Neutrino masses and mixings in gauge models with spontaneous parity violation

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(Received 8 August 1980)

Unified electroweak gauge theories based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, in which the breakdown of parity invariance is spontaneous, lead most naturally to a massive neutrino. Assuming the neutrino to be a Majorana particle, we show that smallness of its mass can be understood as a result of the observed maximality of parity violation in low-energy weak interactions. This result is shown to be independent of the number of generations and unaffected by renormalization effects. Phenomenological consequences of this model at low energies are studied. Observation of neutrinoless double-β decay will provide a crucial test of this class of models. Implications for rare decays such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$, etc. are also noted. It is pointed out that in the realm of neutral-current phenomena, departure from the predictions of the standard model for polarized-electron–hadron scattering, forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$, and neutrino interactions has a universal character and may be therefore used as a test of the model.

I. INTRODUCTION

The nature of weak interactions appears to be intimately connected with the properties of the neutrino. The celebrated $V-A$ theory[1] of charged-current weak interactions, which enjoys resounding phenomenological success at low energies, was motivated on the basis of γ_5 invariance of the Weyl equation of a massless neutrino and its generalization to charged fermions. The standard $SU(2)_L \times U(1)$ gauge model of weak and electromagnetic interactions[2] provides a sound mathematical basis for the $(V-A)$ theory of charged-current weak interactions and predicts the existence of neutral-current weak interactions which have also been confirmed[3] within present experimental accuracies. In the standard electroweak model, as in the current-current $V-A$ theories, a massless neutrino and a maximally parity-violating weak Lagrangian seem to go hand in hand.

In recent years, an alternative approach[4] to electroweak interactions has been proposed according to which the basic weak Lagrangian is invariant under space reflections, as are electromagnetic and strong-interactions. It therefore involves both $V-A$ as well as $V+A$ charged currents. The observed predominance of left-handed weak interactions[4] at low energies is understood as a consequence of the fact that vacuum is not symmetric under space reflection. More precisely, the weak Lagrangian prior to symmetry breakdown is given by

$$\mathcal{L}_{\rm wk} = \frac{g}{\sqrt{2}} (\vec{J}_{\mu L} \vec{W}_L^{\mu} + \vec{J}_{\mu R} \vec{W}_R^{\mu}),$$

where $\vec{J}_{\mu L} = \vec{J}_{\mu R}(\gamma_5 \to -\gamma_5)$ and $\vec{W}_L$ and $\vec{W}_R$ are the left- and right-handed gauge bosons, respectively. The noninvariance of the vacuum under space reflection results in $m_{W_R} \gg m_{W_L}$ and, as a result, all low-energy weak processes appear the same as in the $SU(2)_L \times U(1)$ theory, with small corrections [proportional to $(m_{W_L}/m_{W_R})^2$], undetectable in experiments performed to date.

In the left-right-symmetric models,[5] since both left- and right-handed helicities of the neutrino are included, the neutrino naturally has a mass.[6] The important question can then be raised as to why the neutrino mass is so small.[7] There appears to be a growing conviction among many physicists that a satisfactory understanding of small neutrino mass requires the neutrino to be a Majorana particle.[8-10] This point of view was, in the context of left-right-symmetric theories, advocated by us in a previous paper,[10] where we have shown that for neutrino being a Majorana particle, one can obtain the following qualitative relation:

$$m_{\nu_e} \simeq O\left(\frac{1}{m_{W_R}}\right). \tag{1.1}$$

The precise form of (1.1) depends, as we shall see, on the unknown, free parameters of the Lagrangian. We suggest a class of models where Eq. (1.1) takes naturally an interesting form

$$m_{\nu_e} = \text{const} \times \frac{m_e^2}{m_{W_R}} \tag{1.2}$$

relating the mass of the neutrino to the mass of the electron. We believe, however, that the importance of Eq. (1.1) [and (1.2)] lies not so much in the precise value of m_ν, but rather in the fact that it provides a deeper physical insight into the connection between the small neutrino mass and the maximality of parity violation. In particular, *note that in the limit of* $m_{W_R} \to \infty$, $m_{\nu_e} \to 0$ *and weak interactions become pure* $V-A$ *type*. If the recent experimental results from Irvine[11] and the Soviet Union[12] indicating a nonvanishing neutrino mass are confirmed, they would provide a support for the line of reasoning presented above.

In this paper, we analyze in detail the question of small neutrino masses and mixings in the left-right-symmetric models in which Eq. (1.1) holds true. We study the naturalness of Eq. (1.1) and extend it to include the neutrinos at higher generations, i.e., ν_μ and ν_τ. We also study the implications of our model for various low-energy weak interactions. In particular, we discuss the predictions for rare decays such as $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ and for neutrinoless double-β decay,[13] whose observation, we argue, would be a crucial test of the model.

The rest of this paper is then organized as follows: Section II describes the basic ideas behind this work, i.e., emergence of neutrino masses and their natural smallness in the particular $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory. In Sec. III we discuss the phenomenology of the model, paying special attention to the realm of neutral-current phenomena. Section IV deals with the generalization of the model to the case of three generations of fermions. Section V presents the estimates for rare processes that violate lepton number, in particular neutrinoless double-β decay, $\mu \to e\gamma$, and $\mu \to ee\bar{e}$ decays. It turns out that neutrinoless double-β decay is the most interesting prediction of the model, since its observability requires two main features of our model to hold true: Majorana character of the neutrino and reasonably small value for m_{W_R}. Finally, we summarize our work in Sec. VI. Some of the technical details of the paper are left for two appendices: in Appendix A we show how the particular choice of the Higgs sector forces neutrinos to be two-component Majorana particles. In Appendix B we discuss a major aspect of symmetry breaking in our model; we show how parity gets broken spontaneously, and more than that, how the vacuum expectation values of left-handed Higgs scalars which give masses to neutrinos are necessarily small ($\sim m_{W_R}^{-1}$). That in turn leads to the main conclusion of this paper: *in the limit* $m_{W_R} \to \infty$, *neutrino masses vanish naturally*.

II. LEFT-RIGHT SYMMETRY AND SMALL NEUTRINO MASS

In this section, we will derive Eq. (1.1) relating the small neutrino mass with the strength of the $V+A$ charged currents in left-right-symmetric models. For the purpose of simplicity, we will work in an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model.[4,10] Here, we have used the recent observation[14] that in contrast with the U(1) generator of the standard model, that of the left-right-symmetric models can be interpreted as the $B-L$ quantum number. As we show below, this observation provides physical insight[10,15] into Eq. (1.1). To see this, note that in left-right-symmetric models, the formula for the electric charge reads as follows:

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}\,. \tag{2.1}$$

Since $\Delta Q = 0$ and if we are above 100–200 GeV, $\Delta I_{3L} \simeq 0$, Eq. (2.1) leads to

$$\Delta I_{3R} \simeq -\tfrac{1}{2}\Delta(B-L). \tag{2.2}$$

This implies that breakdown of parity and breaking of local $B-L$ symmetry are related. Since for the neutrino $B=0$, Eq. (2.2) makes it clear why the neutrino ought to be a Majorana particle (since then $\Delta L \neq 0$) and in particular why its mass must have something to do with $m_{W_R}^{-1}$. An explicit realization of this intuitive picture is provided by specifying the Higgs-particle and fermion content of the left-right-symmetric model. We illustrate our procedure with one generation of fermions and extend it subsequently to include higher generations.

Fermions are assigned to left-right-symmetric representations[4] of the group as follows:

$$\begin{aligned} &\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \ (\tfrac{1}{2}, 0, -1), \quad \psi_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \ (0, \tfrac{1}{2}, -1), \\ &Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \ (\tfrac{1}{2}, 0, \tfrac{1}{3}), \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \ (0, \tfrac{1}{2}, \tfrac{1}{3}). \end{aligned} \tag{2.3}$$

To implement the physical picture outlined in Eq. (2.2) we will choose a particular set of Higgs multiplets,[16] some of which carry $B-L$ quantum numbers. The minimal set with this property is (see Appendix B for a detailed discussion of the Higgs sector)

$$\begin{aligned} &\phi \equiv (\tfrac{1}{2}, \tfrac{1}{2}, 0), \\ &\Delta_L \equiv (1, 0, 2), \quad \Delta_R = (0, 1, 2). \end{aligned} \tag{2.4}$$

Note, incidentally, that the above Higgs multiplets have the same representation content as the following bilinears in fermionic fields: $\phi \equiv \bar{\psi}_L \psi_R$ or $\bar{Q}_L Q_R$ and $\Delta_{L,i} \equiv \psi_L^T C \tau_2 \tau_i \psi_L$ and $\Delta_{R,i} \equiv \psi_R^T C \tau_2 \tau_i \psi_L$. Therefore, the conclusions of this paper are likely to remain valid even if symmetry breaking is dynamical and there are no elementary Higgs scalars.

The various stages of symmetry breaking are

$$SU(2)_L \times SU(2)_R \times U_{B-L}(1) \xrightarrow[\langle\Delta_R\rangle \neq 0,\ \langle\Delta_L\rangle = 0]{} SU(2)_L \times U(1).$$

At this stage parity as well as local $B-L$ symmetry are broken. The subsequent breakdown of $SU(2)_L \times U(1)$ down to $U_{em}(1)$ is achieved via $\langle\phi\rangle \neq 0$. Switching an $\langle\phi\rangle \neq 0$ induces a nonzero[17,18] value for $\langle\Delta_L\rangle$, but as we show in Appendix B, $\langle\Delta_L\rangle \simeq O(\langle\phi\rangle^2/V_R) \ll \langle\phi\rangle$ (also see Ref. 17).

We now proceed to discuss the fermion spectrum, paying special attention to neutrinos. One can imagine two, physically distinct situations:

(i) There is only one ϕ in the theory, whose vacuum expectation value sets the mass scale for both left-handed gauge mesons and fermions. One then attributes tiny fermion mass ($m_f \ll m_{W_L}$) to the arbitrarily chosen small Yukawa couplings. Although being the simplest alternative, we don't find this particularly appealing. As we shall see later, neutrino mass then tends to be somewhat larger than acceptable for reasonably light m_{W_R}.

