Absence of induced interaction terms in the Federbush model

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In a perturbative calculation we show that no new quadrilinear counterterms are necessary to define the Federbush model if one imposes gauge symmetry of the first kind and asymptotic γ^5 invariance. A subtraction scheme satisfying these conditions is constructed and renormalization-group properties of the Green's functions are analyzed.

I. INTRODUCTION

Historically, the Federbush model was proposed in 1961 as a prototype of a solvable field-theoretical model involving massive spinors.[1] It is a two-dimensional model with a Lagrangian density given by

$$L=\sum_{j=1}^{2}\left(\tfrac{1}{2}i\bar{\psi}_j\gamma^\mu\overleftrightarrow{\partial}_\mu\psi_j-M_j\bar{\psi}_j\psi_j\right)+g\epsilon_{\mu\nu}(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma^\nu\psi_2)\,, \quad (1.1)$$

where the indices $j=1,2$ denote the fermion field type.

The original formulation was made more rigorous after the work of Wightman in 1963.[2] More recently there has been some controversy about the perturbative characterization of the model.[3-5] The basic question is that, as graphs with four external fermion lines are superficially logarithmically divergent, counterterms having forms different from those already present in (1.1) [e.g. $(\bar{\psi}_1\psi_1)(\bar{\psi}_2\psi_2)$] could be necessary in order to define finite Green's functions. However, if, owing to some asymptotic symmetry, there is a cancellation of the divergences of the various graphs involved then counterterms are not necessary. The existence of such cancellations, up to second order in g, has been shown in Refs. 4 and 5.

In this paper we will show that in any order of perturbation the renormalization problem for the Lagrangian (1.1) is perfectly compatible with the asymptotic symmetries of the model. If asymptotic γ^5 invariance is imposed then the only quadrilinear counterterms necessary are those needed to make both vector currents conserved.

Furthermore, we will construct a modified Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction scheme preserving such symmetry and show that the resulting Green's functions obey a renormalization-group equation of the usual form. The paper is organized as follows. In Sec. II we prove that no new quadrilinear counterterms are necessary if the vector currents are conserved and asymptotic γ^5 invariance is imposed. In Sec. III we construct renormalized Green's functions and derive Ward identities for the axial-vector currents explicitly satisfying the requirements of Sec. II. Finally, in Sec. IV, we discuss renormalization-group aspects related to the approach used.

II. ASYMPTOTIC γ^5 INVARIANCE AND CANCELLATION OF DIVERGENCES

In this section we want to show how current conservation and asymptotic γ^5 invariance can be used to explain the mechanism of the cancellation of some of the divergences of graphs with four external legs, constructed using the Lagrangian (1.1). As mentioned before, graphs with four external fermion lines are logarithmically divergent. If we adopt the graphical notation of Fig. 1, these graphs can be classified into two groups, I and II, according to whether they can or cannot be separated into disjoint pieces by cutting only one wavy line. In the first case the graphs have the structure of a product with the factors corresponding either to a contribution to the proper Green's function of one of the formal currents $\bar{\psi}_1\gamma^\mu\psi_1$ or $\bar{\psi}_2\gamma^\mu\psi_2$, with two external fermion lines, or to a contribution to the two-point function of the currents. As these factors are (logarithmically) divergent we add counterterms or, equivalently, make subtractions such that the expression obtained has the same structure as before but with the factors now identified with some contribution

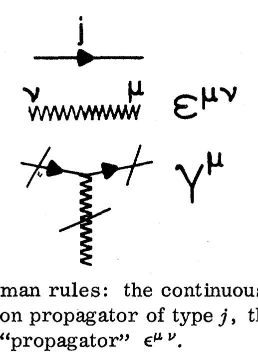

FIG. 1. Feynman rules: the continuous line represents the fermion propagator of type j, the wavy line represents the "propagator" $\epsilon^{\mu\nu}$.

to the corresponding renormalized Green's functions.

We suppose that this renormalization program, which must be carried out starting in lowest order in g, is such that these current Green's functions satisfy Ward identities of the usual form, showing current conservation and asymptotic γ^5 invariance. Since the formally breaking terms of γ^5 invariance occurring in the Lagrangian (1.1) are soft, we think that these requirements can be naturally met. In the next section we will construct explicitly a subtraction scheme satisfying these requirements.

Now consider graphs with four external fermion lines that cannot be separated into disjoint pieces by cutting only one wavy line. In a given order of g these graphs will contain subgraphs of the following type:

(i) Subgraphs with two external fermion lines. These graphs have been made finite by the addition of the corresponding bilinear counterterms.

(ii) Subgraphs with four external fermion lines, belonging to group I. Owing to our previous discussion, the divergences of these subgraphs are canceled with the contributions of the added counterterms.

(iii) Subgraphs with four external fermion lines belonging to group II. These subgraphs have already appeared in lower order. The application of our result below leads to the conclusion that they do not need counterterms.

