Dynamical model for light composite fermions

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A simple *dynamical* model for the internal structure of the three light lepton and quark generations (ν_e, e, u, d), (ν_μ, μ, c, s), and (ν_τ, τ, t, b) is proposed. Each generation is constructed of only one fundamental massive generation $F = (L^0, L^-, U, D)$ with the same $(SU_3)_c \times SU_2 \times U_1$ quantum numbers as the light generations, bound to a core of one or more massive Higgs bosons H, where H is the single physical Higgs boson necessary for spontaneous symmetry breaking in the standard model. For example, $e^- = [L^- H]$, $\mu^- = [L^- HH]$, $\tau^- = [L^- HHH]$. It is shown that the known binding force due to H exchange is attractive and strong enough to produce light bound states. Dynamical calculations for the bound-state composite fermions using the Bethe-Salpeter equation, together with some phenomenological imput, suggest $M_H \sim 16$ TeV and $M_F \sim 100$ GeV. It is likely that such bound states can have properties compatible with the up to now apparently elementary appearance of known fermions, for example, their Dirac magnetic moments and absence of intergeneration radiative decays (such as $\mu \to e\delta$). Phenomenological consequences and tests of the model are discussed.

I. INTRODUCTION

The discovery in recent years of the charmed and bottom quarks and the τ lepton has led to the expectation that all of our apparently fundamental fermions will appear in recurring generation multiplets of successively higher mass but similar $(SU_3)_{color} \times (SU_2) \times (U_1)$ quantum-number structure.[1] In this scheme the leptons and quarks belong to the generation multiplets

$$[I] = [\nu_e, e, u_i, d_i],$$
$$[II] = [\nu_\mu, \mu, c_i, s_i], \qquad (1.1)$$
$$[III] = [\nu_\tau, \tau, t_i, b_i],$$

where the subscript i denotes SU_3 color, and the t quark's existence is presumed but not yet observed.[2] This proliferation of flavors or generations seems to demand a unification more comprehensive than that of the standard $(SU_3)_{color} \times (SU_2 \times U_1)_{electroweak}$ gauge model, yet even the SU_5 grand unified gauge theory[3] [or its O(10) extension] naturally interrelates only quarks and leptons within a generation, but not different generations. Some still larger group[4] is necessary to encompass all generations within one irreducible representation, and the distended size of such groups makes the fundamental nature of such an approach to the generation problem somewhat questionable.

Faced with this, it seems increasingly attractive to believe instead that this recurrent structure betrays a latent compositeness to quarks and leptons, with successive generations corresponding to either the excitation levels of some internal degree of freedom (e.g., a radial quantum number of some bound-state wave function), or perhaps to the addition of successive identical constituents to the lowest-lying generation (analogous to the formation of the periodic table by electron addition). Models of this type, and their discussion, have become increasingly popular,[5-26] and if correct they save grand unified theories from the problem arising from successive generations, that of enlarging the group as more are discovered.

In this paper I propose and discuss a composite model for leptons and quarks which requires no new particles or groups or gauge bosons beyond those already required for other good reasons. Some of this work has been previously outlined,[21] and the same model has been independently considered by Veltman.[19] The model assumes the existence of only *one* very massive fundamental lepton and quark generation, routinely embedded in the standard[27] gauge model $(SU_3)_{color} \times (SU_2 \times U_1)_{electroweak}$, together with the one heavy neutral Higgs boson H that emerges from the standard model's single Higgs-boson doublet in the unitary gauge. The lowest-mass generation [I in Eq. (1.1)] is formed by binding one heavy Higgs boson H to the fundamental generation. Successive generations II, III, . . . , are constructed by adding extra Higgs bosons. I shall show below that the binding of such Higgs bosons H to the fundamental generation is strongly attractive for the mass of H large enough (~ several TeV), and can therefore produce light bound composite fermions. Since the Higgs boson H is neutral with respect to weak, electromagnetic, and colored interactions, it is clear that successive generations will all have similar strong, electromagnetic, and weak interactions. I shall show that it is plausible that despite their compositeness such fermions can appear to be pointlike at probing energies less than 100 GeV—i.e., they can have constant elec-

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tric and magnetic form factors, Dirac magnetic moments corresponding to their bound-state mass, and suppressed radiative generation-to-generation decays, or, in short, behave like the leptons and quarks we know.

An advantage of this model is its economy in utilizing the already required Higgs boson to explicitly (and in principle calculably) account for binding. The only undetermined ingredient in the standard model is the mass of the Higgs boson H, M_H. Since Higgs-boson couplings to themselves and to fermions depend upon the Higgs-boson mass and the fermion mass,[27] and since these couplings must be strong enough to produce *light* composite known fermions, the approximate mass M_H and the approximate mass of the one fundamental generation M_F can be determined. This will be done below, and I shall show that $M_H \sim 16$ TeV and $M_F \sim 100$ GeV are likely values.

Most composite-fermion models[5-26] have little to say about the dynamics of binding, and concentrate mostly on schematics and quantum numbers (exceptions are the paper by Adler[20] and that of Veltman,[19] the latter of which is similar to the present paper and some earlier work on the same model[21]). In contrast the model discussed here has the binding dynamics (due to the Higgs boson) explicitly displayed, and so some approximate dynamical calculations can be executed. Since the treatment of relativistic strong binding is extremely difficult, the approximations made are necessarily crude. The best attitude to take to the calculations below (within the framework of the model) is to regard them as a theoretical laboratory for starting to examine relativistic bound states in the model, but not to take their numerical predictions too literally.

Finally, it is interesting to note that the value $M_H \sim 16$ TeV is the approximate value required for the Higgs-boson mass in dynamically broken gauge models.[28] It is possible that the Higgs boson utilized in this paper is in reality itself a composite of heavier hyperquarks[28] held together by gauge forces, and that the calculations below utilizing an elementary Higgs boson should just be regarded as a phenomenologically viable way of tackling the bound-state problem in such a dynamically broken gauge model.[26]

The paper proceeds as follows. Section II outlines some of the fundamental difficulties of all composite models and discusses their possible solution. In Sec. III the model proposed here is defined and its qualitative features discussed. Section IV contains a potential-theory treatment of the bound states that is intuitively straightforward although of uncertain validity. Nevertheless the results it produces are compatible with those of Sec. V, where the bound states are treated via the ladder-approximation Bethe-Salpeter equation. In both cases similar predictions are obtained for the Higgs-boson mass. In Sec. VI I discuss the phenomenology of the model using some results and assumptions from Sec. V about the solution for the wave function of the composite fermions. In particular, I show how the composite fermions may appear pointlike due to their heavy constituents, and propose some qualitative tests of the model.

II. DIFFICULTIES OF COMPOSITE MODELS

Assuming that one has some notion of the nature of the constituents and perhaps even the binding mechanism, the fundamental stumbling block for composite models of quarks and leptons is their manifest elementarity, particularly in the case of leptons. For example, the measured cross sections for $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, and $\tau^+\tau^-$ agree well enough with pure quantum-electrodynamic predictions so as to imply that "all the known charged leptons are pointlike particles to a distance $\lesssim 10^{-16}$ cm."[29] In momentum variables, this corresponds to constant form factors up to probing momenta ~100 GeV/c. Furthermore, the measured muon anomalous magnetic moment agrees with theory to a part in 10^8, the theory being essentially that of a pointlike Dirac fermion. In addition, the miniscule upper limit[30] of 1.9×10^{-10} on the branching ratio $\mu^+ \to e^+\gamma$, and the suppression of strangeness-changing neutral currents (represented at the quark level by, e.g., $d \to sZ^0$ where Z^0 is the massive neutral gauge boson in the standard model) testify further to the apparent lack of structure of the fermions, since in composite models such transitions should correspond to radiative decays analogous to $2P \to 1S + \gamma$ in hydrogen. Such signatures of elementarity will have to emerge as only low-energy (i.e., $\lesssim 100$ GeV) behavior in any viable composite model.

