PHYSICAL REVIEW D VOLUME 23, NUMBER 7 1 APRIL 1981

CP violation in B-meson decays

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(Received 27 June 1980)

The pattern of CP violation in the bottom sector is discussed. We introduce general techniques to expose new CP-violating effects in the cascade decays of B mesons. In the Kobayashi-Maskawa (KM) model, the CP asymmetries so obtained range from 2–20 % for plausible values of the model parameters. This is to be compared with the small effects, of order 10^{-3}–10^{-4}, previously exhibited within this model. Effects of this size should be observable in upcoming experiments. Our approach stresses the on-shell transitions which make up the cascade decays of heavy mesons to ordinary hadrons, as opposed to the off-shell transitions which occur in the analogs of K^0-$\bar{K}^0$ mixing. The CP asymmetries generated by our techniques are of order $\sin\delta$, where δ is the KM phase angle, and thus represent the maximum effects obtainable in this model.

I. INTRODUCTION

Observation of CP violation has so far been confined to the strange-neutral-meson system. There the symmetry-violating effects are attributable solely to the presence of CP impurities in the K_L and K_S. There is *a priori* no reason to suppose that symmetry violation will be exhibited in such a limited manner in other systems. On the contrary, general expectation holds that as new-flavor thresholds are passed, CP violation will reveal itself more pervasively and more boldly, extending to on-shell transitions and exceeding the asymmetries, of order 10^{-3}, observed in the kaon system. These expectations are particularly keen for transitions involving the bottom (b) quark.

Within the six-quark Kobayashi-Maskawa (KM) model,[1] one expects these general expectations to be fulfilled. In this model CP violation cannot occur in on-shell processes below charm threshold. The only way for CP-violating phases to enter the strange sector is through off-shell transitions to heavy flavors. In this way the model accounts naturally for the isolation of symmetry violation to the K^0-$\bar{K}^0$ system.

Above charm threshold, but below bottom threshold, symmetry violation in on-shell transitions is possible, but the asymmetries generated are necessarily of order $\sin\theta_2\sin\theta_3\sin\delta$, where θ_i and δ are the familiar KM mixing angles and phase, respectively. Phenomenological analysis of the CP-violating parameter ϵ_K indicates that this combination of angles is small: $\sin\theta_2\sin\theta_3\sin\delta \sim 10^{-3}$–$10^{-4}$. Thus CP effects in on-shell charm transitions are small. Virtual effects, through D^0-$\bar{D}^0$ transitions,[8] can be present, but the KM model suggests that these transitions might be too infrequent to exhibit the CP impurities. Experimental data, although not yet very restrictive, do not contradict this suggestion.

Above bottom threshold on-shell effects can be large, of order $\sin\delta$. Most authors[2-4] who have previously studied CP violation in the bottom sector, however, have confined themselves to virtual effects, i.e., to estimates of CP admixture in the B^0-$\bar{B}^0$ system, in rather strict analogy to the K^0-$\bar{K}^0$ system. Yet it appears[4] that the observable effects due to CP impurities in the mixed B^0-$\bar{B}^0$ states are in fact rather small.

In this paper we emphasize the importance of studying on-shell transitions in the bottom sector, as opposed to virtual mixing effects. The former can, we argue, produce CP asymmetries $\sim 10^{-1}$–10^{-2}, whereas the latter produces only asymmetries $\sim 10^{-3}$–10^{-4}. One method of exhibiting the KM phase is to produce, through mixing, a coherent beam of B^0 and $\bar{B}^0$ and observe a hadronic decay channel to which both components contribute. The total rate samples interference between the two decay amplitudes, displaying the CP-violating KM phase δ. This method makes use of the mixing, which might be strong, but does not rely solely upon CP impurity in the mixing. Earlier suggestions for detecting CP violation in the B^0-$\bar{B}^0$ system involve searches for charge asymmetries in semileptonic decays. Since the semileptonic-decay Hamiltonian conserves CP, these methods are only sensitive to the mixing Hamiltonian, which by itself is not believed to give rise to large symmetry-violating effects. In contrast, observation of purely hadronic channels, as we suggest here, allows one to take advantage of potentially large-CP-violating phases in the quark-cascade-decay amplitudes. This method produces symmetry-violating effects proportional to the parameters characterizing the mixing and to the relative phase of B^0 and $\bar{B}^0$ decay amplitudes to the same final state. In the KM model both of these factors are expected to be large.

A second method we discuss involves seeking two different, coherent cascades of a bottom meson to ordinary hadrons. Interference of the two cascade amplitudes depends upon the KM phase δ and changes sign in going from meson to antimeson decay. A difference of decay widths in the CP-conjugate channels results. This method does not depend upon mixing and can be applied to charged as well as neutral mesons, but it involves measurement of decay modes suppressed by one power of the quark mixing angles.

In both methods, detection of K_S's in the final state is essential. These particles, because of their simple CP properties, arise naturally in symmetry-violating processes.

We shall give examples of both methods. We hope in this longer version of our results[5] to provide a discussion sufficiently general to be useful to experimentalists planning searches for CP violation in B-meson decay.

We need not belabor the obvious fact that the merit of testing CP symmetry in bottom decay does not reside in testing the KM model. The vital issue is whether the elusiveness of CP violation to date reflects merely our limited sampling of the weak Hamiltonian. In this view CP symmetry is in no way fundamental to the weak interactions. Against this prospect must be weighted the usual alternative, the appeal of which has not improved with aging, that exotic and extraordinarily discreet new interactions are to be invoked. The KM model aside, it is natural to suppose that if CP violation is widespread in the weak interactions, then it emerges more and more plainly as new flavor thresholds are passed. This is simply because the number of terms and number of combinations of terms sampled from the weak Hamiltonian increase and because, with a richer spectrum of states available, the constraints of CPT symmetry are weaker. The likelihood that CP violation will be found in the bottom sector therefore seems quite high and offers a refuge to those annoyed by the apparent monotony of the flavor spectrum.

Last, we would like to point out that the spirit of our discussion, emphasizing on-shell effects, extends easily to top quarks and beyond, although the details will of course be different.