Taking these observations into account, we are going to show now that, in the order considered, no more quadrilinear counterterms are necessary. To prove this fact note that a generic graph of the type II has the structure shown in Fig. 2. In that figure the vertex V corresponds to the full renormalized current vertex with at most two external wavy lines. Let q be the momentum through the indicated wavy line. We use the identity

$$\epsilon^{\mu\nu} = (\tilde{q}^{\mu} q^{\nu} - q^{\mu}\tilde{q}^{\nu})/q^2 \tag{2.1}$$

and transfer the q_μ and $\tilde{q}_\mu$ factors from the line to the vertex V and apply current conservation and asymptotic γ^5 invariance, respectively. We will obtain graphs that have the same form as before but with V replaced by a soft vertex (in the case of the rotational of the vector current) besides the usual contributions (Fig. 3) that come from the fact that we are considering proper time-ordered functions. Since the original graphs were only logarithmically divergent, it is therefore clear that the result is ultraviolet finite.

The above discussion shows that the only sources of quadrilinear counterterms are the graphs of type I. As will be argued in Sec. III these counterterms correspond to the vertices $(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_1\gamma_\mu\psi_1)$, $(\bar{\psi}_2\gamma_\mu\psi_2)(\bar{\psi}_2\gamma^\mu\psi_2)$, and $(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma_\mu\gamma^5\psi_2)$.

The discussion also shows what one should do to construct a graph-by-graph subtraction scheme satisfying the above requirements: the subtractions for proper graphs with four external lines must be done at the value zero of the masses M_1 and M_2. Possible infrared divergences of the subtraction terms can be eliminated if, in these terms, one replaces $\epsilon^{\mu\nu}$ by $\epsilon^{\mu\nu}q^2/(q^2-\mu^2)$.

III. GREEN'S FUNCTION AND THE SUBTRACTION SCHEME

The observations made at the end of the previous section lead us to define the following effective Lagrangian density for the Federbush model:

$$\begin{aligned} L = {} & N_2\left[\sum_{j=1}^{2}\left(\tfrac{1}{2}i\bar{\psi}_j\gamma^\mu\partial_\mu\psi_j - M_j\bar{\psi}_j\psi_j\right)\right] \\ & + g\epsilon_{\mu\nu}N_2[(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma^\nu\psi_2)] \\ & + a_1N_1(\bar{\psi}_1\psi_1) + a_2N_1(\bar{\psi}_2\psi_2) = L_0 + L_{\rm int}\,, \\ L_{\rm int} = {} & g\epsilon_{\mu\nu}N_2[(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma^\nu\psi_2)] \\ & + a_1N_1(\bar{\psi}_1\psi_1) + a_2N_1(\bar{\psi}_2\psi_2)\,. \end{aligned} \tag{3.1}$$

The finite counterterms a_1 and a_2 were added in order to fix the physical mass of the fields ψ_1 and ψ_2 at the values M_1 and M_2, respectively.

We adopt the graphical notation of Fig. 1. For subtraction purposes a graph will be called proper if it cannot be separated into disjoint pieces by cutting only one line (either a fermion or a wavy line). By power counting the degree of superficial divergence of a proper graph is given by

$$d(\gamma) = 2 - \tfrac{1}{2}N_\gamma - A_\gamma - B_\gamma\,, \tag{3.2}$$

where N_γ = No. of external fermion lines of γ, B_γ = No. of external wavy lines of γ, and A_γ = No. of vertices of type a_1 and a_2 in γ. Without the inser-

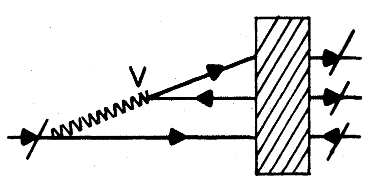

FIG. 2. General structure for the four-point function.

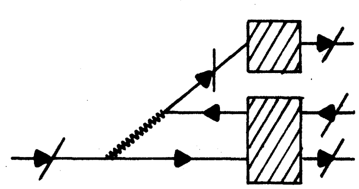

FIG. 3. Here the indicated wavy line has a momentum factor q^ν/q^2 or $\tilde{q}^\nu/q^2$.

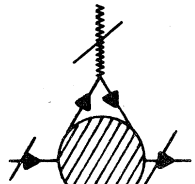

FIG. 4. Generic graph for the vertex function of the current with two external fermion lines.

tion of the counterterms (i.e., $A_\gamma = 0$) the graphs to be subtracted are then the following:

(1) Fermion self-energy graphs, $\delta(\gamma) = 1$.

(2) Vertex functions of the currents with two external fermion lines (see Fig. 4), $\delta(\gamma) = 0$.

(3) Proper functions of two currents (Fig. 5), $\delta(\gamma) = 0$.

(4) Four-point proper functions, $\delta(\gamma) = 0$.