The pointlike behavior of quarks and leptons at ~100-GeV probing energies implies that any constituents are extremely strongly bound, so that a multi-GeV jolt barely jiggles them. Such immense binding energies within light, almost massless, leptons means that the constituents too must have multi-GeV masses.

If the internal constituents of a light composite fermion of mass m have mass M, with magnetic moment of order $e\hbar/2Mc$, it is difficult to see how the light fermion can have a Dirac magnetic moment $e\hbar/2mc$. This problem, especially severe in static models, has recently been emphasized by Lipkin[14] and Glück.[12] The strong-binding and heavy-constituent masses indicated in the paragraph above, however, make the static approximation a bad one for quark and lepton structure, so that the difficulty is not necessarily prohibitive.

There are several hints that the magnetic-moment problem may be soluble. Lipkin and Tavkhelidze[31] have shown that fermions bound in a strong *scalar* relativistic potential obtain a magnetic moment whose magneton is indeed determined by the bound-state mass rather than the elementary fermion mass. In brief, their argument considers a Dirac particle in an external potential constant over all space, in the presence of an external magnetic field. The Dirac equation for a mass M spinor ψ in a scalar potential V and electromagnetic potential A_μ is

$$i\gamma^\mu(\partial_\mu - ieA_\mu)\psi = (M+V)\psi\,. \qquad (2.1)$$

Defining $m = M + V$, this is just the Dirac equation for a free particle of effective mass m in an external magnetic field, and taking the usual nonrelativistic limit[32] leads to a magnetic moment $e\hbar/2mc$ characterized by the effective mass which has been determined by the binding.

For a static *vector* potential[31] V, V instead appears in the Dirac equation with a matrix coefficient γ^0. The resultant Dirac equation then describes a particle of original mass M but with energy shifted by V, so that the magnetic moment emerging from the nonrelativistic limit is $e\hbar/2Mc$, unaltered by the binding, and unsuitable for describing a magneton scale determined by the light bound mass.

This proof is of course only suggestive, since it assumes static potentials and a single-particle Dirac equation. Ciafaloni and Menotti[33] have shown that two-boson bound states, in the limit of large single-boson mass and large binding energy, obtain magnetic moments with magneton scale given by the bound-state mass. Their proof makes use of the ladder approximation to the Bethe-Salpeter equation, and also requires a scalar interaction, as required above. The prognosis with regard to obtaining the correct Dirac magnetic moments for composite fermions in the model discussed in this paper[19,21] therefore seems promising, since the binding here is precisely due to Higgs-boson exchange, and therefore scalar in nature. The way in which this can occur[34] in the present model is discussed further in Sec. VI below.

Another means of examining the magnetic moment of a composite fermion (say the muon) is to write a dispersion relation for the anomalous magnetic moment $F_2(q^2)$, as has been recently done by Shaw *et al.*[23,35] The relation[36]

$$F_2^c(q^2) = \frac{1}{\pi}\int_{M_c{}^2}^{\infty} \frac{dq'^2}{q'^2 - q^2 - i\epsilon}\,\mathrm{Im}F_2^c(q'^2) \qquad (2.2)$$

describes the contribution to $F_2(q^2)$ from the photon dissociating into real constituents of the muon, over and above the usual quantum-electrodynamic and quantum-chromodynamic contributions. The integral in (2.1) therefore starts above the threshold M_c for producing free constituents. As explained by Shaw *et al.*,[23] $\mathrm{Im}F_2^c \sim mM_c/q'^2$, where m, the muon mass, is involved in the definition of F_2, and M_c, the constituent mass, appears due to the helicity-flip nature of the $\sigma_{\mu\nu}$ Dirac matrix involved in defining F_2. Equation (2.1) then yields $F_2^c(0) \sim m/M_c$, which is small for large constituent masses. They also show that for a muon (or other light fermion) composed of a massive fermion F and boson B, with masses M_F, M_B, and with $M_B \gg M_F$,

$$F_2^c \sim mM_F/M_B{}^2\,, \qquad (2.3)$$

an even greater suppression. This formula will be of use in Secs. IV and VI when I examine F_2^c in the present model, where light fermions are composed of one massive fermion and one Higgs boson. Dispersion relations analogous to Eq. (2.2) can be used to similarly show[35] that electric form factors vary slowly with q^2 for $q^2 \ll M_c{}^2$.

These dispersion-relation proofs involve no real dynamic input for the nature of the bound states, and should perhaps best be viewed as demonstrating the consistency between (i) no observed low-energy strong interactions of leptons and (ii) Dirac magnetic moments for leptons. Since (i) and (ii) seem almost tautologous, the significance of such proofs is unclear.

I shall show in Sec. VI how all the difficulties listed at the start of this section *may* find a solution in this model. The assumption of a simple scaling behavior in the Bethe-Salpeter relativistic bound-state fermion wave function as a function of the relative momentum [Eq. (5.34) below] leads to quasipointlike fermion form factors, Dirac magnetic moments with the correct mass scale despite the ultramassive constituents, suppressed radiative decays such as $\mu \to e\gamma$, and perhaps even the suppression of strangeness-changing neutral currents. This scaling wave-function behavior is at present only conjectured from approximate Bethe-Salpeter solutions below; its proof is being examined.[34]

III. THE HIGGS-BOSON BINDING MODEL FOR COMPOSITE LEPTONS AND QUARKS: QUALITATIVE FEATURES

The motivating principle of this model is to construct all known leptons and quarks without necessarily going beyond the standard $(\mathrm{SU}_3)_{\mathrm{color}} \times (\mathrm{SU}_2 \times \mathrm{U}_1)_{\mathrm{electroweak}}$ model.[27,37] I shall try to show that all leptons and quarks can be constructed from the Higgs boson, plus only *one* fundamental massive lepton and quark generation,

$$[F]=(L^0, L^-, U_i, D_i)\,, \tag{3.1}$$

where L^0 and L^- are neutral and charged leptons and U_i and D_i are tricolored up and down quarks, exactly analogous to the known generations in Eq. (1.1). In terms of their $SU_3\times SU_2\times U_1$ quantum numbers, they are routinely embedded into the standard model as left-handed SU_2 doublets $(L^0, L^-)_L$, $(U_i, D_i)_L$ and right-handed SU_2 singlets $(L^0)_R$, $(L^-)_R$, $(U_i)_R$, $(D_i)_R$, with subscript i denoting an SU_3 triplet representation of color for the quarks. The color index will be henceforth suppressed.

These members of $[F]$ have the standard gauge-theory couplings to the charged and neutral gauge bosons that ultimately become the $W^\pm$, Z^0, and γ. They also couple routinely to the single Higgs-boson doublet $\phi=(\phi^+, \phi^0)$ which gives them mass via its vacuum expectation value in the standard model. In the unitarity gauge,[27] where only one neutral Higgs boson H survives as an observable particle or field, the self-couplings of the Higgs boson are given by the interaction Lagrangian

$$\mathcal{L}_H=\frac{-\sqrt{G_F}M_H{}^2}{2^{3/4}}H^3-\frac{G_F M_H{}^2}{4\sqrt{2}}H^4\,, \tag{3.2}$$

where G_F is the weak-interaction Fermi constant, H the Higgs field, and M_H its mass. The interaction of H with a fundamental generation fermion field f (a quark or a lepton) of mass M_f is given by

$$\mathcal{L}_{fH}=-(2^{1/4}\sqrt{G_F}M_f)\bar{f}fH\,. \tag{3.3}$$

The Feynman vertices for Higgs-boson and fermion interactions are displayed in Fig. 1.