In Sec. II we review the general pattern of CP violation in the KM model. Section III reexamines CP violation in the neutral conjugate mesons B^0-$\bar{B}^0$, where the issue has been most extensively discussed. We shall follow others[4] in arguing that CP impurities lead by themselves to small effects, although the mixing might be strong. We conclude that symmetry-violating effects which depend upon these impurities, such as charge asymmetries in semileptonic decays of B^0-$\bar{B}^0$ mesons produced in e^+e^- annihilation, are not fruitful grounds for experimental search. Section IV turns instead to the prospects for real transitions from bottom mesons to ordinary hadrons. We lay down a general program for exhibiting symmetry-violating effects of order $\sin\delta$, which can be 2 orders of magnitude greater than previously exhibited effects. In Sec. V we analyze in detail several explicit examples of these methods. Numerical magnitudes of a typical CP asymmetry generated by these methods, for plausible ranges of the KM parameters, are presented in Sec. VI. Last, Sec. VII contains some concluding remarks.

II. AN OVERVIEW OF CP VIOLATION IN THE KM MODEL

The representation of the KM matrix which we shall use is[1]

$$U=\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}=\begin{pmatrix} U_{du} & U_{su} & U_{bu} \\ U_{dc} & U_{sc} & U_{bc} \\ U_{dt} & U_{st} & U_{bt} \end{pmatrix}, \tag{2.1}$$

where $s_i=\sin\theta_i$, $c_i=\cos\theta_i$. The matrix U relates the $I_Z=-\frac{1}{2}$ members d', s', b' of the $SU(2)_L$ doublets to the eigenstates d, s, b of strong flavor

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L . \tag{2.2}$$

The phenomenology of the model requires, in addition to determination of the angles θ_i and δ, knowledge of the top-quark mass. The angle θ_1 is determined from Cabibbo fits,[6] and s_2 and δ are determined as functions of the remaining unknowns s_3 and m_t by theoretical analysis[7] of the K_L-K_S mass difference and the CP-violating parameter ϵ_K. Although better constraints on the angles are clearly required, the data are at present consistent with all angles being small, of the same order or less than $\theta_1 \cong \theta_C$, where θ_C is the Cabibbo angle, $\sin\theta_C=0.23$. In this paper we shall often present results in the approximation $s_i \ll c_i \cong 1$. This approximation will not mislead as to order of magnitude and avoids an inessential clutter of angular factors. We make no assumptions about δ.

Recall that one arrives at the KM parametrization (2.1) by maximal reduction, through rotation of quark field phases, of the number of complex phases in the weak current. The persistence of the phase angle δ represents the impossibility of reducing, using the quark-phase freedom, the unitary matrix U to a real orthogonal matrix. It follows from this analysis that in order to exhibit the CP-violating phase, a physical process must involve no less than four U-matrix elements, no three of which are drawn from the same row or column. This means that on-shell processes below charm threshold cannot be CP violating, since these processes involve only U_{su} and U_{du}. Above charm threshold, CP violation can occur in on-shell processes, but the effect is necessarily proportional to

$$\frac{\mathrm{Im}\, U_{sc}U_{su}U_{dc}U_{du}}{\mathrm{Re}\, U_{sc}U_{su}U_{dc}U_{du}} \sim s_2 s_3 \sin\delta \,. \tag{2.3}$$

Thus the symmetry-violating effect is suppressed by the angles θ_2 and θ_3 ($s_2 s_3 \sin\delta \sim |\epsilon_K| \sim 10^{-3}$). Of course, decays of strange and charmed mesons can exhibit CP asymmetries through virtual effects in the preparation of the initial states, as in the mixing of K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$.[8] The off-shell transitions implicate more of the U-matrix elements, those coupling the light to heavy quarks, with the potential to introduce large CP-violating relative phases.

Above bottom threshold, large phases can appear in on-shell transitions, giving rise to CP-violating effects proportional to, e.g.,

$$\frac{\mathrm{Im}\, U_{bc}U_{bu}U_{sc}U_{su}}{\mathrm{Re}\, U_{bc}U_{bu}U_{sc}U_{su}} \sim \left(\frac{s_2}{s_3 + s_2\cos\delta}\right) \sin\delta \,. \tag{2.4}$$

Thus symmetry violation in b decays can be as much as 2 orders of magnitude larger than in charm decay, or about 10%. By contrast, as we shall show in the following section, off-shell processes in the B^0-$\bar{B}^0$ mixing yield much smaller symmetry-violating effects, of order 10^{-3}. For this reason we are led to propose in this paper new experimental searches for CP violation which emphasize the on-shell effects.

If any of the mixing angles θ_i should vanish, there can be no CP violation in the KM model. Setting any of the mixing angles equal to zero effectively decouples one of the doublets. The submatrix which couples the remaining two doublets can then be reduced to real Cabibbo form by a quark-phase rotation. Thus formally all CP asymmetries must vanish like $s_1 s_2 s_3 \sin\delta$. However, the fractional CP asymmetry—i.e., the ratio of the CP-asymmetric rate to the symmetric rate—need not vanish in this way, since the $\sin\theta_i$ factors can cancel out in the ratio. It is of course not necessary to consider subdominant channels to effect this cancellation: The dominant decay modes of bottom mesons are already nominally suppressed by s_2 and s_3. Thus both the fractional CP asymmetry *and* the overall rate can be large. In charm decay, as we have emphasized, it is only possible to cancel s_1 in the fractional asymmetry, and the percentage of the total rate which is CP violating is necessarily $O(s_2 s_3 \sin\delta)$ [see Eq. (2.3)].

If any two $I_Z = \frac{1}{2}$ or $-\frac{1}{2}$ quarks are degenerate, CP violation is again impossible in the KM model. In this case the strong interactions of the degenerate pair possess an SU(2) symmetry rather than a U(1) $\times$ U(1) symmetry, and the extra freedom to define the flavor eigenfields is just sufficient to remove the phase δ from the KM matrix. This observation often proves useful in calculations, as does the symmetry of the KM Hamiltonian under the interchanges $(u,c,t) \leftrightarrow (d,s,b)$ and $\theta_2 \leftrightarrow \theta_3$.

