Calculation of proton decay in the nonrelativistic quark model

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We compute two-body branching ratios in proton decay in the SU(5) and SO(10) grand unification schemes. Using nonrelativistic approximation for quarks we can obtain *algebraic* relations between the matrix elements coming from spin-flavor symmetries. The absolute magnitude can be fixed from nonleptonic decays of hyperons, a similar phenomenon. We find branching ratios significantly different from previous works. Pion modes are very important due essentially to phase space ($\pi^0 e^+$, 37%, and $\pi^+\bar{\nu}_e$, 15%), and $\omega^0 e^+$ is quite sizable ($\sim 18\%$) due to its large matrix element. We find also an important $K^0\mu^+$ fraction (19%). Bound-state effects are quite crucial since we get, from two-body modes, a lifetime at the edge of the experimental limits, $\tau(p)\simeq 5\times 10^{29}$ or 8×10^{30} yr from $M_X = 3\times 10^{14}$ or 6×10^{14} GeV.

I. INTRODUCTION

The idea of a grand unification of color SU(3) and electroweak SU(2) $\times$U(1), proposed by Pati and Salam[1] and Georgi and Glashow,[2] has important implications both in cosmology[3] and in particle physics at low energies. Predictions for parameters such as $\sin^2\theta_W$ and fermion masses can be made once one takes into account the renormalization effects coming from extrapolation from the grand unification mass to the energies where the effective theory is SU(3) $\times$SU(2) $\times$U(1).[4] A most interesting prediction of these schemes is the instability of the proton through very weak interactions violating both lepton and baryon number. To know if such a prediction is testable, it is important to have information on the total decay rate and the branching ratios of the expected dominant modes.

Several authors[5] have estimated the total decay rate of the proton within the SU(5) theory from rough calculations of inclusive modes $p\to e^+X^0$, $p\to\bar{\nu}_e X^+$ and various approximations in the determination of the leptoquark mass. The total lifetime obtained by Buras *et al.*[5], $\tau_p\sim 10^{37}$–10^{38} yr, goes down to 10^{30}–10^{31} yr after taking into account calculation of β functions at the two-loop approximation and threshold effects, which lead to a value as low as $M_X = 2.7\times 10^{14}$ GeV.[6]

More precise studies have been done recently to estimate the branching ratios of the exclusive modes. These calculations predict $p\to e^+\pi^0$ roughly dominant but do not agree with each other in the relative rates. Although using essentially SU(6) (spin-flavor) wave functions, these approaches differ in their treatment of the relativistic effects of the *quark* motion. Machacek[7] treats the outgoing antiquark relativistically while the other quarks are taken at rest. The authors using the bag-model wave functions[8] disagree on some branching ratios (although the details of their approaches might be different). In any case, in our opinion these calculations do not show clearly the origin of the relative magnitude of the different matrix elements.

In principle, static-SU(6) wave functions should lead to *algebraic* relations between them. We think that, although relativistic corrections of the *quark* motion can somehow break these simple algebraic predictions, a necessary step to clarify the situation is to compute the matrix elements in the familiar nonrelativistic quark model. After all, the predictions for $\omega\to\pi^0\gamma$, $\Delta^+\to p\gamma$, magnetic moments, $\rho,\omega\to e^+e^-$, etc, are in good agreement with experiment. Encouraging results are also obtained in the nonleptonic decays of baryons,[9] a process not very different in nature from proton decay. We will thus make the assumption of nonrelativistic motion of quarks but we will of course treat the lepton as ultrarelativistic, as was the case for the quarks and the photon in $\omega\to\pi^0\gamma$ or for the quarks and the pion in a process like $\Lambda\to p\pi^-$. The calculation is approximate but free from the ambiguities of the relativistic effects of the quark and the hadron center-of-mass motions (Lorentz contraction and Wigner rotations). We will see that we obtain simple expressions of the matrix elements in terms of the baryon wave function when two quarks are at the same point, which can be extracted from nonleptonic hyperon decays or mass hyperfine splittings.[9] Our SU(6) approximation means that, in the calculation of the matrix elements, ρ,ω,π,η are assumed to be degenerate. We will break the degeneracy in the phase-space factors as is usually done.

Since our branching ratios differ from recent literature [in particular the ratio $\Gamma(p\to\omega e^+)/\Gamma(p\to\rho^0 e^+)$], we will be very explicit. In Sec. II we describe the general framework restricting ourselves for the moment to SU(5). We deduce the first quantization operators contributing to the two-body modes. In Sec. III we write its nonrelativistic limit and compute the matrix elements. In Sec. IV we comment on the algebraic relations between the rates and give the final numerical results. In Sec. V we extend our results to SO(10) and in Sec. VI we conclude.

II. GENERAL FRAMEWORK

For the sake of simplicity, let us first start from the "standard" SU(5) theory in the limit $M_X = M_Y$ (X, Y have charges $|Q| = \frac{4}{3}, \frac{1}{3}$, respectively), neglecting Cabibbo-forbidden modes,[10]

$$\mathcal{L} = \frac{g^2}{2M_X^2}\left[(\epsilon_{\alpha\beta\gamma}\bar{u}^c_{\gamma L}\gamma_\mu u_{\beta L})(2\bar{e}^+_L\gamma^\mu d_{\alpha L} + \bar{\mu}^+_L\gamma^\mu s_{\alpha L} + \bar{e}^+_R\gamma^\mu d_{\alpha R} + \bar{\mu}^+_R\gamma^\mu s_{\alpha R}) + (\epsilon_{\alpha\beta\gamma}\bar{u}^c_{\gamma L}\gamma_\mu d_{\beta L})(\bar{\nu}^c_{eR}\gamma^\mu d_{\alpha R} + \bar{\nu}^c_{\mu R}\gamma^\mu s_{\alpha R})\right]. \quad (1)$$

The relative sign between R and L leptons differs from the one of Ref. 10; we will explain this point in Sec. V.

In Sec. V we will give a prescription for computing branching ratios when $M_X \neq M_Y$ in the SU(5) and in the right-left-symmetric theory SO(10).[11] In (1) α, β, γ denote color indices, $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$. Let us ignore for the moment the muonic-strange decays. We have then three types of operators:

$$O_{e^+_L} = \epsilon_{\alpha\beta\gamma}\left[\bar{u}^c_\gamma\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u_\beta\right]\left[\bar{e}^+_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)d_\alpha\right],$$
$$O_{e^+_R} = \epsilon_{\alpha\beta\gamma}\left[\bar{u}^c_\gamma\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u_\beta\right]\left[\bar{e}^+_R\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)d_\alpha\right], \quad (2)$$
$$O_{\nu^c_{eR}} = \epsilon_{\alpha\beta\gamma}\left[\bar{u}^c_\gamma\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)d_\beta\right]\left[\bar{\nu}^c_{eR}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)d_\alpha\right].$$

We have kept the helicity label for leptons and not for quarks in view of the nonrelativistic approximations we will make in Sec. III. Developing the fields in creation and annihilation operators ($\psi = u a_q + v b^\dagger_{\bar{q}}$, $\bar{\psi}^c = \bar{v} a_q + \bar{u} b^\dagger_{\bar{q}}$), we will have for each operator (2) 16 combinations, half of which will correspond to processes involving an incident e^- (or ν_e), and half to processes emitting an e^+ (or $\bar{\nu}_e$). From these, we do not consider the mechanism $uud \to e^+$ (Fig. 1) since it will be involved in $p\to e^+\eta$ or $p\to e^+\pi\pi$ via Zweig-rule-forbidden processes (as in charmonium $\psi'\to\psi\eta$, $\psi'\to\psi\pi\pi$). The remaining processes will contribute to two-body (Fig. 2) or three-body (Fig. 3) decays. We think that three-body decays are not really small relative to two-body decays, since up to algebraic factors they are of the order[12] (neglecting the pion mass),

$$\Gamma(p\to e^+\pi\pi) \sim \left(\frac{g^2}{2M_X^2}\right)^2 \frac{M_p^5}{192\pi^3}.$$

We will discuss the calculation of these modes in another publication and we will concentrate here on the two-body decays.