(ii) It has been speculated that the small fermion mass may originate from a different mass scale than the masses of gauge bosons. This is achieved by simply postulating the existence of two ϕ's, with one of them coupling to the fermions and providing their small masses through its small vacuum expectation value. We would then have ϕ_W and ϕ_f with $\langle\phi_W\rangle \sim m_W/g$ providing the gauge-meson mass and $\langle\phi_f\rangle \sim m_f/h$ giving the masses to fermions. In this case, we can imagine $\langle\phi_f\rangle \simeq 100$ MeV, so that h need not be much smaller than g. Of course, if such models are right one still would have to explain why $\langle\phi_f\rangle \ll \langle\phi_W\rangle$. In this case, however, we obtain more reasonable values for the neutrino mass, on the order of $m_e{}^2/m_{W_R}$.

Below we analyze the implications of these two cases on the neutrino-mass question.

Case (i). The pattern of symmetry breaking that follows from the minimization of the potential takes the form (see Appendix B for details)

$$\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \quad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix},$$
$$\langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}, \tag{2.5}$$

with $\kappa' \ll \kappa$ in order to suppress $W_L - W_R$ mixing[19] and $\Delta S = 2$ Higgs-particle-induced processes[20]

$$v_R \gg \kappa,$$
$$v_L = \gamma \frac{\kappa^2}{v_R}, \tag{2.6}$$

where γ is the ratio of Higgs-particle self-couplings determined from (B13) in Appendix B.

We postpone the discussion of the gauge-meson sector to Sec. III and go directly to fermions, paying of course special attention to neutrinos. The most general Yukawa couplings are given by

$$\mathcal{L}_Y = h_1 \bar\psi_L \phi \psi_R + h_2 \bar\psi_L \tilde\phi \psi_R + h_3 \bar Q_L \phi Q_R + h_4 \bar Q_L \tilde\phi Q_R$$
$$+ i h_5 (\psi_L^T C \tau_2 \Delta_L \psi_L + \psi_R^T C \tau_2 \Delta_R \psi_R) + \text{H.c.}, \tag{2.7}$$

where $\tilde\phi \equiv \tau_2 \phi^* \tau_2$ and C is the Dirac charge-conjugation matrix. This gives rise to the following masses for charged fermions:

$$m_e = h_1 \kappa' + h_2 \kappa,$$
$$m_u = h_3 \kappa + h_4 \kappa', \tag{2.8}$$
$$m_d = h_3 \kappa' + h_4 \kappa.$$

For the ν_L, ν_R sector, we obtain

$$\mathcal{L}^{\nu}_{\text{mass}} = h_5 [v_L (\nu_L^T C \nu_L + \nu_L^\dagger C^\dagger \nu_L^*) + v_R (\nu_R^T C \nu_R + \nu_R^\dagger C^\dagger \nu_R^*)]$$
$$+ (h_1 \kappa + h_3 \kappa')(\bar\nu_L \nu_R + \bar\nu_R \nu_L). \tag{2.9}$$

The Eq. (2.9) looks like a mixture of Majorana and Dirac mass terms. The situation becomes much simpler if we rewrite (2.9) in terms of two-component spinors $\nu \equiv \nu_L$ and $N \equiv C(\bar\nu_R)^T$. Using the properties of charge-conjugation matrix

$$C^T = -C, \quad C^2 = -1,$$
and
$$C \gamma_\mu C^T = -\gamma_\mu^T \tag{2.10}$$

we easily obtain

$$\nu_R^\dagger C^\dagger \nu_R^* = -N^T C N,$$
$$\bar\nu_R \nu_L = N^T C \nu = \nu^T C N, \tag{2.11}$$

so that Eq. (2.9) can be rewritten as

$$\mathcal{L}^{\nu}_{\text{mass}} = h_5 (v_L \nu^T C \nu - v_R N^T C N)$$
$$+ (h_1 \kappa + h_2 \kappa') \nu^T C N + \text{H.c.} \tag{2.12}$$

The above expression is a significant simplification: now all the mass terms are of the Majorana type; remember that ν and N are effectively two-component complex spinors—that is most simply seen in the representation of Dirac matrices where

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We now summarize the situation with the following form of mass matrix:

$$\mathcal{L}_{\text{mass}} = (\nu^T N^T) M C \begin{pmatrix} \nu \\ N \end{pmatrix} + \text{H.c.},$$

where

$$M = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

and

$$a = h_5 v_L\,, \quad b = -h_5 v_R\,, \quad c = \tfrac{1}{2}(h_1\kappa + h_2\kappa'). \tag{2.13}$$

The eigenstates of this matrix are therefore given by

$$\nu_e = \nu \cos\xi + N \sin\xi\,,$$
$$N_e = -\nu \sin\xi + N\cos\xi\,,$$

with

$$\tan 2\xi = \frac{2c}{b-a} \simeq 2c/b\,. \tag{2.14}$$

In Appendix A we show that ν_e and N_e are Majorana spinors, i.e., they satisfy the equations that the spinors defined through the abbreviation $\psi^c \equiv C(\bar\psi)^T = \psi\; do$.[19] There we also discuss some of the useful properties of Majorana fields (the discussion there is presented in terms of manifest two-component spinors).

We now study the eigenvalues of (2.13), assuming as before $\kappa' \ll \kappa$, only for the purpose of simplicity. Now, since $b \gg a, c$, we obtain

$$m_{\nu_e} \simeq a - c^2/b\,,$$
$$m_{N_e} \simeq b\,. \tag{2.15}$$

Using Eqs. (2.6) and (2.13) one obtains for the light and heavy Majorana neutrino masses,

$$m_{\nu_e} \simeq \left(h_5\gamma + \frac{1}{4}\frac{h_1^{\,5}}{h_5}\right)\frac{\kappa^2}{v_R}\,,$$
$$m_{N_e} = -h_5 v_R\,. \tag{2.16}$$

We remind the reader that $\kappa \sim m_{W_L}/g$ and $v_R \sim m_{W_R}/g$. An important feature of the above expression is noteworthy stressing: in the limit $m_{W_R} \to \infty$ (i.e., $v_R \to \infty$), obviously $m_{N_e} \to \infty$ and $m_{\nu_e} \to 0$, in which case the weak interactions become purely left-handed. That is the main result of our paper, promised in the introduction: *the $V-A$ limit of this theory leads naturally to vanishing neutrino mass*, thus providing (at least qualitatively) a rational for the smallness of the neutrino mass. Unfortunately, the quantitative character of Eq. (2.16) is definitely less clear: h_5, h_1, and γ are free parameters of the Lagrangian. Namely, γ is an unknown ratio of various Higgs-particle self-couplings [see Eq. (B.13)], h_1 is not determined by the electron mass ($m_e \simeq h_2\kappa$ in the limit $\kappa \gg \kappa'$), and h_5 would only be determined by the value of m_{N_e}. To be specific, let us for simplicity assume the natural value $\gamma \simeq 1$ and $h_1 \lesssim h_5$. In this case it is easy to show that the ratio of m_{ν_e} and m_{N_e} is approximately given by

$$\frac{m_{\nu_e}}{m_{N_e}} \simeq \left(\frac{m_{W_L}}{m_{W_R}}\right)^2. \tag{2.17}$$

Now, even if we choose $m_{N_e} \simeq 1$ GeV, in order to obtain $m_{\nu_e} \lesssim 10$ eV one requires $m_{W_R} \gtrsim 10^4 m_{W_L}$. This is admittedly a rather large value of m_{W_R}. However, this still leads to interesting predictions for N-$\bar N$ oscillations[15] (i.e., $t_{N\text{-}\bar N} \simeq 10^5$ sec, which is well within the accessible range of present experiments).

In conclusion, in this case, for reasonably light m_{W_R} ($m_{W_R} \gtrsim 3m_{W_L}$), the natural value for neutrino mass tends to be quite larger than experimentally allowed. We should mention, though, that if γ is small (more precisely if $\gamma \ll h_1^{\,2}/h_5^{\,2}$) and if $h_1 \sim h_2$, we would obtain $m_{\nu_e} \sim m_e^{\,2}/m_{W_R}$, which is definitely a reasonable value. For example, if $m_{W_R} \gtrsim 3m_{W_L}$ (a safe lower bound)[21] we would obtain $m_\nu \lesssim 1$ eV.

Next we turn to case (ii), which, as we shall see, predicts naturally more reasonable values for the neutrino mass.

Case (ii). This is the case where gauge-boson and fermion masses originate from different mass scales. Now,

$$\langle\phi_f\rangle = \begin{pmatrix}\kappa & 0\\ 0 & \kappa'\end{pmatrix}$$

and

$$\langle\phi_W\rangle = \begin{pmatrix}\kappa_W & 0\\ 0 & \kappa_W'\end{pmatrix},$$

where $\kappa_W \sim m_{W_L}/g$ and $\kappa \sim m_f/h$ (we assume for simplicity $\kappa' \ll \kappa$, although now it is not needed since $\langle\phi_f\rangle \ll \langle\phi_W\rangle$). The fact that in this case ϕ_W doesn't couple to the fermions, means that the form of neutrino mass matrix [Eq. (2.13)] is unchanged, since $v_L \sim \kappa^2/v_R$ as before (see Appendix B). The main difference from the previous case is that a natural value for κ is now in the 100 MeV region, and will therefore lead to much smaller values for m_{ν_e}. For example, if we assume $h_1 \simeq h_2 \simeq h_5 \equiv h$ in order to be specific, we obtain

$$m_{N_e} \simeq \frac{h}{g}\, m_{W_R}\,,$$
$$m_{\nu_e} \simeq \frac{h}{g}\,\frac{m_e^{\,2}}{m_{W_R}}\,. \tag{2.18}$$

If $h/g \simeq 1$, substituting again $m_{W_R} \gtrsim 3m_{W_L}$, leads to the predictions

$$m_{N_e} \gtrsim 230\ \text{GeV}, \quad m_{\nu_e} \lesssim 1\ \text{eV}. \tag{2.19}$$

Clearly, this case leads naturally to a small m_{ν_e}. Also, the value for m_{N_e} is safe concerning the somewhat stringent bounds coming from the neutrinoless double-β decay (see Sec. V).

