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# Anomaly in the nonlocal quantum charge of the $CP^{n-1}$ model

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We calculate the quantized nonlocal charge of the $CP^{n-1}$ model in the framework of renormalized $1/n$ perturbation theory and prove that it is not conserved.

## I. INTRODUCTION

Classically the $CP^{n-1}$ model is known to possess an infinite number of conservation laws and to be classically integrable.[1] At the quantum level, one would naively expect the same behavior as in the $O(n)$ nonlinear $\sigma$[2] and Gross-Neveu[3] models, in which the amplitude of pair production is suppressed as a consequence of the infinite number of conservation laws. In the $1/n$ expansion, however, this model allows pair production, and the $S$ matrix does not belong to class II of Ref. 4. In this paper, we show that in spite of some hints from the coupling-constant perturbation theory at high energy,[5] the infrared phase, governed by the $1/n$ expansion, has anomalies in the conservation of the quantum nonlocal charge, vitiating the usual constraints on the $S$ matrix elements. The model and some of its basic properties are reviewed in Sec. II. In Sec. III we discuss the short-distance behavior of the product of two currents, a necessary step for the construction of the quantum analog of the classical nonlocal charge. Subsequently, we show that, owing to the presence of anomalous terms, this quantum (nonlocal) charge is no longer conserved.

## II. DEFINITION OF THE MODEL

The $CP^{n-1}$ model is defined by the Lagrangian

$$\mathcal{L} = \overline{D_\mu z} D_\mu z \tag{2.1}$$

with

$$D_\mu z = \partial_\mu z + A_\mu z, \quad A_\mu = -\frac{f}{n} \bar{z} \overleftrightarrow{\partial}_\mu z \tag{2.1a}$$

and the constraint

$$\bar{z} z = n/2f, \tag{2.1b}$$

where $z$ is a complex $n$-component field $z = (z_1, \ldots, z_n)$. If the index $i$ does not appear, it is summed over.

This model is known to possess instant solutions, to be asymptotically free, and to possess a $1/n$ expansion.[6] In the framework of the $1/n$ expansion, the model describes partons and its $S$ matrix does not factorize, in spite of the classical integrability of the model. It has recently been shown that for a model to be classically integrable it is necessary and sufficient[7] to be defined on a symmetric space. In this case there is an internal-symmetry current $j_\mu^{ij}$ whose conservation is equivalent to the equations of motion and which satisfies

$$\partial_\mu j_\nu^{ij} - \partial_\nu j_\mu^{ij} + 2\,\frac{2f}{n}[j_\mu, j_\nu]^{ij} = 0. \tag{2.2}$$

Using (2.2) it is easily verified that the nonlocal charge

$$Q = \int dy_1 dy_2 \epsilon(y_1 - y_2) j_0(t, y_1) j_0(t, y_2) - \frac{n}{2f} \int dy\, j_1(t, y) \tag{2.3}$$

is conserved.

In the $CP^{n-1}$ model, the current $j_\mu^{ij}$ is given by

$$j_\mu^{ij} = z_i \overleftrightarrow{\partial}_\mu z_j + 2A_\mu z_i \bar{z}_j . \tag{2.4}$$

At the quantum level the charge (2.3) is not well defined, since it involves a product of currents at the same point. The (nonintegrable) singularity of this product must be analyzed in order to obtain a renormalized charge. For the $O(n)$ nonlinear $\sigma$ model this was done in Ref. 8 where it was shown that finiteness and conservation of the charge can be achieved by simply adjusting the coefficient of the second term in (2.3).