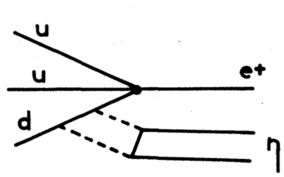

FIG. 1. Example of Zweig-rule-forbidden process involving interaction within the proton of the three valence quarks. The broken lines are gluons.

The second-quantization operators (2) will give, if we keep only the three combinations of Fig. 2 contributing to two-body modes, the following operator:

$$\begin{aligned} O_{e^+_L} = \epsilon_{\alpha\beta\gamma}\Big\{&\left[\bar{v}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u\right]a_{u_\gamma}a_{u_\beta}\left[\bar{u}_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)v\right]b^\dagger_{e^+}b^\dagger_{\bar{d}_\alpha} \\ &+\left[\bar{v}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)v\right]a_{u_\gamma}b^\dagger_{\bar{u}_\beta}\left[\bar{u}_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{d_\alpha} \\ &+\left[\bar{u}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{\bar{u}_\gamma}a_{u_\beta}\left[\bar{u}_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{d_\alpha}\Big\} \end{aligned} \quad (3)$$

and analogously for $O_{e^+_R}$ and $O_{\bar{\nu}_{eR}}$. The notation for spinors and creation and annihilation operators is the standard Bjorken and Drell one. We have re-

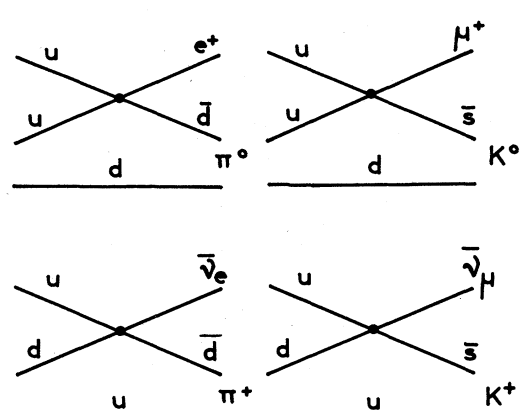

FIG. 2. Examples of proton two-body decay processes computed in the text.

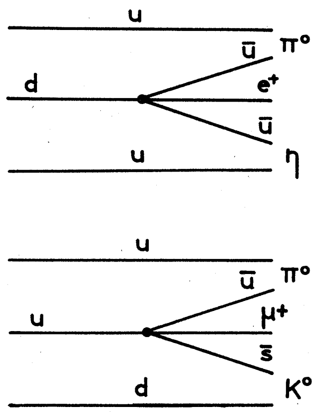

FIG. 3. Examples of three-body proton decays.

spected in (3) the order between creation and annihilation operators and the corresponding spinors. We will now order (3) under the form

$$\epsilon_{\alpha\beta\gamma}(\cdots)b^\dagger_{\bar q_\gamma}a_{q_\beta}(\cdots)b^\dagger_{\bar l}a_{q_\alpha}\,, \tag{4}$$

where $(\cdots)$ means the corresponding spinor matrix element and

$$q=\begin{bmatrix}u\\ d\end{bmatrix},\quad \bar q=\begin{bmatrix}\bar d\\ -\bar u\end{bmatrix},\quad l=\begin{bmatrix}\nu_e\\ e^-\end{bmatrix},\quad \bar l=\begin{bmatrix}e^+\\ -\bar\nu_e\end{bmatrix}. \tag{5}$$

We adopt this isospin notation irrespectively of the helicity [in SU(2)×U(1) we have doublets $q_L, l_L, \bar q_R, \bar l_R$, but this will be useful more generally in the right-left-symmetric theory SO(10)]. The minus sign in (5) affecting the antiparticle is the usual minus sign of antiparticle isodoublets, $\binom{u}{d}$, $\binom{\bar d}{-\bar u}$. To order (3) under the form (4) we need to make a Fierz transformation of the first term [the form $(V-A)\cdot(V-A)$ is invariant under Fierz transformations]:

$$O^{(1)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left[\bar v\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)v\right]a_{u_\gamma}b^\dagger_{\bar d_\alpha}\times\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{u_\beta}\,. \tag{6}$$

We must now exchange the creation and annihilation operators (using $v=C\bar u^T$, $u=C\bar v^T$)

$$\left[\bar v\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)v\right]a_{u_\gamma}b^\dagger_{\bar d_\alpha}=-\left[\bar u\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)u\right]b^\dagger_{\bar d_\alpha}a_{u_\gamma}\,. \tag{7}$$

Changing now the name of the color labels (we have an even combination) the first term in (3) finally gives

$$O^{(1)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{-\left[\bar u\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)u\right]b^\dagger_{\bar d_\gamma}a_{u_\beta}\times\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}\right\}. \tag{8}$$

In the second term of (3) we must first apply (7) which gives

$$O^{(2)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{-\left[\bar u\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)u\right]b^\dagger_{\bar u_\beta}a_{u_\gamma}\times\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{d_\alpha}\right\} \tag{9}$$

and now make a Fierz transformation to (9) to keep the same order defined by (4) and (5):

$$O^{(2)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{-2\left[\bar u\left(\frac{\gamma_5-1}{2}\right)u\right]b^\dagger_{\bar u_\beta}a_{d_\alpha}\times\left[\bar u_L\left(\frac{\gamma_5+1}{2}\right)u\right]b^\dagger_{e^+}a_{u_\gamma}\right\}, \tag{10}$$

since by Fierz rearrangement the Lorentz combination $(V+A)\times(V-A)$ gives $2(P-S)\times(P+S)$. Changing the name of the color labels (even combination) and affecting $\bar u$ with a minus sign to keep to the isospin notation (5), we get

$$O^{(2)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{2\left[\bar u\left(\frac{\gamma_5-1}{2}\right)u\right]b^\dagger_{(-\bar u_\gamma)}a_{d_\beta}\times\left[\bar u_L\left(\frac{\gamma_5+1}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}\right\}. \tag{11}$$

For the third term in (3) we must make a Fierz transformation, affect $\bar u$ with a minus sign and change the color labels (odd combination),

$$O^{(3)}_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{\left[\bar u\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{(-\bar u_\gamma)}a_{d_\beta}\times\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}\right\} \tag{12}$$

and finally we get

$$O_{e^+_L}=\epsilon_{\alpha\beta\gamma}\left\{-\left[\bar u\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)u\right]b^\dagger_{\bar d_\gamma}a_{u_\beta}\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}+2\left[\bar u\left(\frac{\gamma_5-1}{2}\right)u\right]b^\dagger_{(-\bar u_\gamma)}a_{d_\beta}\left[\bar u_L\left(\frac{\gamma_5+1}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}+\left[\bar u\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{(-\bar u_\gamma)}a_{d_\beta}\left[\bar u_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u\right]b^\dagger_{e^+}a_{u_\alpha}\right\}. \tag{13}$$