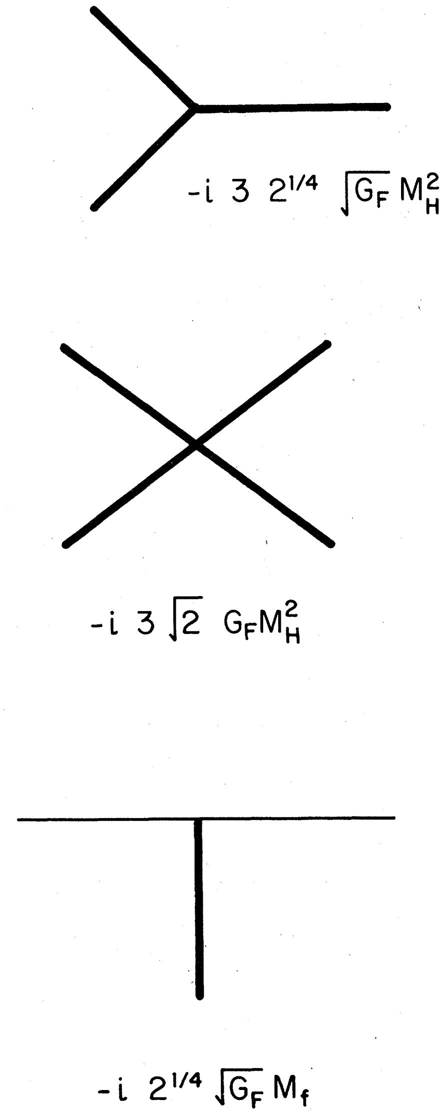

FIG. 1. Feynman vertices for Higgs-boson self-interactions and fermion–Higgs-boson interactions in the standard model. Heavy lines denote the Higgs boson H, mass M_H; light lines denote the fermion f, mass M_f. See Eqs. (3.2) and (3.3) for the Lagrangians which generate these vertices.

A few remarks are appropriate here. I have specified only the $SU_3\times SU_2\times U_1$ quantum numbers of $[F]$. It is possible that $[F]$ is in fact a multiplet of some grand unified gauge theory, e.g., SU_5.[3] This does not negate the approach adopted here, since SU_5 still faces the generation problem and, since its low-energy limit contains $SU_2\times U_1$, with a similar Higgs boson, the same bound-state solution to the generation problem is possible. In fact, the attractive standard SU_5 prediction of $\sin^2\theta_W\simeq 0.2$ is relatively insensitive[38] to the number of fermion generations, and so would still hold in a one-generation model of the type proposed here, in rough agreement with experiment. The generation-number-sensitive predictions of SU_5, such as the mass of the b quark,[39] are irrelevant in the present bound-state model, since there *is no* fundamental b quark whose effective "running" mass can be studied, since the b is a bound state. For dynamically broken gauge models,[28] the use of a fundamental Higgs boson to construct successive generations can perhaps be regarded as a crude way of mocking up the effects of a condensate in the vacuum. Finally, it is important to note that the neutral L^0 member of $[F]$ in Eq. (3.1) is necessarily massive in order that it couple to the Higgs boson H [see Eq. (3.3)] and form a bound-state neutrino below. Neutrinos should therefore be four-component and massive, as perhaps implied by recent results on neutrino oscillations.[40]

The schematic composition of the light fermions is now described. The interactions in Eqs. (3.2) and (3.3) lead to attractive forces due to H exchange between two Higgs bosons, and between a Higgs boson and a fermion; in addition, these forces are proportional to M_H or M_H^2, and to the relevant fermion mass M_f, where $f = L^-$, L^0, U, or D. For large enough M_H and M_f, these binding forces become strong enough to overcome the constituent masses, so that light bound states can occur. Generation [I] in Eq. (1.1) is then identified with bound states of the fundamental generation [F] and a single Higgs boson H, viz,

$$\nu_e = (L^0 H), \quad e^- = (L^- H), \quad u = (UH), \quad d = (DH). \quad (3.4)$$

Since H is (to lowest order) neutral with respect to the gauge-boson interactions of the standard model, the quantum numbers of the bound states are the same as those of [F], and hence appropriate to the known ν_e, e, u, and d. The scalar binding potential due to H exchange is, as discussed in Secs. II and VI, advantageous in solving the magnetic-moment problem. The contruction of generations [II] and [III], as also suggested by Veltman,[19] involves adding one or two extra neutral Higgs bosons H to [I] in Eq. (3.4). Since these extra H's couple strongly to an H for large enough M_H,[41] generations [II] and [III] can also be light, although the addition of extra H's would be expected to make successive generations relatively heavier. The masses of generations [II] and [III], using nonrelativistic three-body methods and the relativistic H-H Bethe-Salpeter equations, will be discussed in a subsequent paper.[34] As is clear, more than three generations should be expected to be found.[42]

The dual requirements that generations [I], [II], and [III] all be light compared to M_f and M_H requires that the HH couplings and the fH couplings be strong enough to cancel their masses in the bound states. In Secs. IV and V I shall show that this condition implies (very roughly) that $M_H \sim 16$ TeV and $M_f \sim 100$ GeV. This lower limit on the constituent mass of 100 GeV is just at the threshold of current experimental searches for lepton structure.[29] If this model is correct, the next order-of-magnitude increase in the resolution of such experiments should start to show signs of compositeness. These values of M_H and M_f will also be shown adequate to plausibly suppress lepton anomalous magnetic moments (due to compositeness) below current experimental bounds.

Before proceeding to calculations within the model, I stress once again that, due to the large masses and strong couplings necessarily involved, the treatment of dynamics is highly approximate. Accurate numerical results should not be expected, only a zeroth or perhaps first-order attempt at showing that the composite fermions in the model can be identified with the ones we know.

IV. BOUND-STATE FERMIONS VIA POTENTIAL THEORY

In this section I shall examine the Higgs-boson binding model of Sec. III by employing nonrelativistic potential theory in the static approximation for the Higgs-boson-exchange potential. I shall show that the potential is attractive, that binding can occur, and I shall estimate the masses M_H of the Higgs boson and M_f of the fundamental generation [F] for which light composite fermions can occur. These masses and binding energies will turn out to be so large as to invalidate the potential theory approximations. Nevertheless, the order-of-magnitude values may be correct, since similar results occur in the relativistic treatment of Sec. V. Further, the potential-theory arguments are more intuitively accessible and useful in estimating the binding energy's dependence on M_f and M_H.

As an example, consider $e^- = (L^- H)$, $\mu^- = (L^- HH)$, and $\tau^- = (L^- HHH)$ in this model. Since e is light (almost massless), the binding energy of L^- and H must approximately cancel their masses. Since μ is light, the addition of an extra H to e must result in additional binding energy that cancels its mass (approximately); this will occur provided the H-H interaction is strong enough to form light bound states. The dual requirements that both $(L^- H)$ and (HH) form light bound states will be implemented below to roughly determine M_L (i.e., M_f) and M_H.

Consider, therefore, H-H scattering via H exchange in the nonrelativistic Born approximation. The Feynman diagrams for this process are displayed in Fig. 2 and lead to the nonrelativistic (static) limit for the amplitude

$$\mathfrak{M}_{HH} = \frac{9\sqrt{2}\, i G_F M_H^4}{\vec{q}^2 + M_H^2} - 3\sqrt{2}\, i G_F M_H^2 . \quad (4.1)$$

The first term corresponds to the Higgs-boson-exchange graph in Fig. 2, with $\vec{q}$ the three-momentum transfer, the second to the seagull-type four-boson interaction. The corresponding nonrelativistic potential is obtained from the Fourier transform

$$V_{HH}(r) = i \int \frac{d^3q}{(2\pi)^3} \frac{\mathfrak{M}_{HH}}{4M_H^2} e^{-i\vec{q}\cdot\vec{r}}$$

$$= \frac{-9 G_F M_H^2}{8\sqrt{2}\,\pi} \frac{e^{-M_H r}}{r} + \frac{3\sqrt{2}\, G_F}{4} \delta^3(\vec{r}) . \quad (4.2)$$

This corresponds to an attractive Yukawa potential of range M_H^{-1} with a repulsive δ-function core. For large M_H, the approximation

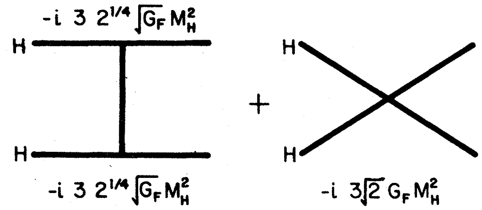

FIG. 2. Feynman diagrams for lowest-order Higgs-boson–Higgs-boson scattering. G_F denotes the Fermi constant.

$$M_H{}^2 \frac{e^{-M_H r}}{r} \simeq 4\pi\delta^3(\vec{r}) \tag{4.3}$$

leads to

$$V_{HH}(r) \simeq -\frac{3G_F}{\sqrt{2}}\delta^3(\vec{r}) \simeq -\frac{3}{4\sqrt{2}\pi} G_F M_H{}^2 \frac{e^{-M_H r}}{r}. \tag{4.4}$$

For large M_H the *attractive Yukawa potential therefore overcomes the repulsive core*, leading to a potential of range $\sim M_H{}^{-1}$ and average height $\sim G_F M_H{}^3$ increasing with M_H.