Conservation of this charge has far-reaching consequences for the theory.[8,9] Because of its nonlocal character, the dynamics will be much constrained. In terms of asymptotic fields, Lüscher[8] showed that in the $O(n)$ nonlinear $\sigma$ model it forbids pair production. Furthermore, this charge is only compatible with a nontrivial $S$ matrix. However, in that case, renormalization is trivial because of the reduced number of composite operators compatible with the symmetry and dimension, which are the current $j_\mu$ itself and its derivative $\partial_\mu j_\nu$, whereas in the $CP^{n-1}$ case, we have many other composite operators for the Wilson expansion, e.g., $z_i \bar{z}_j F_{\mu\nu}$ (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$), $z_i \bar{z}_j D_\mu z\, \overline{D_\nu z}$, $\partial_\mu (z_i \bar{z}_k) j_\nu^{kj}$, etc. We will show that in fact, one of these terms gives rise to an anomaly, destroying conservation of the would-be charge. We will use the $1/n$ expansion so that the possible instanton contributions will not be taken into account. Nevertheless, we expect such contributions to be small, thereby preserving the main argument.

## III. FEYNMAN RULES AND WILSON EXPANSION IN LOWEST ORDER

The $1/n$ expansion of the model was treated in great detail by d'Adda *et al.*[6] The expansion has only two free parameters: the perturbation parameter $1/n$ and the mass $m$ of the $z_i$ fields. The parameter $f$ is completely absorbed into this mass and in the $A_\mu$ field. We could still write $A_\mu$ in terms of $f$ and the $z_i$ fields as in (2.1a) but there is no advantage in doing so: If we consider the regulated theory, then, as the cutoff is removed, $\bar{z}\overleftrightarrow{\partial}_\mu z$ diverges and $f$ goes to zero, so that only the product $f\bar{z}\overleftrightarrow{\partial}_\mu z$ is well defined. In what follows we will use the Feynman rules given in Ref. 6, to which the reader is referred for more details. All calculations will be performed in the Euclidean region.

We are interested in the short-distance behavior of the product

$$j_\mu^{ik}(x)j_\nu^{kj}(y)-j_\nu^{ik}(y)j_\mu^{kj}(x), \tag{3.1}$$

or, more precisely in the singular terms (as $\epsilon$ tends to zero) of

$$j_\mu^{ik}(x+\epsilon)j_\nu^{kj}(x-\epsilon)-j_\nu^{ik}(x-\epsilon)j_\mu^{kj}(x+\epsilon). \tag{3.2}$$

For the product (3.2), we have a sum of the following terms:

$$-\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon), \tag{3.3a}$$

$$\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)\partial_\nu z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3b}$$

$$z_i(x+\epsilon)\partial_\mu\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon), \tag{3.3c}$$

$$-z_i(x+\epsilon)\partial_\mu\bar{z}_k(x+\epsilon)\partial_\nu z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3d}$$

$$2A_\mu(x+\epsilon)z_i(x+\epsilon)\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon), \tag{3.3e}$$

$$-2A_\mu(x+\epsilon)z_i(x+\epsilon)\bar{z}_k(x+\epsilon)\partial_\nu z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3f}$$

$$2z_i(x+\epsilon)\partial_\mu\bar{z}_k(x+\epsilon)A_\nu(x-\epsilon)z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3g}$$

$$-2\partial_\mu z_i(x+\epsilon)\bar{z}_\mu(x+\epsilon)A_\nu(x-\epsilon)z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3h}$$

$$4A_\mu(z+\epsilon)z_i(x+\epsilon)\bar{z}_k(x+\epsilon)A_\nu(x-\epsilon)z_k(x-\epsilon)\bar{z}_j(x-\epsilon), \tag{3.3i}$$

minus the symmetric terms (s.t.) obtained from those above by making the substitutions $\epsilon\leftrightarrow-\epsilon$, $\mu\leftrightarrow\nu$.