Using the same technique, we obtain for $O_{e^+_R}$ the same expression as for $O_{e^+_L}$, but with the substitution $\gamma_5\to-\gamma_5$. For $O_{\bar\nu_{eR}}$ we get the same expression as for $O_{e^+_R}$, but with $e^+_R\to-\bar\nu_{eR}$, $u_\alpha\to d_\alpha$:

$$O_{e^+_R}=O_{e^+_L}(\gamma_5\to-\gamma_5)\,,\qquad O_{\bar\nu_{eR}}=O_{e^+_R}(e^+_R\to-\bar\nu_{eR},u_\alpha\to d_\alpha)\,. \tag{14}$$

These symmetries have been outlined by Weinberg and Wilczek and Zee,[13] and can be seen easily from the original interaction (2) by making the appropriate Fierz and C operations. They come simply from the $SU(3)_c \times SU(2) \times U(1)$ invariance of the interaction and from the broken left-right symmetry specific to SU(5) which allows the embedding into SO(10). One has,[13] for e_R^+ and ν_{eR}^c the operator,

$$\epsilon_{\alpha\beta\gamma}\epsilon_{ij}(\overline{u}^c_{\alpha L}\, d_{\beta R})(\overline{l}^c_{iR} q_{j\gamma L})$$

and (15)

$$\epsilon_{\alpha\beta\gamma}\epsilon_{ij}(\overline{q}^c_{i\alpha R}\, q_{j\beta L})(\overline{l}^c_L u_{\gamma R})$$

for e_L^+. In these expressions Greek indices mean color and i,j components of flavor isodoublets.

III. NONRELATIVISTIC APPROXIMATION FOR QUARKS

Using the nonrelativistic limit for quark spinors and the ultrarelativistic one for leptons,

$$u_L = \frac{1}{\sqrt{2}}\begin{bmatrix}\chi_L\\ -\chi_L\end{bmatrix}, \quad u_R = \frac{1}{\sqrt{2}}\begin{bmatrix}\chi_R\\ \chi_R\end{bmatrix},$$

where $\chi_{L,R} = \frac{1}{2}(1 \mp \vec{\sigma}\cdot\hat{p})\chi$. We get, in terms of Pauli spinors, the following simple operators:

$$O_{e_L^+}(1,2) = -\epsilon_{\alpha\beta\gamma}\frac{1}{\sqrt{2}}\left(\frac{1+\vec{\sigma}_1\cdot\vec{\sigma}_2}{2}\right)[\hat{v}_1(u_\alpha \to e_L^+)\hat{v}_2(u_\beta \to \overline{d}_\gamma) + \hat{v}_1(u_\alpha \to e_L^+)\hat{v}_2(d_\beta \to -\overline{u}_\gamma)] ,$$

$$O_{e_R^+}(1,2) = -\epsilon_{\alpha\beta\gamma}\frac{1}{\sqrt{2}}\left(\frac{1+\vec{\sigma}_1\cdot\vec{\sigma}_2}{2}\right)[\hat{v}_1(u_\alpha \to e_R^+)\hat{v}_2(u_\beta \to \overline{d}_\gamma) + \hat{v}_1(u_\alpha \to e_R^+)\hat{v}_2(d_\beta \to -\overline{u}_\gamma)] , \qquad (16)$$

$$O_{\overline{\nu}_{eR}}(1,2) = -\epsilon_{\alpha\beta\gamma}\frac{1}{\sqrt{2}}\left(\frac{1+\vec{\sigma}_1\cdot\vec{\sigma}_2}{2}\right)[\hat{v}_1(d_\alpha \to -\overline{\nu}_{eR})\hat{v}_2(d_\beta \to -\overline{u}_\gamma) + \hat{v}_1(d_\alpha \to -\overline{\nu}_{eR})\hat{v}_2(u_\beta \to \overline{d}_\gamma)] ,$$

where the labels 1,2 mean the quarks 1,2 in the nucleon wave function, and $\hat{v}_i(a \to b)$ are flavor operators transforming a into b. *Isospin* in the sense of (5) is *conserved.* This gives for *strong isospin* the $\Delta I = \frac{1}{2}$ rule remarked by Machacek.[7,13] What is remarkably simple in these formulas is that the spin operator is just $(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ *in all three cases*. The fact that it is the same is just a translation to the nonrelativistic limit of the symmetries expressed in (15). Note that in the case of nonleptonic hyperon decays we had instead a $(V-A)\times(V-A)$ interaction involving quark fields (and not fields and charge-conjugate fields as in the present case), giving in the same nonrelativistic approximation an operator $(1-\vec{\sigma}_1\cdot\vec{\sigma}_2)$ instead of $(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$.

It is now easy to estimate the matrix elements of the decay $p \to \overline{l}M$ where M is any ground-state meson of isospin $I=0$ or 1. Color will give a factor $\sqrt{2}$ since the proton and meson are color singlets (with normalization factors $1/\sqrt{6}$ and $1/\sqrt{3}$, respectively, and we have six color combinations in the interaction (16)

$$6\frac{1}{\sqrt{3}}\frac{1}{\sqrt{6}} = \sqrt{2} . \qquad (17)$$

The rest of the proton wave function can be written as

$$\frac{1}{\sqrt{2}}(\chi'\varphi' + \chi''\varphi'')\psi^s , \qquad (18)$$

where ψ^s is a symmetric space wave function, and χ', χ'' and φ', φ'' are mixed symmetric spin and isospin wave functions, respectively, antisymmetric and symmetric relative to quarks 1 and 2:

$$\varphi' = \frac{1}{\sqrt{2}}(duu - udu) ,$$
$$\varphi'' = -\left(\frac{2}{3}\right)^{1/2}[uud - \tfrac{1}{2}(udu + duu)] \qquad (19)$$

and $\chi'^{+1/2}$, $\chi''^{+1/2}$ are identical to those with $u \to \uparrow$ and $d \to \downarrow$.

The second-quantization operators (13) and (14) will give rise to operators (16) summed over all six (ij) pairs

$$\sum_{i \neq j} O(ij) .$$

Because of the full symmetry of (18) it is sufficient to compute the matrix element of one operator $O(1,2)$ and multiply the result by 6 (number of Wick contractions). The result must be divided by $\sqrt{6}$ because, in the second-quantization formalism, the proton state is written down as the product of the antisymmetrized wave function times the product of the quark creation operators divided by $\sqrt{3!}$. Note that such factors 6 and $1/\sqrt{6}$ do not appear for the quark-antiquark-antilepton final state since we have not adopted an antisymmetrized wave function in this case of unidentical particles.

The flavor matrix elements are given by [we denote the flavor operator in (16) without the color indices by $\cdots$]

$$\langle e_1^+ \Phi_I(2,3)|\cdots|\varphi'\rangle = \frac{1}{2}(\delta_{I0} + \delta_{I1}) ,$$
$$\langle e_1^+ \Phi_I(2,3)|\cdots|\varphi''\rangle = -\frac{\sqrt{3}}{2}\delta_{I0} + \frac{1}{2\sqrt{3}}\delta_{I1} , \qquad (20)$$
$$\langle \overline{\nu}_{e_1} \Phi_I(2,3)|\cdots|\varphi'\rangle = -\frac{1}{\sqrt{2}}\delta_{I1} ,$$
$$\langle \overline{\nu}_{e_1} \Phi_I(2,3)|\cdots|\varphi''\rangle = -\frac{1}{\sqrt{6}}\delta_{I1} .$$

In these relations Φ_I denotes the outgoing meson isospin wave function.