Before closing this section, we rewrite the new left- and right-handed doublets in terms of physical fields (mass eigenstates) ν_e and N_e,

$$\begin{pmatrix} \nu_e - \xi N_e \\ e_L \end{pmatrix} \text{ (left-handed doublet)}, \quad \begin{pmatrix} N_e + \xi \nu_e \\ e_R \end{pmatrix} \text{ (right-handed doublet)}, \tag{2.20}$$

where we have assumed Majorana condition $N = C(\bar{N})^T$ (since N satisfies Majorana equation) and the tiny mixing between ν_e and N_e is given by ($\xi \simeq \tan \xi \ll 1$)

$$\xi \simeq \frac{c}{b} \simeq -\frac{1}{2} \frac{h_1 \kappa}{h_5 v_R}. \tag{2.21}$$

It is clear from (2.20) that the right-handed currents are very small until very high energies $E > m_N$. Thus, the analysis at low-energy charged-current data[19] ceases to be useful in determining bounds on m_{W_R}. We would like to add that in this model, since the right-handed neutrino is extremely heavy, the astrophysical considerations[22] do not restrict the mass of the right-handed charged gauge boson.

III. NEUTRAL-CURRENT SIGNALS OF THE MODELS AND CONSTRAINTS ON m_{W_R} AND m_{Z_R}

Whether our approach can be experimentally distinguished from the standard model in the near future depends on the right-handed gauge-boson masses. In this section, we therefore analyze the mass spectrum for $W_L^\pm$, Z_L, $W_R^\pm$, and Z_R, the eigenstates of the gauge-boson mass matrices and remark on the constraints that follow from the available neutral-current data. As we mentioned before, charged-current data does not prove helpful in this regard due to the large mass of the heavy neutral leptons (N), which is the right-handed counterpart of ν.

Below we give the gauge-meson eigenstates and their masses. First, in the charged sector,

$$\begin{aligned} W_1 &= W_L \cos \epsilon + W_R \sin \epsilon , \\ W_2 &= -W_L \sin \epsilon + W_R \cos \epsilon , \end{aligned} \tag{3.1}$$

with

$$\begin{aligned} m_{W_1}{}^2 &\simeq \tfrac{1}{2} g^2 (\kappa^2 + \kappa'^2 + 2 v_L{}^2), \\ m_{W_2}{}^2 &\simeq \tfrac{1}{2} g^2 (\kappa^2 + \kappa'^2 + 2 v_R{}^2). \end{aligned} \tag{3.2}$$

We shall, in what follows, assume negligible W_L-W_R mixing(i.e., $\epsilon \ll 1$ or $\kappa' \ll \kappa$). In that approximation, W_L and W_R become the eigenstates of the mass matrix.

As for the neutral-gauge-meson sector, we obtain (in the limit $\kappa \ll v_R$)

$$\begin{aligned} A_\mu &= \sin \theta_W (W_{L\mu}^3 + W_{R\mu}^3) + (\cos 2\theta_W)^{1/2} B_\mu , \\ Z_{L\mu} &\simeq \cos \theta_W W_{L\mu}^3 - \sin \theta_W \tan \theta_W W_{R\mu}^3 \\ &\quad - \tan \theta_W (\cos 2\theta_W)^{1/2} B_\mu , \\ Z_{R\mu} &\simeq \frac{(\cos 2\theta_W)^{1/2}}{\cos \theta_W} W_{R\mu}^3 - \tan \theta_W B_\mu , \end{aligned} \tag{3.3}$$

where $\tan \theta_W = g'/(g^2 + g'^2)^{1/2}$ and

$$\begin{aligned} m_A &= 0, \\ m_{Z_L}{}^2 &\simeq \frac{g^2}{2} \frac{1}{\cos^2 \theta_W} (\kappa^2 + \kappa'^2 + 4 v_L{}^2), \\ m_{Z_R}{}^2 &\simeq 2(g^2 + g'^2) v_R{}^2. \end{aligned} \tag{3.4}$$

In the above expression θ_W has been defined in such a way that it can be identified with the Weinberg angle of $SU(2)_L \times U(1)$ (hence, the subscript W) i.e., $e^2 = g^2 \sin^2 \theta_W$. Also, note that the celebrated relation of the standard model $m_{W_L}{}^2 = m_{Z_L}{}^2 \times \cos^2\theta_W$ is preserved to the lowest order [compare (3.2) and (3.4)]; it gets corrections of order $\kappa^2/v_R{}^2$ and $v_L{}^2/\kappa^2$, but these corrections will be small.

Now, let us proceed to analyze the structure of effective neutral-current Hamiltonian in this model. We will show that there exists a remarkable feature of universality of strength in various neutral-current processes, which may be used as a test of the model, once the desired accuracy of experiments is reached.

To make computations simpler, we employ the method of Georgi and Weinberg.[23] Let us briefly recall their result. The effective neutral-current Hamiltonian can be written in the following form:

$$\mathcal{H}_N = \tfrac{1}{2} \sum_{ij} (\bar{f} \gamma_\mu n_i f)(\bar{f}' \gamma^\mu n_j f') M_{ij}{}^{-2}, \tag{3.5}$$

where f, f' stands for any fermions; i, j counts all the neutral generators but one (arbitrary) corresponding to any U(1) subgroup of an original gauge group;

$$n_i \equiv \frac{g_i}{C_i} \left(C_i T_i - \frac{e^2}{g_i^2} C_i{}^2 Q \right), \tag{3.6}$$

where C_i are defined from the expression for the charge

$$Q = \sum C_\alpha T_\alpha . \tag{3.7}$$

In (3.7) $\{T_\alpha\} = \{T_0, T_i\}$ (T_0 being left out of computations).

We are now equipped to present our results. We will apply the above method to the three distinct, important classes of neutral-current phenomena: neutrino interactions, parity violation in electron-quark scattering (applicable to polarized-electron-hadron scattering and parity violation in atoms), and forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$ processes.

A. Neutrino scattering

In this case one obtains for the relevant piece of the neutral-current Hamiltonian

$$\mathcal{H}^\nu = \sqrt{2} G_F \bar{\nu}\gamma_\mu(1+\gamma_5)\nu \times[(1+\alpha)\bar{\psi}\gamma^\mu(T_3 - 2Q\sin^2\theta)\psi + (1+\beta)\bar{\psi}\gamma^\mu\gamma_5 T_3\psi], \tag{3.8}$$

where

$$\alpha = -\frac{2v_L^2}{\kappa^2} + \frac{\kappa^2}{2v_R^2}, \quad \beta = -\frac{2v_L^2}{\kappa^2} \tag{3.9}$$

(recall that we work in the approximation $\kappa' \ll \kappa$).

B. Parity-violating electron-quark scattering amplitude

Using the same technique, we find that a piece of the Hamiltonian responsible for parity violation in atoms and in the SLAC experiment on polarized-electron–hadron scattering can be written as

$$\mathcal{H}^{\rm PV} = \frac{G_F}{\sqrt{2}}(1+\beta)[\bar{e}\gamma_\mu(-1+4\sin^2\theta_W)e\bar{q}\gamma^\mu\gamma_5 T_3 q - \bar{e}\gamma_\mu\gamma_5 e\bar{q}\gamma^\mu(T_3 - 2Q\sin^2\theta_W)q]. \tag{3.10}$$

Note that $(1+\beta)$ denotes the departure from the predictions of the standard model and is the same factor as the one accompanying the axial-vector piece in neutrino scattering [Eq. (3.8)].

C. Forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$

In this case, the relevant piece is the $\bar{e}\gamma_\mu\gamma_5 e\bar{\mu}\gamma^\mu\gamma_5\mu$ four-fermion interaction. A simple calculation gives

$$\mathcal{H}^{e\bar{e}\to\mu\bar{\mu}} = \frac{G_F}{2\sqrt{2}}(1+\beta)\bar{e}\gamma_\mu\gamma_5 e\bar{\mu}\gamma^\mu\gamma_5\mu. \tag{3.11}$$

Again, the same departure from the standard-model prediction.

It is therefore clear that once the desired experimental accuracy is reached, the above universality of coupling strength may be used as a test of left-right-symmetric models. Let us now analyze it slightly more quantitatively. We have seen that the departures from the standard model are of the form v_L^2/κ^2 and κ^2/v_R^2. The latter ratio is directly related to the ratio of left-handed and right-handed gauge-meson masses, and so its measure would determine the relevant parameter of the model. The situation v_L/κ term is more complicated and it depends on whether case (i) or case (ii) introduced in Sec. II are realized. Namely, in case (i) $\kappa \sim m_{W_L}/g$ and so v_L/κ again measures the ratio of m_{W_L}/m_{W_R}. (Recall that $v_L \sim \kappa^2/v_R$). In case (ii), however, $\kappa \sim m_f/h$ and therefore is expected to be of order 1–100 MeV and therefore completely negligible and so $v_L \sim \kappa^2/v_R$ can be taken practically to be vanishing.

Let us now, in passing, describe the situation in terms of heavy right-handed gauge-meson eigenstates. From (3.4) we have the following relation between W_R and Z_R masses (we ignore for simplicity the tiny mixing between W_L and W_R):

$$m_{Z_R}^{\,2} = 2\,\frac{\cos^2\theta_W}{\cos 2\theta_W} m_{W_R}^{\,2}. \tag{3.12}$$

Analysis of the neutral-current state including these effects have been carried out by several groups. Ecker[21] gives the following bound (using 1 standard deviation):

$$m_{Z_L}/m_{Z_R} \leq 0.29\,. \tag{3.13}$$

Taking $m_{Z_L} \simeq 90$ GeV, (3.13) implies $m_{Z_R} \gtrsim 300$ GeV. Now, for $\sin^2\theta_W \simeq 0.22$, from (3.12) we obtain

$$m_{W_R}^{\,2} = 0.7\, m_{Z_R}^{\,2} \tag{3.14}$$

which then gives a lower bound on m_{W_R},

$$m_{W_R} \gtrsim 180 \text{ GeV} \tag{3.15}$$

or $m_{W_R} \gtrsim 2.25 m_{W_L}$.

