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# Relativistic wave equations for antisymmetric tensor gauge fields

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Relativistic wave equations for totally skew-symmetric tensor gauge fields are regarded as generalized Maxwell equations. The spin content is determined for both the massive and massless case. In particular, we find that an antisymmetric tensor gauge field $A_{\mu\nu\rho}(x)$ represents a cosmological constant when massless while in the massive case it describes a spinless particle without violating the gauge invariance. This phenomenon is interpreted as a generalized Higgs mechanism.

## I. INTRODUCTION

Antisymmetric tensor gauge fields appear in a natural way in the study of dual resonance models,[1] in the dynamical theory of relativistic strings and membranes[2,3] in quantum gravity,[4,5] and in various formulations of extended supergravity.[6,7] In particular it was observed by several authors[1,2,8] that an antisymmetric tensor field subject to a gauge transformation $\delta A_{\mu\nu}(x) = \partial_\mu \Lambda_\nu(x) - \partial_\nu \Lambda_\mu(x)$, where $\Lambda_\mu(x)$ is an arbitrary vector gauge function, is suitable to describe a spin-0 particle of zero rest mass. When a mass term is present, the field $A_{\mu\nu}(x)$ describes instead a massive spin-1 particle obeying a relativistic wave equation first derived by Takahashi and Palmer.[9]

Here we wish to discuss briefly some properties of the obvious extensions of the field $A_{\mu\nu}(x)$, namely a totally antisymmetric tensor gauge field $A_{\mu\nu\rho}(x)$ subject to the gauge transformation

$$\delta A_{\mu\nu\rho}(x) = \partial_\mu \Lambda_{\nu\rho}(x) + \partial_\nu \Lambda_{\rho\mu}(x) + \partial_\rho \Lambda_{\mu\nu}(x) \qquad (1.1)$$

with $\Lambda_{\mu\nu}(x) + \Lambda_{\nu\mu}(x) = 0$. In Sec. II we discuss the simple properties of the tensor potential $A_{\mu\nu\rho}(x)$. In the massless case it propagates no degrees of freedom and the associated Maxwell tensor (2.1) represents a "cosmological constant" disguised as a gauge field. When a mass term is present, $A_{\mu\nu\rho}(x)$ describes instead a massive spin-0 particle while preserving the invariance under the gauge transformation (1.1). In Sec. III this property is dynamically interpreted as a modified Higgs mechanism not unlike the Schwinger mechanism in two-dimensional space-time. We suggest that this mechanism provides a natural explanation for the experimental absence of a Goldstone boson associated with the U(1) axial-vector current in quantum chromodynamics (QCD) and for the observed existence of a massive pseudoscalar meson in that same channel.

## II. THE GENERALIZED MAXWELL EQUATION

Consider the generalized Maxwell field strength

$$F_{\mu\nu\rho\sigma}(x) = \partial_\mu A_{\nu\rho\sigma}(x) - \partial_\nu A_{\rho\sigma\mu}(x) + \partial_\rho A_{\sigma\mu\nu}(x) - \partial_\sigma A_{\mu\nu\rho}(x) , \qquad (2.1)$$

where $A_{\mu\nu\rho}(x)$ is a totally antisymmetric tensor potential transforming according to Eq. (1.1). Unlike the electromagnetic case, the Bianchi identity $\partial_\alpha F_{\mu\nu\rho\lambda}$ + cyclic permutations $\equiv 0$ imposes no restrictions on $F_{\mu\nu\rho\sigma}(x)$ and Eq. (2.1) must be regarded as an independent definition. Moreover, in this case the dual field strength is a total divergence

$$*F(x) \equiv \frac{i}{4!}\epsilon_{\mu\nu\lambda\rho} F_{\mu\nu\lambda\rho}(x) = \frac{i}{3!}\epsilon_{\mu\nu\lambda\rho}\partial_\mu A_{\nu\lambda\rho}(x) \equiv \partial_\mu {}^*A_\mu(x) \qquad (2.2)$$

which, *in vacuo*, represents a constant background field on account of Maxwell's equation

$$\partial_\mu F_{\mu\nu\rho\sigma}(x) = 0 \rightarrow {}^*F(x) = \text{constant} . \qquad (2.3)$$