The action of the spin operator over the nucleon wave function is very simple:

$$\left(\frac{1+\vec{\sigma}_1\cdot\vec{\sigma}_2}{2}\right)\frac{1}{\sqrt{2}}(\chi'\varphi'+\chi''\varphi'')=\frac{1}{\sqrt{2}}(\chi''\varphi''-\chi'\varphi')\,, \tag{21}$$

since in χ' and χ'' the quarks (1, 2) are in a spin singlet or triplet, respectively,

$$\vec{\sigma}_1\cdot\vec{\sigma}_2\,|\chi'\rangle=-3\,|\chi'\rangle,\quad \vec{\sigma}_1\cdot\vec{\sigma}_2\,|\chi''\rangle=|\chi''\rangle.$$

Clearly this spin operator turns a $\underline{56}$ representation of SU(6) into a $\underline{70}''$. Since the outgoing lepton is a helicity eigenstate, it is convenient to quantize the quark spin in the direction of the lepton momentum. In this way the spin-matrix elements are very simple. For spin-zero mesons (π,η),

$$\begin{aligned}\langle\chi_{1/2}^{\pm1/2}(1)\chi_0(2,3)\,|\,\chi_{12,3}'^{(\pm1/2)}\rangle&=\frac{1}{2}\,,\\ \langle\chi_{1/2}^{\pm1/2}(1)\chi_0(2,3)\,|\,\chi_{12,3}''^{(\pm1/2)}\rangle&=-\frac{\sqrt{3}}{2}\,.\end{aligned} \tag{22a}$$

For spin-one mesons (ω,ρ),

$$\begin{aligned}\langle\chi_{1/2}^{\pm1/2}(1)\chi_1^0(2,3)\,|\,\chi_{12,3}'^{(\pm1/2)}\rangle&=\mp\frac{1}{2}\,,\\ \langle\chi_{1/2}^{\pm1/2}(1)\chi_1^0(2,3)\,|\,\chi_{12,3}''^{(\pm1/2)}\rangle&=\mp\frac{1}{2\sqrt{3}}\,,\\ \langle\chi_{1/2}^{\pm1/2}(1)\chi_1^{\mp1}(2,3)\,|\,\chi_{12,3}'^{(\mp1/2)}\rangle&=\mp\frac{1}{\sqrt{2}}\,,\\ \langle\chi_{1/2}^{\pm2}(1)\chi_1^{\mp1}(2,3)\,|\,\chi_{12,3}''^{(\mp1/2)}\rangle&=\mp\frac{1}{\sqrt{6}}\,,\end{aligned} \tag{22b}$$

where $\chi_{1/2}^{\pm1/2}$ means the lepton with helicity $\pm\frac{1}{2}$ (respectively right or left handed), $\chi_{12,3}$ is the nucleon-spin wave function with mixed symmetry [well-defined symmetry in the (1, 2) labels], and the spin meson wave functions are denoted by χ_J^m, $\chi_{1,0}^0=(1/\sqrt{2})(\uparrow\downarrow\pm\downarrow\uparrow)$, $\chi_1^{+1}=\uparrow\uparrow$.

For muonic-strange Cabbibo-allowed modes we only have one operator instead of three as in (3), since we can only create an $\bar{s}$ and not annihilate an s, in the valence approximation. Ordering as in (4), we get for the left-handed muon,

$$\begin{aligned}O_{\mu_L^+}=\epsilon_{\alpha\beta\gamma}\bigg\{2\left[\bar{u}\left(\frac{\gamma_5-1}{2}\right)u\right]b_{\bar{s}\gamma}^\dagger a_{u_\beta}\\ \times\left[\bar{u}_L\left(\frac{\gamma_5+1}{2}\right)u\right]b_{\mu^+}^\dagger a_{u_\alpha}\bigg\}\end{aligned} \tag{23}$$

and the corresponding operators for the right-handed muon and antineutrino

$$\begin{aligned}O_{\mu_R^+}&=O_{\mu_L^+}(\gamma_5\to-\gamma_5)\,,\\ O_{\bar{\nu}_{\mu R}}&=O_{\mu_R^+}(\mu_R^+\to-\bar{\nu}_{\mu R},u_\alpha\to d_\alpha)\,.\end{aligned} \tag{24}$$

These operators give, in the nonrelativistic limit,

$$\begin{aligned}O_{\mu_L^+}(1,2)&=-\epsilon_{\alpha\beta\gamma}\frac{1}{\sqrt{2}}[\hat{v}_1(u_\alpha\to\mu_L^+)\hat{v}_2(u_\beta\to\bar{s}_\gamma)]\,,\\ O_{\mu_R^+}(1,2)&=O_{\mu_L^+}(1,2)(\mu_R^+\to\mu_L^+)\,,\\ O_{\bar{\nu}_{\mu R}}(1,2)&=-\epsilon_{\alpha\beta\gamma}\frac{1}{\sqrt{2}}[\hat{v}_1(d_\alpha\to-\bar{\nu}_{\mu R})\hat{v}_2(u_\beta\to\bar{s}_\gamma)]\,.\end{aligned} \tag{25}$$

The spin operator is just a constant, instead of $(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ for the electronic cases. As we will see, this will lead to a selection rule.

We will give the matrix elements in the simplified situation of neglecting the muon mass. Of course, this is not a good approximation as for the electron, but it is interesting to know the relation to the leptonic modes in this limit. The spin matrix elements are the same as before, and we get, for the flavor matrix elements,

$$\begin{aligned}\langle\mu_1^+\Phi_{K^0}(2,3)\,|\cdots|\,\varphi''^+\rangle&=-\left(\frac{2}{3}\right)^{1/2}\,,\\ \langle\mu_1^+\Phi_{K^0}(2,3)\,|\cdots|\,\varphi'^+\rangle&=0\,,\\ \langle\bar{\nu}_{\mu1}\Phi_{K^+}(2,3)\,|\cdots|\,\varphi''^+\rangle&=-\frac{1}{\sqrt{6}}\,,\\ \langle\bar{\nu}_{\mu1}\Phi_{K^+}(2,3)\,|\cdots|\,\varphi'^+\rangle&=-\frac{1}{\sqrt{2}}\,.\end{aligned} \tag{26}$$

All of these calculations give the rates of Table I (phase-space factor is not included, sum over meson polarizations and average over nucleon polarizations is made). The calculations of Table I include: (i) the flavor and spin matrix elements (20), (22), (ii) the factor $6/\sqrt{6}$, number of quark order-

TABLE I. The spin-averaged rates defined in the text (Sec. IV).