To close this section, we repeat some remarks due to Pais and Treiman[8] concerning the constraints imposed by CPT symmetry and their implications for observing CP violation. Let P and $\bar{P}$ denote a particle of definite lifetime and its antiparticle, F_i final states into which P can decay, and $\bar{F}_i$ the corresponding CP-conjugate states, obtained from F_i by changing particles to antiparticles and reversing momenta and spins. To test CP symmetry one wishes to compare the decay $P \to F_i$ to the conjugate decay $\bar{P} \to \bar{F}_i$. CPT of course requires the *total* widths of P and $\bar{P}$ to be equal:

$$\sum_{\text{all } i} \Gamma(P \to F_i) \underset{CPT}{=} \sum_{\text{all } i} \Gamma(\bar{P} \to \bar{F}_i) \,. \tag{2.5}$$

Further, is we organize the states F_i into subgroups $\{F_i\}_\alpha, \{F_i\}_\beta, \ldots$ such that the states within each subgroup couple to one another via strong and electromagnetic final-state interactions, but there are no such interactions between states in different subgroups, then the *partial* width of P to each subgroup must be equal by CPT (*independent* of CP symmetry) to the width of $\bar{P}$ to the corresponding subgroup of conjugate states:

$$\begin{aligned} \sum_{i \in \{\,\}_\alpha} \Gamma(P \to F_i) &\underset{CPT}{=} \sum_{i \in \{\,\}_\alpha} \Gamma(\bar{P} \to \bar{F}_i) \,, \\ \sum_{i \in \{\,\}_\beta} \Gamma(P \to F_i) &\underset{CPT}{=} \sum_{i \in \{\,\}_\beta} \Gamma(\bar{P} \to \bar{F}_i) \,, \\ &\ \vdots \end{aligned} \tag{2.6}$$

The subgroups $\{F_i\}_{\alpha,\beta,\ldots}$ are labeled by quantum numbers conserved by the strong and electromagnetic interactions. We are of course ignoring weak final-state interactions. If a subgroup contains only a single state, then comparison of decays in that channel and its conjugate can provide no information about *CP* symmetry. The equality of the widths $\Gamma(K^+\to\pi^+\pi^0)$ and $\Gamma(K^-\to\pi^-\pi^0)$ is an example: it is required by *CPT*, independent of *CP*. On the other hand, whereas *CPT* requires the total three-pion widths of K^+ and K^- to be equal, examination separately of $\Gamma(K^+\to 2\pi^+\pi^-)$ vs $\Gamma(K^-\to 2\pi^-\pi^+)$ and $\Gamma(K^+\to\pi^+2\pi^0)$ vs $\Gamma(K^-\to\pi^-2\pi^0)$ can furnish tests of *CP* symmetry.

In general, the constraints of *CPT* symmetry become weaker as one progresses from strange to charm to bottom decays. For instance, the widths of $D^\pm$ to $\pi^\pm\pi^0$ need not be equal as must the widths of $K^\pm$ to the same final states. As the number of possible final states F_i increases, the probability that a subgroup $\{F_i\}_\alpha$ contains only a single state or a small number of states decreases. For decays of bottom mesons, the spectrum of hadronic channels is so rich that, as a practical matter, the constraints of *CPT* symmetry rarely frustrate searches for *CP* asymmetries. For semileptonic channels, however, this is not so. Proposed searches for lepton charge asymmetries in the decays of B_0 and $\bar{B}_0$ mesons are sensitive to *CP* violation in the Hamiltonian only insofar as mixing occurs in the initial state; the subsequent semileptonic-decay process is *CP*-invariant. In the following section we shall see that *CP* impurities in the mixing are small. One is therefore led to consider purely hadronic, as opposed to semileptonic, decay modes. In the former case *CP* violation can occur in the decay cascade itself, as opposed to the initial-state mixing. The resulting *CP* asymmetries in the hadronic final state are large.

III. MIXING AND *CP* IMPURITIES IN THE NEUTRAL CONJUGATE MESONS

Most studies of *CP* violation in the bottom sector[2-4] have focused upon impurities in the mixing of the conjugate neutral mesons B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$. (We shall follow the standard system of nomenclature for the neutral conjugate mesons of the six-quark model: $D_0\equiv c\bar{u}$, $B_s^0\equiv b\bar{s}$, $B_d^0\equiv b\bar{d}$, $T_u^0=t\bar{u}$, $T_c^0=t\bar{c}$. B_0 will stand generically for B_s^0 or B_d^0; likewise T_0 for T_u^0 or T_c^0.) This is a natural focus, since the formal analogies to the K^0-$\bar{K}^0$ system are clear. Where it is necessary to adjust the phenomenology to the shorter lifetimes and more diverse decay modes of the heavier mesons, guidance has already been given in studies of the D^0-$\bar{D}^0$ system.[8] In this section we will review arguments[4] suggesting that although the mixing might be strong in the B^0-$\bar{B}^0$ system, the *CP* impurities in the mixed states are probably small. Searches for *CP* violation which rely upon exposing these impurities, e.g., by measuring charge asymmetries in semileptonic decays of B^0 and $\bar{B}^0$, are thus unlikely to prove fruitful. The strong mixing has other uses, however: Coherent admixtures of B^0 and $\bar{B}^0$ so produced can reveal in their nonleptonic cascade decays the *CP*-violating phases discussed in the previous section. The overall prospects for observing *CP* violation in the B^0-$\bar{B}^0$ system are thus far from grim, as we shall show in the next section.

The qualitative pattern of mixing among the neutral conjugate mesons is easily understood within the KM model and has been discussed by several authors.[2,3,9] The time evolution of a neutral conjugate system is described in the basis of strong interaction eigenstates (denoted generically by $P^0,\bar{P}^0$) by a Hamiltonian

$$H\begin{pmatrix}P^0\\ \bar{P}^0\end{pmatrix}=\begin{pmatrix}M-\frac{i}{2}\Gamma & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & M-\frac{i}{2}\Gamma\end{pmatrix}\begin{pmatrix}P^0\\ \bar{P}^0\end{pmatrix}. \quad (3.1)$$

The states of definite mass $M_{1,2}$ and lifetime $\Gamma_{1,2}$ are

$$P_1=\frac{(1+\epsilon)P^0+(1-\epsilon)\bar{P}^0}{[2(1+|\epsilon|^2)]^{1/2}}\equiv pP^0+q\bar{P}^0,$$
$$P_2=\frac{(1+\epsilon)P^0-(1-\epsilon)\bar{P}^0}{[2(1+|\epsilon|^2)]^{1/2}}\equiv pP^0-q\bar{P}^0. \quad (3.2)$$