Hence there is no radiation field associated with $A_{\mu\nu\rho}(x)$ and therefore no propagation of physical particles. What is then the significance of the dual field strength $*F$? In general the constant $*F$ is not zero: There is a static effect represented by $\frac{1}{2}(*F)^2$. Indeed, in terms of $*F$ the field contribution to the energy-momentum tensor is simply

$$T^{\mu\nu} = -\tfrac{1}{2} g^{\mu\nu} (*F)^2 . \qquad (2.4)$$

Hence the tensor $F_{\mu\nu\rho\sigma}(x)$ carries no momentum but contributes to the background energy density. In particular, in the presence of a gravitational field the net result of $T^{\mu\nu}(x)$ is the addition of a cosmological term to the right-hand side of the Einstein equation.[10] An application of this interpretation was recently suggested in the context of $N=8$ supergravity[7] theory where the cosmological constant arises through the use of the gauge field $A_{\mu\nu\rho}(x)$ and is due to the spontaneous breakdown of supersymmetry.

Of course, the interpretation of the constant $\frac{1}{2}(*F)^2$ as a cosmological constant in no way restricts the coupling of the gauge field $A_{\mu\nu\rho}(x)$ to gravity alone. For instance, in the bag formulation of hadron dynamics one can consistently introduce the so-called vacuum pressure term in the Lagrangian of the quark-gluon system through the use of the gauge field $A_{\mu\nu\rho}(x)$.[11] In this case $*F(x)$ contributes to the vacuum energy density of the bag and the cosmological constant $\frac{1}{2}(*F)^2$ plays the role of the bag constant $B$.

So far we have considered the massless case. Turning to the massive field equation

$$\partial_\mu F_{\mu\nu\rho\sigma}(x) - m^2 A_{\nu\rho\sigma}(x) = 0 \tag{2.5}$$

we note that, with the definition (2.1), Eq. (2.5) is equivalent to the set of equations

$$(\Box - m^2)A_{\mu\nu\rho}(x) = 0\ , \tag{2.6}$$

$$\partial_\mu A_{\mu\nu\rho}(x) = 0\ . \tag{2.7}$$

The condition (2.7) imposes three further constraints on the four independent components of $A_{\mu\nu\rho}(x)$ leaving only one propagating degree of freedom. We will argue that this transition from a nonpropagating mode in the massless case to a propagating single degree of freedom in the massive case can be interpreted as a Higgs-type mechanism. In this connection we observe that Eq. (2.5) is not invariant under the gauge transformation (1.1). However, in terms of the dual potential $*A_\mu(x)$ defined in Eq. (2.2), Eq. (2.5) becomes

$$\partial_\alpha \partial_\mu *A_\mu(x) - m^2 *A_\alpha(x) = 0 \tag{2.8}$$

from which it follows

$$(\Box - m^2)\partial_\mu *A_\mu(x) = 0 \tag{2.9}$$

which is a gauge-invariant equation for the massive spinless field $\partial_\mu *A_\mu(x)$.

## III. DYNAMICAL MASS AND THE HIGGS MECHANISM

We can now give a dynamical interpretation to the field equation (2.9) according to the following scheme. Consider the coupling of the Maxwell tensor $F_{\mu\nu\rho\sigma}(x)$ to a matter field described by a totally antisymmetric tensor current $J_{\nu\lambda\rho}(x)$ with an arbitrary coupling strength $f$,

$$\partial_\mu F_{\mu\nu\lambda\rho}(x) = f J_{\nu\lambda\rho}(x)\ . \tag{3.1}$$

The case in which $J_{\nu\lambda\rho}(x)$ describes an extended classical source was discussed elsewhere.[3,11] The explicit form or dimensionality of the tensor current $J_{\nu\lambda\rho}(x)$ is immaterial at present but we envisage a dynamical framework whereby the current is conserved:

$$\partial_\mu J_{\mu\nu\lambda}(x) = 0 = \partial_\nu *J_\lambda(x) - \partial_\lambda *J_\nu(x)\ , \tag{3.2}$$

$$*J_\mu(x) \equiv \frac{i}{3!}\epsilon_{\mu\nu\lambda\rho} J_{\nu\lambda\rho}(x)\ , \tag{3.3}$$

in consistency with the field equation (3.1) and satisfies the "anomaly equation"

$$\partial_\mu *J_\mu(x) = \frac{i}{4!} f \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}(x) = f\partial_\mu *A_\mu(x)\ . \tag{3.4}$$