Process	$\langle\lvert M\rvert^2\rangle$	Process	$\langle\lvert M\rvert^2\rangle$
$p\to e_L^+\pi^0$	$\frac{9}{24}$	$p\to e^+\pi^0$	$\frac{45}{96}$
$p\to e_R^+\pi^0$	$\frac{9}{96}$		
$p\to e_L^+\eta$	$\frac{3}{24}$	$p\to e^+\eta$	$\frac{15}{96}$
$p\to e_R^+\eta$	$\frac{3}{96}$		
$p\to e_L^+\rho^0$	$\frac{3}{24}$	$p\to e^+\rho^0$	$\frac{15}{96}$
$p\to e_R^+\rho^0$	$\frac{3}{96}$		
$p\to e_L^+\omega$	$\frac{27}{24}$	$p\to e^+\omega$	$\frac{135}{96}$
$p\to e_R^+\omega$	$\frac{27}{96}$		
$p\to\bar{\nu}_{eR}\pi^+$	$\frac{9}{48}$		
$p\to\bar{\nu}_{eR}\rho^+$	$\frac{3}{48}$		
$p\to\mu_R^+K^0$	$\frac{9}{48}$	$p\to\mu^+K^0$	$\frac{9}{24}$
$p\to\mu_L^+K^0$	$\frac{9}{48}$		
$p\to\bar{\nu}_{\mu R}K^+$	0		

ings and normalization factor, (iii) the normalization of the nucleon wave function (18), $1/\sqrt{2}$, (iv) the factor $\sqrt{2}$ of color (17), (v) the factor $1/\sqrt{2}$ due to the normalization factor in front of (16), (vi) the factor $\frac{1}{2}$ in front of the interaction (1) (our coupling g at the grand unification mass is the one defined by Buras *et al.*[5]), and (vii) the factor 2 affecting the operator $O_{e_L^+}$ in the original Lagrangian (1). In the second column of Table I we have simply added the R and L lepton polarizations, neglecting the difference in renormalization due to $SU(2)\times U(1)$ from the unification mass to low energy in the R and L operators (2). We will take it into account in the numerical results, as we will see in the next section.

IV. ALGEBRAIC RELATIONS BETWEEN RATES. NUMERICAL RESULTS

Let us now comment on the algebraic results of Table I. We will use the notation $\overline{\Gamma}=\Gamma/\text{phase space}$ when the modes that we compare have very different Q values. We get the following relations between rates:

(i) Relation between R and L positron modes:

$$\Gamma(e_L^+X^0)=4\Gamma(e_R^+X^0)\,, \tag{27}$$

where X^0 is any neutral nonstrange meson. This relation is general (i.e., not specific of the approximations)[13] and comes from the R-L symmetry (15) and the factor 2 affecting the $O_{e_L^+}$ operator.

(ii) Relation between e_R^+ and $\bar{\nu}_{eR}$ modes:

$$2\Gamma(e_R^+X^0)=\Gamma(\bar{\nu}_{eR}X^+)\,. \tag{28}$$

If X^0, X^+ are nonstrange mesons belonging to the same isospin triplet. This comes from the $\Delta I=\frac{1}{2}$ rule of interaction (16).

(iii) Relation between isosinglet and isotriplet mesons:

$$\overline{\Gamma}(e^+\pi^0)=3\overline{\Gamma}(e^+\eta)\,, \tag{29}$$

$$\Gamma(e^+\omega^0)=9\Gamma(e^+\rho^0) \tag{30}$$

for e_R^+ or e_L^+. Relation (29) comes simply from flavor SU(3), and (30) from the quark model with ideal mixing of ω and ϕ. Note that the factor 9 comes just in the opposite way as in $\Gamma(\rho^0\to e^+e^-)=9\Gamma(\omega\to e^+e^-)$. This relation comes from the particular correlation between spin and isospin in the nucleon wave function (18). To our knowledge, this result is new.

(iv) Relation between pseudoscalar and vector-meson modes:

$$\overline{\Gamma}(e^+\pi^0)=3\overline{\Gamma}(e^+\rho^0)\,. \tag{31}$$

This relation is also a consequence of the spin-flavor SU(6) structure of the baryon and meson wave functions and the spin-flavor operators (16).

Relations (27), (28), and (29) are completely general, independent of the corrections to our exact-SU(6) limit. On the contrary, relations (30) and (31) are particular to our approximations and could be corrected by relativistic effects. We do expect, however, that the qualitative features will remain in view of our experience in other domains of particle physics.

We turn now to the Cabibbo-allowed strange-muonic decays. Within our drastic approximation of neglecting the muon mass, we get the following relations:

(v) Relation between R and L muons:

$$\Gamma(p\to\mu_L^+K^0)=\Gamma(p\to\mu_R^+K^0)\,. \tag{32}$$

(32) comes simply from (25) plus $m_\mu=0$.

(vi) Comparing muonic and electronic modes in our limit we get

$$\begin{aligned}\overline{\Gamma}(p\to\mu_L^+K^0)&=\tfrac{1}{2}\overline{\Gamma}(p\to e_L^+\pi^0)\,,\\ \overline{\Gamma}(p\to\mu_R^+K^0)&=2\overline{\Gamma}(p\to e_R^+\pi^0)\,,\\ \overline{\Gamma}(p\to\mu^+K^0)&=\tfrac{4}{5}\overline{\Gamma}(p\to e^+\pi^0)\,.\end{aligned} \tag{33}$$

These relations come from the relative factor of 2 between e_L^+ and e_R^+ in (1) and $1/\sqrt{2}$ coming from the π^0 wave function. Note that we have a different spin operator for μ^+, $\bar{\nu}_\mu$ production than for e^+, $\bar{\nu}_e$ [1 instead of $\frac{1}{2}(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$] but in the case of μ^+, the q_1q_2 state being uu, it must be in the $\varphi''\chi''$ state implying that 1 and $\frac{1}{2}(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ are equivalent.

(vii) The spin structure (25) leads to the selection rule

$$\Gamma(p\to\bar{\nu}_{\mu R}K^0)=0\,, \tag{34}$$

as we can see from the flavor (26) and spin (22a) matrix elements.

All these relations can be somewhat modified by the nonvanishing muon mass, but we expect they are qualitatively true. On this basis we do not understand why some authors find very small muonic modes. The estimations of phase space (Table II) and corrections due to the muon mass (we find from preliminary estimations about a 30% correction) do not explain in our opinion these small rates. Note that relation (34) does not have this lepton mass correction, and could be more general.

Let us comment now on the spatial matrix elements. It can be intuitively seen that the *amplitude* $p\to\bar{l}M$ is proportional to the spatial nucleon wave when two quarks are at the same point, and the rate proportional to the square of it,[5]

$$\langle\psi^s|\,\delta(\vec{r}_1-\vec{r}_2)\,|\psi^s\rangle\,. \tag{35}$$

Remember that in nonleptonic decays of hyperons, the *amplitude* was proportional to (35), since we had the mean value of $\delta(\vec{r}_1-\vec{r}_2)$ between the initial

TABLE II. Algebraic branching ratio from Table I (without phase space), phase-space factor $\rho(k)=kE_lE_m/M_p^3$, and proton partial decay rates as a function of the gauge-boson mass M_X.

