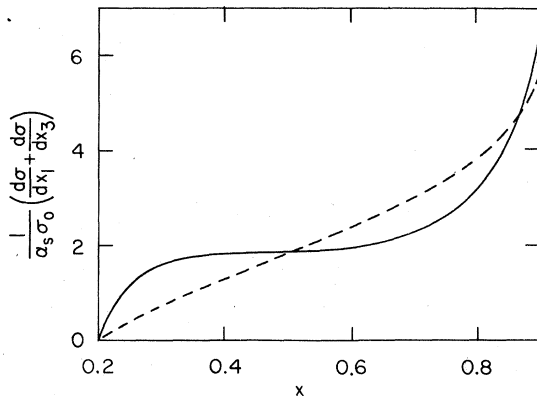


FIG. 3. The sum of the quark and gluon energy spectra. The solid dashed line corresponds to QCD (scalar gluon model).

TABLE II. The values of A_i for the q jet, g jet, and the sum of both given separately for the total kinematic region as well as for its lower and upper half. The values P_q and P_g show the percentage of events which lie in the respective regions.

		P_q	A_q	P_g	A_g	A_{q+g}
$\epsilon = 0.1,$ $\delta = 0.2$	Total region	100	0.63	100	-0.03	0.31
	Lower half	13	0.08	58	-0.20	-0.14
	Upper half	87	0.71	42	0.20	0.51
$\epsilon = 0.1,$ $\delta = 0.3$	Total region	100	0.60	100	-0.01	0.29
	Lower half	19	0.18	62	-0.15	-0.04
	Upper half	80	0.70	38	0.21	0.43

ability of 77% for the gluon to be the x_L jet. If we furthermore restrict the considered x_1 range [e.g., demand that $x_1 = U(T)$, which happens to be the case in 38% (45%) of the events] the gluon probability in the slow x_L jet could be enhanced up to 90% (which by the way would not be possible in a model

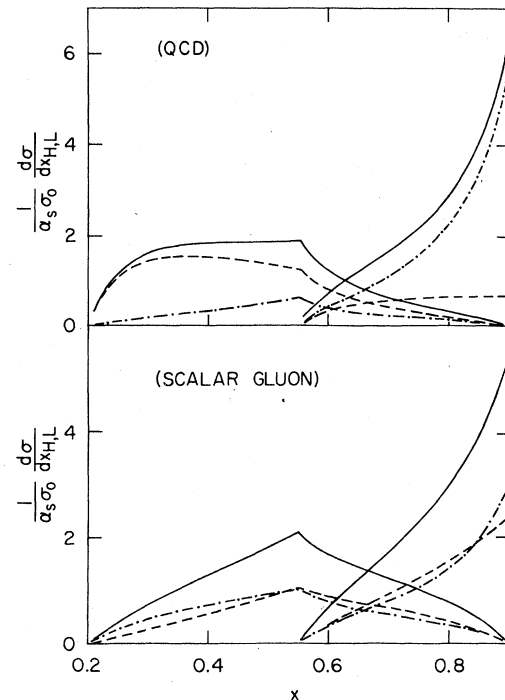


FIG. 4. Energy distributions of the fast (x_H) and slow (x_L) jet (solid line) as defined in the text. The dashed (dashed-dotted) curves show the contributions of gluons (quarks).

with scalar gluons). Results are summarized in Tables III and IV.

With these numbers checked experimentally one can now determine properties of gluon jets, such as multiplicity and transverse momentum distributions.

A. Transverse momentum

The average p_{\perp} of a quark jet $\langle |p_{\perp}| \rangle_q$ has been measured in low-energy ($\sqrt{s} \leq 10$ GeV) e^+e^- annihilation.² This quantity has turned out to be constant¹⁴ in that energy range and is assumed to reflect the fragmentation of colored quarks into hadrons. It should coincide with the average p_{\perp} of the x_1 jet. From

$$\langle |p_{\perp}| \rangle_L = a \langle |p_{\perp}| \rangle_g + (1-a) \langle |p_{\perp}| \rangle_q \quad (4)$$

the average transverse momentum of the gluon jet could be extracted and one could decide whether the different color charges of quarks¹⁵ and gluons have an influence on that quantity. a denotes the probability that the x_L jet is a gluon jet (see Tables III and IV). It is obvious that regions with large a ($0.8 \leq a \leq 1$) are most suitable for such considerations. The average $|p_{\perp}|$ of the x_L jet in Eq. (4) is understood in the way that one first performs the p_{\perp} average of hadrons in the x_L jet of each event and then further averages this quantity over all events. This ensures that each event gets the same weight, regardless of the number of hadrons it contains.

