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Neutron charge form factor in a quark model with hyperfine interactions

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It is known that the charge radius of the neutron can be quantitatively understood as being due to color hyperfine interactions which mix nonsymmetric components into the nucleon spatial wave function. We calculate the electric form factor of the neutron in this model and show that it compares favorably with the data on $G_E^n(q^2)$.

The close analogy between quantum chromodynamics (QCD) and quantum electrodynamics has led to the expectation that strong forces will in some instances resemble familiar electromagnetic forces. In particular, it has been conjectured that the short-distance behavior of the interquark potential will be of the Breit-Fermi type.[1] Quark models incorporating the kind of dynamics expected from QCD which have consequently been applied to the study of both mesons and baryons have indeed had considerable success, and at least some features of the conjectured behavior seem to be verified.[2] The feature with which this note is principally concerned is that piece of the expected interquark potential analogous to the magnetic-dipole–magnetic-dipole (or hyperfine) interactions of electromagnetism.[3]

The successful applications of color hyperfine interactions to "soft" hadronic properties are too numerous to recount here.[2] One particularly incisive test, however, arises from studying the effect of such interactions on the nucleons. In this case, the relevant sector of the Hamiltonian (namely the ${}^2S_S\frac{1}{2}^+$ ground state and the nearby states ${}^2S'_S\frac{1}{2}^+$, ${}^2S_M\frac{1}{2}^+$, and ${}^4D_M\frac{1}{2}^+$, which are connected to ${}^2S_S\frac{1}{2}^+$ by first-order perturbation theory) is SU(6) invariant except for the hyperfine interactions. (We use the notation ${}^{2S+1}L_\pi J^P$, where $\pi = S,M,A$ is the permutation symmetry of the spatial wave function.) Thus SU(6) violations which arise from the resultant "impurities" in the nucleon wave function are good tests for hyperfine interactions (spin-orbit terms can, for example, not contribute).

The admixture of ${}^4D_M\frac{1}{2}^+$ in the nucleon (which arises from the tensor part of the hyperfine interactions) is predicted to be quite small. The admixture of the radial excitation ${}^2S'_S\frac{1}{2}^+$, while substantial, is difficult to detect; it corresponds only to a slight difference in the sizes of the nucleon and Δ. The admixture of ${}^2S_M\frac{1}{2}^+$, i.e., $(70,0^+)$ with an amplitude of about $-\frac{1}{4}$ has, in contrast, some very dramatic consequences which are already borne out by experiment[4]: It gives rise to the observed charge radius of the neutron, to the violations of the Moorehouse selection rules $A_{3/2}(N\,{}^4P_M\frac{5}{2}^- \to p\gamma)=0$ and $A_{1/2}(N\,{}^4P_M\frac{5}{2}^- \to p\gamma)=0$, and to the observed violations of the Faiman-Plane selection rule $A(\Lambda\,{}^4P_M\frac{5}{2}^- \to \bar{K}N)=0$.

It is the purpose of this note to make a simple extension of the observation that the ${}^2S_M$ admixture gives the correct neutron charge radius[5] by calculating the full neutron form factor and comparing it to the experimental data on $G_E^n(q^2)$. The nonzero neutron charge distribution arises in this picture in a very simple way. The two identical d quarks in the ${}^2S_S$ neutron must have $S=1$ to satisfy the Pauli antisymmetrization principle; they therefore repel each other (as evidenced by the fact that the quarks with parallel spins in the Δ repel each other). On the other hand, on the average, the quarks in the $S=\frac{1}{2}$ neutron attract each other ($M_N < M_\Delta$) so we can see that the neutron wave function will be distorted away from symmetry: The down-quark pair repel each other while the two up-down pairs attract; this pushes the down quarks to the periphery of the neutron and pulls the up quark into the center, thus accounting for the negative sign of the neutron's mean squared charge radius $\langle \sum_i e_i r_i^2 \rangle$.

Clearly this picture also predicts in more detail the neutron charge distribution. To leading nonvanishing order in the mixing coefficients (i.e., to order α_s), we find using the harmonic-oscillator model[4] that

$$G_E^n(q^2) = -\frac{1}{6}\left\langle \sum_i e_i r_i^2 \right\rangle_n q^2 e^{-q^2/6\alpha^2}\ , \tag{1}$$

$$G_E^p(q^2)=e^{-q^2/6\alpha^2}\,. \qquad (2)$$

A Gaussian of essentially the form (1) has in fact been used previously[6] on phenomenological grounds in computations of the electric-charge distributions in nuclei. In a nonrelativistic model like the one under discussion here, one should not place much faith in the calculation of a form factor beyond $q\sim m_{\text{quark}}$ (see Ref. 7). It is, in fact, beyond such values that the observed $G_E^p(q^2)$ deviates substantially from the Gaussian shape of Eq. (2); at $q=0.5$ GeV, however, the deviation is only 10%. Since the maximum of $G_E^n(q^2)$ according to Eq. (1) occurs at a value of $q\simeq 0.6$ GeV, we see that we may nevertheless expect to obtain reliable predictions from the model for more than just the initial slope $\frac{1}{6}\langle\sum_i e_i r_i^{\,2}\rangle_n$ of G_E^n.

Taking $\langle\sum_i e_i r_i^{\,2}\rangle_n$ as previously calculated in Ref. 4 and using a Gaussian with the slope of the usual dipole form factor for $G_E^p(q^2)$ (Ref. 8) we compare Eq. (1) with the data on $G_E^n(q^2)$ from electron scattering experiments[9] in Fig. 1. The agreement is clearly satisfactory even somewhat beyond the expected range of validity of (1). We take this as further support for the attribution of the nonzero value of the neutron charge radius to $N\,^2S_M$ components mixed into the nucleon by color hyperfine interactions.

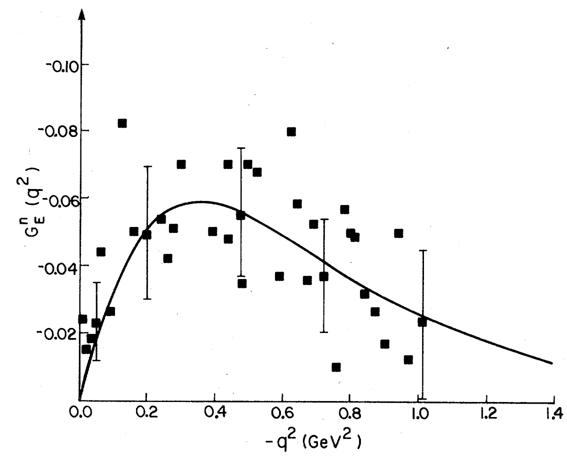

FIG. 1. The predicted $G_E^n(q^2)$ compared with electron scattering data. Typical error flags are shown.

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[8]Namely, $G_E^p(q^2)=(1-q^2/0.71)^{-2}$. This gives a different $\langle\Sigma_i e_i r_i^{\,2}\rangle_p$ than F. Borkowski *et al.* [Nucl. Phys. A222, 269 (1974)], but a better fit to G_E^p for the q^2 values of interest here.

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