Neutrino-oscillation thought experiment

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We propose a neutrino-oscillation experiment in order to demonstrate the significance of new *CP*-violating phases present in the lepton mixing matrix with massive Majorana neutrinos.

The possibility that neutrinos may have mass is in the air.[1] A large set of theories when classified according to the weak-electromagnetic gauge group SU(2)× U(1) may be designated by (n, m). Here n is the number of "generations" [number of neutrinos belonging to SU (2) doublets] and m is the number of SU(2)-singlet neutrinos. An important question for understanding the pattern of leptonic weak interactions is: What is the analog of the Kobayashi-Maskawa-Cabibbo hadronic matrix for leptons? In Ref. 2, a parametrization of this leptonic matrix for the general case was given. When the number m of singlet neutrinos is nonzero there is the amusing possibility[2,3] that neutral-current reactions induced by neutrino beams would oscillate with distance from the source. It was also pointed out[4] that even when $m=0$ there is a new feature in that the theory possesses more CP-violating phases than are expected on the basis of analogy to the hadronic theory. This $m=0$ case is a very interesting one because most treatments of grand unified models use the Gell-Mann–Ramond–Slansky mechanism[5] which effectively freezes out the singlet neutrino fields by making them very heavy. Then one has approximately a theory similar to (n, o) at low energies.

In an (n, o) model the free-neutrino Lagrangian is

$$\mathcal{L}_{\text{free}} = -\sum_{\alpha=1}^{n} \left[i\nu_\alpha^\dagger \sigma_\mu \partial_\mu \nu_\alpha + \tfrac{1}{2}(\nu_\alpha^T \sigma_2 \nu_\alpha m_\alpha + \text{H.c.})\right] \qquad (1)$$

and the charged leptonic weak interaction is

$$\mathcal{L}_{\text{int}} = ig2^{-1/2} W_\mu^- \sum_{\alpha,\beta=1}^{n} \bar{e}_{L\alpha}\gamma_\mu K_{\alpha\beta} \begin{pmatrix} \nu_\beta \\ 0 \end{pmatrix} + \text{H.c.}. \qquad (2)$$

The notation and detailed discussion is given in Sec. III. of Ref. 2. Note that the ν_α are taken to be van der Waerden spinors and we are working always in a γ_5-diagonal representation of the Dirac algebra. m_α is the mass of the neutrino of type α. The lepton mixing matrix K is a unitary $n\times n$ matrix which can be simplified by multiplying the electron fields in (2) by arbitrary phases (noting that the free-electron Lagrangian is invariant under this transformation) and using these phases to cancel phases in K. However, the free-neutrino Lagrangian (1) is not invariant under phase transformation of the neutrino fields. This means that fewer phases in K can be canceled than in the hadronic case. In particular K has $n(n-1)/2$ angles and $n(n-1)/2$ phases [$(n-1)$ of these would not be present in the hadronic case] and may be explicitly written[2] as

$$K = \prod_{a<b} \omega(\eta_{ab}). \qquad (3)$$

The $\omega(\eta_{ab})$'s are "complex rotations" in the (ab) planes. For two generations, with $\eta_{12}=\eta e^{i\theta}$ we have

$$K = \begin{pmatrix} \cos\eta & e^{i\theta}\sin\eta \\ -e^{-i\theta}\sin\eta & \cos\eta \end{pmatrix}. \qquad (4)$$

The presence of the phase θ in (4) means that CP-violation effects[6] are to be expected already in a leptonic theory with only two generations.

The most immediate question, which we shall discuss in the present note, is whether these new phases are (at least in principle) physically observable or whether they could somehow be shown to cancel out of the predictions for observable processes. Where can one look for new effects? The new phases come about because the symmetry corresponding to individual-lepton-number conservation is broken by the Majorana mass term. Therefore, we should look at lepton-number-nonconserving processes. Imagine the following "thought experiment:" A beam of positive leptons of type $\bar{a}$ ($\bar{1}=e^+$, $\bar{2}=\mu^+$, etc.) is incident upon a neutron target. We are interested in the W-boson-mediated reaction $e_a^+ + n \to \nu_\alpha + p$. After heavy shielding, which discriminates against particles other than neutrinos, the neutrinos emitted in a given direction are allowed to hit another neutron target at time t. Here we are interested in the reaction $\nu_\alpha + n \to e_b^- + p$. Then the amplitude for the overall process would be proportional to

$$A_{\bar{a}b}(t) = \frac{1}{E}\sum_{\alpha=1}^{n} K_{a\alpha}K_{b\alpha}m_\alpha e^{-iE_\alpha t}, \qquad (5)$$

where E is the beam energy and $K_{a\alpha}$ is the mixing matrix in (2). The derivation of (5) will be discussed later. One can imagine a similar experiment with an incident negative lepton of type a and a final positive lepton of type $\bar{b}$. This should be described by an amplitude factor

$$A_{a\bar{b}}(t)=\frac{1}{E}\sum_{\alpha=1}^{n}K^{*}_{a\alpha}K^{*}_{b\alpha}m_{\alpha}e^{-iE_{\alpha}t} \quad . \tag{6}$$