IV. HIGHER LEPTON GENERATIONS AND LEPTON MIXING

In Sec. II, we discussed the neutrino mass for one generation only and showed that its smallness is related to the smallness of $V+A$ charged-current couplings. In this section, we extend this result for neutrinos of all three generations and discuss the implications of neutrino mixing. To begin we denote the leptonic doublets, which are weak eigenstates,

$$\psi_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \psi_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}. \tag{4.1}$$

Working with the same set of Higgs mesons (i.e., Δ_L, Δ_R, and ϕ), we obtain the following most general Yukawa coupling allowed by renormalizability:

$$\mathcal{L}_Y = \sum_{i,j=1}^{3} i h_{ij}(\psi_{iL}^T \tau_2 \Delta_L C\psi_{jL} + \psi_{iR}^T\tau_2\Delta_R C\psi_{jR}) + \sum_{i,j=1}^{3}(f_{ij}\bar{\psi}_{iL}\phi\psi_{jR} + \tilde{f}_{ij}\bar{\psi}_{iL}\tilde{\phi}\psi_{jR}) + \text{H.c.} \tag{4.2}$$

Notice that nothing would change if there are both ϕ_f and ϕ_W, as in the case (ii) of Sec. II, since only ϕ_f couples to the fermions and so (4.2) follows again. If we assume $\kappa' \ll \kappa$ and introduce fields $\nu_i \equiv \nu_{iL}$ and $N_i = C(\nu_{iR})^T$ as before, one obtains the following mass matrix for the $(\nu_e, \nu_\mu, \nu_\tau)$ sector:

$$M = \begin{pmatrix} m_{\nu\nu} & M_{\nu N} \\ M_{\nu N}^\dagger & \mathfrak{M}_{NN} \end{pmatrix}, \quad (4.3)$$

where $m_{\nu\nu}$, $M_{\nu N}$, and $\mathfrak{M}_{NN}$ are 3×3 matrices given by

$$\begin{aligned} (m_{\nu\nu})_{ij} &\simeq \gamma \frac{\kappa^2}{v_R} h_{ij}, \\ (M_{\nu N})_{ij} &\simeq \kappa \tilde{f}_{ij}, \\ (\mathfrak{M}_{NN})_{ij} &\simeq v_R h_{ij}. \end{aligned} \quad (4.4)$$

Again, we see that all three neutrino masses vanish separately in the limit $v_R \to \infty$ (or $m_{W_R} \to \infty$). This is the generalization of the main result of this paper for higher generations. This, in particular, implies that the predominant left-handed nature of the leptonic weak currents as well as the corresponding hadronic currents at low energies is due to the smallness of neutrino masses.

We now give the general method for diagonalizing[24] Eq. (4.3) and present rough estimates of the various mixing angles in our case. To do this, we write the mass eigenstates as

$$\begin{pmatrix} \nu_M \\ N_M \end{pmatrix} = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3 \end{bmatrix} \quad \text{(for three generations)}.$$

They are related to the weak eigenstates, described by

$$\begin{pmatrix} \nu_W \\ N_W \end{pmatrix} = \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ N_e \\ N_\mu \\ N_\tau \end{bmatrix},$$

as follows

$$\begin{pmatrix} \nu_M \\ N_M \end{pmatrix} = \begin{pmatrix} X & Y \\ Z & U \end{pmatrix} \begin{pmatrix} \nu_W \\ N_W \end{pmatrix}. \quad (4.5)$$

Using Eq. (4.3), we can write

$$\begin{aligned} mX + MZ &= XD_\nu, \\ mY + MU &= YD_N, \\ MX + \mathfrak{M}Z &= ZD_\nu, \\ MY + \mathfrak{M}U &= UD_N, \end{aligned} \quad (4.6)$$

where

$$D_\nu = \begin{pmatrix} m_{\nu_1} & & \\ & m_{\nu_2} & \\ & & m_{\nu_3} \end{pmatrix}; \quad D_N = \begin{pmatrix} m_{N_1} & & \\ & m_{N_2} & \\ & & m_{N_3} \end{pmatrix}. \quad (4.7)$$

Of course, in our model $m_{N_i} \gg m_{\nu_j}$. Since we expect X_{ij} and U_{ij} to be, in general, of order 1 (or at least not small), Eq. (4.6) then implies that $|Z_{ij}| \lesssim (m_\nu/m_l)|X_{ij}|$, $|U_{ij}|$ and $|Y_{ij}| \lesssim (m_l/m_N)|U_{ij}|$, where m_l is a typical parameter in the mass matrix of charged leptons, expected to be of order $(m_e, m_\mu$ or $m_\tau)$. As a result, in general, we expect very small mixing between the heavy and the light Majorana lepton.

V. LEPTON-NUMBER-VIOLATING PROCESSES

In this section we discuss the rare processes which do not conserve the lepton number. We divide such processes in two categories: processes which violate the total lepton number and those which conserve the total lepton number, but violate electron, muon, or τ lepton number. The example of the first class of processes is the neutrinoless double-β decay $(n + n \to p + p + e + e)$ in which the lepton number is charged by two units, and the second class of processes are the often discussed muon decays: $\mu \to e\gamma$, $\mu \to 3e$, etc. for which $\Delta L(\text{total}) = 0$. We shall discuss both classes in some detail.

A. Neutrinoless double-β decay [$(\beta\beta)^0$ process]

This process can take place in the second order in the Fermi coupling if neutrinos are Majorana particles (see Fig. 1). To see this we write the charged-weak-current Lagrangian in terms of physical lepton fields (for simplicity we first discuss the case of one generation and generalize it subsequently)

$$\mathcal{L}_{\text{wk}}^{\text{CC}} = \frac{g}{\sqrt{2}} (J_\mu^{(1)} W_{1\mu}^- + J_\mu^{(2)} W_{2\mu}^{(-)} + \text{H.c.}), \quad (5.1)$$

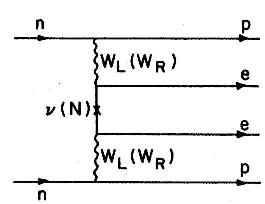

FIG. 1. The dominant diagrams which lead to neutrinoless double-β decay through exchange of W_L and ν_i $(i = e, \mu, \tau)$ or W_R and N_i. The cross on a neutrino internal line denotes a (Majorana) mass insertion, since it is clearly a term $\bar{\nu}^C \nu (\bar{N}^C N)$ which can lead to a production of two electrons in a final state.

where

$$J_\mu^{(1)} = \bar{e}\gamma_\mu \frac{1+\gamma_5}{2}(\nu_e + \xi N_e) + \epsilon\,\bar{e}\gamma_\mu \frac{1-\gamma_5}{2}(N_e - \xi\nu_e)$$

and

$$J_\mu^{(2)} = \bar{e}\gamma_\mu \frac{1-\gamma_5}{2}(N_e - \xi\nu_e) - \epsilon\bar{e}\gamma_\mu \frac{1+\gamma_5}{2}(\nu_e + \xi N_e)\,, \tag{5.2}$$

where $\xi \simeq m_e/m_N$ [see (2.22)] and ϵ is $W_L - W_R$ mixing, i.e., $W_1 = W_L + \epsilon W_R$, $W_2 = -\epsilon W_L + W_R$.

We first note that in the lowest order the $(\beta\beta)_0$ process has to go through mass insertions of the type $\bar{\nu}^c\nu$ or $\bar{N}^c N$ or through currents of opposite chirality. From (5.1) and (5.2) it is immediately clear that the latter type of contributions are proportional to $\epsilon\xi$. Now, from the usual β and μ decay, we know that ϵ is small ($\epsilon \lesssim 10^{-2}$) in order not to conflict with the predominantly left-handed character of such processes. We will present our estimates in terms of the usual parametrization of the $(\beta\beta)^0$ process, that is, in terms of the admixture of left- and right-handed neutrino-electron currents (denoted η hereafter), i.e., $\bar{e}\gamma_\mu[(1+\gamma_5)/2 + \eta(1-\gamma_5)/2]\nu$. We then obtain for the contribution involving $W_L - W_R$ and ν-N mixing

$$\eta = |\epsilon\xi| \leqslant 10^{-2}\frac{m_e}{m_N} \lesssim 10^{-7} \tag{5.3}$$

for $m_N \gtrsim 100$ GeV. To compare this with experiment, we use the analysis of Halprin *et al.*[25] who obtain

$$\eta \lesssim 5\times 10^{-4} \tag{5.4}$$

on the basis of the present data in ^{48}Ca $\to$ ^{48}Ti, ^{76}Ge $\to$ ^{76}Se and ^{86}Se $\to$ ^{86}Kr. For illustration, note that the bound in (5.4) corresponds to the half-life of ^{86}Se: $T_{1/2}^{(\beta\beta)^0} \gtrsim 4\times 10^{21\pm 2}$ years. Clearly then, such effects are extremely small in our model and in what follows, we ignore the tiny $W_L - W_R$ (ϵ) and ν-$N(\xi)$ mixing.