Instead of a separation in x_H and x_L jets, a separation in fat and slim jets might be useful to consider. The fat (slim) jet in an event is that jet which has larger (smaller) $\langle |p_{\perp}| \rangle$ averaged over the hadrons in that jet, $\langle |p_{\perp}| \rangle_{\text{fat (slim)}}$ is the aver-

TABLE III. Probabilities for the H jet (L jet) to be a gluon jet P_H^g (P_L^g) as well as the angular parameters A_H and A_L for three kinematic regions ($P_{H,L}^g = 100 - P_{H,L}^q$). The P column gives the percentage of events in the respective regions.

		P	P_H^g	P_L^g	A_H	A_L
$\epsilon = 0.1,$ $\delta = 0.2$	Total region	100	23	77	0.63	-0.04
	Lower half	24	38	81	0.39	-0.15
	Upper half	76	19	66	0.71	0.21
$\epsilon = 0.1,$ $\delta = 0.3$	Total region	100	25	75	0.59	-0.03
	Lower half	27	38	78	0.39	-0.10
	Upper half	73	20	66	0.67	0.22

age of that quantity over all fat (slim) jets. The p_{\perp} distribution of the jets is assumed to be²

$$\frac{1}{\sigma_0} \frac{d\sigma^{q(g)}}{dp_{\perp}} = \frac{1}{\sqrt{2\pi} \sigma_{q(g)}} \exp(-p_{\perp}^2 / 2\sigma_{q(g)}^2). \quad (5)$$

For the mean values one obtains

$$\begin{aligned} \langle |p_{\perp}| \rangle_{\text{fat}} &= \left[\frac{2}{\pi} (\sigma_q^2 + \sigma_g^2) \right]^{1/2}, \\ \langle |p_{\perp}| \rangle_{\text{slim}} &= \left(\frac{2}{\pi} \right)^{1/2} [\sigma_q + \sigma_g - (\sigma_q^2 + \sigma_g^2)], \\ \langle |p_{\perp}| \rangle_{q(g)} &= \left(\frac{2}{\pi} \right)^{1/2} \sigma_{q(g)}. \end{aligned} \quad (6)$$

It is an amusing property of distribution (5) that

TABLE IV. The coefficients a, b, c, d as defined in Eq. (9) for $\epsilon = 0.1$ and $\delta = 2\epsilon$ in various kinematical regions. a (b) gives the probability for quarks (gluons) in the respective jets. Again, the P column gives the percentage of events in the considered kinematical region.

		P	H jet				P	L jet			
			a_H	b_H	c_H	d_H		a_L	b_L	c_L	d_L
All events		100	0.77	0.23	0.93	0.25	100	0.23	0.77	0.22	0.59
$x_1 = U$	Total region	38	0.85	0.15	1.23	0.21	38	0.15	0.85	0.12	0.58
	Lower half	5.5	0.69	0.31	0.90	0.40	21	0.10	0.90	0.17	0.61
	Upper half	32.5	0.88	0.12	1.28	0.18	17	0.21	0.79	0.06	0.54
$x_1 = T$	Total region	44.5	0.73	0.27	0.94	0.34	44.5	0.27	0.73	0.22	0.50
	Lower half	21.5	0.62	0.38	0.74	0.45	22.5	0.17	0.83	0.10	0.42
	Upper half	23	0.83	0.17	1.15	0.23	22	0.38	0.62	0.36	0.59

$$\langle |p_{\perp \text{ fat}} p_{\perp \text{ slim}}| \rangle = \langle |p_{\perp}| \rangle_q \langle |p_{\perp}| \rangle_g \quad (7)$$

allows a direct measurement of $\langle |p_{\perp}| \rangle_g \langle |p_{\perp}| \rangle_{\text{fat}} \times p_{\perp \text{ slim}}| \rangle$ is obtained by first performing the average of $|p_{\perp i} p_{\perp j}|$ over the hadrons i (j) of the fat (slim) jet in a single event and finally averaging over all events.