Here

$$\frac{p}{q}=\frac{1+\epsilon}{1-\epsilon}=\left(\frac{M_{12}-\frac{i}{2}\Gamma_{12}}{M_{12}^*-\frac{i}{2}\Gamma_{12}^*}\right)^{1/2} \quad (3.3)$$

and we define

$$\Gamma\equiv\Gamma_1+\Gamma_2,$$
$$\Delta\Gamma\equiv\Gamma_1-\Gamma_2=-4\,\mathrm{Im}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}, \quad (3.4)$$
$$\Delta m=m_1-m_2=2\,\mathrm{Re}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}.$$

A state of pure P^0 prepared by the strong interactions becomes at a time t later the superposition

$$|\psi(t)\rangle=f_+(t)\,|P^0\rangle+\frac{q}{p}f_-(t)\,|\bar{P}^0\rangle, \quad (3.5)$$

where

$$f_{\pm}=\tfrac{1}{2}\{\exp[it(-m_1+i\Gamma_1/2)]\pm\exp[it(-m_2+i\Gamma_2/2)]\}\,.$$

Experimental measurements are performed essentially at $t=\infty$, so a convenient measure of the mixing is

$$\rho\equiv\frac{\int_0^\infty dt\,|f_-(t)|^2}{\int_0^\infty dt\,|f_+(t)|^2}=\frac{4(\Delta m/\Gamma)^2+(\Delta\Gamma/\Gamma)^2}{2+4(\Delta m/\Gamma)^2-(\Delta\Gamma/\Gamma)^2}\,. \tag{3.6}$$

The strength of the mixing thus depends upon the ratios $\Delta m/\Gamma$ and $\Delta\Gamma/\Gamma$. Let us consider the quantity $\Delta m/\Gamma$ for all the neutral conjugate mesons of the six-quark KM model. Arguments towards the same qualitative pattern can be made for $\Delta\Gamma/\Gamma$.[3]

(We do not intend to address here the thorny questions of strong-interaction corrections to the quark amplitudes,[10-11] detailed dependence upon the mixing angles, the effects of Higgs-boson exchange,[12] and the like. Our purpose throughout is merely to indicate general patterns. Clearly careful treatment of these issues is important, but we believe that such treatment will not vitiate the conclusions we present.)

Consider a doublet consisting of a lighter charge $-\frac{1}{3}$ quark L and its heavier charge $+\frac{2}{3}$ partner H:

$$\begin{pmatrix}H\\L\end{pmatrix}=\begin{pmatrix}c\\s\end{pmatrix},\begin{pmatrix}t\\b\end{pmatrix},\;\cdots\,.$$

The L quarks must decay out of their doublets, so their widths are suppressed by Cabibbo-type mixing angles, which we assume to be small. The widths of L quarks are further suppressed relative to the heavier H quarks by phase space. Thus in general

$$\Gamma_L<\Gamma_H\,.$$

On the other hand, if the mixing angles are small compared to the ratios of quark masses in succeeding doublets, then the main contribution to Δm for conjugate meson pairs P_H-$\overline{P}_H$, P_L-$\overline{P}_L$ containing H or L quarks comes from exchange of doublet partners in the familiar box diagram (Fig. 1). The resulting Δm is proportional to the mass squared of the exchanged quark

$$(\Delta m)_{P_L}\propto M_H{}^2\,,$$

$$(\Delta m)_{P_H}\propto M_L{}^2\,.$$

It follows from these simple considerations that

$$\left(\frac{\Delta m}{\Gamma}\right)_{P_H}<\left(\frac{\Delta m}{\Gamma}\right)_{P_L}\,,$$

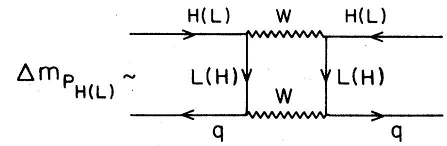

FIG. 1. Box diagram for mixing of a neutral conjugate meson (P_H^0 or P_L^0) containing a heavy (H) or light (L) quark. If the mixing angles are small compared to quark mass ratios, then mixing of P_H^0 (P_L^0) is dominated by exchange of an L (H) quark.

i.e.,

$$\left(\frac{\Delta m}{\Gamma}\right)_{T^0}<\left(\frac{\Delta m}{\Gamma}\right)_{B^0}$$

and

$$\left(\frac{\Delta m}{\Gamma}\right)_{D^0}<\left(\frac{\Delta m}{\Gamma}\right)_{K^0}\,.$$

Mixing in the B^0-$\overline{B}^0$ system might therefore be very strong, as it is in the K^0-$\overline{K}^0$ system. Prospects for strong mixing in the D^0-$\overline{D}^0$ and T^0-$\overline{T}^0$ systems seem correspondingly dimmer. Indeed, there is some hint of this pattern from the study of D^0-$\overline{D}^0$ mixing, which gives[13] [see Eq. (3.6)] $\rho_D<16\%$.

If the mixed B^0-$\overline{B}^0$ states contain large CP impurities, then the symmetry violation should be possible to detect by observing charge asymmetries in the semileptonic branching ratios. Following Pais and Treiman,[8] one imagines preparing by strong or electromagnetic interaction a state of pure B^0.[14] The ratio of negative to positive leptons produced through the primary semileptonic transitions $b\to l^-\overline{\nu}_l c$, $\overline{b}\to l^+\nu_l\overline{c}$ obtained from Eq. (3.5) by integrating over the history of the mixed state:

$$\frac{N(B^0\to l^-)}{N(B^0\to l^+)}\equiv r=\left|\frac{1-\epsilon}{1+\epsilon}\right|^2\rho\,. \tag{3.7}$$

Similarly for an initial state of pure $\overline{B}_0$:

$$\frac{N(\overline{B}^0\to l^+)}{N(\overline{B}^0\to l^-)}\equiv\overline{r}=\left|\frac{1+\epsilon}{1-\epsilon}\right|^2\rho\,. \tag{3.8}$$

The departure from unity of $r/\overline{r}$ would signal the presence of CP impurities in the mixed states. (Distortions in the lepton counts due to the cascades

$$b\to l^-\overline{\nu}_l c,\quad c\to l^+\quad\text{and}\quad b\to\overline{u}dc,\quad c\to l^+$$

could be eliminated by a spectral cut requiring large lepton energies.[3])