Let I_G be the unsubtracted Feynman integrand associated with the graph G. In order to construct the subtracted Feynman integrand we first substitute I_G by $\bar{I}_G$ which is obtained from I_G by replacing the $\epsilon^{\mu\nu}$ factor associated with a wavy line in which flows the momentum q by

$$\epsilon^{\mu\nu} \frac{q^2}{q^2 - s^2 + i\epsilon(\vec{q}^2 + s^2)} . \tag{3.3}$$

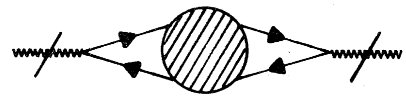

FIG. 5. Proper functions of two currents.

The renormalized integrand associated with I_G is then given by the forest formula

$$R_G = S_G \sum_{U \in F_G} \prod_\gamma (-\tau^{d(\gamma)} S(\gamma)) \bar{I}_G(U) , \tag{3.4}$$

where the sum is over all G forests [i.e., families of proper, nonoverlapping subgraphs of G, having $d(\gamma) \geq 0$] and where $\tau^{d(\gamma)}$ is a generalized Taylor operator defined as follows:

(a) $\tau^{d(\gamma)}$ is the Taylor operator of order $d(\gamma) = 0$ in the external momenta p^γ of γ and in M^γ at $p^\gamma = 0$ and $M^\gamma = \bar{\mu}$ if γ is the graph of Fig. 6: $\tau_\gamma^0 I_\gamma(p^\gamma, M^\gamma) = I_\gamma(0, \bar{\mu})$ if $\gamma =$ Fig. 6.

(b) For the remaining graphs $\tau^{d(\gamma)}$ is a Taylor operator of order $d(\gamma)$ in p^γ, M_1^γ, M_2^γ at $p^\gamma = 0$, $M_1^\gamma = M_2^\gamma = 0$ and, besides, in the last subtraction of $\tau^{d(\gamma)}$ s^γ is replaced by μ. Explicitly, in these cases we have

(i) $\tau_\gamma^0 I_\gamma(p^\gamma, M_1^\gamma, M_2^\gamma, s^\gamma) = I_\gamma(0,0,0,\mu)$ for logarithmically divergent graphs,

(ii) $$\tau_\gamma^1 I_\gamma(p^\gamma, M_1^\gamma, M_2^\gamma, s^\gamma) = I_\gamma(0,0,0,s^\gamma) + p_\nu^\gamma\left(\frac{\partial}{\partial p_\nu^\gamma} I_\gamma\bigg|_{p^\gamma=0, M^\gamma=0, s^\gamma=\mu}\right) + M_1^\gamma\left(\frac{\partial}{\partial M_1^\gamma} I_\gamma\bigg|_{p^\gamma=0, M^\gamma=0, s^\gamma=\mu}\right) + M_2^\gamma\left(\frac{\partial}{\partial M_2^\gamma} I_\gamma\bigg|_{p^\gamma=0, M^\gamma=0, s^\gamma=\mu}\right)$$ for linearly divergent graphs.

S_γ is a substitution operator writing the variables of $\lambda \subset \gamma$ in terms of those of γ; S_G does the additional job of setting $s = 0$ and of replacing $\bar{\mu}$ either by M_1 or M_2 according to the type of current in Fig. 6.

Notice that at $s = 0$ the "propagator" of the wavy line is $\epsilon_{\mu\nu} q^2 (q^2 + i\epsilon\vec{q}^2)$ so that, in all places where generalized Taylor operators did not act, this becomes simply $\epsilon_{\mu\nu}$ as $\epsilon \to 0$. Therefore, subtractions for graphs of type 2 and 3 in the list after (3.2) will correspond to quadrilinear counterterms in the fermion fields.

We remark that the proposed subtraction scheme does not create infrared divergences. Some of the subtractions are done at $s^\gamma = 0$ and $M^\gamma = 0$ but the last subtractions, which if done at $s^\gamma = 0$ and $M^\gamma = 0$ would lead to infrared divergences, are actually done at $s^\gamma = \mu$. The special treatment given to the graph of Fig. 6 comes from the fact that, in this case, there is no internal wavy line to provide a natural infrared cutoff.

The scheme so constructed is a simple modification of those employed in Refs. 6 and 7 and both ultraviolet and infrared finiteness can be proven using similar arguments of those references. For completeness, a sketch of a proof of the infrared convergence is given in the Appendix.