Let us add a technical remark. It might be worthwhile to measure the p_{\perp} distributions with respect to the jet axis *and* the $q\bar{q}g$ plane, since this quantity is less sensitive to a possible wrong assignment of the hadrons to the different jets.¹⁶

B. Multiplicity

Multiplicity distributions depend on the energy of the jets. The low-energy results show a logarithmic increase of the mean multiplicity and above 13 GeV an even faster increase is observed.² We here adopt the point of view that this faster than logarithmic increase is mostly due to the occurrence of three-jet events in which the gluon is emitted at a larger angle, so that a possible color screening might be avoided, resulting in an independent fragmentation of all three partons. In the considered energy region (three-jets at $\sqrt{s} = 40$ GeV) we assume therefore that *jet* multiplicities can be parametrized as

$$\langle n \rangle_{q(g)} = \alpha_{q(g)} + \beta_{q(g)} \ln \left(\frac{x}{2\epsilon} \frac{\sqrt{s}}{30 \text{ GeV}} \right), \quad (8)$$

where x is the energy fraction of the jet ($x = 2E/\sqrt{s}$). Again α_q and β_q can be determined either from low-energy data or from the multiplicity of the x_1 jet, so that α_g and β_g can be obtained from the multiplicities of the x_H, x_L jets,

$$\begin{aligned} \langle n \rangle_{H,L} = & a_{H,L} \left[\alpha_q + \beta_q \ln \left(\frac{\sqrt{s}}{30 \text{ GeV}} \right) \right] \\ & + b_{H,L} \left[\alpha_g + \beta_g \ln \left(\frac{\sqrt{s}}{30 \text{ GeV}} \right) \right] \\ & + c_{H,L} \beta_q + d_{H,L} \beta_g. \end{aligned} \quad (9)$$

The values of the coefficients a, b, c, d for the different kinematical regions are given in Table IV.

It will certainly be possible to determine by this procedure whether the gluon-jet properties are different from those of quark jets. A large difference in these properties could lead to an isolation of gluon jets on an event-by-event basis, perhaps even in the case where none of the jets is identified.

IV. REMARKS

A. Cuts

The first step in the experimental procedure should be the selection of three-jet events, which

could be done with rather loose thrust¹⁷ and triplicity¹⁸ cuts. Having determined the jet axes and assigned the hadrons to the three jets one should measure the energies E_i of the jets, giving $x_i = 2E_i/E_{\text{vis}}$. (In case of complete reconstruction, $E_{\text{vis}} = \sqrt{s}$.) In the next step one imposes the cut $\max(x_1, x_2, x_3) = 1 - \epsilon$, which automatically induces $\min(x_i) = 2\epsilon$. A cut of $\epsilon = 0.1$ avoids infrared and collinear singularities, assures the validity of first-order perturbation theory ($\alpha_s \ln^2 \epsilon < 1$) and forces the smallest angle between any two of the three jets⁴ to be larger than 70° . Since E_{min} can be as small as 4 GeV at $\sqrt{s} = 40$ GeV, it might be necessary to introduce an additional cut $\min(x_i) = \delta > 2\epsilon$. This eliminates only a few events, e.g., 10% in changing δ from 0.2–0.3. We repeated the calculations including these two cuts. The minor differences can be inspected in the tables.

In the sense as discussed above, the quantities should be rather insensitive to fragmentation since $x_1 + x_2 + x_3 = 2$, in contrast to the case where one only considers longitudinal momenta. Effects of second-order QCD leading to separated four-jet events are expected to be small in the considered energy range.¹⁹

B. Leptons

The probability for a lepton (e or μ) to be produced in a charmed-quark jet²⁰ is of the order of 20%. The respective probability in a gluon jet is expected to be orders of magnitude smaller. It can be estimated using e^+e^- annihilation data at low energy²¹ ($\sqrt{s} \leq 3.6$ GeV) since $c\bar{c}$ production in a gluon jet below 20 GeV is shown to be tiny.²²

C. b quarks

Jets with b flavors might have larger semileptonic branching ratios than c quarks but are only produced at a rate of a quarter of the c quarks. Three-jet events including b quarks can probably be distinguished from those of u, d, s, c quarks.^{4, 23} Spectra in the massive case^{4, 24} and lepton multiplicities²⁵ are available in the literature.