By power counting, the Green's functions which diverge in the $\epsilon\to 0$ limit will have either two or four external $z$ lines and zero, one, or two external $A$ lines. A term with two $z$'s and one $\alpha$ (the Lagrange multiplier field which enforces the constraint $\bar{z}z$ = constant—see Ref. 6) external line is forbidden, in this expansion, by parity and time-reversal symmetry. Furthermore, possible divergences of graphs with more than four external $z$ lines actually cancel among themselves, as a consequence of the constraint (2.1b). Thus, to first order in $1/n$, the divergent piece receives contributions only from the proper parts of the following Green's functions:

$$\langle 0|T z_\alpha\bar{z}_\beta(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle, \tag{3.4}$$

$$\langle 0|T z_\alpha\bar{z}_\beta A_\gamma(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle, \tag{3.5}$$

$$\langle 0|T z_\alpha\bar{z}_\beta A_\gamma A_\delta(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle, \tag{3.6}$$

$$\langle 0|T z_\alpha\bar{z}_\beta z_\lambda\bar{z}_\sigma(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle, \tag{3.7}$$

$$\langle 0|T z_\alpha\bar{z}_\beta z_\lambda\bar{z}_\sigma A_\gamma(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle, \tag{3.8}$$

$$\langle 0|T z_\alpha\bar{z}_\beta z_\lambda\bar{z}_\sigma A_\gamma A_\delta(j_\mu(x+\epsilon)j_\nu(x-\epsilon)-j_\nu(x-\epsilon)j_\mu(x+\epsilon))|0\rangle. \tag{3.9}$$

We are going to consider only the case $i\neq j$ so that the $z_i$ and $\bar{z}_j$ fields in the product of the currents are always contracted with some of the external fields. The graphical structure of the first three terms is shown in Fig. 1. As we will see, the divergent parts of these graphs combine (as they should) to form gauge-invariant objects. The calculation will be done to lowest order [so that only (3.4) and (3.5) contribute], and we begin by considering contributions from the Green's function (3.4). In this case, only (3.3a)–(3.3d) contribute, and we have

$$\begin{aligned}&-\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon)\\&\quad=-\partial_\mu z_i(x+\epsilon){:}\bar{z}_k(x+\epsilon)z_k(x-\epsilon){:}\,\partial_\nu\bar{z}_j(x-\epsilon)-\partial_\mu z_i(x+\epsilon)\partial_\nu\bar{z}_j(x-\epsilon)\langle 0|T\bar{z}_k(x+\epsilon)z_k(x-\epsilon)|0\rangle.\end{aligned} \tag{3.10}$$

The last term corresponds to part of graph (a) of Fig. 1. Now

$$\langle 0|T\bar{z}_k(x+\epsilon)z_k(x-\epsilon)|0\rangle = \frac{n}{2\pi}K_0(2\epsilon m)\,. \tag{3.11}$$

$K_0$ is a modified Bessel function of zeroth order

$$K_0(zm) \simeq -\frac{1}{2}\left[1+\left(\frac{zm}{2}\right)^2\right]\ln\frac{m^2z^2}{4} - \gamma + (1-\gamma)\frac{z^2m^2}{4}\,, \tag{3.12}$$

where $\gamma$ is the Euler-Mascheroni constant, $\gamma = 0.5777\ldots$. In order to simplify the notation, we will indicate the second (non-Wick-ordered) term in (3.10) by

$$\mathrm{Div}(-\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon))\,. \tag{3.13}$$

We have then

$$\mathrm{Div}(-\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon)) = \frac{-n}{2\pi}\,\partial_\mu z_i(x)\partial_\nu\bar{z}_j(x)K_0\,, \tag{3.14}$$

where we made a Taylor expansion around $x$ and dropped terms which go to zero with $\epsilon$. Analogously,

$$\mathrm{Div}(\partial_\mu z_i(x+\epsilon)\bar{z}_k(x+\epsilon)\partial_\nu z_k(x-\epsilon)\bar{z}_j(x-\epsilon)) = \frac{-n}{4\pi}\partial_\mu z_i(x+\epsilon)\bar{z}_j(x-\epsilon)\frac{\partial}{\partial\epsilon^\nu}K_0\,, \tag{3.15}$$

$$\mathrm{Div}(z_i(x+\epsilon)\partial_\mu\bar{z}_k(x+\epsilon)z_k(x-\epsilon)\partial_\nu\bar{z}_j(x-\epsilon)) = \frac{n}{4\pi}z_i(x+\epsilon)\partial_\nu\bar{z}_j(x-\epsilon)\frac{\partial}{\partial\epsilon^\mu}K_0\,, \tag{3.16}$$