In the same way as done in Sec. II we can prove that the global subtractions for the proper four-point graphs [case (4) in the list after (3.2)] are actually redundant. To verify this we write the

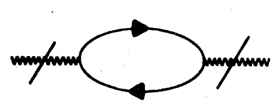

FIG. 6. In this graph the fermion mass is not modified by the action of the subtraction operator.

contribution of order n as

$$R_n^{(4)} = \sum_i (1 - \tau^0_{G_i})\bar{R}_{G_i}, \tag{3.5}$$

where the sum is over all proper four-point graphs and $\bar{R}_{G_i}$ is defined by the forest formula but without the subtractions associated with the graph G_i. We will show that $\sum_i \tau^0_{G_i}\bar{R}_{G_i} = 0$. To this end let us assume that this result has already been proven up to order $n-1$. Using this fact we see that $\bar{R}_{G_i}$ only contains subtractions for subgraphs of the types (1), (2), and (3) in the list after Eq. (3.2). Note that G_i has the structure shown in Fig. 7. As before, we apply the identity

$$\frac{q^\mu \tilde{q}^\nu - \tilde{q}^\mu q^\nu}{q^2 + i\epsilon \vec{q}^2} = \epsilon^{\mu\nu} + O(\epsilon) \tag{3.6}$$

to the line indicated linking V_1 to V_2. We obtain a sum of graphs where $\tilde{\not{q}}$ in V_2 is substituted by $2M\gamma^5$ or appears, in a factorized form, the contribution of a graph which vanishes at $P_{G_i} = M_{G_i} = 0$. In both cases the application of the last subtraction operator gives zero.

Similarly, one can also show that the overall subtractions for logarithmically divergent graphs containing a closed fermion loop with more than two vertices are also redundant. This kind of argument can be employed to determine formally the type of the quadrilinear counterterms associated with the subtraction procedure. From the above discussion these counterterms can come only from the overall subtractions (more precisely, from the product of the global subtractions associated with each proper part) for the graphs in Fig. 8. Using Lorentz covariance and PT invariance the only possible forms of the counterterms are then

(i) $(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_2\gamma_\mu\gamma^5\psi_2)$

if the square box in Fig. 8 with two external wavy lines contains an even number of subgraphs of the type of Fig. 6, and

(ii) $(\bar{\psi}_1\gamma^\mu\psi_1)(\bar{\psi}_1\gamma_\mu\psi_1)$ and $(\bar{\psi}_2\gamma_\mu\psi_2)(\bar{\psi}_2\gamma_\mu\psi_2)$

if the square box contains an odd number of subgraphs of the type of Fig. 6.

Another interesting property concerns graphs that have two closed fermion loops linked by at least one wavy line. In this case the graph will not contribute. This can be seen by the same reasoning as above. We simply use (3.6) to one of the lines linking the two closed loops and apply current conservation.

Green's functions containing normal products $N_{\delta_i}(\theta_i)$, $\delta_i \le 2$, where δ_i = (operator dimension of θ + number of mass parameters in θ_i), are defined by (3.4), but using

$$d(\gamma) = 2 - \tfrac{1}{2}N_\gamma - A_\gamma - B_\gamma - \sum_{V_i \in \gamma} (2 - \delta_i). \tag{3.7}$$

The axial-vector currents $N_1(\bar{\psi}_1\gamma^\mu\gamma^5\psi_1)$ satisfy Ward identities which can be derived in the usual way. With the notation

$$X = \prod_{j=1}^{N_1} \psi_1(x_j)\bar{\psi}(y_j) \prod_{j=1}^{N_2} \psi_2(z_j)\bar{\psi}_2(w_j)$$

we have, for example,

$$\begin{aligned}\partial_x^\mu \langle TN_1(\bar{\psi}_1\gamma^\mu\gamma^5\psi_1)(x)X\rangle &= i\langle T\{N_2[\bar{\psi}_1(-i\overleftarrow{\not\partial} - M_1)\gamma^5\psi_1](x) + N_2[\bar{\psi}_1\gamma^5(i\not\partial - M_1)\psi_1](x) + 2N_2(M\bar{\psi}_1\gamma^5\psi_1)(x)\}X\rangle \\ &= 2i\langle TN_2(M_1\bar{\psi}_1\gamma^5\psi_1)(x)X\rangle - 2ia_1\langle TN_1(\bar{\psi}_1\gamma^5\psi_1)(x)X\rangle - \sum_{i=1}^{N_1}[\delta(x-x_i)\gamma^5_{x_i} + \delta(x-y_i)\gamma^{5\,T}_{y_i}]\langle TX\rangle .\end{aligned} \tag{3.8}$$

Observing that the only difference between $N_2(M_1\bar{\psi}\gamma^5\psi_1)$ and $M_1N_1(\bar{\psi}_1\gamma^5\psi_1)$ comes from the subtraction for the graph of Fig. 9, we can write

$$2iN_2(M_1\bar{\psi}_1\gamma^5\psi_1) = 2iM_1N_1(\bar{\psi}_1\gamma^5\psi_1) + a\partial^\mu N_1(\bar{\psi}_2\gamma^\mu\psi_2),$$
$$a = -g/\pi . \tag{3.9}$$

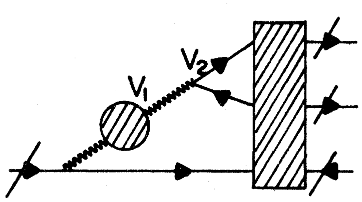

FIG. 7. The bubble represents possible contributions to the wavy line "propagator" (the two fermion lines at V_2 do not give further contributions to the propagator).

