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Clue to the unification of gravitation and particle physics

Paul S. Wesson
Department of Physics, University of Alberta, Canada
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Astrophysical data reveal the existence of a (broken) symmetry of the gravitational interaction implying the existence of a new dimensional constant $p \simeq 8\times 10^{-16}$ $\mathrm{g^{-1}\,cm^2\,sec^{-1}}$. The existence of this constant provides a clue to the unification of gravitation and particle physics.

I. INTRODUCTION

The currently accepted criterion for the true unification of two or more of the four known interactions of physics is that the resulting unified theory should contain *one dimensionless* coupling constant.[1] This criterion has been applied to the interactions of particle physics in a convincing way. By connecting together a series of reasonable hypotheses, it has been shown[1] that the strong, electromagnetic, and weak interactions are probably[2] describable in terms of the group SU(5)[3,4]. The unique dimensionless coupling constant is the fine-structure constant $\alpha \equiv e^2/\hbar c$ (e = electron charge, $\hbar$ = Planck's constant divided by 2π, and c = velocity of light). The remaining interaction, gravitation, has not yet been included in the picture, although many attempts have been made on the problem.[5] The object of what follows is to discuss a facet of this problem that can most aptly be described as a clue to the unification of particle physics and gravitation. The arguments that follow at best suggest a way of achieving unification and at worst lead to the recognition of a numerical coincidence which indicates that unification is possible.

The treatment adopted is similar to that used for the particle-physics case.[1] That is, it will be heuristic (intended to guide rather than prove) and proceed by connecting together a series of hypotheses to formulate a concrete test (Sec. II). The tested relation appears to be valid (Sec. III), and the numbers indicate that unification is realizable (Sec. IV). The conclusion (Sec. V) gives a short account of the implications of the results of previous sections.

II. PARTICLES AND GRAVITATION

Particle physics involves three constants with the dimensions $[e]=M^{1/2}L^{3/2}T^{-1}$, $[h]=ML^2T^{-1}$, and $[c]=LT^{-1}$. These form a complete set in the sense that only one dimensionless number $[\alpha]=0$ can be formed from them. Gravitation as it has been developed so far involves two constants, the Newtonian gravitational parameter $[G]=M^{-1}L^3T^{-2}$ and c. These two constants do not by themselves form a set, since no dimensionless constant can be formed from less than three dimensional constants, provided the latter have different dimensionalities.

It is a fact that all physics can be described in terms of quantities having dimensions composed of the three basic dimension-defining parameters M, L, and T. This is usually presented as an empirical fact of physics, but it can also be viewed as a realization in Nature of a property of elementary group theory.[6] There is nothing fundamental about the choice of the dimension-defining parameters to be a length, a time, and a mass. They can be equally well defined as parameters with compound dimensions, and the physical constants e, h, c, and G then have different dimensions from the ones noted above. But if one does change the three basic parameters (perhaps for reasons of mathematical convenience), and so changes the dimensions of e, h, c, and G, then the physical content of these constants does not change. In particular, changing the dimension-defining parameters does not change the number of physical constants. Thus, for particle physics and gravitation, there are two irreducible sets of three and two constants, respectively (e, h, c and G, c). Together, they give a set of four distinct dimensional constants. In what follows, the conventional dimensions of these constants (noted above) will be retained.

Given four distinct constants, it is possible *in principle* to combine some of them so as to form constants with the simple dimensions of M, L, or T. For example, the two constant masses formable from the four constants noted above are $(e^2/G)^{1/2}$ and $(\hbar c/G)^{1/2}$. These would be expected to play fundamental roles in a classical unified theory of electromagnetism and gravity, and a quantum theory of gravity, respectively. At present, no generally accepted theory of either kind exists, although it is widely assumed that acceptable theories could be formulated if one knew how. But, this does *not* mean that the two constant masses just mentioned will play the most fundamental roles in a grand unified theory that combines the forces of particle physics and gravity. (This and the other comments made in this paragraph apply also to constant lengths and times formable from the four constants noted above.) On the contrary, progress so far towards a grand unified theory indicates[1,5] that constants with simple dimensions like those just mentioned will *not* play the most fundamental roles in such a theory. Rather, the *dimensionless* ratio of the two mentioned constant masses (in the form $\alpha \equiv e^2/\hbar c$) is expected to play the most fundamental role. In other words, if one is to find a way to a grand unified theory, it is expected to involve combinations of dimensional physical constants that do not yield other dimensional constants but rather dimensionless constants.