Early estimates[2,3] of CP impurity in B^0-$\overline{B}^0$ mixing were based upon calculations within the KM model of the quantity $|\epsilon|$. These calculations gave

$$|\epsilon|_{B_d^0\text{-}\overline{B}_d^0}\cong\tan 2\delta\,,$$

$$|\epsilon|_{B_s^0\text{-}\overline{B}_s^0}\cong\left(\frac{s_2}{s_3+s_2\cos\delta}\right)\tan 2\delta\,, \tag{3.9}$$

to be compared with

$$|\epsilon|_{K^0\text{-}\bar{K}^0} \sim s_2 s_3 \sin\delta\,. \tag{3.10}$$

On this evidence, CP asymmetries in B^0 decay would be enormous, since $|\epsilon|_B$ would be several orders of magnitude greater than $|\epsilon|_K \sim 10^{-3}$. Accompanied by strong mixing $[\rho_B \approx \rho_K \approx O(1)]$, the ratios r and $\bar{r}$ would be easily measured and their deviation from equality exposed.

Unfortunately, the absolute value of ϵ is without physical significance.[15] The conjugate states P^0 and $\bar{P}^0$ are defined by the strong interactions. But since no strong-interaction amplitude can transform P^0 into $\bar{P}^0$, the relative phase of the two states is left undetermined by this definition. Fixing this phase is entirely a matter of arbitrary convention; no physics can depend upon it. Referring to Eq. (3.2), it is clear that a quantity independent of this arbitrary relative phase is $|(1-\epsilon)/(1+\epsilon)|$, which is precisely what appears in the physical quantities r and $\bar{r}$. Defining

$$\frac{r}{\bar{r}} = \left|\frac{1-\epsilon}{1+\epsilon}\right|^4 \equiv \frac{1-Z}{1+Z}, \tag{3.11}$$

we obtain in Z a physical, phase-choice-independent measure of CP impurity:

$$Z = \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1+\frac{1}{4}|\Gamma_{12}/M_{12}|^2}. \tag{3.12}$$

Simple arguments from quark diagrams within the KM model indicate[4] that Z_B is in fact quite small, of order 10^{-3}. Thus CP impurities in the B^0-$\bar{B}^0$ systems are unlikely to be any more dramatic than in the K^0-$\bar{K}^0$ system.[16]

It is very easy and quite instructive to understand the smallness of Z_B in the KM model. We shall give the flavor of the argument by proving that Z_B vanishes in the limit $(m_u^2 - m_c^2)/m_b^2 \ll 1$, $m_u, m_c \ll m_t$; from here it is a small matter to set in the mass corrections. From (3.12) it is clear that Z_B depends upon the relative phase of the absorptive and dispersive parts of the B^0-$\bar{B}^0$ mixing amplitude. The quark model should be a reliable guide to the relative phase of M_{12} and Γ_{12}, if not necessarily to their magnitudes. M_{12} is obtained from the box diagrams of Fig. 2 and Γ_{12} by taking the discontinuity along the dashed lines. The KM U-matrix couplings for the box diagram

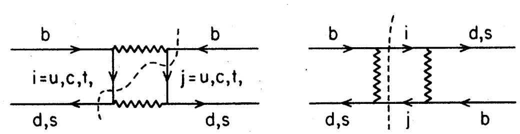

FIG. 2. Diagrams contributing to M_{12}, and their discontinuities (dashed lines), contributing to Γ_{12}, in the $B^0-\bar{B}^0$ systems.

with quarks $i,j = u,c,t$ exchanged are

$$U^*_{s,di} U_{bi} U^*_{s,dj} U_{bj} \equiv V_{ij}\,. \tag{3.13}$$

Note that unitarity of the KM matrix gives

$$\sum_i V_{ij} = \sum_j V_{ij} = 0\,. \tag{3.14}$$

The loop integration gives a term A independent of the quark masses and a term B_{ij} which depends upon the quark masses. A is eliminated by the unitarity of U [generalized Glasbow-Iliopoulous-Maiani (GIM) mechanism], and the largest remaining contribution comes when both exchanged quarks i and j are top quarks. Thus

$$M_{12} = \sum_{i,j} V_{ij}(A + B_{ij}) \cong B_{tt} V_{tt}\,. \tag{3.15}$$

The absorptive part Γ_{12} does not include the top quark since it is above threshold:

$$\Gamma_{12} = S_{cc}V_{cc} + S_{uu}V_{uu} + S_{cu}V_{cu} + S_{uc}V_{uc}\,, \tag{3.16}$$

where S_{ij} are the absorptive amplitudes. In the limit $(m_u^2 - m_c^2)/m_b^2 \ll 1$ these S_{ij} are equal to a common amplitude S, and

$$\Gamma_{12} \cong S(V_{cc} + V_{uu} + V_{cu} + V_{uc}) = SV_{tt}\,, \tag{3.17}$$

where in the last equality we have used (3.14). Thus in the approximation used, M_{12} and Γ_{12} share a common phase, that of V_{tt}, and $Z_B = 0$.

To conclude, it appears that although the mixing in the B^0-$\bar{B}^0$ systems might be quite strong, the CP impurities generated are probably small. Thus searches for CP asymmetry, which, like measurement of $r/\bar{r}$, depend solely upon the magnitude of symmetry violation in the *virtual* transitions of B^0 to $\bar{B}^0$, seem unlikely to yield dramatic results. In the following section we demonstrate that *real* transitions of bottom mesons to decay final states offer more fertile ground for symmetry violation.

IV. METHODS TO EXPOSE *CP* VIOLATION IN *B* DECAY

This section aims to present in general and clear terms our methods for exposing, as CP-violating physical effects, the phases which the KM model predicts in bottom decay. In the following section we turn these generalities to concrete purposes, giving examples of each of the methods proposed.

Consider a given cascade decay of the b quark and the corresponding conjugate cascade of the $\bar{b}$ quark, obtained from the former by changing all quarks to antiquarks. Associated with each cascade is a certain product of elements of the KM matrix. For instance, the cascade $b \to c\bar{u}d$, $c \to su\bar{d}$

has amplitude proportional to $G \equiv U_{bc}U_{sc}^{*}|U_{du}|^2$. CPT requires that the corresponding factor for the conjugate cascade $\bar{b} \to \bar{c}u\bar{d}$, $\bar{c} \to \bar{s}\bar{u}d$ be G^*. CP is violated if G and G^* differ in phase, i.e., if $\text{Im}(G/G^*) \neq 0$. We have repeatedly emphasized that in the KM model the relative phase of G and G^* can be large for b decays; e.g., in this case

$$\text{Im}(G/G^*) = \left[\frac{2s_2(s_3 + s_2\cos\delta)}{s_3^2 + s_2^2 + 2s_2s_3\cos\delta}\right]\sin\delta .$$

For a broad range of mixing angles, the factor in brackets is order unity.