$$\mathrm{Div}(-z_i(x+\epsilon)\partial_\mu\bar{z}_k(x+\epsilon)\partial_\nu z_k(x-\epsilon)\bar{z}_j(x-\epsilon)) = \frac{n}{8\pi}z_i(x+\epsilon)\bar{z}_k(x-\epsilon)\frac{\partial^2}{\partial\epsilon^\mu\partial\epsilon^\nu}K_0\,. \tag{3.17}$$

Performing a further Taylor expansion of the operators, we obtain for graphs (3.3a)–(3.3d) of Fig. 1

$$(3.3\mathrm{a}) = \frac{-n}{2\pi}K_0(\partial_\mu z_i\partial_\nu\bar{z}_j - \partial_\nu z_i\partial_\mu\bar{z}_j)\,,$$

$$(3.3\mathrm{b}) = \frac{-n}{4\pi}\partial_\nu K_0\partial_\mu z_i\bar{z}_j + \frac{n}{4\pi}\partial_\mu\partial_\nu z_i\bar{z}_jK_0 - \frac{n}{4\pi}\partial_\mu z_i\partial_\nu\bar{z}_jK_0 + \frac{n}{8\pi}\partial_\mu\partial_\beta z_i\bar{z}_j\partial_\nu\partial_\beta K_1 - \frac{n}{8\pi}\partial_\mu z_i\partial_\beta\bar{z}_j\partial_\nu\partial_\beta K_1 - (\mathrm{s.t.})\,,$$

$$(3.3\mathrm{c}) = \frac{n}{4\pi}\partial_\mu K_0 z_i\partial_\nu\bar{z}_j - \frac{n}{4\pi}\partial_\mu z_i\partial_\nu\bar{z}_jK_0 + \frac{n}{4\pi}z_i\partial_\mu\partial_\nu\bar{z}_jK_0 - \frac{n}{8\pi}\partial_\beta z_i\partial_\nu\bar{z}_j\partial_\mu\partial_\beta K_1 + \frac{n}{8\pi}z_i\partial_\nu\partial_\beta\bar{z}_j\partial_\mu\partial_\beta K_1 - (\mathrm{s.t.})\,,$$

$$(3.3\mathrm{d}) = \frac{-n}{4\pi}\partial_\mu z_i\bar{z}_j\partial_\nu K_0 + \frac{n}{4\pi}z_i\partial_\mu\bar{z}_j\partial_\nu K_0 + \frac{n}{8\pi}\partial_\beta z_i\bar{z}_j\partial_\mu\partial_\nu\partial_\beta K_1\,,$$

where

$$K_1(m\epsilon) = -\frac{\partial}{\partial m^2}K_0(m\epsilon)\,.$$

Graph (b) can be handled in the same way; we have only contributions from (3.3e) and (3.3f):

$$(3.3\mathrm{e}) = \frac{n}{2\pi}[2K_0(A_\mu z_i\partial_\nu\bar{z}_j) - z_i\bar{z}_jA_\mu\partial_\nu K_0 + (-\partial_\nu z_iA_\mu\bar{z}_j - \partial_\nu A_\mu z_i\bar{z}_j + z_iA_\mu\partial_\nu\bar{z}_j)K_0$$
$$+\tfrac{1}{2}(z_iA_\mu\partial_\beta\bar{z}_j - \partial_\beta z_iA_\mu\bar{z}_j - \partial_\beta A_\mu z_i\bar{z}_j)\partial_\nu\partial_\beta K_1] - (\mathrm{s.t.})\,,$$

$$(3.3\mathrm{f}) = \frac{n}{2\pi}[2K_0(-A_\nu\partial_\mu z_i\bar{z}_j) - z_i\bar{z}_jA_\nu\partial_\mu K_0 + (z_i\partial_\mu A_\nu\bar{z}_j + z_i\partial_\mu\bar{z}_jA_\nu - \partial_\mu z_iA_\nu\bar{z}_j)K_0$$
$$+\tfrac{1}{2}(z_i\partial_\beta A_\nu\bar{z}_j + z_i\partial_\beta\bar{z}_jA_\nu - \partial_\beta z_iA_\nu\bar{z}_j)\partial_\mu\partial_\beta K_1] - (\mathrm{s.t.})\,.$$