This inference gains most of its support from particle physics. (For example, the constant α can be formed as the ratio of two masses, as above, but the result does not involve the G typical of gravitational physics; see below.) Theoretically, more work has been done towards a unification of the three forces of particle physics than towards a unification of these with gravity. Empirically, the data which support the inference just outlined consist of scaling laws and similar relations whose validity is undisputed. However, the inference that a grand unified theory should have as its most fundamental parameters not dimensional but dimensionless constants gains support also from gravitational physics. Theoretically, there has recently been very significant progress towards a scale-invariant theory of gravity (see Ref. 5, which is a detailed account of scale-invariance in gravitation and particle physics and how these relate to each other). Empirically, the data which support the inference consist of indications that the Universe over long distances has a scale-free property. From observations, many different systems of astrophysical and cosmological size are known, but there is no evidence for a unique, constant length. According to general relativity, the Universe might in principle have a finite radius of curvature (R_*, where $[R_*] = L$); or be influenced in its dynamical evolution by a force due to a finite cosmological constant (Λ, where $[\Lambda] = L^{-2}$ or T^{-2} depending on how Einstein's field equations are set up). However, observations show that the values of R_*^{-1} and Λ are both close to, and probably equal to, zero.[5,7] This is another way of saying that there is no evidence for a unique constant with the dimension L in gravitation. Indeed, in cosmology the absence of a unique constant L is one way of paraphrasing the cosmological principle.[8] Similar comments can be given regarding the absence of unique constants with the dimensions M or T. Thus, the observed Universe appears to be scale-free, which means that it can be described in terms of dimensionless parameters, as indeed it has been.[8]

The comments of the two preceding paragraphs are important in the search for a way to unify particle physics and gravitation. The conclusion one draws from then is that the most promising way to unification, as far as the physical constants are concerned, is the following: *Combine the dimensional constants of particle physics and gravitation together in such a manner that all relevant constants are involved but only in terms of combinations that are dimensionless.*

It is to be hoped, of course, that this can be done in such a manner that only one dimensionless (combined) constant results.[1] It was noted in the first paragraph of this section that the three known constants of particle physics form a complete set (in that one dimensionless constant can be formed from them), but that the two known constants of gravitation do not. This difference suggests the following as a possible way to unification: By analogy with the case of particle physics, introduce a *new* constant for the case of gravitation to complete the set of constants for the gravitational interaction.

Let this new constant be denoted p. The dimensions of G and c, in conjunction with the requirement that G, p, and c shall form a complete set (i.e., shall be combinable to form one dimensionless constant), fix the dimensions of p. Thus $[p] = M^{-1}L^2T^{-1}$. So far the argument is abstract. But if p really exists it will have to make itself known as an observable symmetry property in physics, whose nature will be connected with the dimensions of p. (This situation applies for the other constants of physics.) In the present case, the existence of p is equivalent to saying that the following relation holds:

$$J = pM^2 . \tag{1}$$

Here, J is the angular momentum and M the mass of a rotating system in which gravity is the only interaction.

The relation (1) is a concrete prediction based on a series of plausible hypotheses. The fact that the dimensions of G and c (which initiated the argument) are actually arbitrary does not affect the main results of this line of reasoning, which are that a new constant such as p can be introduced and that its existence implies a relation such as (1). It should, though, be noted that if p exists then the theory of gravity which incorporates the full set of constants (G, p, c) must be a broader one than general relativity (which would be recovered in the limit $p \to 0$). While no theory which explicitly incorporates G, p, and c is yet known, it is not difficult to see what kind of theory will be involved.[9] The Einstein-Cartan-Weyl theory of gravity[10] may provide a framework for the incorporation of three constants.

To sum up: (i) There are two sets of dimensional constants, one for particle physics (e, h, c) and one for gravitation (G, c); (ii) theoretical and observational considerations imply that *these sets cannot be mixed;* (iii) to connect the two subjects, one can argue by analogy with the particle physics set that the gravitational set needs to be completed by the addition of a new dimensional constant p, where $[p] = M^{-1}L^2T^{-1}$; (iv) the existence of a constant p with the noted dimensions implies the rule $J = pM^2$ for the angular momenta J of rotating systems with masses M. The validity of this line of argument can obviously be decided by investigating the prediction $J = pM^2$ which is its end point.

III. THE $J = pM^2$ RELATION

The relation (1) can be tested. Before doing this, two comments are in order about this relation.

Firstly, (1) does not *conflict* with established gravitational theory (Newtonian or Einsteinian). Within the framework of established theory, (1) is a statement of a symmetry property which picks out the form $J = pM^2$ from the infinite number of choices for $J = J(M)$ that are allowed. However, as noted in Sec. II above, the existence of a relation such as (1) implies a need for a widening of gravitational theory, beyond that which is established, if (1) is to be incorporated into the theory at a basic level.[9]

Secondly, (1) will only hold exactly when gravity is the only force present. Most systems are governed in their physical characteristics by a mixture of forces. Gravity is the main interaction for systems of planetary and larger size, but even so the effects of solid-state forces, viscosity, pressures, etc., are often not negligible.[7] Such nongravitational forces will tend to camouflage the simple $J = pM^2$ relation. In a plot of $\log_{10}J$ vs $\log_{10}M$ data, one will therefore expect to find a line of slope 2 but with a finite spread due to the influence of nongravitational forces as well as observational uncertainties. The relation will emerge most clearly for systems where gravity is dominant, meaning systems of astrophysical size.