From Eq. (4.4), two Higgs bosons bound within radius $M_H{}^{-1}$ by V_{HH} have a binding energy $\sim 3G_F M_H{}^2/(4\sqrt{2}\pi)$. The net mass of such a bound state is therefore

$$M_{HH} \simeq 2M_H - \frac{3G_F M_H{}^3}{4\sqrt{2}\pi}. \tag{4.5}$$

As discussed above, one requires $M_{HH} \simeq 0$ in order that μ and τ as well as e be light. This necessitates

$$G_F M_H{}^2 \simeq 8\pi\sqrt{2}/3. \tag{4.6}$$

Consider also the mass of an $(L^- H)$ bound state, corresponding to the almost massless electron. The lowest-order Feynman diagram for L^--H scattering is displayed in Fig. 3, and yields the nonrelativistic amplitude

$$\mathfrak{M}_{LH} = \frac{3\sqrt{2}\, i G_F M_L M_H{}^2}{\vec{q}^{\,2} + M_H{}^2}, \tag{4.7}$$

where $\vec{q}$ is the three-momentum transfer. The corresponding potential here is

$$\begin{aligned} V_{LH}(r) &= i \int d^3q \frac{\mathfrak{M}_{LH} e^{-i\vec{q}\cdot\vec{r}}}{16\pi^3 M_H} \\ &= -\frac{3}{4\pi\sqrt{2}} G_F M_L M_H \frac{e^{-M_H r}}{r}, \end{aligned} \tag{4.8}$$

again an *attractive* potential of range $M_H{}^{-1}$ and average height $\sim G_F M_L M_H{}^2$ increasing with M_H, so that for large M_L (or in general M_f) and M_H one expects strongly bound fermionic states of L^- and H that can have small radii.

The binding energy of an $(L^- H)$ state confined to radius $M_H{}^{-1}$ is, from Eq. (4.8), $\sim 3G_F M_L M_H{}^2/(4\sqrt{2}\pi)$. The net bound-state mass is

$$M_{LH} \simeq M_L + M_H - \frac{3G_F M_L M_H{}^2}{4\sqrt{2}\pi}, \tag{4.9}$$

which, since the electron is almost massless, is approximately zero, implying

$$G_F M_L M_H \simeq \frac{4\sqrt{2}\pi}{3}\left(1 + \frac{M_L}{M_H}\right). \tag{4.10}$$

Equations (4.6) and (4.10) embody in a crude way the conditions on M_L^- and M_H such that e, μ, and τ all be light. Analogous constraints apply to M_{L^0}, M_U, M_D from requiring that all generation members be light. Equations (4.6) and (4.10) yield

$$M_F \simeq M_L \simeq M_H \simeq 1 \text{ TeV}. \tag{4.11}$$

This value of M_H (in the TeV range) is similar to that obtained from unitarity bounds on H-H scattering,[41] and of course corresponds to the value for which perturbation theory fails due to strong coupling.

The above approximations made these 1-TeV values only crude estimates. It is more likely that $M_H \sim 16$ TeV and $M_L \simeq M_F \sim 0.1$ TeV, so that $M_L/M_H \ll 1$. The main reason for this is that, as shown by Shaw *et al.*[23] [see Eq. (2.3) above], the contribution of the muon's constituents to its anomalous magnetic moment in models of the present type is given by $F_2^c \sim m_\mu M_L/M_H{}^2$. For $M_L \sim M_H \sim 1$ TeV, $F_2^c \sim 10^{-4}$, small but still too large compared to the eight significant figure accuracy agreement of the muon's moment with quantum electrodynamics. If $M_L/M_H \ll 1$, say for $M_L \sim 0.1$ TeV and $M_H \sim 16$ TeV, F_2^c can be suppressed to more appropriate values. Note that $M_F < 0.1$ TeV is unacceptable because of the 100-GeV lower limit on the momentum transfer below which lep-

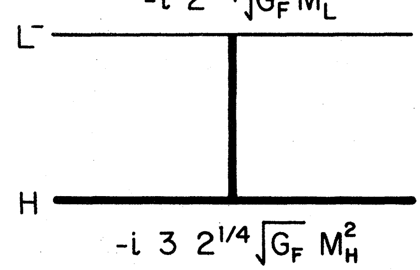

FIG. 3. Feynman diagram for lowest-order L^--H scattering. G_F denotes the Fermi constant.

tons appear pointlike.[29] I shall also show in the relativistic treatment of e as an (LH) bound state below that if $M_L \simeq 0.1$ TeV, then $M_H \simeq 16$ TeV.

For these reasons these values will be taken as the most likely ones for M_f and M_H in the model. They imply that no more than an order-of-magnitude increase in the accuracy of lepton-structure experiments should reveal their compositeness.

V. RELATIVISTIC LIGHT COMPOSITE FERMIONS

As shown above, the binding energies and masses of the constituents of leptons and quarks in the Higgs-boson binding model must be comparable, so that a relativistic treatment of the bound state is essential. In this section I shall treat the bound state of one massive fermion and one Higgs boson by means of the relativistic Bethe-Salpeter equation.[43] In order to be specific the fermion will be taken to be the charged massive L^- of the fundamental generation $[F]$ in Eq. (3.1), so that the bound state corresponds to the electron. Exactly analogous equations could apply to Higgs-boson bound states of L^0, U, and D corresponding to ν_e, u, d. Since $M_H \gtrsim 1$ TeV and the H-H coupling is large, there is no naturally small coupling constant to employ in perturbatively expanding the total Bethe-Salpeter kernel. I shall simply work in the ladder approximation for no reason other than that it is tractable. Although this is still a crude approximation, it provides a theoretical laboratory for obtaining relativistically covariant bound-state wave functions and thus goes a step beyond the static nonrelativistic treatment of Sec. IV. Because of the approximated kernel, numerical results should not be taken literally. *The main aim here is to show the qualitative feasibility of the model.* Further assumptions about the solutions below are explained as they are invoked.

A. The Bethe-Salpeter equation for composite fermions

Figure 4 displays the graphical form of the Bethe-Salpeter equation for the bound-state vertex function of an (L^-H) bound-fermion state in the ladder approximation of H exchange. The vertex function $\psi_P(k)$ satisfies

$$\psi_P(k) = 3\sqrt{2}iG_F M_L M_H^2 \int \frac{d^4k'}{(2\pi)^4} \frac{1}{(\eta \not{P} + \not{k}' - M_L)} \frac{1}{[(1-\eta)P - k']^2 - M_H^2} \frac{1}{(k-k')^2 - M_H^2} \psi_P(k'), \tag{5.1}$$

where the vertex factors for H-H and L^--H coupling can be obtained from Fig. 1. P^μ is the bound-state electron's total four-momentum, and k^μ (or k'^μ) its relative momentum; η is an arbitrary parameter $(0<\eta<1)$ involved in defining the notion of a relative momentum.