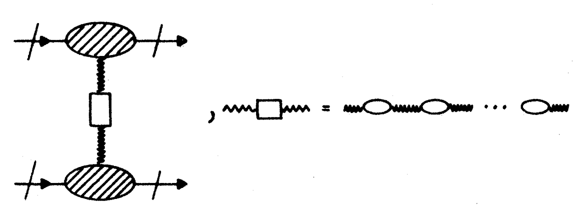

FIG. 8. Quadrilinear counterterms can be generated by the global subtractions for each proper part of this graph.

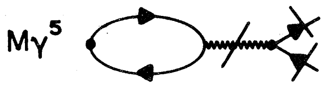

FIG. 9. Only contribution to the anomaly of the axial-vector current.

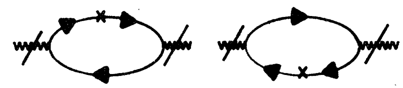

FIG. 10. Fermion loop with a mass insertion.

Now, by using

$$\partial^{\mu}\langle TN_1(\bar{\psi}_2\gamma_{\mu}\psi_2)(x)X\rangle=\sum_{j=1}^{N_2}[\delta(x-w_j)-\delta(x-z_j)]\langle TX\rangle \tag{3.10}$$

and (3.9) in (3.8) we obtain

$$\partial_x^{\mu}\langle TN_1(\bar{\psi}_1\gamma_{\mu}\gamma^5\psi_1)(x)X\rangle=2i(M_1-a_1)\langle TN_1(\bar{\psi}_1\gamma^5\psi_1)(x)X\rangle+a\sum_{j=1}^{N_2}[\delta(x-w_j)-\delta(x-z_j)]\langle TX\rangle$$
$$-\sum_{i=1}^{N_1}[\delta(x-x_i)\gamma_{x_i}{}^5+\delta(x-y_i)\gamma_{y_i}^{5T}]\langle TX\rangle\,. \tag{3.11}$$

Note the mild form (proportional to the divergence of the vector current of the ψ_2 field) of the anomalous term. If, instead of the proposed scheme, we had employed the usual BPHZ procedure,[8] we would get a hard breaking of the axial-vector current's conservation.

IV. THE RENORMALIZATION-GROUP EQUATION

The Green's functions defined in the previous section satisfy a renormalization-group equation which can be derived by using the following differential vertex operations[9] (DVO's):

$$\Delta_{1j}=i\int d^2x\,N_1(\bar{\psi}_j\psi_j)\,,\quad \bar{\Delta}_{1j}=i\int d^2x\,N_2(M_j\bar{\psi}_j\psi_j)(x)\,,$$
$$\Delta_{2j}=-\tfrac{1}{2}\int d^2x\,N_2(\bar{\psi}_j\partial\!\!\!/\psi_j)(x)\,,\quad j=1,2 \tag{4.1}$$
$$\Delta_3=i\int d^2x\,N_2[\epsilon_{\mu\nu}(\bar{\psi}_1\gamma^{\mu}\psi_1)(\bar{\psi}_2\gamma^{\nu}\psi_2)](x)\,.$$

The operators Δ_{1i} and $\bar{\Delta}_{1i}$ are not independent. Their only difference is due to the subtractions for the subgraphs of Fig. 10. However, a straightforward calculation shows that the sum of these subtractions is actually zero, and we have

$$M_i\Delta_{1i}\Gamma^{(N_1,N_2)}=\bar{\Delta}_{1i}\Gamma^{(N_1,N_2)}\,. \tag{4.2}$$

In (4.2) and hereafter $\Gamma^{(N_1,N_2)}$ denotes the one-particle-irreducible (1PI) vertex function (i.e., those corresponding to graphs that cannot be separated into disjoint pieces by cutting only one fermion line) of N_1 fields of type 1 and N_2 fields of type 2.

Using (4.1) and (4.2) we can derive, in the usual way, the relations

$$N_i\Gamma^{(N_1,N_2)}=[\Delta_{2i}-(M_1-a_i)\Delta_{1i}+g\Delta_3]\Gamma^{(N_1,N_2)}\,, \tag{4.3}$$

$$\frac{\partial}{\partial g}\Gamma^{(N_1,N_2)}=\left(\frac{\partial a_1}{\partial g}\Delta_{11}+\frac{\partial a_2}{\partial g}\Delta_{12}+\Delta_3\right)\Gamma^{(N_1,N_2)}\,. \tag{4.4}$$