A large amount of data for J and M for astrophysical systems was collated in Ref. 11. The class of data for smallest M related to asteroids. But since the original asteroid data were collected, it has become clear[12] that individual asteroids are notably irregular in shape and are not near hydrostatic equilibrium like the planets. In other words, solid-state forces play a significant role in holding asteroids together, and the force of gravity is not dominant to the same degree as it is for larger-M systems. In view of this situation and the second comment above, asteroids will be left out of the present compilation. [A statement about the validity of relation (1) and the appearance of Fig. 1 below when asteroids are included is given in Ref. 13.] Confining attention to systems for which gravity is dominant, currently available observations allow one to test (1) with a set of 26 data on J, M. These data are presented in Fig. 1.

The relation $J = pM^2$ of (1) is confirmed reasonably well by the data. The slope of the plot of $\log_{10}J$ vs $\log_{10}M$ is 1.98 (±0.04, mean error), in excellent agreement with the predicted value of 2. The value of p is more uncertain, being given by $\log_{10}p = -15.1$ (±0.9, mean error). For the purpose of Sec. IV below the mean value

$$p = 8\times10^{-16}\ \mathrm{g^{-1}cm^2sec^{-1}} \qquad (2)$$

will be taken.

The fact that the relation (1) predicted by the arguments of Sec. II is obeyed by gravity-dominated systems represents strong support for the correctness of those arguments. But even if those arguments should prove invalid, the relation $J = pM^2$ revealed by Fig. 1 still represents a significant result for astrophysics.

IV. UNIFICATION

To say something meaningful about unification, it is necessary to have values of dimensionless

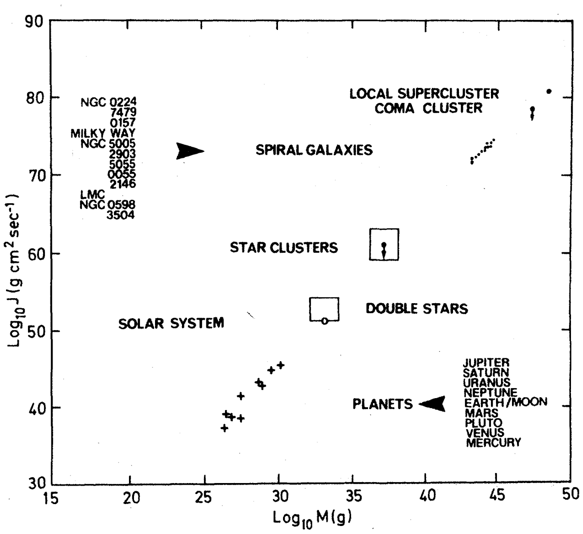

FIG. 1. A test of the $J = pM^2$ relation. The data are taken from Ref. 11.

coupling constants for particle physics and gravitation (Sec. I). Let the group for particle physics be denoted G_p. It is not established what G_p actually is [although SU(5) is favored[1,2]], but its dimensionless constant is certainly $\alpha \equiv e^2/\hbar c \simeq 1/137 \simeq 7\times10^{-3}$. Let the group for gravitation be denoted G_g. It is not known what G_g actually is [although it will almost surely contain the group GL(4, R) of general coordinate transformations as a subgroup,[10] in order to incorporate covariance and yield general relativity in some limit], but its dimensionless constant is certainly $\beta \equiv G/pc$ if one admits the existence of the new constant p (Sec. III). The value of p is uncertain by about an order of magnitude, but, taking the mean value given by (2), the value of β is $\beta \simeq 3\times10^{-3}$. Thus there is a numerical coincidence

$$\alpha \simeq \beta, \quad \text{to an order of magnitude.} \tag{3}$$

This near coincidence involves empirical values of α and β. Factors of order unity (such as π) connected with group theory might of course have to be included when the group structure underlying α and β becomes understood; but this will not upset the near equality (3). In view of this latter possibility and the observational uncertainty in the value of p, (3) may be an exact equality for the group constants of G_p and G_g.

What (3) shows, therefore, is that the dimensionless coupling constant for particle physics is the *same* as the dimensionless coupling constant for gravitation, to within present uncertainties. If it were not for (3), there would be "too many" constants in physics: There would be four distinct constants (e, h, c, and G) instead of two equivalent sets having three constants each (e, h, c and G, p, c), and it would then be difficult to understand why the Universe is scale-free in the sense discussed in Sec. II. The result (3) is remarkable in that it indicates that Nature is, after all, economical of constants: There appears to be only *one* dimensionless coupling constant in an absolute (i.e., numerical) sense, which just happens to turn up in two different (physical) guises.