It is often convenient to deal with the fermion wave function $\Phi_P(k)$ defined by

$$\Phi_P(k) = \frac{1}{\eta\not{P} + \not{k} - M_L} \frac{1}{[(1-\eta)P - k]^2 - M_H^2} \psi_P(k). \tag{5.2}$$

In terms of Φ_P, the Bethe-Salpeter equation is

$$(\eta\not{P} + \not{k} - M_L)\{[(1-\eta)P - k]^2 - M_H^2\} \Phi_P(k) = 3\sqrt{2}\, iG_F M_L M_H^2 \int \frac{d^4k'}{(2\pi)^4} \frac{1}{(k-k')^2 - M_H^2} \Phi_P(k'). \tag{5.3}$$

Equation (5.3) is not easily solvable. Since $P^\mu P_\mu = m_e^2$ for an electron bound state, the equation is an eigenvalue problem for M_H and/or M_L. In the center-of-momentum frame $\vec{P} = 0$, it becomes

$$(\eta m_e \gamma^0 + \not{k} - M_L)[(1-\eta)^2 m_e^2 - 2(1-\eta) m_e k^0 + k^2 - M_H^2]\Phi_P(k) = 3\sqrt{2}\, iG_F M_L M_H^2 \int \frac{d^4k'}{(2\pi)^4} \frac{1}{(k-k')^2 - M_H^2} \Phi_P(k'). \tag{5.4}$$

Since the nonrelativistic treatment of Sec. IV suggested $M_L \sim M_H \sim 1$ TeV, I shall proceed by ignoring m_e in Eq. (5.4) compared to all internal masses and relative momenta. In this limit of $m_e = 0$, Eq. (5.4) becomes

$$(\not{k} - M_L)(k^2 - M_H^2)\Phi_0(k) = 3\sqrt{2}\, iG_F M_L M_H^2 \int \frac{d^4k'}{(2\pi)^4} [(k-k')^2 - M_H^2]^{-1} \Phi_0(k'), \tag{5.5}$$

an O(3,1)-symmetric eigenvalue equation for M_H, with $\Phi_0(k)$ denoting the zero-mass bound-state wave function.

By expanding $\Phi_0(k)$ in terms of spin and momentum eigenstates one can in general write

$$\Phi_0(k) = \Gamma(k) U_0, \quad \Gamma(k) = A\underline{1} + B\frac{\not{k}}{M_H}, \tag{5.6}$$

where U_0 is a free Dirac four-component rest-frame spinor,[44] Γ is a 4×4 matrix, and A and B are functions of k^0 and $|\vec{k}|$. Substituting Eq. (5.6) into (5.5) leads to integral eigenvalue equations for A and B. Performing the standard Wick rotation[45] to Euclidean space one obtains the coupled equations

$$(k^2+M_H^2)(M_LA+Bk^2/M_H)=3\sqrt{2}\,G_FM_LM_H^2\int\frac{d^4k'}{(2\pi)^4}\frac{A(k')}{(k-k')^2+M_H^2},$$
$$-k^2(k^2+M_H^2)(A-M_LB/M_H)=3\sqrt{2}\,G_FM_LM_H\int\frac{d^4k'}{(2\pi)^4}\frac{(k\cdot k')B}{(k-k')^2+M_H^2}. \tag{5.7}$$

In Eq. (5.7) the Wick rotation has resulted in converting the O(3, 1) symmetry to an O(4) symmetry, with $k^2=k_0^2+\vec{k}^2$.

I now assume that the ground state of an L^-H system (i.e., the electron), being of lowest energy, has an O(4) spherically symmetric eigenfunction, i.e., A and B are functions of Euclidean k^2 only. In that case the angular integrals in Eq. (5.8) can be performed to yield

$$(s+M_H^2)(M_LA+sB/M_H)=-\frac{3\sqrt{2}\,G_FM_LM_H^2}{8\pi^2}\int_0^\infty dt\,\frac{tA(t)}{s+t+M_H^2+[(s+t+M_H^2)^2-4st]^{1/2}},$$
$$(s+M_H^2)(A-M_LB/M_H)=\frac{-3\sqrt{2}\,G_FM_LM_H}{8\pi^2}\int_0^\infty dt\,\frac{t^2B(t)}{\{s+t+M_H^2+[(s+t+M_H^2)^2-4st]^{1/2}\}^2}, \tag{5.8}$$

where $s=k^2$ and $t=k'^2$. Defining new dimensionless variables

$$x=s/M_H^2,\qquad y=t/M_H^2, \tag{5.9}$$

one obtains the integral equations

$$(x+1)(M_LA+xM_HB)=\lambda M_H\int_0^\infty dy\,\frac{yA}{x+y+1+[(x+y+1)^2-4xy]^{1/2}},$$
$$(x+1)(A-M_LB/M_H)=-\lambda\int_0^\infty dy\,\frac{y^2B}{\{x+y+1+[(x+y+1)^2-4xy]^{1/2}\}^2}, \tag{5.10}$$

where

$$\lambda=\frac{3\sqrt{2}\,G_FM_LM_H}{8\pi^2} \tag{5.11}$$

is a dimensionless eigenvalue.

In order to obtain an approximate value for λ, I employ the argument presented in the last paragraph of Sec. IV above that requires $M_L/M_H \ll 1$ in order that the bound-state magnetic moment be of the Dirac type. I shall therefore neglect terms in Eq. (5.10) that are proportional to M_L compared to those $\sim M_H$; the solutions for A and B to Eq. (5.10) will then strictly only be valid for $x\gg M_L/M_H$. With this approximation, Eqs. (5.10) become

$$x(x+1)B=\lambda\int_0^\infty dy\,\frac{y}{(x+y+1)+[(x+y+1)^2-4xy]^{1/2}}A,$$
$$(x+1)A=-\lambda\int_0^\infty dy\,\frac{y^2}{\{x+y+1+[(x+y+1)^2-4xy]^{1/2}\}^2}B. \tag{5.12}$$

The transformation

$$u(x)=x^{2/3}(1+x)^{1/3}A(x),\qquad v(x)=x^{4/3}(1+x)^{2/3}B(x) \tag{5.13}$$

reduces Eq. (5.12) to the convenient final form

$$v(x)=\lambda\int_0^\infty dyK(x,y)u(y), \tag{5.14a}$$

$$u(x)=-\lambda\int_0^\infty dyK^2(x,y)v(y), \tag{5.14b}$$

where

$$K(x,y)=\left[\frac{xy}{(1+x)(1+y)}\right]^{1/3}\times\frac{1}{x+y+1+[(x+y+1)^2-4xy]^{1/2}} \tag{5.15}$$

is a positive kernel.

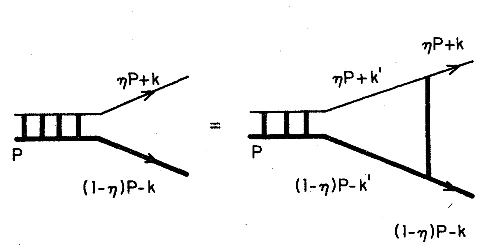

FIG. 4. Graphical form of the Bethe-Salpeter equation for the (L^-H) bound-state vertex function $\psi_b(k)$ in the ladder approximation. Four-momenta labels are those used in Eqs. (5.1)–(5.3).