The calculation for graph (c) is more involved. We formally have

$$\langle 0|T(j_\mu(x+\epsilon)j_\nu(x-\epsilon) - j_\nu(x-\epsilon)j_\mu(x+\epsilon))A_\beta(y)|0\rangle$$
$$= \langle 0|TN_2(j_\mu(x)j_\nu(x) - j_\nu(x)j_\mu(x))A_\beta(y)|0\rangle + R_{\mu\nu\beta}(x,y,\epsilon)\,, \tag{3.18}$$

where the symbol $N_2$ denotes the normal product defined by Zimmermann,[10] making the minimum number of subtractions necessary to render the formal product of currents at the same point well defined. $R_{\mu\nu\beta}(x,y,\epsilon)$ are these subtraction terms which will diverge as $\epsilon\to 0$. Now the first term does not contribute to the operator expansion, since $\bar{z}z = 0$ in our subtraction procedure. Classically, if $\bar{z}z = a$ = constant, we have the following

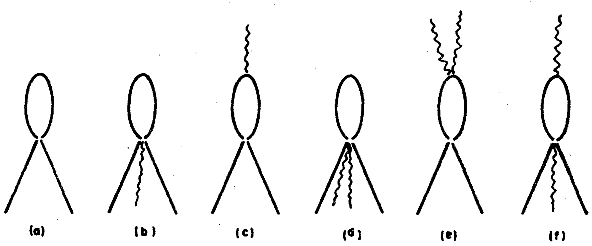


FIG. 1. Lowest-order graphs contributing to the short-distance expansion of the product of the currents.

equality:

$$2(j_\mu j_\nu - j_\nu j_\mu) = a(\partial_\nu j_\mu - \partial_\mu j_\nu)\,. \tag{3.19}$$

In a subtraction scheme preserving this equality, we should have

$$2\langle 0|TN(j_\mu j_\nu - j_\nu j_\mu)\mathrm{X}\,|\,0\rangle = a\langle 0|T(\partial_\nu j_\mu - \partial_\mu j_\nu)\mathrm{X}|0\rangle + \delta \text{ terms}, \tag{3.20}$$

and therefore, for $a = 0$, we have the desired result. This statement can be explicitly verified in a lowest-order calculation. The divergent part of the second term in (3.18) is

$$\mathrm{Div}[(3.3\mathrm{a}) + A_\beta] = 0\,, \tag{3.21a}$$

$$\mathrm{Div}[(3.3\mathrm{b}) + A_\beta] = -\tfrac{1}{2}\partial_\nu\partial_\beta K_1\partial_\mu z_i(x)\bar{z}_j(x)A_\beta(x) + \tfrac{1}{2}\partial_\mu\partial_\beta K_1\partial_\nu z_i(x)\bar{z}_j(x)A_\beta(x)\,, \tag{3.21b}$$

$$\mathrm{Div}[(3.3\mathrm{c}) + A_\beta] = \tfrac{1}{2}\partial_\mu\partial_\beta K_1 z_i(x)\partial_\nu\bar{z}_j(x)A_\beta(x) - \tfrac{1}{2}\partial_\nu\partial_\beta K_1\partial_\mu\bar{z}_j(x)z_i(x)A_\beta(x)\,, \tag{3.21c}$$

$$\mathrm{Div}[(3.3\mathrm{c}) + A_\beta] = \tfrac{1}{2}z_i(x)\bar{z}_j(x)\partial_\mu A_\beta(x)\partial_\nu\partial_\beta K_1 - \tfrac{1}{2}z_i(x)\bar{z}_j(x)\partial_\nu A_\beta(x)\partial_\mu\partial_\beta K_1\,. \tag{3.21d}$$