When these conditions are satisfied, the field equation (3.1) combined with the anomaly equation (3.4) leads immediately to the gauge-invariant massive Klein-Gordon equation which we previously discussed:

$$(\Box - f^2)\partial_\mu *A_\mu(x) = 0. \tag{3.5}$$

A simple realization of Eqs. (3.1), (3.2), and (3.4) is given by the Lagrangian density

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}F_\mu(x)F_\mu(x) - F_\mu(x)\partial_\mu\rho(x) + \frac{1}{2}\frac{1}{4!}F_{\mu\nu\lambda\rho}(x)F^{\mu\nu\lambda\rho}(x) \\ & - \frac{1}{4!}F_{\mu\nu\lambda\rho}(x)[\partial_\mu A_{\nu\lambda\rho}(x) - \partial_\nu A_{\lambda\rho\mu}(x) + \partial_\lambda A_{\rho\mu\nu}(x) \\ & \qquad - \partial_\rho A_{\mu\nu\lambda}(x)] \\ & + f\frac{i}{3!}\epsilon_{\mu\nu\lambda\rho}A_{\nu\lambda\rho}(x)\partial_\mu\rho(x)\end{aligned} \tag{3.6}$$

from which it follows

$$\begin{aligned}\partial_\mu F_\mu(x) = \Box\rho(x) &\equiv \frac{fi}{3!}\partial_\mu \epsilon^{\mu\nu\lambda\rho}A_{\nu\lambda\rho}(x) \\ &= f\partial_\mu *A_\mu(x)\end{aligned} \tag{3.7}$$

and

$$\partial_\mu F_{\mu\nu\lambda\rho}(x) = -fi\epsilon_{\mu\nu\lambda\rho}\partial_\mu\rho(x)\ , \tag{3.8}$$

with $F_\mu(x) = \partial_\mu\rho(x)$ and $F_{\mu\nu\rho\sigma}(x)$ as given by Eq. (2.1). It is immediately seen that the coupled equations (3.7) and (3.8) lead again to Eq. (2.9) showing that the spin-0 field $\partial_\mu *A_\mu(x)$ becomes massive as a result of the $\rho$-$A$ mixing.

In principle the tensor current $J_{\mu\nu\rho}(x)$ could be expressed in terms of spinor variables as well. This interesting possibility is at present under consideration. In this connection, however, it is worth observing that the anomaly condition (3.4) can be replaced by the more familiar requirement that ${}^*J_\mu(x)$ satisfies the canonical equal-time commutator

$$K^{-2}[{}^*J_0(x), {}^*J_k(x')] = -i\partial_k \delta(\vec{x}-\vec{x}') \quad \text{at } t=t' . \qquad (3.9)$$

Here $K$ is a standard regularization parameter with dimensions of mass which is necessary to interpret the spinor current in terms of the point-separation technique with spacelike separation. It has been shown by Takahashi[12] that the point-separation technique can be consistently accommodated in the canonical formalism. Once the currents are properly defined as point-separated currents, the result of the form (3.9) of the canonical equal-time commutator is the following form of the anomaly equation:

$$\partial_\mu {}^*J_\mu(x) = fK^2 \partial_\mu {}^*A_\mu(x) \qquad (3.10)$$

and $f$ is now a dimensionless coupling constant. Hence, when ${}^*J_\mu(x)$ is expressed in terms of fermionic variables, the underlying dynamical framework is defined either by Eqs. (3.1) and (3.9) or by Eqs. (3.1) and (3.10). Of course, the result of combining Eq. (3.1) with Eq. (3.10) is again the gauge-invariant massive Eq. (2.9).

All we have said so far is a straightforward (albeit unusual) generalization from two dimensions where the ordinary vector potential $A_\mu(x)$ plays the same role as the tensor potential $A_{\mu\nu\rho}(x)$ in four dimensions. Indeed, in 1+1 dimensions Eqs. (3.1)–(3.5) and the commutator (3.9) all reduce to the well-known relations of the Schwinger model. As a matter of fact the Lagrangian system (3.6) *is* a four-dimensional generalization of the Schwinger model in the boson formulation

$$\begin{aligned}\mathcal{L}(2\,\text{dim}) = &-\tfrac{1}{2}\partial_\mu\rho(x)\partial_\mu\rho(x) + \frac{1}{2\times 2!}F_{\mu\nu}(x)\\ &-\frac{1}{2!}F_{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)\\ &+fi\epsilon_{\mu\nu}A_\nu(x)\partial_\mu\rho(x)\end{aligned} \qquad (3.11)$$