To expose this potentially large symmetry-violating phase, two approaches are proposed. The simplest[17] involves finding two different cascades of the b quark (within, say, a B^- meson) to the same final state X. Let G_1 and G_2 be the products of KM matrix elements associated with the two cascades. Then the amplitude to produce the final state X is $F = G_1A_1 + G_2A_2$, where A_1 and A_2 are matrix elements of products of four-field Fermi operators and contain phases due to the strong interactions which organize the hadronic final state. All the dependence on the KM angles, including the CP-violating phase δ, is contained in G_1 and G_2. The $\bar{b}$ quark (e.g., B^+ meson) decays to the state $\bar{X}$, conjugate to X, with amplitude $\bar{F} = G_1^*A_1 + G_2^*A_2$. Here A_1 and A_2 appear as before, since the strong interactions respect CP and the CP violation in the weak interactions has been isolated in $G_{1,2}$. In order that a CP-violating asymmetry exists between the b and $\bar{b}$ decays ($|F|^2 \neq |\bar{F}|^2$), it is clearly necessary that A_1 and A_2 possess a nontrivial relative phase. This phase is provided by the strong final-state interactions,[17] since in general the two cascades differ in their isospin structures. Thus this method produces CP asymmetries which depend upon the difference of strong-interaction phase shifts.

In the other method we propose, one produces, through mixing, a coherent beam of b and $\bar{b}$ in the form of $B^0 = (b\bar{q})$ and $\bar{B}^0 = (\bar{b}q)$. Suppose that both B^0 and $\bar{B}^0$ can produce the same final state X through cascades with associated KM factors G and G^*, respectively. (In general X will contain one or more K_S's.) Then the amplitude for $B^0 \to X$ is GA and the amplitude for $\bar{B}^0 \to X$ is $G^*\bar{A}$. Here $A = -\bar{A}$ if we take $CP|B^0\rangle = -|\bar{B}^0\rangle$; the relative phase of the amplitudes is subject to an arbitrary choice, as we will discuss below. A state of pure B^0 prepared at time $t = 0$ becomes at a later time $|B^0(t)\rangle = a(t)B^0 + b(t)\bar{B}^0$ and has amplitude $c = aGA + bG^*\bar{A}$ to produce state X. A state of pure $\bar{B}^0$ at $t = 0$ develops according to $|\bar{B}^0(t)\rangle = \bar{a}(t)B^0 + \bar{b}(t)\bar{B}^0$ and has amplitude $\bar{c} \equiv \bar{a}GA + \bar{b}G^*\bar{A}$ to produce X. The CP-violating asymmetry $(|c|^2 - |\bar{c}|^2/|c|^2 + |\bar{c}|^2)$ is completely calculable in the KM model and involves, as well as the phase δ, the parameters characterizing B^0-$\bar{B}^0$ mixing. It is easy to construct more realistic applications of this method, in which one considers B^0-$\bar{B}^0$ pairs produced, e.g., off Υ-like resonances in e^+e^- colliding beams.

At this point let us digress to discuss an annoying but very important issue of phase conventions.[15] The states B^0 and $\bar{B}^0$ are defined by their strong interactions. Since the strong interactions conserve flavor and can never turn a B^0 into a $\bar{B}^0$, the relative phase of these states is a matter of arbitrary convention. This freedom of phase choice is well known from the analogous K^0-$\bar{K}^0$ system.[18] In quark language, freedom to choose the phase between $B^0_{s,d}$ and $\bar{B}^0_{s,d}$ amounts to choice of the relative phase of the b and the d or s quark fields. The important point to note is that by choosing the particular representation (2.1) for the KM matrix, one *fixes* the relative phases of the quarks. There remains a freedom to choose the phase of the CP operator which relates conjugate states, e.g., $CP|B^0\rangle = -|\bar{B}^0\rangle$. From this point on, the relative phase of K^0-$\bar{K}^0$, B^0-$\bar{B}^0$, etc., the phase of the CP operator which relates the conjugate states, and the real and imaginary parts of ϵ_K, ϵ_B, etc., are no longer a matter of choice, but of consistent calculation. The relative phases of $A, \bar{A}, a, \bar{a}, b, \bar{b}$ are fixed as functions of the KM parameters, even as G and G^* are.

V. EXAMPLES

As an example of the first method discussed in the previous section, consider the following decays, which are the initial stages of cascade decays of the b quark: $B^- \to D^0K_SX^-$, $B^- \to \bar{D}^0K_SX^-$. Here X^- is some nonstrange collection of hadrons. Although these decays are expected to be suppressed relative to the dominant decay modes by the (assumed) small angles s_2 and s_3, they are useful to demonstrate the mechanism of CP violation. Since the lifetimes of D^0 and $\bar{D}^0$ are long compared to the strong-interaction time scale, we can consider these decays to produce a coherent beam of D^0 and $\bar{D}^0$, free from strong interactions with K_S and X^-. Excitation of D^* states, with subsequent rapid decay to D will also contribute to this beam.

The two channels can be made to interfere by selecting a common decay mode of D^0 and $\bar{D}^0$, that to $K_S + X'$. If s_2 and s_3 are of the same order (and barring a pathological cancellation when $\cos\delta < 0$), the two amplitudes are of the same order, and interference can be strong. The final result of the decays is thus $B^- \to K_SK_SX_n^-$, where n labels the various possible exclusive final states.