Graphs (d), (e), and (f) are easily shown to vanish by symmetry. Now we collect all terms and obtain

$$\begin{aligned} j_\mu(x+\epsilon)j_\nu(x-\epsilon) - j_\nu(x-\epsilon)j_\mu(x+\epsilon) = {} & \frac{n}{2\pi}\left[\frac{-\delta_{\mu\nu}\epsilon_\rho}{2\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\nu}{2\epsilon^2} + \frac{\delta_{\nu\rho}\epsilon_\mu}{2\epsilon^2} + \frac{\epsilon_\mu\epsilon_\nu\epsilon_\rho}{(\epsilon^2)^2}\; j_\rho\right. \\ & + \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \tfrac{1}{4}\ln m^2\epsilon^2\right)(\delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\nu\sigma}\delta_{\mu\rho}) + \frac{\delta_{\nu\sigma}\epsilon_\mu\epsilon_\rho}{2\epsilon^2} - \frac{\delta_{\mu\sigma}\epsilon_\nu\epsilon_\rho}{2\epsilon^2}\right]\partial_\sigma j_\rho \\ & + \frac{n}{2\pi}\left(2\delta_{\mu\rho}\frac{\epsilon_\nu\epsilon_\sigma}{\epsilon^2} + 2\delta_{\nu\sigma}\frac{\epsilon_\mu\epsilon_\rho}{\epsilon^2}\right)z_i\bar{z}_jF_{\rho\sigma}\,, \end{aligned} \tag{3.22}$$

or

$$\begin{aligned} j_\mu(x+\epsilon)j_\nu(x) - j_\nu(x)j_\mu(x+\epsilon) = {} & \frac{n}{2\pi}\left[\frac{-\delta_{\mu\nu}\epsilon_\rho}{\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\nu}{\epsilon^2} + \frac{\delta_{\nu\rho}\epsilon_\mu}{\epsilon^2} + \frac{2\epsilon_\mu\epsilon_\nu\epsilon_\rho}{(\epsilon^2)^2}\right] j_\rho \\ & + \frac{n}{2\pi}\left[\left(\frac{\gamma}{2} + \tfrac{1}{4}\ln\frac{m^2\epsilon^2}{4}\right)(\delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\nu\sigma}\delta_{\mu\rho}) + \frac{\delta_{\nu\sigma}\epsilon_\mu\epsilon_\rho}{2\epsilon^2} - \frac{\delta_{\mu\sigma}\epsilon_\nu\epsilon_\rho}{2\epsilon^2}\right. \\ & \left. - \frac{\delta_{\mu\nu}\epsilon_\rho\epsilon_\sigma}{2\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\nu\epsilon_\sigma}{2\epsilon^2} + \frac{\delta_{\mu\rho}\epsilon_\mu\epsilon_\sigma}{2\epsilon^2} + \frac{\epsilon_\mu\epsilon_\nu\epsilon_\rho\epsilon_\sigma}{(\epsilon^2)^2}\right]\partial_\sigma j_\rho \\ & + \frac{n}{2\pi} + 2\delta_{\mu\rho}\frac{\epsilon_\nu\epsilon_\sigma}{\epsilon^2} + 2\delta_{\nu\sigma}\frac{\epsilon_\mu\epsilon_\rho}{\epsilon^2}\;\; z_i\bar{z}_jF_{\rho\sigma}\,. \end{aligned} \tag{3.23}$$

## IV. DEFINITION OF THE QUANTUM NONLOCAL CHARGE

As remarked above, due to the singular nature of the product of the currents at the same point, some care must be exercised in constructing the quantum analog of nonlocal charge (2.3). Similarly to Ref. 8, we define the quantum version of the nonlocal charge as

$$Q = \lim_{\delta\to 0} Q_\delta\,,$$

where

$$Q_\delta^{ij} = \frac{1}{n}\int_{|y_1-y_2|\geq\delta} dy_1dy_2\epsilon(y_1-y_2)j_0^{ik}(t,y_1)j_0^{kj}(t,y_2) - \frac{z}{n}\int dy\, j_1^{ij}(t,y)\,, \tag{4.1}$$

where the dependence of $z$ on the cutoff $\delta$ is such as to cancel the aforementioned divergences.