Process	Algebraic branching ratio	Phase-space factor $\rho(k)$	Partial rate $[10^{-28}\text{yr}^{-1}(M_X/10^{14}\text{GeV})^{-4}]$
$p\to e^+\pi^0$	$\frac{45}{270}\simeq 0.17$	0.122	0.62
$p\to e^+\eta$	$\frac{15}{270}\simeq 0.05$	0.072	0.12
$p\to e^+\rho^0$	$\frac{15}{270}\simeq 0.05$	0.022	0.04
$p\to e^+\omega$	$\frac{135}{270}=0.50$	0.019	0.29
$p\to \bar\nu_e\pi^+$	$\frac{18}{270}\simeq 0.07$	0.122	0.25
$p\to \bar\nu_e\rho^+$	$\frac{6}{270}\simeq 0.02$	0.022	0.02
$p\to \mu^+K^0$	$\frac{36}{270}\simeq 0.13$	0.081	0.32
$p\to \bar\nu_\mu K^+$	0		0
Total			1.65

and the final baryons.[9] But the rate is not exactly given by (35) since the two outgoing quarks must recombine to give a meson. To have an idea of this effect, let us use harmonic-oscillator baryon and meson wave functions, the only ones which allow the separation of the center of mass:

$$\psi_N^s(\vec r_1,\vec r_2,\vec r_3)=\left(\frac{1}{\sqrt3\,\pi R_N^2}\right)^{3/2}\exp\left[-\sum_{i<j}\frac{(\vec r_i-\vec r_j)^2}{6R_N^2}\right],$$
$$\psi_M(\vec r_1,\vec r_2)=\left(\frac{1}{\sqrt{2\pi}R_M}\right)^{3/2}\exp\left[-\frac{(\vec r_1-\vec r_2)^2}{4R_M^2}\right] \quad (36)$$

normalized to unit with respect to the center-of-mass measure

$$\prod_i d\vec r_i\,\delta\left(\frac1n\sum_j\vec r_j\right),$$

with $n=3$ or 2. Using the fact that the mean value (35) is given, in the harmonic oscillator, by

$$\langle\psi^s|\,\delta(\vec r_1-\vec r_2)\,|\psi^s\rangle=\left(\frac{1}{\sqrt{2\pi}R_N}\right)^3, \quad (37)$$

we get, for the overlap integral (the outgoing lepton wave function is just a plane wave),

$$|S|^2=\frac{(\frac34 R_N^2R_M^2)^{3/2}}{[\frac12(R_M^2+\frac34R_N^2)]^3}\langle\psi^s|\,\delta(\vec r_1-\vec r_2)\,|\psi^s\rangle \times\exp\left(-\frac{6R_N^2R_M^2\vec k^2}{12R_N^2+16R_M^2}\right). \quad (38)$$

This expression shows the dependence in baryon and meson radii and the momentum transfer k. The factor in front of (38) is just equal to one if the meson and baryon radii satisfy the relation

$$4R_M^2=3R_N^2. \quad (39)$$

Moreover, this factor hardly deviates significantly from one. For example, if the qq in a baryon and the $q\bar q$ in a meson are bound by a harmonic confining potential with strength proportional to the color Casimir operator, $-\frac43$ for a $q\bar q$ color singlet, and $-\frac23$ for a qq color $\bar 3$, we get, with the definitions (36) instead of (39),

$$\frac{R_M^2}{R_N^2}=\frac{\sqrt3}{2} \quad (40)$$

which gives, for the factor in front of (38), 0.97. We think then that it is reasonable to take it equal to one. On the other hand, the exponential is also very close to unity even in the most unfavored situation of pion transitions since the baryon radius is close to $R_N^2=6\ \text{GeV}^{-2}$, as we have discussed elsewhere.[14] We will then take it also equal to one. Finally, we only keep the square of the wave function of the baryon at small distances (35),

$$|S|^2=\langle\psi^s|\,\delta(\vec r_1-\vec r_2)\,|\psi^s\rangle. \quad (41)$$

Taking into account all previous calculations plus phase space and renormalization from the grand unification mass (GUM) to present energies, the rates are given by

$$\Gamma=\left(\frac{g^2}{M_X^2}\right)^2\frac1\pi\left(\frac{kE_ME_l}{M_p}\right)|S|^2|A_3|^2 \times(\langle|M_L|^2\rangle|A_{12}^L|^2+\langle|M_R|^2\rangle|A_{12}^R|^2). \quad (42)$$

In this expression g is defined according to Buras *et al.*,[5] we assume $M_X=M_Y$, $|S|^2$ is simply given by (41), $\langle|M|^2\rangle$ are given in Table I, and A_3 and $A_{12}^{L,R}$ are, respectively, the $SU(3)_c$ (Ref. 5) and $SU(2)\times U(1)$ (Refs. 10, 12) renormalization factors:

$$A_3=\left[\frac{\alpha_s(\mu^2)}{\alpha(\mathrm{GUM})}\right]^{6/(33-4n)}, \tag{43}$$

$$A_{12}^L=\left[\frac{\alpha_2(M_Z^2)}{\alpha(\mathrm{GUM})}\right]^{27/(86-16n)}\left[\frac{\alpha_1(M_Z^2)}{\alpha(\mathrm{GUM})}\right]^{-69/(6+80n)}, \tag{44}$$

$$A_{12}^R=\left[\frac{\alpha_2(M_Z^2)}{\alpha(\mathrm{GUM})}\right]^{27/(86-16n)}\left[\frac{\alpha_1(M_Z^2)}{\alpha(\mathrm{GUM})}\right]^{-33/(6+80n)}, \tag{45}$$

where n is the number of generations ($2n=f$, f being the number of quark flavors), and

$$\alpha_2(M_Z^2)=\alpha_{\mathrm{em}}/\sin^2\theta_W\,,\quad \alpha_1(M_Z^2)=\frac{5\alpha_{\mathrm{em}}}{3\cos^2\theta_W}\,. \tag{46}$$

We need now a determination of (42). We will use for it the rather model-independent estimation from nonleptonic hyperon decays[9] whose absolute magnitude is given in terms of the SU(3) coupling,

$$F=\tfrac{3}{2}\sin\theta_C\cos\theta_C\langle\psi^s|\,\delta(\vec{r}_1-\vec{r}_2)\,|\psi^s\rangle c_-\,. \tag{47}$$

We have taken into account the quantum-chromodynamic short-distance factor c_- to be consistent with the previous calculation where we have considered this type of corrections

$$c_-=\left[\frac{\alpha_s(\mu^2)}{\alpha_s(M_W^2)}\right]^{12/(33-4n)}. \tag{48}$$

F is just a combination of matrix elements of the weak Hamiltonian (the parity-conserving part of it) between baryon states,

$$F=-\frac{1}{4}\left(\frac{1}{\sqrt{2}}\langle p|H_w^{\mathrm{PC}}|\Sigma^+\rangle+\sqrt{3}\langle n|H_w^{\mathrm{PC}}|\Lambda\rangle\right)(2\pi)^3\,. \tag{49}$$

We see that c_- enters the F expression just as $|A_3|^2$ in the one of the width (42), multiplying the square of the wave function at the origin. From (48) and (43) we see moreover that the anomalous dimension in c_- is just twice the one in the factor A_3. Then,

$$\frac{|A_3|^2}{c_-}=\left[\frac{\alpha_s(\mu^2)}{\alpha(\mathrm{GUM})}\right]^{12/(33-4n)}\left[\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)}\right]^{12/(33-4n)}=\left[\frac{\alpha_s(M_W^2)}{\alpha(\mathrm{GUM})}\right]^{12/(33-4n)}. \tag{50}$$

We see that in the ratio (50) the strong coupling $\alpha_s(\mu^2)$ at the scale μ^2—where both phenomena, nonleptonic decays and proton decay, take place—cancels out in this expression. We can then express the product

$$|A_3|^2\langle\psi^s|\,\delta(r_1-r_2)\,|\psi^s\rangle=\left[\frac{\alpha_s(M_W^2)}{\alpha(\mathrm{GUM})}\right]^{12/(33-4n)}\frac{2F}{3G\sin\theta_C\cos\theta_C} \tag{51}$$

in terms of the magnitude of the nonleptonic amplitude F and an enhancement factor at the M_W^2 scale, just as the electroweak factors (44) and (45).