For comparison we also give the lepton-number-conserving (usual neutrino-oscillation) amplitude factors computed in the same way:

$$A_{ab}(t)=\sum_{\alpha=1}^{n}K^{*}_{a\alpha}K_{b\alpha}e^{-iE_{\alpha}t}, \tag{7}$$

$$A_{\bar{a}\bar{b}}(t)=\sum_{\alpha=1}^{n}K_{a\alpha}K^{*}_{b\alpha}e^{-iE_{\alpha}t} \quad . \tag{8}$$

The amplitude factors in (5) and (6) have been divided by E to make them dimensionless and to show that they are suppressed by $m_\alpha/E \lesssim 10^{-5}$ compared to (7) and (8). In each case the probability factors are given by

$$I(t)=|A(t)|^2 \quad . \tag{9}$$

It has been previously pointed out[7] that the new phases will not affect (7) and (8). This is clear since the new phases in K arise from the impossibility in the Majorana case, of rephasing the neutrino fields, or equivalently of transforming

$$K_{a\alpha}\rightarrow K_{a\alpha}e^{i\phi_\alpha} \quad . \tag{10}$$

The replacement (10) leaves (7) and (8) unchanged but clearly affects the new oscillations given by (5) and (6). As an explicit example let us consider the two generation model with K given by (4). Then the probability factor for a positron to produce an electron is

$$I_{\bar{1}1}(t)=\frac{m_1m_2}{E^2}\left\{\frac{m_1}{m_2}c^4+\frac{m_2}{m_1}s^4 +2s^2c^2\cos[(E_1-E_2)t+2\theta]\right\} \quad , \tag{11}$$

while the probability factor for an e^+ to produce a μ^- is

$$I_{\bar{1}2}=\frac{m_1m_2}{E^2}c^2s^2\left\{\frac{m_1}{m_2}+\frac{m_2}{m_1} -2\cos[(E_1-E_2)t+2\theta]\right\} \quad . \tag{12}$$

In (11) and (12) c stands for $\cos\eta$ and s for $\sin\eta$. It is very clear that the CP-violating phase θ is an *observable* since it enters directly into the argument of the oscillating factor. Equations (5) to (8) display the following properties[8] (only the ones involving $A_{\bar{a}b}$ and $A_{\bar{a}b}$ are new):

$$\begin{aligned}&A_{\bar{a}\bar{b}}(t)=A_{ba}(t),\quad A_{a\bar{b}}(t)=A_{b\bar{a}}(t),\quad A_{\bar{a}b}(t)=A_{\bar{b}a}(t) \quad ,\\ &A_{\bar{a}\bar{b}}(t)=A^{*}_{ab}(-t),\quad A_{\bar{a}b}(t)=A^{*}_{a\bar{b}}(-t) \quad .\end{aligned} \tag{13}$$

Equations (13) hold as a result of CPT invariance. If the assumption of T or CP invariance is made giving a real K we have

$$\begin{aligned}&A_{ab}(t)=A_{ba}(t)=A_{\bar{a}\bar{b}}(t)=A_{\bar{b}\bar{a}}(t) \quad ,\\ &A_{a\bar{b}}(t)=A_{\bar{a}b}(t)=A_{b\bar{a}}(t)=A_{\bar{b}a}(t) \quad .\end{aligned} \tag{14}$$

Let us now discuss the derivation of (5), for example. This amplitude is, for a given configuration of the final particles, the product of three factors: (i) the amplitude for an incoming type $\bar{a}$ positive lepton to produce a neutrino of type α momentum p, and spin label r, (ii) the amplitude for this neutrino to travel for a time t, and (iii) the amplitude for a type α, momentum p, and spin label r neutrino to produce a negative lepton of type b. We must sum over the intermediate neutrino types and spins. Using (2) above and Eq. (A4) of Ref. 2, we find for (i)

$$K_{a\alpha}S_\mu\bar{v}\gamma_\mu\frac{1+\gamma_5}{2}v^r(\vec{p}) \quad , \tag{15}$$