Besides (4.3) and (4.4) we also have

$$\frac{\partial\Gamma^{(N_1,N_2)}}{\partial\mu}=\sum_{i=1}^{2}(\lambda_i\Delta_{1i}+\delta_i\Delta_{2i})\Gamma^{(N_1,N_2)}+\gamma\Delta_3\Gamma^{(N_1,N_2)}\,, \tag{4.5}$$

where λ_i, δ_i, and γ are power series in the coupling constant g. Equation (4.5) is obtained by noting that $(\partial/\partial\mu)\Gamma^{(N_1,N_2)}$ receives contributions only of the subtraction terms and by noting the following:

(i) Renormalization parts corresponding to contributions from the proper function of two currents do not contribute. This follows because the graph of Fig. 6 does not depend on μ and graphs of this type of higher order will have closed fermion loops linked by at least one wavy line and, as argued before, they identically vanish.

(ii) The subtractions for proper graphs with four external fermion lines cancel among themselves.

Thus the only contributions for $\partial\Gamma/\partial\mu$ come from subtractions for the vertex and fermion's self-energy graphs. Concerning the self-energy graphs, observe that they do not give contributions to DVO's of the type $\int d^2x\,N_1(\bar{\psi}\gamma^5\psi)$ or $\int d^2x\,N_2(\bar{\psi}\gamma^{\mu}\gamma^5\partial^{\mu}\psi)(x)$. To see this consider, for example, possible contributions to $\int d^2x\,N_1(\bar{\psi}\gamma^5\psi)(x)$ coming from graphs of the type shown in Fig. 11

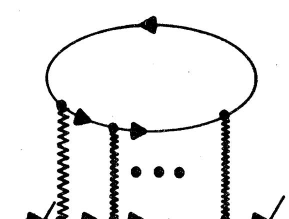

FIG. 11. A contribution to the fermion self-energy.

(in this case the diagram must be of odd order in g). These arise from the subtraction term

$$M\frac{\partial}{\partial\mu}\frac{\partial}{\partial M}I_G\bigg|_{p=M=0,s^2=\mu^2}.$$

Now if $\partial/\partial M$ acts on the propagators outside the fermion loop we obtain zero (as result of vector and axial-vector-current conservation applied to any of the vertices in the loop). On the other hand, if $\partial/\partial M$ is applied in the lines of the fermion loop the result is also zero since now the loop will have an odd number of γ matrices and the trace gives zero (graphs of this type with insertion of mass counterterms will not contribute by the same argument).

That $\int N(\bar\psi\gamma^\mu\gamma^5\partial_\mu\psi)d^2x$ does not give a contribution to $\partial\Gamma/\partial\mu$ can be seen most easily by choosing the routing of the external momentum so that it does not flow throughout the lines of the fermion loop and applying the same reasoning as before.

Using (4.3), (4.4), and (4.5) we can write the renormalization-group equation as

$$\left(\mu\frac{\partial}{\partial\mu}+\sigma\frac{\partial}{\partial g}-N_1\tau_1-N_2\tau_2\right)\Gamma^{(N_1,N_2)}=0\,. \tag{4.6}$$

The proof of (4.6) is standard.[9] We follow the usual argument: by substituting (4.3), (4.4), and (4.5) in (4.6) and by equating to zero the coefficient of each DVO we get

$$\begin{aligned}
&\mu\lambda_1+\sigma\frac{\partial a_1}{\partial g}+(M_1-a_1)\tau_1=0\,,\\
&\mu\lambda_2+\sigma\frac{\partial a_2}{\partial g}+(M_2-a_2)\tau_2=0\,,\\
&\mu\delta_1-\tau_1=0\,,\\
&\mu\delta_2-\tau_2=0\,,\\
&\mu\gamma+\sigma+g(\tau_1+\tau_2)=0\,.
\end{aligned} \tag{4.7}$$

The last three equations can be used to determine τ_1, τ_2, and σ. To show that the first two equations are then identically satisfied we use the fact that

$$\Gamma^{(2,0)}|_{\not p=M_1}=0 \text{ and } \Gamma^{(0,2)}|_{\not p=M_2}=0$$

so that

$$\begin{aligned}
&\left(\mu\frac{\partial}{\partial\mu}+\sigma\frac{\partial}{\partial g}-2\tau_1\right)\Gamma^{(2,0)}\bigg|_{\not p=M_1}=0\,,\\
&\left(\mu\frac{\partial}{\partial\mu}+\sigma\frac{\partial}{\partial g}-2\tau_2\right)\Gamma^{(0,2)}\bigg|_{\not p=M_2}=0\,,
\end{aligned} \tag{4.8}$$

and therefore

$$\begin{aligned}
&C_1\Delta_{11}\Gamma^{(2,0)}|_{\not p=M_1}+C_2\Delta_{12}\Gamma^{(2,0)}|_{\not p=M_1}=0\,,\\
&C_1\Delta_{11}\Gamma^{(0,2)}|_{\not p=M_2}+C_2\Delta_{12}\Gamma^{(0,2)}|_{\not p=M_2}=0\,,
\end{aligned} \tag{4.9}$$

where C_1 and C_2 are the left-hand side of the first two equations in (4.7), respectively. As the determinant of the coefficients in (4.9) is different from zero, then necessarily $C_1=C_2=0$.