From

$$\alpha_s(Q^2)=\frac{1}{\left(\frac{33-2f}{12\pi}\right)\ln\left(\frac{Q^2}{\Lambda^2}\right)}, \tag{52}$$

we get, with $\Lambda^2=0.3$ GeV^2 and $f=6$, $\alpha_s(M_W^2)=0.18$. From the value for the coupling at the grand unification mass[6,10]

$$\alpha(\mathrm{GUM})=\tfrac{1}{42}\,, \tag{53}$$

we get $|A_3|^2/c_-=3.20$. From the value of F fitted from P waves[15] (see Ref. 16 for a discussion on this point), we obtain

$$\frac{|A_3|^2\langle\psi^s|\,\delta(\vec{r}_1-\vec{r}_2)\,|\psi^s\rangle}{M_p^3}=0.046. \tag{54}$$

To clarify the origin of all numerical factors, and to express Γ as a function of M_X (normalized to 10^{14} GeV), we rewrite (42) under the form

$$\Gamma=\left[\frac{M_p^5}{(10^{14}\ \mathrm{GeV})^4}\right]\left(\frac{10^{14}\ \mathrm{GeV}}{M_X}\right)^4\rho(k)16\pi[\alpha(\mathrm{GUM})]^2\left[\frac{|A_3|^2\langle\psi^s|\,\delta(\vec{r}_1-\vec{r}_2)\,|\psi^s\rangle}{M_p^3}\right][\langle|M_L|^2\rangle|A_{12}^L|^2+\langle|M_R|^2\rangle|A_{12}^R|^2]\,. \tag{55}$$

We have factorized (54), and the dimensionless expression

$$\rho(k)=\frac{kE_lE_m}{M_p^3} \tag{56}$$

coming from phase space and varying from mode to mode. The factor in front of (55) gives the dimensions

$$\frac{M_p^5}{(10^{14}\ \mathrm{GeV})^4}=3.47\times10^{-25}\ \mathrm{yrs}^{-1}\,. \tag{57}$$

For the SU(2) × U(1) renormalization factors we adopt the values of Ellis *et al.*[10]

$$|A_{12}^L|^2=2.49\,,\quad |A_{12}^R|^2=2.20\,. \tag{58}$$

The values of $\langle|M_{R,L}|^2\rangle$ averaged over the nucleon and summed over the final-meson polarization are given in Table I.

In order to show the origin of all numerical values, we give in Table II the algebraic branching ratio (without phase space) expected from the results of Table I, the phase space factors $\rho(k)$ for each mode, and finally the partial widths in years^{-1} for various values of M_X. In Table III we give the branching ratios relative to the *total two-body decay rate*. There are then *upper bounds* on these branching ratios—close to the actual values since

TABLE III. Branching ratios of nucleon decays relative to the total two-body decay rate.

Process	Branching ratio	Process	Branching ratio
$p\to e^+\pi^0$	37%	$n\to e^+\pi^-$	74%
$p\to e^+\eta$	7%	$n\to e^+\rho^-$	4%
$p\to e^+\rho^0$	2%	$n\to \bar\nu_e\pi^0$	7.5%
$p\to e^+\omega$	18%	$n\to \bar\nu_e\rho^0$	0.5%
$p\to \bar\nu_e\pi^+$	15%	$n\to \bar\nu_e\eta$	1.5%
$p\to \bar\nu_e\rho^+$	1%	$n\to \bar\nu_e\omega$	3.5%
$p\to \mu^+K^0$	19%	$n\to \bar\nu_\mu K^0$	9.5%
$p\to \bar\nu_\mu K^+$	0		

three-body decays cannot be a large fraction.

Concerning the proton lifetime we obtain an upper bound—close to the actual value—

$$\tau(p)\approx 6\times\left(\frac{M_X}{10^{14}\text{ GeV}}\right)^4. \tag{59}$$

TABLE IV. Proton lifetime as a function of the gauge-boson mass M_X. This includes only two-body modes.

M_X (GeV)	$\tau(p)$ (yr)
10^{14}	6×10^{27}
3×10^{14}	4.8×10^{29}
6×10^{14}	7.8×10^{30}
10^{15}	6×10^{31}
10^{16}	6×10^{35}

We get, for various values of M_X within the range considered by Goldman and Ross,[6] the lifetimes of Table IV, and the neutron lifetime equal to the proton's within 1%.

We will comment on all these results in Sec. VI.

V. EXTENSION TO SO(10)

In the case of SO(10)[11] we have to replace the effective Lagrangian (1) by

$$\begin{aligned}\mathcal{L}=\frac{g^2}{2}\epsilon_{\alpha\beta\gamma}\Big\{&(\bar u^c_{\gamma L}\gamma_\mu u_{\beta L})\left[\left(\frac{1}{M_X^2}+\frac{1}{M_Y^2}\right)(\bar e^+_L\gamma^\mu d_{\alpha L})+\frac{1}{M_X^2}(\bar\mu^+_L\gamma^\mu s_{\alpha L})+\frac{1}{M_X^2}(\bar e^+_R\gamma^\mu d_{\alpha R})+\frac{1}{M_X^2}(\bar\mu^+_R\gamma^\mu s_{\alpha R})\right]\\&+(\bar u^c_{\gamma L}\gamma_\mu d_{\beta L})\left[\frac{1}{M_Y^2}(\bar\nu^c_{eR}\gamma^\mu d_{\alpha R})+\frac{1}{M_Y^2}(\bar\nu^c_{\mu R}\gamma^\mu s_{\alpha R})\right]\\&+(\bar d^c_{\gamma L}\gamma_\mu u_{\beta L})\left[\left(\frac{1}{M_{Y'}^2}+\frac{1}{M_{X_D}^2}\right)(\bar\nu^c_{eL}\gamma^\mu d_{\alpha L})+\frac{1}{M_{Y'}^2}(\bar\nu^c_{\mu L}\gamma^\mu s_{\alpha L})+\frac{1}{M_{Y'}^2}(\bar e^+_R\gamma^\mu u_{\alpha R})\right]\\&+(\bar d^c_{\gamma L}\gamma_\mu d_{\beta L})\frac{1}{M_{X_D}^2}(\bar\nu^c_{eR}\gamma^\mu u_{\alpha R})\Big\}.\end{aligned} \tag{60}$$

Clearly, when we take $M_X^2=M_Y^2$, and $M_{Y'}^2=M_{X_D}^2=\infty$, we recover (1). $\nu^c_{e_L}$ is the left-handed antineutrino, in the same irreducible $\underline{16}$ representation as all other left-handed fermions of one family. Note that now the reason for the sign between the R and L fermions we have found for the SU(5) case, Eq. (1), is clear. This relative sign +, differs from the one found by other authors,[5,8] and agrees with the one found by Machacek.[7] Indeed, SO(10) is a left-right-symmetric theory which asymptotically has a *pure vector* coupling. Recall that it contains the electroweak pure vector theory $SU(2)_L\times SU(2)_R\times U(1)$. With the opposite relative sign this coupling would be purely axial. Technically this relative sign comes from the following prescription in contracting the tensors $\langle\overline{10}|24|10\rangle$: $\bar b_{ik}(T^a)^k_j b^{ij}$, where b^{ij} is the rank-two antisymmetric tensor $\underline{10}$, $(\bar b)_{ij}=(b^{ij})$, and T is the $\underline{24}$ adjoint tensor. Only this prescription gives *the same charge* to e^+_R and e^+_L, d_R and d_L, and so on.