B. The Higgs-boson mass

In order to *estimate* λ and hence M_H from Eqs. (5.14), it is unnecessary to solve it numerically. Instead one can combine the two equations to obtain

$$v(x) = -\lambda^2 \int_0^\infty H(x,z)v(z)dz\,,$$
$$H(x,z) = \int_0^\infty dy K(x,y)K^2(y,z)\,. \quad (5.16)$$

This is a homogeneous integral equation with a positive Hilbert-Schmidt kernel H, as shown by the bound on the norm $\|H\| < 1$ proved in the Appendix. From Eq. (5.16),

$$\int_0^\infty v^2(x)dx = -\lambda^2 \iint_0^\infty v(x)H(x,z)v(z)dx\,dz\,.$$

Schwartz inequalities applied to the above double integral can then be used to show

$$|\lambda|^2 \geq \|H\|^{-1} > 1\,, \quad (5.17)$$

or from Eq. (5.11)

$$G_F M_L M_H > 8\pi^2/3\sqrt{2}\,. \quad (5.18)$$

This lower bound on $G_F M_L M_H$ agrees qualitatively with the nonrelativistic result of Eq. (4.10) for $M_L \ll M_H$. Assuming that the lower bound gives the correct order of magnitude, one obtains

$$G_F M_L M_H \sim 8\pi^2/3\sqrt{2}\,, \quad (5.19)$$

so that

$$M_L M_H \sim 1.6 \times 10^6 \text{ GeV}^2\,. \quad (5.20)$$

Since $M_L \gtrsim 100$ GeV from the experimental lack of lepton structure,

$$M_H \lesssim 16 \text{ TeV}\,. \quad (5.21)$$

This is of course still a crude estimate in view of the approximations made above, but it will suffice here, and it agrees roughly with the previous nonrelativistic results.

Not that larger values of M_L in Eq. (5.20) lead to lower values of M_H, and therefore larger ratios M_L/M_H. If M_L is appreciably greater than the experimental lower bound of 100 GeV obtained from experiments of lepton structure,[29] the anomalous magnetic moment of the muon $F_2^c \sim m_\mu M_L/M_H^2$ discussed at the end of Sec. IV becomes much too large. Although all these estimates are qualitative only, I therefore expect M_L to lie close to the 100-GeV lower limit, so that more accurate experiments stand a good chance of detecting the L structure inside electrons or muons.

C. The zero-mass wave function

With M_H estimated, it is interesting to examine the zero-mass wave function $\Phi_0(k)$, or its components $u(x)$ and $v(x)$ defined by Eqs. (5.6) and (5.13), since these (at least in the small bound-state mass case to which I hope the zero-mass limit is a reasonable approximation) determine the bound-state structure, form factors, etc. Note firstly that since $H(x,z)$ in Eqs. (5.16) is positive, and since λ must be real for a solution corresponding to a stable electron, $v(x)$ [and similarly $u(x)$] must change sign at least once in $[0,\infty]$.[46]

Although Eqs. (5.14) and (5.15) can be solved numerically, it is more interesting to examine their analytic form qualitatively. The $x=0$ limit of Eqs. (5.14) shows that

$$v(x) \underset{x\to 0}{\sim} x^{1/3}\,,$$
$$u(x) \underset{x\to 0}{\sim} x^{2/3}\,. \quad (5.22)$$

For large x, if one assumes $\int_0^\infty dy u(y)$ is convergent, the kernel in Eq. (5.14a) may be extracted from the integral to yield $v(x) \sim x^{-1}$. Similarly, assuming $\int_0^\infty v(x)dx$ is convergent, the kernel in Eq. (5.14b) may be extracted, leading to $u(x) \sim x^{-2}$ for large x. This latter result is invalid since $\int_0^\infty v(x)dx$ is logarithmically divergent, but a slightly more careful treatment leads to the consistent results

$$u(x) \underset{x\to\infty}{\sim} x^{-2}\ln x\,,$$
$$v(x) \underset{x\to\infty}{\sim} x^{-1}\,. \quad (5.23)$$

Combining Eqs. (5.13), (5.22), and (5.23) one can write A and B in Eq. (5.6) as

$$A(x) = -\frac{F(x)\ln(1+x)}{x(1+x)^2}\,,$$
$$B(x) = \frac{G(x)}{x(1+x)^2}\,, \quad (5.24)$$

where $F(x)$ and $G(x)$ are finite in $[0,\infty]$ and can in principle be solved for numerically, though this is unnecessary here. Equation (5.24) shows that as $x\to 0$, $A \sim x^0$, $B \sim x^{-1}$; as $x\to\infty$, $A \sim x^{-3}\ln x$, $B \sim x^{-3}$.

Combining Eqs. (5.6), (5.9), and (5.24) gives the general form of the zero-mass bound-state spinor wave function

$$\Phi_0(k) = \Gamma(k/M_H)U_0\,, \quad (5.25)$$

where

$$\Gamma(k/M_H) = \frac{M_H^2}{k^2(1-k^2/M_H^2)^2}\left[F(k^2/M_H^2)\ln(1-k^2/M_H^2) + \frac{\not{k}}{M_H}G(k^2/M_H^2)\right]. \quad (5.26)$$

Here the solution has been analytically continued back to Minkowski space and U_0 is a free Dirac rest-frame spinor. The nonzero-mass bound-state wave function will be discussed below.

$\Phi_P(k)$ in Eq. (5.3) represents the Bethe-Salpeter wave function for an incoming bound state, as does $\Phi_0(k)$ above. For an outgoing state one requires the conjugate solution $\overline{\Phi}_P(k)$, which is not simply given by $\Phi_P^\dagger(k)\gamma_0$ as would be the case for a free elementary spinor. This is so because the Bethe-Salpeter equation for an outgoing state $\overline{\Phi}_P(k)$ is given by

$$\overline{\Phi}_P(k)(\eta \not{P} + \not{k} - M_L)\{[(1-\eta)P-k]^2 - M_H{}^2\} = 3\sqrt{2}iG_F M_L M_H{}^2 \int \frac{d^4k'}{(2\pi)^4} \frac{1}{(k-k')^2 - M_H{}^2} \overline{\Phi}_P(k'), \tag{5.27}$$

analogous to Eq. (5.3) with the *same* sign of the imaginary coefficient on the right-hand side, and with the same complex poles in the propagators, *not* those corresponding to the complex conjugate of Eq. (5.3).[47] Although $\Phi_P^\dagger\gamma_0$ does not satisfy Eq. (5.27), it is straightforward to show that

$$\overline{\Phi}_P = \tilde{\Phi}_P K^{-1} \tag{5.28}$$

does, where $\tilde{\Phi}_P$ denotes the transpose of Φ_P, and

$$K = \gamma^3\gamma^1 \tag{5.29}$$

is the time-reversal operator for the Dirac equation[48] and satisfies

$$K\tilde{\gamma}_\mu K^{-1} = \gamma_\mu\,. \tag{5.30}$$

So, given a solution Φ_P, the conjugate wave function $\overline{\Phi}_P$ can be constructed from Eq. (5.28). For a *free* Dirac spinor U_P of momentum P, the conjugate spinor given by Eq. (5.28) is $\overline{U}_P = \tilde{U}_P K^{-1} \equiv U_P^\dagger\gamma_0$, as can be proved from the properties of the Dirac-equation free-particle solutions.

The Lorentz covariance of the Bethe-Salpeter equation is derived analogously to that of the standard Dirac equation. Requiring that the equation have the same form in different Lorentz frames leads to the standard Lorentz boost operator S for spinors,[49] where $S^{-1}\gamma^\mu S = a^\mu_\nu\gamma^\nu$ with a^μ_ν the 4×4 Lorentz boost matrix. In a frame where the bound-state momentum $P'^\mu = a^\mu_\nu P^\nu$, the bound-state spinor is

$$\Phi'_P(k') = S\Phi_P(k)\,, \tag{5.31}$$

and $k'^\mu = a^\mu_\nu k^\nu$.