The renormalization-group equation (4.6) shows that, although our scheme employs an auxiliary mass μ, the value taken by μ is irrelevant in the sense that changes in this parameter can be absorbed in coupling-constant and wave-function renormalizations.

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APPENDIX

In this appendix we will give a sketch of the proof of the infrared convergence of the subtracted Feynman amplitudes. We follow the notation of Ref. 6 adapted to the present situation. Let $u_1,\ldots,u_a$, $v_1,\ldots,v_{m-a}$ be an arbitrary basis of $L(\Gamma)$, the space of linear forms in p and k of a connected graph Γ, with $(\partial/\partial k)\,(u,v)\neq 0$. Furthermore, let C be a Γ forest which is complete with respect to S, the subspace of $L(\Gamma)$ spanned by $u_1,\ldots,u_a$. Then we will show that

$$\underline{\deg}_u R_\Gamma(C)+2a>0\,, \tag{A1}$$

where $\underline{\deg}_u R$ denotes the lower degree in u of R:

$$R_\Gamma(C)=(1-\tau_\Gamma^{d(\Gamma)})Y_\Gamma\,, \tag{A2}$$

with

$$Y_\Gamma(C)=I_{\bar\gamma(C)}S_\gamma\prod_\alpha f_{\gamma_\alpha}Y_{\gamma_\alpha}(C)\,, \tag{A3}$$

$$\bar\gamma(C)=\gamma/\gamma_i\cdots\gamma_n\,, \tag{A4}$$

$\{\gamma_1,\ldots,\gamma_n\}$ = set of maximal elements of C contained in γ and

$$f_{\gamma_\alpha}=\begin{cases}1-\tau_{\gamma_\alpha}^{d(\gamma_\alpha)} & \text{if } \bar\gamma_\alpha(C)\not\parallel S\,,\ \ \bar\gamma(C)\parallel S\,,\\[2ex] -\tau_{\gamma_\alpha} & \text{otherwise}\,.\end{cases} \tag{A5}$$

Since τ_γ differs from a Taylor operator only by the last subtraction (see Sec. III), i.e.,

$$\tau_\gamma^{d(\gamma)}=\tau_\gamma^{d(\gamma)-1}+x\,, \tag{A6}$$

the lemmas of Ref. 6 are somehow modified. We have the following.

Lemma 1:

(a) If $\tau^{d(\gamma)}Y_\gamma\neq 0$ then

$$\underline{\deg}_u\tau_\gamma^{d(\gamma)}Y_\gamma\geq\underline{\deg}_{\,\mathrm{up}\gamma_M\gamma}-d(\gamma)\quad\text{if }\bar\gamma\parallel S\,, \tag{A7}$$

$$\underline{\deg}_u\tau_\gamma^{d(\gamma)}Y_\gamma\geq\underline{\deg}_u Y_\gamma\text{ if }\bar\gamma\not\parallel S\,, \tag{A8}$$

$$\underline{\deg}_{\mathrm{up}\gamma_M\gamma}\tau_\gamma^{d(\gamma)}Y_\gamma\geq\underline{\deg}_{\mathrm{up}\gamma_M\gamma}Y_\gamma\,. \tag{A9}$$

(b) $\underline{\deg}_{up\gamma_M\gamma}(1-\tau_\gamma^{d(\gamma)})Y_\gamma \geq \underline{\deg}_{up\gamma_M\gamma}Y_\gamma + \max\{d(\gamma),0\}$ if $\bar{\gamma}\not\parallel S$.

Lemma 2:

$$\underline{\deg}_{up\gamma_M\gamma}Y_\gamma \geq d(\gamma)-M(\gamma) \text{ if } \bar{\gamma}\parallel S , \quad (A10)$$

$$\underline{\deg}_u Y_\gamma \geq -M(\gamma) \text{ if } \bar{\gamma}\not\parallel S . \quad (A11)$$

Lemma 3: Let λ be a maximal element of C properly contained in $\gamma\subset\Gamma$. Then the following inequalities hold:

(a) If $\tau_\lambda^{d(\lambda)}Y_\lambda \neq 0$ then

$$\underline{\deg}_u S_\gamma\tau_\lambda^{d(\lambda)}Y_\lambda \geq -M(\lambda) , \quad (A12)$$

$$\underline{\deg}_{up\gamma_M\gamma}S_\gamma\tau_\lambda^{d(\lambda)}Y_\lambda \geq d(\lambda)-M(\lambda) \text{ if } \bar{\lambda}\parallel S . \quad (A13)$$