As we shall see next, the situation is quite different with the first type of processes which go through $\bar{\nu}^c\nu$ and $\bar{N}^c N$ mass insertions. It turns out that the model predicts amplitudes which ought to be observable in the next generation of experiments. Due to the tiny neutrino masses, the exchange of heavy right-handed leptons N_i ($i = e, \mu, \tau$) will obviously dominate and we discuss it first. It can be shown that in this case η will be given by[25]

$$\eta \lesssim \left(\frac{m_{W_L}}{m_{W_R}}\right)^4 \frac{1}{m_N} f_{\text{nuc}}\,, \tag{5.5}$$

where f_{nuc} is the nuclear factor, estimated by Halprin *et al.*[25] for $A \simeq 100$ $(\beta\beta)$ nuclei to be about 0.35 GeV. If we take $(m_{W_L}/m_{W_R})^2 \lesssim \frac{1}{10}$, as experiment dictates, and $m_N \gtrsim 100$ GeV, Eq. (5.5) then yields

$$\eta \lesssim (3.5)\times 10^{-5}\,. \tag{5.6}$$

Detection of this effect would require measuring $T_{1/2}^{(\beta\beta)^0}$ to an accuracy better than $10^{24\pm 2}$ years, which can hopefully be reached in the next generation of experiments. In all fairness we should admit that the above estimate depends sensitively on m_{W_R}. However, if m_{W_R} is light, as we believe, then the estimation is fairly good, since N cannot be much heavier than what we take; otherwise Yukawa couplings would be too large and the perturbation theory would break down. Namely, using $m_N = hv_R$, $m_{W_R} = gm_{W_R}$, we obtain $m_N = h/gm_{W_R}$. Requiring $h \lesssim 1$ leads to the estimate $m_N \lesssim 2m_{W_R}$, since $g \simeq 240$ GeV, then clearly $m_N \lesssim 480$ GeV and we have a strict bound $\eta \lesssim 7\times 10^{-6}$. In conclusion we have shown that for low m_{W_R} our model predicts neutrinoless double-β decay with $\eta \simeq (10^{-4}$–$10^{-5})$. We believe that it makes future experiments even more called for.

In passing, we comment on the upper bounds on neutrino masses which can follow from the analysis of $(\beta\beta)^0$ decay. Namely, the requirement that $\eta \lesssim 5\times 10^{-4}$ can be shown[25] to lead to the bound

$$\sum_i (O_{L1i})^2 m_{\nu_i} < 1\ \text{keV}\,, \tag{5.7}$$

where O_L is the Cabibbo-type rotation in left-handed leptonic currents. To see what (5.7) implies, let us first recall the laboratory upper limits on m_{ν_i}[7]: $m_{\nu_e} < 35$ eV, $m_{\nu_\mu} < 500$ keV, and $m_{\nu_\tau} < 250$ MeV. We should also mention the cosmological bounds[26] which result from the requirement that neutrinos do not dominate the matter content of the universe: $\Sigma m_\nu \lesssim 50$ eV, where the above limit (quoted somewhat conservatively) applies to stable ($\tau_{\nu_i} > 10^3$ sec) neutrinos only. Now one can easily be convinced that ν_μ is practically stable. Namely, from $m_{\nu_\mu} < 500$ keV, it's possible decay $\nu_\mu \to \nu_e + \gamma$ leads to at least $\tau_{\nu_\mu} > 10^{10}$ sec. Therefore the cosmological bound applies to ν_μ and we conclude that $m_{\nu_\mu} < 40$ eV, in which case $(\beta\beta)^0$ doesn't provide any new limit on m_{ν_μ}. However, since ν_τ could be rather heavy (250 MeV), it can sufficiently fast decay into $\nu_e e\bar{e}$, so that the cosmological bound doesn't apply to it. Therefore, we can conclude that

$$(O_{L13})^2 m_{\nu_\tau} < 1\ \text{keV}\,, \tag{5.8}$$

which for heavy ν_τ then implies a rather small mixing angle with the first generation of leptons.

B. Electron- and muon-number-nonconserving processes

We present here simple order-of-magnitude estimates of the various muon- and electron-number-changing processes: $\mu\to e\gamma$, $\mu\to 3e$, muon capture by nucleus, etc.[27] We start the discussion by analyzing $\mu\to e\gamma$ decay. The dominant diagrams for such processes are depicted in Fig. 2. To set up the notation, we write the most general form of the amplitude for $\mu\to e\gamma$,

$$m(\mu\to e\gamma)=\bar{e}\,(f+f_5\gamma_5)im_\mu\sigma_{\lambda\nu}q^\nu\mu\epsilon^\lambda, \tag{5.9}$$

where m_μ is the muon mass. From (5.9) one derives the decay width

$$\Gamma(\mu\to e\gamma)=\frac{m_\mu^{\,5}}{8\pi}\left(|f|^2+|f_5|^2\right). \tag{5.10}$$

Our task is then to estimate the leading expressions for f and f_5. We separate the neutrino and heavy-lepton contributions. By the analogy with previous models and computations

$$\begin{aligned} f_\nu &= f_{5\nu}\simeq \frac{e}{16\pi^2}\,\frac{g^2}{8m_{W_L}^{\,2}}\,\delta_\nu,\\ f_N &= f_{5N}\simeq \frac{e}{16\pi^2}\,\frac{g^2}{8m_{W_R}^{\,2}}\,\delta_N, \end{aligned} \tag{5.11}$$

where δ_ν and δ_N are the Glashow-Iliopoulos-Maiani (GIM) factors[28]

$$\begin{aligned} \delta_\nu &= \sum_i (O_L)_{i1}(O_L)_{i2}\frac{m_{\nu_i}^{\,2}}{m_{W_R}^2},\\ \delta_N &= \sum_i (O_R)_{i1}(O_R)_{i2}\frac{m_{N_i}^2}{m_{W_R}^{\,2}}, \end{aligned} \tag{5.12}$$

where $i=e,\mu,\tau$ is the generation index and O_L and O_R are the Cabibbo-type rotations in the leptonic

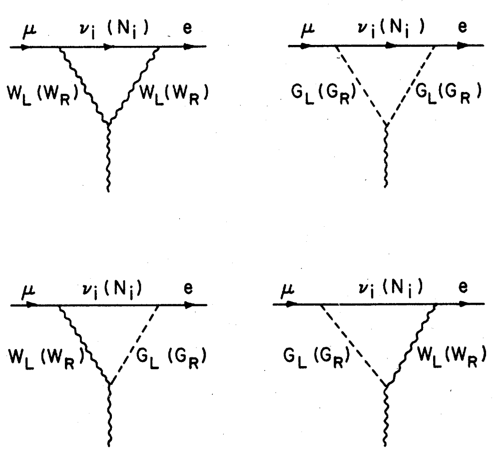

FIG. 2. The leading diagram for a lepton-flavor-changing process $\mu\to e\gamma$. Again, the process goes through the exchange of ν_i and W_L or N_i and W_R. In addition, due to the GIM mechanism, the Goldstone-boson exchanges (denoted by G_L and G_R in obvious notation) are comparable in strength to gauge-boson-mediated amplitudes. We ignore the physical-Higgs-particle exchanges, by assuming $m_H \gg m_W$.

sector introduced in Eq. (5.1). For example, for the case of only two generations

$$\begin{aligned} \delta_\nu &= \sin\theta_L\cos\theta_L\,\frac{m_{\nu_e}^{\,2}-m_{\nu_\mu}^{\,2}}{m_{W_L}^{\,2}},\\ \delta_N &= \sin\theta_R\cos\theta_R\,\frac{m_{N_e}^{\,2}-m_{N_\mu}^{\,2}}{m_{W_R}^{\,2}}, \end{aligned} \tag{5.13}$$

since in this case

$$O_{L,R}=\begin{pmatrix}\cos\theta_{L,R} & \sin\theta_{L,R}\\ -\sin\theta_{L,R} & \cos\theta_{L,R}\end{pmatrix}$$

(we ignore the tiny mixings between ν's and N's, since it doesn't affect the generality of our results). The result stated in (5.12) and (5.13) is the well-known statement of the GIM mechanism: the amplitude vanishes for vanishing neutral-lepton masses or vanishing mixing angles.

Using the formula for the usual lepton-number-changing muon decay

$$\Gamma(\mu\to e\,\nu_\mu\bar{\nu}_e)=\frac{G_F^{\,2}m_\mu^{\,5}}{192\pi^3} \tag{5.14}$$

we obtain for the branching ratio $B(\mu\to e\gamma)\equiv\Gamma(\mu\to e\gamma)/\Gamma(\mu\to e\nu_\mu\bar{\nu}_e)$,

$$\begin{aligned} B(\mu\to e\gamma) &= B^\nu(\mu\to e\gamma)+B^N(\mu\to e\gamma)\\ &\quad + B^{\nu N}(\mu\to e\gamma), \end{aligned} \tag{5.15}$$

where

$$\begin{aligned} B^\nu(\mu\to e\gamma) &\simeq \frac{\alpha}{\pi}\,\delta_\nu^{\,2},\\ B^N(\mu\to e\gamma) &\simeq \frac{\alpha}{\pi}\left(\frac{m_{W_L}}{m_{W_R}}\right)^4\delta_N^{\,2},\\ B^{\nu N}(\mu\to e\gamma) &\simeq \frac{\alpha}{\pi}\left(\frac{m_{W_L}}{m_{W_R}}\right)^2\delta_\nu\delta_N. \end{aligned} \tag{5.16}$$

From the limit in (5.2) coming from double-β decay it is clear that δ_ν is desperately small, so that clearly either $B^N(\mu\to e\gamma)$ dominates in the above equation or the whole amplitude is negligible. Continuing our assumption that W_R is reasonably light, we present some estimates for $B(\mu\to e\gamma)\simeq B^N(\mu\to e\gamma)$. Its precise value is obviously obscured by the lack of knowledge of the GIM factor δ_N even if m_{W_R} is of order of a few hundred GeV. Taking for definiteness $\delta_N\simeq 10^{-2}$–10^{-3}, Eq. (5.16) then gives

$$\begin{aligned} \frac{m_{W_L}^{\,2}}{m_{W_R}^{\,2}} &= \frac{1}{10}:\ B(\mu\to e\gamma)\simeq 10^{-9}-10^{-11},\\ \frac{m_{W_L}^{\,2}}{m_{W_R}^{\,2}} &= \frac{1}{100}:\ B(\mu\to e\gamma)=10^{-11}-10^{-13}. \end{aligned} \tag{5.17}$$

Needless to say, varying δ_N would produce different values for $B(\mu\to e\gamma)$. The above estimate can, therefore, at best be taken as suggestive.