Because of the linearly divergent term in Eq. (3.23), it is easy to see that this coefficient must be equal to

$$\frac{n}{2\pi}\ln\left(\frac{e^{\gamma-1}m}{2}\delta\right)$$

in order to have a well-defined charge $Q$. In (4.1) a factor $1/n$ was introduced in order to guarantee the existence of the limit $n\to\infty$. This charge is the only candidate for the quantum nonlocal charge corresponding to (2.3).

However, this charge is no longer time independent. This is verified as follows: Current conservation and partial integration give

$$\frac{dQ_\delta^{ij}}{dt}=\frac{-1}{n}\int_{-\infty}^{\infty}dy\left\{[j_1^{ik}(t,y+\delta)+j_1^{ik}(t,y-\delta)]j_0^{kj}(t,y)-j_0^{ik}(t,y)[j_1^{kj}(t,y+\delta)+j_1^{kj}(t,y-\delta)]\right.$$
$$\left.+\frac{n}{2\pi}\ln\mu\delta\partial_0 j_1^{ij}(t,y)\right\},$$

where

$$\mu=\frac{e^{\gamma-1}m}{2}\,. \tag{4.2}$$

As $\delta$ goes to zero we use the operator expansion (3.23) to obtain

$$[j_1^{ik}(t,y+\delta)+j_1^{ik}(t,y-\delta)]j_0^{kj}(t,y)-j_0^{ik}(t,y)[j_1^{kj}(t,y+\delta)+j_1^{kj}(t,y-\delta)]$$
$$=\frac{n}{2\pi}\left[\left(\gamma+\tfrac{1}{2}\ln\frac{m^2\delta^2}{4}\right)(\partial_1 j_0-\partial_0 j_1)+\partial_0 j_1+4z_i\bar{z}_jF_{10}\right], \tag{4.3}$$

and we have

$$\frac{dQ_\delta^{ij}}{dt}=\frac{-2}{\pi}\int_{-\infty}^{\infty}z_i\bar{z}_jF_{10}dy\,. \tag{4.4}$$

## V. CONCLUSION

As is well known, the $CP^{n-1}$ model, in the framework of the $1/n$ expansion, does allow production of pairs. This can now be traced back to an anomaly in the quantum nonlocal charge, in contrast to the case of the $O(n)$ nonlinear $\sigma$ model, in which Lüscher quantized the analogous nonlocal charge and this turned out to be conserved. We presume that this connection between pair production and the existence of quantum nonlocal conservation laws has a more general nature. Thus, in view of the proposed $S$ matrices for the Gross-Neveu[3] and chiral Gross-Neveu[11] models, we expect that the corresponding nonlocal conservation laws do survive quantization.

The supersymmetric extension of the $CP^{n-1}$ model has already been studied[12] and proved to factorize.[13] As will be shown in a forthcoming paper, in principle this can be traced back to a cancellation of the anomalies studied here with an Adler-type anomaly coming from the coupling of the chiral Gross-Neveu model to the $CP^{n-1}$ model. In other words, the source of the complementary anomaly is the coupling of the fermion fields to the $A_\mu$ field and not, as one could naively suspect, the chiral Gross-Neveu model by itself. As argued before, the latter must not have any anomalies.

We would like to recall that an alternative explanation for the absence of pair production in the nonlinear $\sigma$ model[14] is provided by Pohlmeyer's local conservation laws.[15] In Ref. 16 the quantum behavior of the local charges was analyzed for a number of models. For the $CP^{n-1}$ model in particular, it seems that these conservation laws are plagued by anomalies, in accordance with our result above.

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