D. Spectra of the leptons

These spectra have to be computed for mainly two reasons. (i) Experimental detection efficiencies will require a certain minimal energy for the leptons to be detected. (ii) The angle between the lepton and the jet axis should be small enough to assure the assignment of the lepton to that jet in which it was produced. The leptons will be mostly decay products of charged D mesons where quark-model calculations seem to be reliable.²⁰ The main uncertainty in the calculation of the lepton spectra

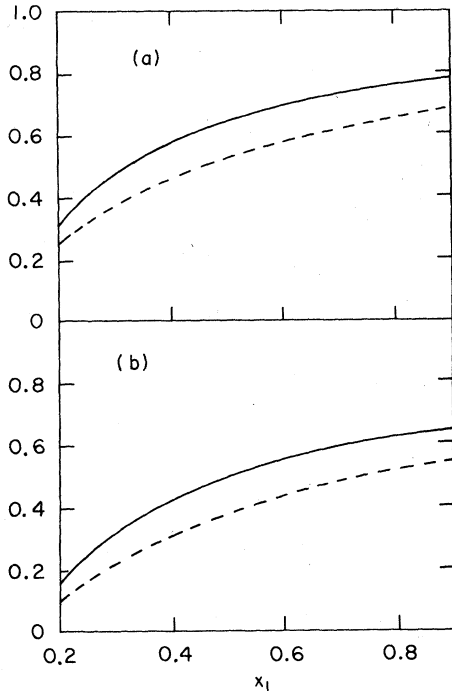


FIG. 5. The multiplicative correction factor for $(1/\sigma_0)/(d\sigma/dx_1)$ including only events where the lepton has an angle of less than 30° with the jet axis and energy (a) $E_l \geq 100$ MeV, (b) $E_l \geq 1$ GeV ($\sqrt{s} = 30$ GeV). The solid [dashed] line corresponds to fragmentation functions $D(z) = 1$ [$D(z) = 2(1-z)$].

comes therefore from the poor knowledge of the fragmentation function $D(z)$, which gives the probability for producing a D meson with energy fraction z from a charmed quark. The correction factor for the quantity $(1/\sigma_0)d\sigma/dx_1$ from the above constraints is shown in Fig. 5 for different $D(z)$. It is obvious that the corrections are large in the small- x_1 region, where the cross section is small. Including all the discussed cuts we expect the number of three-jet single-lepton events (containing c quarks) to be 1% of all hadronic events.

E. Missing neutrino energy

The leptons (e or μ) will be accompanied by neutrinos which escape detection. The computed neutrino spectrum can be used to correct the $(1/\sigma_0)d\sigma/dx_i$ distributions for this missing energy. A Monte Carlo calculation for the \tilde{x}_1 spectrum ($\tilde{x}_1 = x_1 - 2E_\nu/\sqrt{s}$) is shown in Fig. 6. With this information the $d\sigma/dx_i$ and $d\sigma/d\tilde{x}_i$ spectra can be related to each

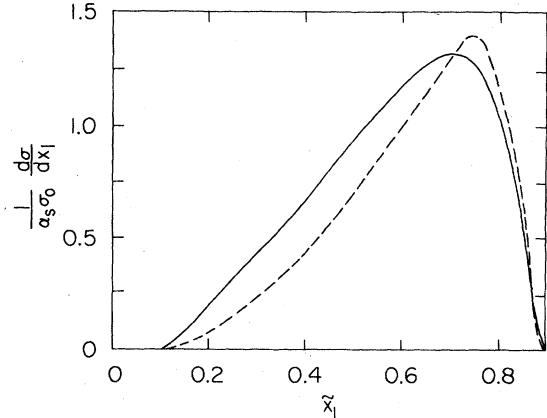


FIG. 6. Quark-jet energy spectra in the case of missing neutrino energy ($\tilde{x}_1 = x_1 - 2E_\nu/\sqrt{s}$) for those events where the electron has energy $E_e \geq 100$ MeV and the electron-jet angle is less than 30° ($\sqrt{s} = 30$ GeV). The solid [dashed] curve corresponds to $D(z) = 1$ [$D(z) = 2(1-z)$].

other. Since the direction of the jet axes are less sensitive to that missing energy a correction on an event-by-event basis may be possible as well. The knowledge of the angles between the different jets allows an independent determination of the jet-energy fractions x_i .

V. CONCLUSIONS

We have investigated the possibility of separating quark and gluon jets and have shown that the subsample of three-jet events in e^+e^- annihilations that contains c quarks will enable us to extract properties of gluon jets such as energy, transverse-momentum, and multiplicity spectra. The measurements will be possible with the number of three-jet events expected in the near future. An unambiguous determination of the spin of the gluon could then be possible. This information will then allow a more detailed investigation of gluon properties on the basis of all three-jet events.

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