D. Finite-nonzero-mass bound-state spinors

Equations (5.18)–(5.26) embodying the results on eigenvalues and wave-function properties were derived for zero-mass bound-state composite fermions, in the ladder approximation, neglecting M_L compared to M_H. For a real (say) electron, the mass is nonzero, M_L may not be negligible, and the ladder approximation is unjustified because of strong coupling, so that I am not honestly in a position to employ the above wave function to calculate its physical properties. A more accurate solution to the Bethe-Salpeter equation for finite bound-state mass is beyond the scope of this paper. Nevertheless, it is interesting to examine qualitatively some bound-state properties using a trial wave function, and so, in order to proceed, I shall assume that Φ_0 in Eq. (5.25) also adequately describes a bound state of small but *nonzero* mass $(P_\mu^2)^{1/2}$ in its center-of-momentum frame, at least in the limit that its constituent masses are much greater than the bound-state mass. Explicitly therefore I *assume that in the limit of large constituent masses,* in the center-of-momentum frame, the correct bound-state wave function is

$$\Phi_{\vec{P}=0} = \Gamma(k/M_H)U_0\,, \tag{5.32}$$

where U_0 is a free Dirac rest-frame spinor and (a crucial assumption for all subsequent phenomenology) Γ is a function *only* of k/M_H. M_H should henceforth be interpreted as a typical extremely large internal mass, usually that of the Higgs boson H or sometimes the heavy internal fermion (say L). Where necessary these will be distinguished.

Employing Eq. (5.31) to boost $\Phi_{\vec{P}=0}$ to a moving frame, one obtains the bound-state spinor of momentum P_μ,

$$\Phi_P(k) = \Gamma(k/M_H)U_P\,, \tag{5.33}$$

where U_P is a free Dirac spinor of momentum P_μ. I shall show below that this assumption of (k/M_H) dependence of Γ, at least in the large-M_H limit, is sufficient to guarantee agreeable behavior (pointlike properties, no radiative decays) for the bound-state generations.

VI. PHENOMENOLOGY AND DISCUSSION

In this section I discuss the physical implications of the above model, often using the assumed composite wave function $\Phi_P(k)$ of Eq. (5.33). The discussion is not exhaustive; rather I examine an assortment of physical processes with the aim of qualitatively showing that leptons and quarks in the model, although composite, can appear almost elementary. I also try to estimate how, and at what energies, structure will manifest itself.

A. Electromagnetic form factors

In terms of the Feynman diagrams of Fig. 5, the matrix element of the electromagnetic current J_μ^{em} between an incoming bound-state electron of

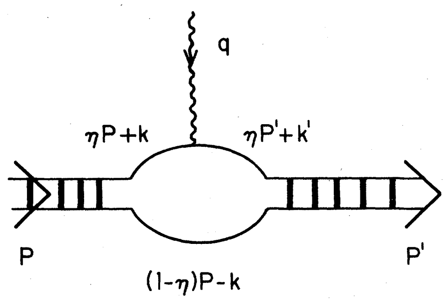

FIG. 5. Feynman diagram for the electromagnetic form factors of the electron in the composite model, corresponding to Eq. (6.1).

momentum P^μ and an outgoing one of momentum $P'^\mu = P^\mu + q^\mu$ is given by[43]

$$\begin{aligned}\langle P' \,|\, J_\mu^{\text{em}}(0) \,|\, P\rangle &\equiv \bar{U}_{P'}[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_e} F_2(q^2)]U_P \\ &= -\int \frac{d^4k}{(2\pi)^4} \{[(1-\eta)P-k]^2 - M_H{}^2\}\bar{\Phi}_{P'}(k')\gamma_\mu \Phi_P(k)\,. \end{aligned} \tag{6.1}$$

Here Φ_P and $\bar{\Phi}_P$ are the electron's wave function and its conjugate, and $k' = k + (1-\eta)q$ is the relative momentum of the final wave function. F_1 and F_2 are the usual electric and magnetic form factors.

Current conservation $q^\mu \langle P' \,|J_\mu^{\text{em}}(0)\,|P\rangle = 0$ follows from Eqs. (5.3) and (5.27); although this involves the ladder approximation, it would have to be true for the exact solutions too. The Bethe-Salpeter normalization condition follows from the charge normalization condition $F_1(0)=1$, and implies

$$-\int \frac{d^4k}{(2\pi)^4} \{[(1-\eta)P-k]^2 - M_H{}^2\}\bar{\Phi}_P(k)\gamma_\mu \Phi_P(k) = P_\mu/m_e \tag{6.2}$$

for a bound state with $P_\mu P^\mu = m_e{}^2$.

Combining Eq. (6.1) with the wave function $\Phi_P(k) = \Gamma(k/M_H)U_P$ in Eq. (5.33) gives

$$\langle P' \,|J_\mu^{\text{em}}(0)\,|P\rangle = -\int \frac{d^4k}{(2\pi)^4} \{[(1-\eta)P-k]^2 - M_H{}^2\} \times \bar{U}_{P'}\Gamma(k'/M_H)\gamma_\mu\Gamma(k/M_H)U_P\,. \tag{6.3}$$

Since k' always appears divided by M_H (the large internal mass) in Eq. (6.3) and since $k' = k + (1-\eta)q$, the limit $M_H \to \infty$ is identical to the limit $q \to 0$. Furthermore, in the limit $M_H \to \infty$, the terms of order $P\cdot k$ and P^2 in Eq. (6.3) are negligible compared to k^2 and $M_H{}^2$. Thus

$$\lim_{M_H\to\infty} \langle P' \,|J_\mu^{\text{em}}\,|P\rangle \to -\int \frac{d^4k}{(2\pi)^4} [k^2 - M_H{}^2]\bar{U}_{P'}\Gamma(k/M_H) \times \gamma_\mu \Gamma(k/M_H)U_P\,. \tag{6.4}$$

The integrand above is a function of k only, and thus the integral must be proportional to γ_μ, so that

$$\lim_{M_H\to\infty} \langle P' \,|J_\mu^{\text{em}}\,|P\rangle \to N_{ee}\bar{U}_{P'}\gamma_\mu U_P\,, \tag{6.5}$$

where N_{ee} is essentially given by the integral in Eq. (6.4). The charge normalization condition (true irrespective of the value of M_H) then ensures $N_{ee} = 1$, so that the electromagnetic vertex of a composite electron, in the limit of infinite constituent mass, is simply γ_μ—that of a Dirac fermion. Thus $F_1(q^2)=1$, $F_2(q^2)=0$, and the composite particle with these assumptions has no anomalous magnetic moment due to its structure. It will presumably[23] have the standard quantum-electrodynamic (QED) perturbative additions to the moment.

For large but *finite* M_H, $F_1(q^2)$ and $F_2(q^2)$ will receive small contributions from their structure. The corrections to $F_1(q^2)=1$ can be of order $q^2/M_H{}^2$, where M_H could correspond to either the Higgs-boson mass or the lighter heavy-lepton mass M_L inside the electron. The corrections to $F_2(q^2)=0$ are $\sim m_e M_L/M_H{}^2$, as suggested by the dispersion argument of Ref. 23, where M_H is the Higgs-boson mass and M_L that of the heavy lepton in this model. For $M_H \sim 16$ TeV and $M_L \sim 100$ GeV, this is much smaller than the uncertainty between QED theory and experiment.

The muon in this model contains two Higgs bosons. Assuming that its wave function is also given by Eq. (5.33) in form, it is also quasipointlike with a composite contribution to its moment $\sim m_\mu M_L/M_H{}^2 \sim 5\times 10^{-8}$. Although this is a little too large not to spoil the agreement between QED theory and experiment for the muon, the estimates above are sufficiently crude that it is easily conceivable that an accurate calculation (were it feasible) could come out an order-of-magnitude smaller, which would be satisfactory.

It is clear, therefore, that provided the present model produces a wave function like Eq. (5.33), it is consistent with experiments on lepton structure, and *a fortiori* on quark structure. It also suggests that another order-of-magnitude increase in the accuracy of $g-2$ experiments on muons, or in $e^+e^- \to \mu^+\mu^-$ experiments, should show deviations from pure QED.