We now briefly comment on some other muon- and electron-number-changing processes. The interesting possible decay model in our model is obviously $\mu \to ee\bar{e}$. This process is suppressed by additional power in the coupling constant, as is clear from a typical graph as depicted in Fig. 3. When compared to the $\mu \to e\gamma$ process, the branching ratio $B(\mu \to ee\bar{e}) = \Gamma(\mu \to ee\bar{e})/\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)$ becomes free of any uncertainties and one simply obtains the order-of-magnitude estimate

$$B(\mu \to ee\bar{e}) \simeq \frac{\alpha}{\sin^2\theta_W} B(\mu \to e\gamma). \tag{5.18}$$

For $\sin^2\theta_W \simeq 0.22$, this would give $B(\mu \to ee\bar{e})/B(\mu \to e\gamma) \simeq (1-10)\%$.[26] This is an important prediction of our model: $\mu \to e\gamma$ and $\mu \to ee\bar{e}$ are tied up to each other and the simultaneous observation of both could be used as a crucial test of the ideas discussed in this paper. Lacking the hint from experiment, we satisfied ourselves by order-of-magnitude estimate given in (5.18). It is clear, however, that a calculation of $B(\mu \to ee\bar{e})/B(\mu \to e\gamma)$ is called for.

Other possible muon-number-changing processes are $e\bar{\mu} \to \mu\bar{e}$ and muon capture by the nucleus: $\mu + (A,Z) \to e + (A,Z)$. It can be readily checked that their evaluation is similar to $\mu \to ee\bar{e}$ and the amplitude for all three processes are of the same order of magnitude.

In conclusion, the model we suggest predicts muon-number-changing processes. In particular we estimated that $B(\mu \to e\gamma) \simeq 10^{-9}-10^{-13}$ for reasonable values of m_{W_R}: $10 \lesssim m_{W_R}^2/m_{W_L}^2 \lesssim 100$ and (somewhat arbitrary) input $\delta_N \simeq 10^{-2}-10^{-3}$. Other processes such as $\mu \to ee\bar{e}$ are also possible, with

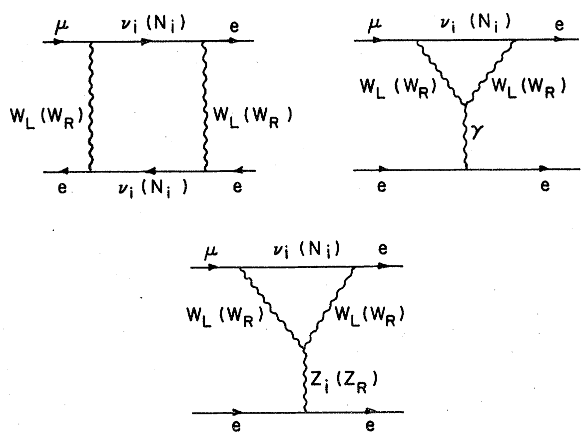

FIG. 3. Some typical diagrams leading to a decay $\mu \to ee\bar{e}$. Clearly, there are quite a few more graphs involving Goldstone bosons, which we didn't display here.

typical ratios $B(\mu \to ee\bar{e})/B(\mu \to e\gamma) \simeq$ a few % (a feature typical of models involving heavy neutral leptons as sources of muon-number-changing decays). Hopefully, future searches for such decays (with improved sensitivity) will be able to serve as a test of this and similar models.

VI. COMMENTS AND CONCLUSION

The main result of our work, as we emphasized repeatedly, is the explicit connection between the smallness of neutrino mass and the maximality of parity violation in low-energy weak interaction. Precisely, *we have shown that the neutrino mass is inversely proportional to the mass of the right-handed charged gauge boson* W_R, *which means that the* $V-A$ *limit of left-right-symmetric theories corresponds to the vanishing of neutrino mass.* The crucial ingredient which was responsible for our result is the Majorana character of neutrinos, i.e., the fact that the left-handed and right-handed neutrinos acquire very small and very large Majorana mass, respectively. That in turn is dictated by the choice of the Higgs sector, which reduces the amount of arbitrariness in the theory. The same choice of Higgs multiplets leads to definite phenomenological predictions in the realm of neutral-current phenomena and therefore ties the nature of neutrino states and the value of their masses with the properties of gauge bosons. We have discussed at length the experimental implications of our model and concluded that the most interesting ones, which also characterize the model most uniquely, are the processes which violate the lepton-number conservation. In particular, the neutrinoless double-β decay (with $\Delta L = 2$) turns out to be the definite prediction of the theory. We found that the relatively small m_{W_R} ($m_{W_R} \gtrsim 3m_{W_L}$) leads to appreciable values for such amplitudes which ought to be observable in the next generation of experiments. Our analysis also showed that the strength of neutrinoless double-β decay is tied up to the strength of various lepton-flavor-changing processes (with total lepton number conserved), such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and others. Again, it is the mass of W_R which affects such processes most and for low W_R we predict $B(\mu \to e\gamma) \simeq 10^{-9}-10^{-13}$ and $B(\mu \to ee\bar{e}) \simeq$ (a few %) $B(\mu \to e\gamma)$. The experiments now in preparation[29] are likely to be able to observe the amplitudes of such strength and combined with neutrinoless double-β decay can serve as crucial tests of the ideas presented in this paper.

Now, what about the actual values for neutrino masses? As we have discussed at length in Sec. II, the precise quantitative predictions of our model are still lacking at this point, mainly due to our lack of knowledge of the values of various

Yukawa couplings and Higgs-particle self-couplings. It turned out that typically expected values of m_ν's depend crucially on whether there is only one mass scale in the theory from which both left-handed gauge bosons and charged fermions receive their mass (in which case the small values of m_e, m_u, m_d, m_μ, m_s is attributed to very small Yukawa couplings) or maybe it is the existence of hierarchy of mass scales which is responsible for rather different values of fermion masses in different generations and gauge-boson masses.[30] The latter case admittedly requires a rather complicated Higgs sector with probably four ϕ's: ϕ_e, ϕ_μ, ϕ_τ, and ϕ_W with subscripts e, μ, τ, and W denoting the fact that they give the mass to the first, second, and third generation of fermions, and W_L and Z_L bosons, respectively However, we find it more appealing on several grounds. First, the smallness of first-generation fermion masses is not attributed to arbitrarily chosen small Yukawa couplings, but rather would be related to the smallness of mass scales $\langle\phi_e\rangle$. It is not inconceivable that one may eventually construct natural hierarchies, in which case the situation $\langle\phi_e\rangle \ll \langle\phi_\mu\rangle \ll \langle\phi_\tau\rangle \ll \langle\phi_W\rangle$ would emerge as a prediction of the theory, rather than to be postulated *ad hoc*. Also, from the point of view of our work, this case leads to much more plausible predictions for neutrino masses as compared to the case of single ϕ, when their natural values tend to get larger than experimentally allowed. Finally, we should add that in this case the Cabibbo-type angles which characterize quark-flavor mixings are necessarily small. Take, for example, the case of two generations and let us for simplicity, concentrate on d and s quarks only. We then expect the following mass matrix:

$$\begin{array}{cc} & \begin{array}{cc} d & s \end{array} \\ \begin{array}{c} d \\ s \end{array} & \begin{pmatrix} \sim\langle\phi_e\rangle & \sim\langle\phi_e\rangle \\ \sim\langle\phi_e\rangle & \sim\langle\phi_\mu\rangle \end{pmatrix} \end{array} .$$

Since $m_d \simeq \langle\phi_e\rangle$, $m_s \simeq \langle\phi_\mu\rangle$, then clearly the Cabibbo angle is bound to be very small (the reasonable value for θ_C can be obtained using the suggestion which forbids the term $\bar{d}d$).

In summary, we have shown how left-right-symmetric theories naturally lead to small m_ν, linking it to the parity violation in nature. These models agree with the predictions of the standard theory at low energies, while at the same time predict small and universal departures in the neutral- current processes. The main characteristic of the model is the Majorana character of neutrinos (dictated by the proposed Higgs sector) which leads to the prediction of neutrinoless double-β decay. We believe, in view of our results, that the experiments devised to search indirectly for low-mass W_R are even more called for.

ACKNOWLEDGMENTS

We thank Tony Kennedy, R. E. Marshak, and Riazuddin for useful discussions. The work of R. N. M. was supported by a National Science Foundation grant and CUNY-PSC-BHE Research Award No. RS13406. The work of G. S. was supported by a grant from the National Science Foundation.