In terms of quarks, the decay cascades are shown in Fig. 3. The interference term contains

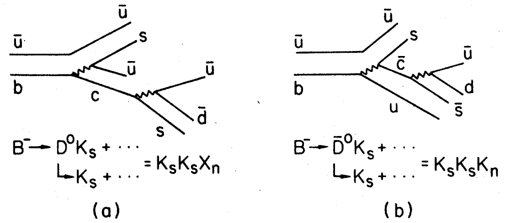

FIG. 3. Two amplitudes which can be made to interfere by selecting final states containing K_S. The CP-violating relative phase between these amplitudes can be measured by observing asymmetries in the widths of B^- and B^+ to conjugate final states.

a factor

$$C = U_{bc} U^*_{bu} U^*_{sc} U^2_{du} U^*_{su} |U_{sc}|^2 . \tag{5.1}$$

C contains the phases of five KM matrix elements, and it is clear that no rotation of quark phases will serve to eliminate all of them. Projection of the strong-interaction eigenstates $\overline{K}^0 = (s\overline{d})$ and $K^0 = (\overline{s}d)$ onto the selected final state K_S produces an additional phase. The product

$$C\langle K_S|\overline{K}^0\rangle\langle K_S|K^0\rangle^* \tag{5.2}$$

which appears in the interference term is phase-choice invariant, since under rotation of all the quarks i by phase angles δ_i, $C \to e^{2i(\delta_d-\delta_s)}C$, $\langle K_S|\overline{K}^0\rangle \to e^{-i(\delta_d-\delta_s)}\langle K_S|\overline{K}^0\rangle$, $\langle K_S|K^0\rangle \to e^{+i(\delta_d-\delta_s)}\langle K_S|K^0\rangle$. With the KM convention, the phases are chosen so that all the factors in $C\langle K_S|\overline{K}^0\rangle\langle K_S|K^0\rangle^*$ are approximately real except U_{bc}. [Keeping track of the other phases contributes only $O(s_2 s_3 \sin\delta)$ relative to the final result and can be neglected.]

Examination of the quark diagrams of Fig. 3 shows that they give rise to final states with different isospin structures, leading to a nontrivial relative phase due to strong interactions. For each exclusive channel $K_SK_SX_n^-$, we can write

$$\text{Amplitude(Fig. 3(a))} = A_n e^{i\eta_n} U_{bc} U^*_{sc} U_{du} U^*_{su} \equiv A_n e^{i\eta_n} f ,$$

$$\text{Amplitude(Fig. 3(b))} = \overline{A}_n e^{i\overline{\eta}_n} U_{bu} U^*_{sc} U_{sc} U^*_{du} \equiv \overline{A}_n e^{i\overline{\eta}_n} g . \tag{5.3}$$

The corresponding amplitudes for B^+ decay to $K_SK_SX_n^+$ (here X_n^+ is related to X_n^- by CP transformation) are obtained by complex conjugation of the U-matrix elements. Writing $\Gamma^\pm \equiv \sum_n \Gamma(B^\pm \to K_SK_SX_n^\pm)$, the CP-violating difference between the semi-inclusive widths in the conjugate channels is exhibited as

$$\frac{\Gamma^+ - \Gamma^-}{\Gamma^- + \Gamma^+} = \frac{A\,\mathrm{Im}(fg^*)}{B|f|^2 + C|g|^2 + D\,\mathrm{Re}(fg^*)} , \tag{5.4}$$

where $A = \sum_n A_n \overline{A}_n \sin(\eta_n - \overline{\eta}_n)$, $B = \sum A_n{}^2$, $C = \sum A_n{}^2$, and $D = \sum A_n \overline{A}_n \cos(\eta_n - \overline{\eta}_n)$. If A, B, C, D are all of the same magnitude and the mixing angles are small and of the same magnitude, then the asymmetry (5.4) is $O(\sin\delta)$. Recall that under the same approximations, previously exhibited CP-violating effects are $O(s_2 s_3 \sin\delta)$.

The second method discussed in Sec. IV produces CP asymmetries proportional to the parameters characterizing $B^0 - \overline{B}^0$ mixing. B^0 or $\overline{B}^0$ states produced at time $t = 0$ evolve into the superpositions

$$\begin{aligned} |B^0(t)\rangle\!\rangle &= f_+(t)|B^0\rangle + \frac{p}{q} f_-(t)|\overline{B}^0\rangle , \\ |\overline{B}^0(t)\rangle\!\rangle &= \frac{p}{q} f_-(t)|B^0\rangle + f_+(t)|\overline{B}^0\rangle , \end{aligned} \tag{5.5}$$

where

$$f_\pm = \tfrac{1}{2}\{\exp[it(-m_1 + i\Gamma_1/2)] \pm \exp[it(-m_2 + i\Gamma_2/2)]\} .$$

For the B^0 systems, this mixing might be substantial, as discussed in Sec. III. Consider now the decays $B^0_d \to D^0 + X$, $\overline{B}^0_d \to \overline{D}^0 + X$, with X a set of hadrons common to both decays. In contrast to the $B^\pm$ decays discussed above, these decays are expected to be among the dominant ones. The result of the mixing and subsequent decays is again to produce a coherent beam of D^0 and $\overline{D}^0$. The components of this beam can be made to interfere by selecting a common decay channel of D^0 and $\overline{D}^0$:

$$B^0_d \to D^0 + X \;\; (D^0 \searrow K_S + Y) , \quad \overline{B}^0_d \to \overline{D}^0 + X \;\; (\overline{D}^0 \searrow K_S + Y) . \tag{5.6}$$

In terms of quark diagrams, these decays are shown in Fig. 4.

Define

$$M = \langle K_S + X + Y|H|B^0\rangle , \quad \overline{M} = \langle K_S + X + Y|H|\overline{B}^0\rangle ,$$

choose the phase of the CP operator such that $CP|B^0\rangle = -|\overline{B}^0\rangle$, and take $CP|K_S + X + Y\rangle = |K_S + X + Y\rangle$. In writing the last equality we have ignored the small CP impurity in the $K^0 - \overline{K}^0$ mixing and used the spherical symmetry of the decay final state. If CP were a good symmetry, we would have $M = -\overline{M}$. Instead, M and $\overline{M}$ differ by the KM U-matrix elements. Referring to Fig. 4, we have

$$M = GA \text{ and } \overline{M} = -G^*A , \tag{5.7}$$

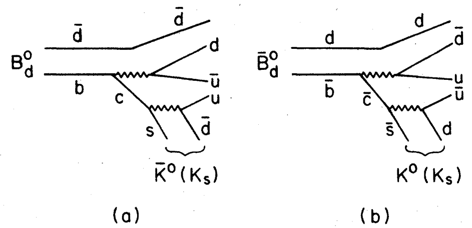

FIG. 4. Amplitudes for cascade decays of B_d^0 and $\bar{B}_d^0$ to a common final state containing a K_S. The CP-violating relative phase of these amplitudes can be measured by preparing, through mixing, a coherent beam of B and $\bar{B}$.

with $G=U_{bc}U_{sc}^*|U_{du}|^2$.