The calculation for the SO(10) case then proceeds in a straightforward way as we have just described. The algebraic relations exposed in Sec. IV generalize very easily to this general case. Relation (27) generalizes to

$$\frac{\Gamma(p\to e^+_R X^0)}{\Gamma(p\to e^+_L X^0)}=\left(\frac{(1/M_X^2)+(1/M_{Y'}^2)}{(1/M_X^2)+(1/M_Y^2)}\right)^2, \tag{61}$$

with a similar result for the ratio between left-handed and right-handed antineutrinos:

$$\frac{\Gamma(p\to\bar\nu_{e_R}X^+)}{\Gamma(p\to\bar\nu_{e_L}X^+)}=\left(\frac{(1/M_Y^2)+(1/M_{X_D}^2)}{(1/M_{Y'}^2)+(1/M_{X_D}^2)}\right)^2. \tag{62}$$

And relation (32) is still true in the general case

$$\frac{\Gamma(p\to\mu^+_R K^0)}{\Gamma(p\to\mu^+_L K^0)}=1, \tag{63}$$

as well as the selection rule we have obtained for the K^+ decay mode,

$$\Gamma(p\to\bar\nu_{\mu R}K^+)=0 \tag{64}$$

within, of course, the same approximations as before.

Relation (28) generalizes to

$$\frac{2\Gamma(p\to e_R^+X^0)}{\Gamma(p\to\bar{\nu}_{e_R}X^+)}=\left(\frac{(1/M_X^2)+(1/M_{Y'}^2)}{(1/M_Y^2)+(1/M_{X_D}^2)}\right)^2 \tag{65}$$

and analogously for e_L^+ and $\bar{\nu}_{eL}$. Flipping now all isospins, we can obtain the corresponding neutron decay modes:

$$\frac{\Gamma(p\to e_R^+X^0)}{\Gamma(n\to\bar{\nu}_{e_R}X^0)}=\left(\frac{(1/M_X^2)+(1/M_{Y'}^2)}{(1/M_Y^2)+(1/M_{X_D}^2)}\right)^2, \tag{66}$$

$$\frac{\Gamma(p\to e_L^+X^0)}{\Gamma(n\to\bar{\nu}_{e_L}X^0)}=\left(\frac{(1/M_X^2)+(1/M_Y^2)}{(1/M_{Y'}^2)+(1/M_{X_D}^2)}\right)^2, \tag{67}$$

$$\frac{\Gamma(p\to\mu_R^+K^0)}{\Gamma(n\to\bar{\nu}_{\mu_R}K^0)}=\left(\frac{1/M_X^2}{1/M_Y^2}\right)^2, \tag{68}$$

$$\frac{\Gamma(p\to\mu_L^+K^0)}{\Gamma(n\to\bar{\nu}_{\mu_L}K^0)}=\left(\frac{1/M_X^2}{1/M_{Y'}^2}\right)^2, \tag{69}$$

$$\frac{\Gamma(p\to e_R^+X^0)}{\Gamma(n\to e_R^+X^-)}=\frac{1}{2}, \tag{70}$$

$$\frac{\Gamma(p\to\bar{\nu}_{eR}X^+)}{\Gamma(n\to\bar{\nu}_{eR}X^0)}=2\,. \tag{71}$$

(In all these relations X^+, X^-, and X^0 are nonstrange systems belonging to the same isomultiplet.)

Note that most of these relations have been outlined by Machacek.[7] From these relations one gets easily, for SU(5), the neutron rates (Table III).

VI. DISCUSSION AND CONCLUSION

Let us now comment on our results summarized in the tables. We obtain, for the range of M_X proposed by Goldman and Ross, $M_X\sim(2-6)\times10^{14}$ GeV, a lifetime which is close or even below the experimental limit, $\tau(p)>2\times10^{30}$ yr. There are, of course, uncertainties in our calculation, but we think that this is an interesting and encouraging result for experimentalists, since this means that maybe we will soon have experimental results on proton decay.

The absolute magnitude of the total rate is given by M_X, essentially, and by the hadronic matrix element. We have eliminated uncertainties on this last factor by relating it to the nonleptonic hyperon amplitudes. Since we have made the same approximations in both cases—proton decay and hyperon decays—we are confident in our normalization.

Concerning our approximations, we think that, *grosso modo*, our prediction of the branching ratios will not be changed significantly if we take into account relativistic effects of the internal quark motion. We know for instance from relativistic effects in the nucleon axial-vector coupling, that this type of corrections to the simple static result are at most of the order of 30%. This can change the absolute magnitude of the rate maybe by about a factor of 2, and the branching ratios by presumably much less.

We do not understand how some authors do not find a significant $e^+\omega$ rate for which we get, *algebraically, half of the total rate* (Table II). Phase space, however, suppresses somehow this very intense mode. Donoghue finds 56% for this decay; this seems too much. We think that, on the other hand, his 9% for $e^+\pi^0$ is too small. This comes from the form-factor suppression by $\frac{1}{3}$ in amplitude of his model for pionic modes and seems correlated to his large ωe^+. The corrective factor in (38) is, for pion modes, about 0.8 *in rate*. Note that other classical phenomena correctly described by the naive quark model, as $\omega\to\pi^0\gamma$, have a similar momentum transfer. On the other hand, it seems (private communication) that Donoghue finds now 14% of $K^0\mu^+$. All these effects go in the direction of a pattern of branching ratios qualitatively close to our results, if one takes off the pionic suppression factor of $\frac{1}{9}$ in rate of Donoghue.

A last comment is to be made about the large branching ratio in μ^+K^0 (19%). This is interesting since muons can be easily detected. Previous works find very small rates for this mode. We think that our algebraic result is correct, we do not see really how the spin-flavor matrix element should be small. Moreover, phase-space suppression relative to the π modes is not very large (Table II). It is true, however, that because of simplicity we have neglected $m_\mu\neq0$ effects which will give an interference between the right and left parts of the interaction. But these corrections even increase the μ^+K^0 by a factor $(E_\mu+m_\mu)/E_\mu=1.3$. This is due to the relative sign we get in the effective Lagrangian (1) between left and right couplings which leads to constructive left-right interference.

In conclusion, we think that our study can clarify the situation concerning proton-decay branching ratios. Our estimation is easily controllable, since we obtain algebraic relations between matrix elements, and their scale is fixed by the gauge-vector-boson masses and by the related phenomenon of the nonleptonic hyperon decays.

Note added. Our static approximation implies that the operator responsible for proton decay is pure S wave ($L=0$). Taking into account the relativistic corrections would induce higher waves' contributions. We do not claim that p and d waves are negligible. We rather guess that realistic inclusion of their contributions would not change the qualitative features we have stressed. In our opinion this guess has been afterwards confirmed by a recent work from Kane and Karl.[17] They use three models, a static one equivalent to ours and two models including intermediate and large relativistic contributions. The qualitative results happen, indeed, to agree in all three models: the dominant channels are the same.

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