APPENDIX A: MAJORANA MASS OF NEUTRINO

In this appendix, we briefly recapitulate some salient features of the theory of a Majorana mass[19] and also remind the reader how in the limit of zero-neutrino mass, both a Majorana and Weyl neutrino are identical. As is well known, for a given four-component spinor ψ transforming under Lorentz transformations as $\psi \to S\psi$, where $S = e^{i/2\epsilon_{\mu\nu}\Sigma_{\mu\nu}}$, (with $\Sigma^\dagger_{\mu\nu} = \Sigma_{\mu\nu}$ and $\epsilon^\dagger_{\mu\nu} = (-)^{\delta_{\nu 4}}(-)^{\delta_{\nu 4}} \epsilon_{\mu\nu}$) there exist two possible Lorentz-invariant bilinears involving the ψ: (a) $\psi^+\gamma_4\psi$ and (b)$\psi^T C^{-1}\psi$ where C is the Dirac charge-conjugation matrix, which satisfies the property $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. [In our basis,

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

$C = \gamma_2\gamma_4$.] The important difference between the type (a) and (b) mass terms is that case (a) term is invariant under a phase transformations of ψ: $\psi \to e^{i\alpha}\psi$, whereas case (b) is not. Therefore, for a spin-$\frac{1}{2}$ particle without any kind of conserved charge associated with it, one may choose either (a) or (b) and in particular it is more economical to choose (b). The reason this is so is that one may then work with only two-component spinors. The reduction from four components to two is usually done by requiring the Majorana condition $\psi^C \equiv C(\bar{\psi})^T = \psi$ in which case (a) and (b) become identical. The above condition yields

$$\psi = \begin{pmatrix} \phi \\ i\sigma_2\phi^* \end{pmatrix} , \tag{A1}$$

where ϕ is a two-component complex spinor. The free-particle equation satisfied by ϕ is[31]

$$(\vec{\sigma}\cdot\vec{\nabla}-\partial t)\phi - m\sigma_2\phi^* = 0 ,$$
$$(\vec{\sigma}\cdot\vec{\nabla}+\partial t)\sigma_2\phi^* + m\phi = 0 . \tag{A2}$$

These two equations have been analyzed in detail in Ref. 31 and the Majorana field ϕ quantized in this paper. The decomposition in terms of the creation and annihilation operators can be written as follows:

$$\phi = \sum_{\substack{\lambda=1,2\\ k}} (1/V)^{1/2}[A_\lambda(\vec{k})U_\lambda(\vec{k})e^{ik\cdot x} + A^*_\lambda(\vec{k})V_\lambda(k)e^{-ik\cdot x}] , \tag{A3}$$

where A_λ and A^*_λ are annihilation and creation operators, respectively, satisfying the canonical anticommutation relation

$$\{A_\lambda(\vec{k}),\ A^*_{\lambda'}(\vec{k}')\} = \delta_{\lambda\lambda'}\delta_{\vec{k},\vec{k}'} . \tag{A4}$$

U_λ and V_λ are two-component spinors which can be expressed in terms of the orthonormal basis: $\alpha(k)$, $\beta(k)$ satisfying the relations: $\alpha^\dagger\alpha = \beta^\dagger\beta = 1$ and $\alpha^\dagger\beta = 0$ and

$$\vec{\sigma}\cdot\vec{k}/|\vec{k}| \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}+\alpha\\ -\beta\end{pmatrix} .$$

In terms of α and β, we can write

$$U_1(\vec{k}) = V_2(\vec{k}) = [N(\vec{k})]^{1/2}\alpha(k)$$

and

$$U_2(\vec{k}) = \frac{m[N(k)]^{1/2}\beta(k)}{k_0+|\vec{k}|} \tag{A5}$$

and

$$V_1(\vec{k}) = \frac{-m[N(k)]^{1/2}\beta(k)}{k_0+|\vec{k}|} ,$$

where

$$N(k)^{-1} = 1 + \frac{m^2}{(k_0+|\vec{k}|)^2} .$$

From Eqs. (A3), (A4), and (A5), it is clear that in the limit of $m\to 0$, we have

$$\phi = \sum_{\vec{k}} (1/V)[A_1(\vec{k})\,U_1(k)e^{ik\cdot x} + A^*_2(\vec{k})V_2(k)e^{-ik\cdot x}] . \tag{A6}$$

It is clear that in this case, particle and antiparticle states are different and they restore lepton number as a conserved quantity. From this it also follows that, for the massive Majorana neutrino, the violation of lepton number is always proportional to the factor (m_ν/E_ν). This in particular implies that[31]

$$\frac{\sigma(\nu + \to e^+ +)}{\sigma(\nu + \to e^- +)} \simeq \left(\frac{m_\nu}{E_\nu}\right) \lesssim 10^{-10} . \tag{A7}$$

The present experimental bound on the ratio of corresponding cross sections obtained by Davis[32] is at the level of 10%.

The discussion of this appendix also makes it clear why double β-decay amplitudes (see Sec. V) are also proportional to the lepton mass.

Next, we would like to show that ν_e and N_e introduced in Sec. II satisfy Majorana equation (A2). We remind the reader that we have defined $\nu \equiv \nu_L$ and $N \equiv C(\overline{N}_R)^T$. In turn, that leads us [see (2.13)] to the following Lagrangian for ν and N:

$$\mathcal{L} = \bar{\nu}\gamma_\mu\partial^\mu\nu + \overline{N}\gamma_\mu\partial^\mu N + (\nu^T N^T)MC\begin{pmatrix}\nu\\ N\end{pmatrix} + \text{H.c.} \tag{A8}$$

In obtaining the above form we had to use the simple equality $\overline{N}_R\gamma^\mu\partial_\mu N_R = N$ which follows from the properties of the charge-conjugation matrix stated before: $C^T = -C = C^{-1}$, $C\gamma_\mu C^T = -\gamma_\mu^T$. Since M is symmetric and real, it can be diagonalized by an orthogonal transformation

$$\begin{pmatrix}\nu\\ N\end{pmatrix} = \begin{pmatrix}\cos\xi & \sin\xi\\ -\sin\xi & \cos\xi\end{pmatrix}\begin{pmatrix}\nu_e\\ N_e\end{pmatrix} .$$

That in turn diagonalizes the kinetic part of the potential, so that we get

$$\mathcal{L} = \bar{\nu}\gamma_\mu\partial^\mu\nu + \tfrac{1}{2}m_\nu(\nu^T C\nu + \nu^\dagger C^\dagger\nu^*) + \overline{N}\gamma_\mu\partial^\mu N + \tfrac{1}{2}m_N(N^T CN + N^\dagger C^\dagger N^*) . \tag{A9}$$

The peculiar factor of $\frac{1}{2}$ is just a definition of m_ν and m_N at the moment; its meaning will be clear from the discussion we now present. Obviously N will satisfy the same equations of motion, so we will analyze just one of them, say ν. From $\nu = \nu_L$, where

$$L = \tfrac{1}{2}(1+\gamma_5) = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

in our representation, we obtain $\nu = \binom{\phi}{0}$, where ϕ is a two-component spinor. Therefore

$$\mathcal{L}_\nu = (\phi^\dagger 0)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\left[\begin{pmatrix}0 & -i\sigma_i\\ i\sigma_i & 0\end{pmatrix}\partial_i + \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\partial_4\right]\begin{pmatrix}\phi\\ 0\end{pmatrix}$$
$$+\tfrac{1}{2}m_\nu(\phi^\dagger 0)\begin{pmatrix}0 & -i\sigma_2\\ i\sigma_2 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\phi\\ 0\end{pmatrix} + \text{H.c.} \quad \text{(A10)}$$

It is then a simple exercise to arrive at the following form of the free Lagrangian for ϕ:

$$\mathcal{L}_\phi = \phi^\dagger\sigma_\mu\partial^\mu\phi + \frac{m_\nu}{2}\,(\phi^T\sigma_2\phi + \phi^\dagger\sigma_2\phi^*)\ , \quad \text{(A11)}$$

where σ_i are the usual Pauli matrices and $\sigma_4 = -i$ as before. Varying the Lagrangian in ϕ^*, we obtain the equation that ϕ satisfies

$$\sigma_\mu\partial^\mu\phi = m_\nu\sigma_2\phi^*\ . \quad \text{(A12)}$$

This completes our proof: ν_e and N_e obviously satisfy the Majorana equation (A2), i.e., free-particle equation satisfied by spinors for which $\psi^C = \psi$.[30] We went through the little exercise described above with an aim to show that we do not impose the conditions that neutrinos are Majorana particles, but rather that such a condition is the consequence of the particular Higgs sector and the pattern of symmetry breaking. Namely, choosing triplets of Higgs scalars Δ_L and Δ_R to break left-right symmetry led automatically to the physics of neutrinos as described in this paper.

APPENDIX B: THE HIGGS POTENTIAL AND THE PATTERN OF SYMMETRY BREAKING

As we saw in Sec. II, the Higgs sector in our model consists of the following types of multiplets:

$$\Delta_L(1,0,2),\quad \Delta_R(0,1,2)\ ,$$
$$\phi(\tfrac{1}{2},\tfrac{1}{2}^*,0),\quad \tilde{\phi}\equiv\tau_2\phi^*\tau_2(\tfrac{1}{2},\tfrac{1}{2}^*,0)\ , \quad \text{(B1)}$$

with the numbers in brackets denoting $SU(2)_L$, $SU(2)_R$, and $U_{B-L}(1)$ quantum numbers, respectively. We give their transformation properties under $SU(2)_L$ and $SU(2)_R$, choosing the matrix form for Δ_L and Δ_R, ($\Delta \equiv 1/\sqrt{2}\ \tau_i\Delta_i$)

$$\Delta_L \to U_L\Delta_L U_L^\dagger,\quad \Delta_R \to U_R\Delta_R U_R^\dagger,$$
$$\phi \to U_L\phi U_R^\dagger,\quad \tilde{\phi} \to U_L\tilde{\phi}U_R^\dagger\ . \quad \text{(B2)}$$

Their charge decomposition is easily shown to be

$$\Delta_{L,R} = \begin{bmatrix}\frac{1}{\sqrt{2}}\delta^+ & \delta^{++}\\ \delta^0 & -\frac{1}{\sqrt{2}}\delta^+\end{bmatrix}_{L,R}\ ,$$
$$\phi = \begin{bmatrix}\phi_1^0 & \phi_1^+\\ \phi_2^- & \phi_2^0\end{bmatrix},\quad \tilde{\phi} = \begin{bmatrix}\phi_2^{0*} & -\phi_2^+\\ -\phi_1^- & \phi_1^{0*}\end{bmatrix}\ . \quad \text{(B3)}$$

We present first the analysis of symmetry breaking for the case of single ϕ [i.e., case (i) of Sec. II]. Towards the end of this section we shall see how the analysis given below simply carries to the case of two ϕ's [case (ii) of Sec. II]: ϕ_W and ϕ_f, with ϕ_W providing the gauge-boson masses and only ϕ_f coupling to the fermions and being responsible for their masses.