and one readily checks that the Lagrangian system (3.11) leads again to Eq. (2.9) in two space-time dimensions. The reader will notice that in both systems (3.6) and (3.11) the dimensional coupling constant $f$ sets a mass scale in the model and therefore it is hardly surprising that we get a mass term in the physical spectrum. However, what seems more significant to us is the fact that in both cases the nonpropagating fields $A_\mu(x)$ and $A_{\mu\nu\rho}(x)$ give rise to a spin-0 particle while at the same time the massless boson $\rho(x)$ is eliminated from the physical sector. This suggests the following Higgs-type mechanism for the gauge field $A_{\mu\nu\rho}(x)$: The system (3.8) not only is invariant under the generalized gauge transformation (1.1) but, in addition, possesses the familiar symmetry

$$\rho(x) \to \rho(x) + \text{const}. \qquad (3.12)$$

The associated conserved current is

$$j^5_\mu(x) = \partial_\mu\rho(x) - \frac{fi}{3!}\epsilon_{\mu\nu\rho\sigma}A_{\nu\rho\sigma}(x) \qquad (3.13)$$

which is gauge variant. Here $\rho(x)$ plays the role of a pseudoscalar Goldstone boson. On the other hand, the gauge-invariant current

$${}^*J_\mu \equiv J^5_\mu(x) = \partial_\mu\rho(x) \qquad (3.14)$$

has a nonvanishing divergence which, on account of the equation of motion (3.7), is identical to the anomaly equation (3.4). Because of the anomaly there is a $\rho$-$A_{\mu\nu\rho}$ mixing and the Lagrangian must be diagonalized before the physical spectrum is obtained. As we have seen, the would-be Goldstone boson is absorbed in the scalar mode of the gauge field ${}^*A_\mu(x)$ and one obtains a massive spin-0 boson thus evading the Goldstone theorem.

The phenomena just described by the simple Abelian model for $A_{\mu\nu\rho}(x)$ bear a close resemblance to the actual phenomena occuring in the U(1) sector of QCD. This was recently observed by Aurilia, Takahashi, and Townsend[13] and here we briefly summarize the points which are relevant for this paper. We refer specifically to the following aspects of the U(1) problem: (i) the nonexistence of the U(1) Goldstone boson required by the Ward-Takahashi identity in the chiral symmetry limit and (ii) the observed existence of a massive pseudoscalar meson, the $\eta'$, in that same U(1) channel.

Since the $\eta'$ is a spin-0 boson the usual Higgs mechanism is not applicable in this case. On the contrary the Higgs mechanism for the gauge field $A_{\mu\nu\rho}(x)$ is perfectly suited to describe the situation at hand. From the point of view advocated in this paper, the Higgs mechanism can be traced back to the single property that, unlike the electromagnetic case, the anomaly terms $\epsilon_{\mu\nu}F_{\mu\nu}(x) = \partial_\mu {}^*A_\mu(x)$ (in two dimensions) and $\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}(x) = \partial_\mu {}^*A_\mu(x)$ (in four dimensions) are in both cases proportional to the dual field strength of the "Maxwell tensor." Although the QCD anomaly term is nonlinear in the Yang-Mills (YM) fields it can be written in the form[14]

$$\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x) \equiv K^2\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}(x), \qquad (3.15)$$

where as usual $F_{\mu\nu\rho\sigma}(x) = \partial_\mu A_{\nu\rho\sigma}(x) + 3$ terms and $A_{\mu\rho\sigma}(x)$ is an antisymmetric Abelian gauge field transforming as in Eq. (1.1) under non-Abelian gauge transformations of the YM potential. The constant $K$ in (3.15) has dimensions of mass so that the field $A_{\nu\rho\sigma}(x)$ also has dimensions of mass. In view of the topological significance of the anomaly term (3.15) there are no degrees of freedom associated with $A_{\mu\nu\rho}(x)$ in perturbation theory. It is, however, an underlying assumption of the effective Lagrangian approach[15] to QCD that $A_{\mu\nu\rho}(x)$ develops a one-particle pole to leading order in the $1/N$ expansion and behaves therefore as a fundamental field in the effective Lagrangian. Under such circumstances the anomaly term in the effective Lagrangian is again linear in $F_{\mu\nu\rho\sigma}(x)$ and the Higgs mechanism is activated in exactly the same way as we have described in this paper.


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