B. Suppression of radiative lepton and quark decays, and flavor-changing neutral currents

Suppose[50] that both μ and e have wave functions given in form by Eq. (5.33). A calculation analogous to Eqs. (6.2)–(6.5) above then shows that, in the limit $M_H \to \infty$, the amplitude for $\mu \to e\gamma$ is given by

$$A(\mu \to e\gamma) = N_{\mu e}\bar{U}(\mu)\gamma_\lambda U(e)\epsilon^\lambda, \quad (6.6)$$

where U denotes the relevant lepton's spinor, ϵ^λ the photon polarization vector, and $N_{\mu e}$ a wave-function overlap integral analogous to that of Eq. (6.4) for N_{ee}.

Electromagnetic gauge invariance necessitates that $N_{\mu e} = 0$. This expresses a sort of orthogonality between the e and μ wave functions (in the large M_H limit) which guarantees the vanishing of radiative amplitudes in the limit $m_\mu/M_H \to 0$. The same applies to $s \to d\gamma$, etc. For finite but small m_μ/M_H, the amplitude is nonzero but small; the exact value depends upon the detailed wave functions and its estimation is beyond the scope of this paper, although it would clearly have to be appreciably suppressed to match the upper limit of 1.9×10^{-10} on the branching ratio[30] $\mu^+ \to e^+\gamma$.

The vanishing or suppression of $N_{\mu e}$ (or N_{sd}) from gauge invariance also automatically ensures the vanishing of the vector part of any flavor-changing neutral current coupled to the standard model's Z^0 boson. However the suppression of the axial part of the flavor-changing neutral current is not an automatic consequence of this; one requires some more general orthogonality of d and s (or μ and e, etc.) wave functions that holds for both γ_μ *and* $\gamma_\mu\gamma_5$ vertices in Eq. (6.4), since the Z^0 couples to both these currents in general. This imposes constraints on the wave functions which should somehow emerge naturally from their bound-state solutions, but how this happens is at present unclear, though under investigation.

Note finally that for weak decays like $u \to sW^-$, $u \to dW^-$, where because of the mass of the W boson gauge invariance *cannot* be invoked to deduce vanishing amplitudes, the ratios of the amplitudes (which determine the Cabibbo angle) are given by wave-function overlap integrals between quarks of different charge. These are therefore in principle calculable; in practice one needs a realistic wave function, at present unavailable.

C. Neutrino masses and mixing angles

Only qualitative remarks can be made. Since $\nu_e = (L^0H)$, $\nu_\mu = (L^0HH)$, and $\nu_\tau = (L^0HHH)$, and since L^0 must be massive to couple to H, I expect all neutrinos to be massive, with a similar hierarchical structure to the e-μ-τ series. Why they are so light is then a puzzle. Weak mixing angles are, as for quarks, in principle, calculable, and neutrino oscillations[40] to be expected.[21] The ν_τ, with three latent Higgs bosons, could have appreciable mass, and better experimental limits would be welcome.

D. Absence of light spin-3/2 leptons

The angular-momentum excitations of the (LH) system necessary to produce higher-spin leptons (or similarly, quarks) would be expected to excite a finite fraction of the binding energy $\sim M_H$. Such leptons or quarks would be too massive to be currently produced.

E. Lepton-number-violating interactions

Since different leptons of the same charge are distinguished by their Higgs-boson content, Higgs-boson rearrangements occurring during weak interactions could lead to small violations of lepton number (which I am unable to estimate numerically). For example, instead of the traditional decay $\mu^- \to e^-\bar{\nu}_e\nu_\mu$, H rearrangements in the final bound states can lead to the rare decay $\mu^- \to e^-\bar{\nu}_\mu\nu_e$, reminiscent of rare decays occurring in gauge theories with a multiplicative lepton number.[51] Experimental bounds[52] on such amplitudes are at the level of 20% of known weak decays and therefore not yet particularly prohibitive. Analogous rearrangements can occur for quarks, leading to a nonzero amplitude for $\nu_\mu + d \to e^- + c$ with flavor changes at both vertices. (Such an amplitude would also occur in the presence of ν_μ-ν_e oscillations.)

F. Higgs-boson–light-lepton couplings: Testing compositeness

Since H is responsible (via its vacuum expectation value) for the masses of the fundamental generation (L^0, L^-, U, D), their couplings to H are proportional to their mass. Thus their bound states, say $e = (L^-H)$, would *not* have Yukawa couplings to H proportional to their light masses, as occurs in the standard model with elementary fermions. This suggests another possible way to ultimately distinguish elementary from composite fermions. Given the expected mass of H though, it is probably only feasible to do so at energies where their structure should already be manifest in other processes mentioned above.

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APPENDIX

In this Appendix, I bound the norm $||H||$ of the kernel H in Eq. (5.17). From Eqs. (5.16) and (5.17),

$$H(x,z)=\left(\frac{x}{x+1}\right)^{1/3}\left(\frac{z}{z+1}\right)^{2/3}\int_0^\infty \frac{y}{y+1}\left(\frac{1}{\{x+y+1+[(x+y+1)^2-4xy]^{1/2}\}}\right)\left(\frac{1}{y+z+1+[(y+z+1)^2-4zy]^{1/2}}\right)^2 dy\,.$$

By means of the inequalities

$$\frac{x}{x+1}<1\,,$$

$$[(x+y+1)^2-4xy]^{1/2}>0\,,$$

one obtains

$$H(x,z)<\int_0^\infty dy\,\frac{1}{(x+y+1)(y+z+1)^2}$$

$$<\int_0^\infty dy\,\frac{1}{x+1}\,\frac{1}{(y+z+1)^2}=\frac{1}{(x+1)(z+1)}\,.$$

Thus,

$$||H||^2=\int\int_0^\infty dx\,dz\,H^2(x,z)<\left[\int_0^\infty \frac{dx}{(x+1)^2}\right]^2=1\,.$$

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[1]For a review of the generation structure of leptons and quarks, see, e.g., H. Harari, Phys. Rep. 42C, 235 (1978).

[2]It is of course possible that no t quark exists, in which case the composite model proposed below, like most others, cannot be correct. For examples of gauge models with elementary (noncomposite) leptons and quarks, but no t quark, see F. Gursey, P. Ramond, and P. Sikivie, Phys. Rev. D 12, 2166 (1975); H. Georgi and A. Pais, Phys. Rev. D 19, 2746 (1979).

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[37]The possibility of embedding the standard model in a grand unified model, and still constructing generations as bound states, is discussed later in this section.

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[41]The possibility of producing light (HH) bound states for large enough $M_H\sim 1$ TeV was first discussed by M. Veltman, Phys. Lett. 70B, 253 (1977).

[42]Some authors (e.g., Ref. 25) have suggested the construction of higher generations via radial excitations of the lowest one. I do not believe this is feasible here. Since $e^-=(L^-H)$ is light, the binding energy is $\gtrsim M_H$. A radial excitation of the electron would presumably involve exciting some finite fraction of the binding

energy. Such a state would therefore have mass $\sim M_H$ and not be identifiable with the muon.

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[46]There are, no doubt, solutions to Eq. (5.16) with complex λ. Similarly there may be tachyonic bound-state solutions to Eq. (5.3). These are probably artifacts of the invalid strong-coupling ladder approximation rather than states of physical interest. In any event, I proceed by looking at real solutions only, in the spirit of treating the ladder-approximated Bethe-Salpeter equation as a laboratory. I assume that the smallest real λ corresponds to the ground state of the *LH* system, with higher values corresponding to radial excitations. For some discussion of the reality and orthogonality of Bethe-Salpeter eigenvalues and eigenfunctions, see N. Nakanishi, Prog. Theor. Phys. Suppl. 43, 1 (1969). Even here though, most results concern the equal-mass scalar-scalar equation, and little seems to be known about the general spinor-scalar case.

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[49]See Ref. 32, p. 22.

[50]An attempted treatment of the relativistic *LHH* system is beyond the scope of this paper. The assumption that $\mu=(L^-HH)$ or $\tau=(L^-HHH)$ have wave functions similar in form to $e=(L^-H)$ in Eq. (5.33) is based, therefore, upon a physical picture of the known leptons as "atoms" with one L^- in orbit around successively heavier tightly bound Higgs-boson "nuclei."

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