where S_μ is a kinematic factor and $v^r(\vec{p})$ is an ordinary Dirac spinor. The amplitude (ii) is $e^{-iE_\alpha t}$ and (iii) is

$$K_{b\alpha}T_\mu\bar{u}\gamma_\mu\frac{1+\gamma_5}{2}u^r(\vec{p}) \quad , \tag{16}$$

where T_μ is another kinematic factor. In forming the product of (15) and (16) and taking the sum over spin labels r we encounter an intermediate factor

$$\frac{1+\gamma_5}{2}(m_\alpha-i\gamma\cdot p)C^T\frac{1+\gamma_5^T}{2}=m_\alpha\frac{1+\gamma_5}{2}C \tag{17}$$

(C is the charge-conjugation matrix of the Dirac theory) which shows the origin of the m_α in Eq. (5). Notice that Eq. (5) contains all the α dependence of the amplitude for *any* given final configuration. Hence (assuming that all the neutrinos have very small masses so there are no kinematic effects) (5) factors out of the expression for the total amplitude. In contrast, for deriving (8) we encounter an intermediate factor

$$\frac{1+\gamma_5}{2}(m_\alpha+i\gamma\cdot p)\frac{1-\gamma_5}{2}=\frac{1+\gamma_5}{2}i\gamma\cdot p \quad , \tag{18}$$

which leads to no m_α factor for the lepton-number-conserving oscillation amplitude. Comparing (17) and (18) demonstrates, furthermore, the m_α/E suppression of the lepton-number-violating amplitudes. Such a suppression can be understood since when the m_α go to zero the lepton-number-violating amplitudes (5) and (6) must vanish. It is, furthermore, easy to see that other lepton-number-nonconserving processes, such as neutrinoless double-β decay, will have amplitudes with the same general structure. This is because our argument above is analogous to the explicit construction of the neutrino propagators.

To sum up, the extra phases [$(n-1)$ of them the n-generation model $(n, 0)$ for example] are genuine parameters of the theory. Their effects will be hard to measure since, as we have seen, they will show up in lepton-number-violating processes suppressed in intensity by about 10^{-10} compared to usual weak interactions. Nevertheless they are there.

Note added in proof. After submitting this report we learned that the subject of "neutrino-antineutrino" oscillations has a long history. However, the contents of the present paper are believed to be new. See B. Pontecorvo, Zh. Eksp. Teor. Fiz. 34, 247 (1958) [Sov. Phys.—JETP 7, 172 (1958)]; J. Bahcall and H. Primakoff, Phys. Rev. D 18, 3463 (1978); S. Pakvasa, Hawaii Report No. UH-511-409-80, 1980 (unpublished).

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[1]An up-to-date survey of the situation will be soon given in the Proceedings of the Wisconsin Mini-Conference on Neutrino Masses, 1980, edited by V. Barger and D. Cline (unpublished).

[2]J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980).

[3]V. Barger, P. Langacker, J. P. Leville, and S. Pakvasa, Phys. Rev. Lett. 45, 692 (1980); M. Gell-Mann, G. Stephenson, and R. Slansky (unpublished); D. D. Wu, Phys. Lett. 96B, 311 (1980).

[4]See Sec. III of Ref. 2. The extra phases were also noted by S. M. Bilenky, J. Hošek, and S. T. Petcov, Phys. Lett 94B, 495 (1980) and by M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasagi, Osaka Report No. OS-GE 80-23, 1980 (unpublished).

[5]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315.

[6]We define the transformation property under *CP* of the two-component spinor ν_α as $\nu_\alpha(\vec{x},t) \to -i\sigma_2\nu_\alpha^*(-\vec{x},t)$. We transform the electron fields as $e(\vec{x},t) \to -i\gamma_4 C\bar{e}^T(-\vec{x},t)$. Then (1) and (2) will both be *CP* invariant when K is real. Note that, as discussed for example in Sec. II and the appendix of Ref. 2, the free Dirac theory can be considered as the special case of Eq. (1) where $n=2$ and $m_1=m_2$. Because of this mass degeneracy, there is greater freedom to make symmetry transformations in the theory and the conventional *CP* operation interchanges $\rho_1 \leftrightarrow -i\sigma_2\rho_2^*$ (ρ_1 and ρ_2 are the two-component spinors). In our notation a three-generation model with pure Dirac neutrinos is called (3,3). However, there is an implied mass degeneracy so the theory is not "natural."

[7]See Bilenky *et al.*, Ref. 4.

[8]N. Cabibbo, Phys. Lett. 72B, 333 (1978).