(b) $$\underline{\deg}_{up\gamma_M\gamma}S_\gamma(1-\tau_\lambda^{d(\lambda)})Y_\lambda \geq \max\{d(\lambda),0\}-M(\lambda) \text{ if } \bar{\lambda}\not\parallel S, \bar{\lambda}\parallel S . \quad (A14)$$

Using the above lemma we can verify that if $\bar{\Gamma}\parallel S$ then

$$\underline{\deg}_u Y_\Gamma > -M(\Gamma), \quad \bar{\Gamma}\parallel S . \quad (A15)$$

However, these estimates are not strong enough to prove the result (A1). With this aim we proceed as follows:

(i) If $\bar{\Gamma}\not\parallel S$ we have

$$\underline{\deg}_u Y_\Gamma = \underline{\deg}_u I_{\bar{\Gamma}} + \sum_\alpha \underline{\deg}_u(f(\gamma_\alpha)I_{\gamma_\alpha}) . \quad (A16)$$

Now, if for some α we have $\gamma_\alpha\parallel S$, then we use

$$\tau_{\gamma_\alpha}I_{\gamma_\alpha} = \tau_1^{d(\gamma_\alpha)}I_{\gamma_\alpha} + \tau_2^{d(\gamma_\alpha)}I_{\gamma_\alpha} , \quad (A17)$$

where $\tau_1^{d(\gamma)}=\tau^{d(\gamma)-1}$ = Taylor operator of order $d(\gamma)-1$ in p^γ and M^γ at $p^\gamma=0$, $M^\gamma=0$ and $\tau_2^{d(\gamma)}$ = last subtraction terms, i.e., $\tau_2^0F(p^\gamma,M^\gamma)=F(0,\bar{\mu})$ if γ is the graph of Fig. 6, $\tau_2^0F(p^\gamma,M^\gamma,S^\gamma)=F(0,0,\mu)$ otherwise,

$$\tau_2^1F(p^\gamma,M^\gamma,S^\gamma) = p^\gamma\frac{\partial}{\partial p^\gamma}F\Big|_{p^\gamma=0,M^\gamma=0,S^\gamma=\mu} + M^\gamma\frac{\partial}{\partial M^\gamma}F\Big|_{p^\gamma=0,M^\gamma=0,S^\gamma=\mu} .$$

As $\tau_1^{d(\gamma)}$ is a Taylor operator then

$$\underline{\deg}_\mu\tau_1Y_{\gamma_\alpha} \geq \underline{\deg}_{\mu p\gamma_M\gamma}Y_{\gamma_\alpha} - d(\gamma_\alpha) + 1 > -M(\gamma_\alpha) \text{ if } \bar{\gamma}_\alpha\parallel S . \quad (A18)$$

In the last inequality lemma 2 was used.

Concerning τ_2 observe that $\tau_2Y_{\gamma_\alpha}=\tau^2\bar{Y}_{\gamma_\alpha}$, where $\bar{Y}_{\gamma_\alpha}=Y_{\gamma_\alpha}|_{s^2=\mu^2}$ only contains Taylor operators. Using $\bar{Y}_{\gamma_\alpha}$, lemmas 2 and 3 will be replaced by lemma 2 of Ref. 6 (in the adaptation of the convergence proof of Ref. 6 to our case, the graph γ_2 of that reference must be replaced by our graph of Fig. 6). In particular we will have $\underline{\deg}_{\mu p}\gamma_{\alpha_M}\gamma_\alpha\bar{Y}_{\gamma_\alpha} > d(\gamma_\alpha)-M(\gamma_\alpha)\gamma_\alpha\parallel S$ so that

$$\underline{\deg}_u\tau_2Y_{\gamma_\alpha} = \underline{\deg}_u\tau_2\bar{Y}_{\gamma_\alpha} \geq \underline{\deg}_{\mu p}\gamma_{\alpha_M}\gamma_\alpha\bar{Y}_{\gamma_\alpha} - d(\gamma_\alpha) > -M(\gamma_\alpha) . \quad (A19)$$

Substituting (A18) and (A19) into (A16) we obtain

$$\underline{\deg}_u Y_\Gamma > -M(\Gamma), \quad \bar{\Gamma}\not\parallel S . \quad (A20)$$

If none of the γ_α lies along S (i.e., if $\gamma_\alpha\not\parallel S$ for all α) then we apply repeatedly (A8) until we find some $\lambda\subset\gamma_\alpha$, $\lambda\parallel S$ where the reasoning above can be applied.

(ii) If $\bar{\Gamma}\parallel S$ we use the same argument as in (A17) to obtain

$$\underline{\deg}_u\tau_\Gamma^{d(\Gamma)}Y_\Gamma > -M(\Gamma) \text{ if } \bar{\Gamma}\parallel S . \quad (A21)$$

Equations (A15), (A20), and (A21) furnish immediately the result (A1).

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