V. CONCLUSION

The particle physics group G_p has a dimensionless coupling constant $\alpha \equiv (e^2/\hbar c)$. Arguing by analogy with particle physics, a case can be made for the existence of a third dimensional constant p for gravitational physics (Sec. II). The existence of p entails the angular momentum/mass law $J = pM^2$, and the validity of this relation as a (broken) symmetry of the gravitational interaction is established by astrophysical data (Sec. III and Fig. 1). The gravity group G_g can now be characterized by a dimensionless coupling constant β ($\equiv G/pc$). The numerical sizes of α and β are equal to within observational uncertainty (Sec. IV). The near equality of α and β can be interpreted as being of either greater or lesser significance for unification.

(i) The fact that $\alpha \simeq \beta$ suggests that G_p and G_g are the same group. If this is so, it means that the arguments of Sec. II provide a way of unifying particle physics and gravitation. These two subjects may, in order words, be different representations of the same group. Their apparent difference may be merely a result of the (substantial) difference in the mathematical languages that have been hitherto used in describing them. The language mismatch at present is considerable[5,10,14], but there are signs that a link might be made,[15] and if G_p and G_g are indeed the same group then it should be feasible to base both particle physics and gravitation on a single representation of it.

(ii) It is possible (but unlikely) that the near equality of α (for G_p) and β (for G_g) is an accident. (Or, that α and β are not really equal, as might be the case if β has been evaluated wrongly due to some unknown systematic error in the data of Fig. 1.) In this instance, G_p and G_g cannot be the same group, and a way of unification has yet to be found. However, the existence of α and β does nevertheless indicate that unification is possible. The reason is that in principle it is possible to find a larger group G which contains G_p and G_g (it may be just the direct-product group $G = G_p \otimes G_g$). This interpretation, though weaker than (i) above, would nevertheless represent an encouraging result. Thus the least that can be said for what has been presented here is that it shows that the unification of particle physics and gravitation is possible.

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[1]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[2]The second most likely group is SU(6), but there are problems with this. For reviews of the status of SU(6) see Ref. 3, pp. 184–194, and Ref. 4, pp. 227–238.
[3]D. B. Lichtenberg, *Unitary Symmetry and Elementary Particles* (Academic, New York, 1970).
[4]A. Pais, Rev. Mod. Phys. 38, 215 (1966).
[5]P. S. Wesson, *Gravity, Particles, and Astrophysics*

(Reidel, Dordrecht, Holland, 1980).
[6]G. Birkhoff, *Hydrodynamics* (Princeton University Press, Princeton, New Jersey, 1950).
[7]P. S. Wesson, *Cosmology and Geophysics* (Hilger, Bristol, England, 1978).
[8]P. S. Wesson, Astron. Astrophys. 68, 131 (1978).
[9]The constant p implies that a theory of gravity which incorporates this constant will be one which attributes a more fundamental importance to rotation (or torsion) than does general relativity. The Einstein-Cartan theory is such a theory. Also, scale invariance will be required, and this can be introduced as a gauge invariance. The Weyl form of gauge invariance is suitable for this. An account of the Einstein-Cartan-Weyl theory is given in Ref. 10.
[10]A. Salam, in *Fundamental Interactions in Physics*, edited by B. Kursunoglu and A. Perlmutter (Plenum, New York, 1973), pp. 55–82.
[11]P. S. Wesson, Astron. Astrophys. 80, 296 (1979).
[12]*Comets, Asteroids, Meteorites* (International Astronomical Union Coll. 39), edited by A. H. Delsemme (University of Toledo Press, Ohio, 1977).
[13]If one includes asteroid data in Fig. 1 of the text, one obtains a diagram essentially the same as Fig. 1 of Ref. 11. A line drawn through the data points then has slope 1.87 (± 0.14) and the value of p is given by $\log_{10} p = -13$ (± 3). Here, the errors are derived from envelope straight lines drawn between the data of highest and lowest M (i.e., they are maximum expected errors). When the data are weighted according to the reliability of the observations concerned, the slope is 1.9 (± 0.05) and $\log_{10} p = -15$ (± 1), where the errors are now mean errors. The latter values were quoted in Ref. 11.
[14]B. Kursunoglu, in *Symmetry Principles at High Energies*, edited by B. Kursunoglu and A. Perlmutter (Freeman, San Francisco, 1964), pp. 20–32.
[15]The relativistic generalization of SU(6), that is, SL(6, C), has been linked with the Einstein-Cartan-Weyl theory (mentioned in Ref. 9 and Sec. II of the text) by Salam (see Ref. 10).