Now, consistent with the transformation properties of Δ_L, Δ_R, ϕ and $\tilde{\phi}$ and left-right symmetry (for simplicity and without losing generality we forbid the trilinear couplings by an appropriate discrete symmetry)

$$V = -\sum_{i,j=1}^{2}\mu_{ij}^2\,\mathrm{tr}\phi_i^\dagger\phi_j + \sum_{i,j,k,l=1}^{2}\lambda_{ijkl}\,\mathrm{tr}(\phi_i^\dagger\phi_j)\mathrm{tr}(\phi_k^\dagger\phi_l + \sum_{i,j,k,l=1}^{2}\lambda'_{ijkl}\,\mathrm{tr}\phi_i^\dagger\phi_j\,\phi_k^\dagger\phi_l - \mu^2\mathrm{tr}(\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R)$$
$$+\rho_1[(\mathrm{tr}\Delta_L^\dagger\Delta_L)^2 + (\mathrm{tr}\Delta_R^\dagger\Delta_R)^2] + \rho_2(\mathrm{tr}\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L + \mathrm{tr}\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R) + \rho_3\mathrm{tr}\Delta_L^\dagger\Delta_L\Delta_R^\dagger\Delta_R$$
$$+\sum_{i,j=1}^{2}\alpha_{ij}\,\mathrm{tr}\phi_i^\dagger\phi_j(\mathrm{tr}\Delta_L^\dagger\Delta_L + \mathrm{tr}\Delta_R^\dagger\Delta_R) + \sum_{i,j=1}^{2}\beta_{ij}(\mathrm{tr}\Delta_L^\dagger\Delta_L\phi_i\phi_j^\dagger + \mathrm{tr}\Delta_R^\dagger\Delta_R\phi_i^\dagger\phi_j) + \sum_{i,j=1}^{2}\gamma_{ij}\,\mathrm{tr}\Delta_L^\dagger\phi_i\Delta_R\phi_j^\dagger, \quad \text{(B4)}$$

where $\phi_1 \equiv \phi$ and $\phi_2 \equiv \tilde{\phi}$. The symmetry of the potential under parity conjugation (left-right) symmetry is recovered by the following constraints on the Higgs-particle couplings (some of them being equivalent to conditions for hermicity of the potential):

$$\mu_{ij} = \mu_{ji},\quad \lambda_{1212} = \lambda_{2121},\quad \lambda_{iijk} = \lambda_{iikj},$$
$$\lambda_{ijkk} = \lambda_{jikk},\quad \lambda'_{ijkl} = \lambda'_{iijk} = \lambda'_{klij} = \lambda'_{jkli}, \quad \text{(B5)}$$
$$\alpha_{ij} = \alpha_{ji},\quad \beta_{ij} = \beta_{ji},\quad \gamma_{ij} = \gamma_{ji}\ .$$

The most general form of vacuum expectation values of the above fields consistent with $U_{\rm em}(1)$ electromagnetic invariance is

$$\langle\Delta_L\rangle=\begin{pmatrix}0&0\\ v_L&0\end{pmatrix},\quad \langle\Delta_R\rangle=\begin{pmatrix}0&0\\ v_R&0\end{pmatrix},$$
$$\langle\phi_1\rangle=\langle\phi\rangle=\begin{pmatrix}\kappa&0\\ 0&\kappa'\end{pmatrix},\quad \langle\phi_2\rangle=\langle\tilde\phi\rangle=\begin{pmatrix}\kappa'&0\\ 0&\kappa\end{pmatrix}. \tag{B6}$$

We shall assume, for simplicity in what follows, that all the vacuum expectation values are real (it can always be made true for a proper range of free parameters of the potential).

Our aim here is to show the relationship between v_L and v_R, without entering the lengthy but otherwise straightforward exercise of proving that the extremizing solution is also an absolute minimum (that was discussed before). We then obviously need to discuss the potential as a function of v_L, v_R, κ, and κ' only. We obtain

$$V(\Delta_L,\Delta_R,\kappa,\kappa')=-\mu^2(v_L^2+v_R^2)+\frac{\rho}{4}(v_L^4+v_R^4)+\frac{\rho'}{2}v_L^2v_R^2$$
$$+(v_L^2+v_R^2)\,(\alpha_{11}+\alpha_{22}+\beta_{11})\kappa^2+(\alpha_{11}+\alpha_{22}+\beta_{22})\kappa'^2+(4\alpha_{12}+2\beta_{12})\kappa\kappa']$$
$$+2v_Lv_R[(\gamma_{11}+\gamma_{22})\kappa\kappa'+\gamma_{12}(\kappa^2+\kappa'^2)]+\text{terms which depend on }\kappa,\kappa'\text{ only}, \tag{B7}$$

where

$$\rho=4(\rho_1+\rho_2),\quad \rho'=2\rho_3\,. \tag{B8}$$

As previously, we will work in the approximation $\kappa'\ll\kappa$, so that (B7) becomes

$$V(\Delta_L,\Delta_R,\kappa)=-\mu^2(v_L^2+v_R^2)+\frac{\rho}{4}(v_L^4+v_R^4)+\frac{\rho'}{2}v_L^2v_R^2+\frac{\alpha}{2}(v_L^2+v_R^2)\kappa^2+\beta v_Lv_R\kappa^2, \tag{B9}$$

with

$$\alpha=2(\alpha_{11}+\alpha_{22}+\beta_{11}),\qquad \beta=2\gamma_{12}\,. \tag{B10}$$

From the extremizing conditions $0=\partial V/\partial v_L=\partial V/\partial v_R$, we obtain

$$\mu^2v_L=\rho v_L^3+\rho' v_Lv_R^2+\alpha\kappa^2v_L+\beta\kappa^2v_R\,,$$
$$\mu^2v_R=\rho v_R^3+\rho' v_Rv_L^2+\alpha\kappa^2v_R+\beta\kappa^2v_L\,. \tag{B11}$$

It is a simple exercise [we multiply the first equation (B11) by v_R and second by v_L and then to subtract them] to obtain

$$[(\rho-\rho')v_Lv_R-\beta\kappa^2](v_L^2-v_R^2)=0\,. \tag{B12}$$

It is clear that the possible solutions to (B12) are

(a) $v_L^2=v_R^2$,

(b) $v_L\neq v_R^2$, in which case

$$v_Lv_R=\frac{\beta}{\rho-\rho'}\kappa^2\,. \tag{B13}$$

Now, under a choice of the parameters of the Lagrangian one can show that solution (b) is a minimum [in that case solution (a) becomes a local maximum]. That is a solution which we seek—we wanted from the beginning parity to be spontaneously broken. The relevance of (B13) is now manifest, if we write

$$v_L=\gamma\frac{\kappa^2}{v_R}, \tag{B14}$$

where $\gamma\equiv\beta/(\rho-\rho')$. Clearly, when $v_R\to\infty$, $v_L\to0$ and so also $m_\nu\to0$ (since $m_\nu\propto v_L$).

Finally, we want to offer some discussion of case (ii) discussed in Sec. II. It is a case of two ϕ's; ϕ_f and ϕ_w, with only ϕ_f coupling to the fermions, which enables us to imagine an interesting situation: $\langle\phi_f\rangle\ll\langle\phi_w\rangle$. We claimed in Sec. II that one can still arrange that $v_L\sim\langle\phi_f\rangle^2/v_R$, i.e., v_L does not depend on large mass scale $\langle\phi_w\rangle$. We now prove that statement. From (B11) and (B13) it is easy to see how to go about it: we should forbid the term $\beta_W\kappa_W^2v_Lv_R$ (if such a term was absent for a single ϕ, we would have obtained $v_L=0$). Now, $\beta_W=2(\gamma_W)_{12}$ so we need only forbid the term $(\gamma_w)_{12}\,\mathrm{tr}\Delta_L^\dagger\phi_w\Delta_R\phi_w^\dagger$ (notice that we cannot forbid $\mathrm{tr}\Delta_L^\dagger\phi_W\Delta_R\tilde\phi_W^\dagger$ term, but it is proportional to $\kappa_W\kappa_W'$ and so can be made arbitarily small). Let us therefore impose the symmetry D:

$$\phi_W\to i\phi_w,\quad \tilde\phi_w\to-i\tilde\phi_w\,,$$
$$\phi_f\to\phi_f,\quad \tilde\phi_f\to\tilde\phi_f\,, \tag{B15}$$
$$\Delta_L\to\Delta_L,\quad \Delta_R\to\Delta_R\,.$$

In this case, the potential is going to have the general form ($\kappa_w\gg\kappa_w'$, $\kappa\gg\kappa'$)

$$V(\Delta_L, \Delta_R, \kappa, \kappa_W) = -\mu^2(v_L^2 + v_R^2) + \frac{\rho}{4}(v_L^4 + v_R^4) + \frac{\rho'}{2} v_L^2 v_R^2 + \tfrac{1}{2}(v_L^2 + v_R^2)(\alpha\kappa^2 + \alpha_w \kappa_w^2) + \beta v_L v_R \kappa^2 + \text{terms which depend on } \kappa, \kappa_w \text{ only}. \tag{B16}$$

The main point is that (B11) now becomes

$$\begin{aligned} \mu^2 v_L &= \rho v_L^3 + \rho' v_R^2 v_L + (\alpha\kappa^2 + \alpha_W \kappa_w^2) v_L + \beta\kappa^2 v_R \\ \mu^2 v_R &= \rho v_R^3 + \rho' v_L^2 v_R + (\alpha\kappa^2 + \alpha_w \kappa_w^2) v_R + \beta\kappa^2 v_L . \end{aligned} \tag{B17}$$

Similarly as before, it is easy to show that

$$v_L v_R = \gamma\kappa^2 , \tag{B18}$$

where $\gamma = \beta/(\rho - \rho')$ (α_w term drops out, as expected). That is a useful result: it justified our claim in subsection (ii) of Sec. II, that $m_\nu \sim m_f^2/m_{W_R}$, since $\kappa \sim m_f/h$, rather than being $\sim m_{W_L}^2/m_{W_R}$.

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