The amplitudes for states initially pure B^0 and $\bar{B}^0$ to decay to K_S+X+Y at time t are

$$\begin{aligned}\langle K_S XY|H|B^0(t)\rangle &= f_+(t)M+f_-(t)\frac{q}{p}\bar{M}\\ &\equiv \frac{q}{p}\bar{M}(\lambda f_+ + f_-)\ ,\\ \langle K_S XY|H|\bar{B}^0(t)\rangle &= f_-(t)\frac{p}{q}M+f_+(t)\bar{M}\\ &\equiv \bar{M}(\lambda f_- + f_+)\ .\end{aligned} \tag{5.8}$$

The widths Γ, $\bar{\Gamma}$ for initial states B^0, $\bar{B}^0$ to decay throughout their lifetimes are obtained by squaring the amplitudes (5.3) and integrating from $t=0$ to $t=\infty$. We obtain

$$\begin{Bmatrix}\Gamma\\ \bar{\Gamma}\end{Bmatrix} = (\text{const})\times\begin{Bmatrix}|p|^2\\ |q|^2\end{Bmatrix} \times[1\pm\alpha+|\lambda|^2(1\mp\alpha)-2\,\mathrm{Re}\lambda(y\pm ix\alpha)]\ , \tag{5.9}$$

where $y=\Delta\Gamma/\Gamma$, $x=2\Delta m/\Gamma$, $\alpha=(1-y^2)/(1+x^2)$, and $\lambda=pM/q\bar{M}$. Referring to the expression for p/q, Eq. (3.3), it is easy to show that

$$\frac{p}{q}=\frac{M_{12}}{|M_{12}|}[1+O(\mathrm{Im}\Gamma_{12}/M_{12})]\ . \tag{5.10}$$

We have shown in Sec. III that

$$\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)=O\left(\frac{m_c{}^2-m_u{}^2}{m_b{}^2}\right)\ , \tag{5.11}$$

and from Eq. (3.15) we have

$$\frac{M_{12}}{|M_{12}|}=\frac{(U_{bt}^*)^2}{|U_{bt}|^2}\ . \tag{5.12}$$

Finally, using the expressions (5.7) for M and $\bar{M}$, we obtain

$$\lambda=\frac{-(U_{bt}^*U_{bc}U_{sc}^*)^2}{|U_{bt}U_{bc}U_{sc}|^2}\equiv -e^{-2i\phi}\ . \tag{5.13}$$

At last we display the CP-violating asymmetry between the decay widths of B_d^0 and $\bar{B}_d^0$:

$$\frac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}}=\frac{-x\alpha\sin 2\phi}{1+y\cos 2\phi}\ . \tag{5.14}$$

In terms of the KM angles, we have

$$\sin\phi\cong\frac{s_3}{(s_2{}^2+s_3{}^2+2s_2s_3\cos\delta)^{1/2}}\sin\delta, \tag{5.15}$$

and we have argued in Sec. III that the mixing parameters for the $B^0\bar{B}^0$ system are in all likelihood such that $x\alpha\sim O(1)$. Thus an effect such as (5.14) can be sizable; we shall analyze its magnitude for a plausible range of mixing parameters in the following section.

This is a result for a particular choice of X and Y in the reactions given in Eq. (5.6) such that $CP\,|K_S+X+Y\rangle=|K_S+X+Y\rangle$. An admixture of state with $CP\,|K_S+X'+Y'\rangle=-|K_S+X'+Y'\rangle$ has the effect of reducing the asymmetry given in Eq. (5.14) by a factor independent of the KM angles. Obviously, the reduction factor depends crucially on the final-state interaction. While some simple-minded quark-model arguments suggest that the final state in Eq. (5.6) satisfies $CP|K_S+Y\rangle=|K_S+Y\rangle$ for the majority of states Y, such an argument is not conclusive. What we want to stress in any event is the importance of examining such an asymmetry and not its exact values. Note that information on the mixing parameter x will become available soon independent of the asymmetry measurement. The complication discussed above will be examined in detail if the parameter x turns out to be favorable for observing the CP asymmetry. We thank Dr. I. Bigi and Dr. M. Peshkin for discussion on the point.

Instead of considering a single B^0 or $\bar{B}^0$ produced at $t=0$, a somewhat more practical situation imagines a B^0-$\bar{B}^0$ pair produced in an e^+e^- colliding beam. For instance, the processes

$$e^+e^-\to\text{“}\Upsilon\text{”}\to B_d^0\bar{B}_d^0+X_1\to K^\pm K_S+X^\mp\ , \tag{5.16}$$

with "Υ" an upsilonlike resonance, can occur frequently since they represent major decay modes of B mesons. Now consider an event in which a K^- (K^+) is produced at time t_0. This means that one of the mesons was a B_d^0 ($\bar{B}_d^0$) at t_0. This in turn determines the other meson as a particular linear combination of B_d^0 and $\bar{B}_d^0$ at that time. The wave function of the second meson is then known for all previous and subsequent times, and we may ask for the probability that it decays to a K_S, as in the previous example.

To be precise, consider the time dependence of a B_d^0-$\bar{B}_d^0$ state with orbital angular momentum l in the rest frame:

$$|B(t),\ \vec{k};\ \bar{B}(t),\ -\vec{k}\rangle + (-1)^l|\bar{B}(t),\ \vec{k};\ B(t), -\vec{k}\rangle\,. \tag{5.17}$$

The time dependence of a single meson state with momentum $-\vec{k}$ which has as its "partner" a state of momentum $\vec{k}$ which decays to $K^- + X^+$ at time t_0 is

$$\langle K^-X^+|H|B^0(t_0),\vec{k}\rangle\,|\bar{B}_0(t), -\vec{k}\rangle + (-1)^l\langle K^-X^+|\bar{B}^0(t_0),\ \vec{k}\rangle|B^0(t),\ -\vec{k}\rangle\,. \tag{5.18}$$

Using Eq. (5.5), we can express the dependence upon t_0 and t explicitly, obtaining for (5.18)

$$\langle K^-X^+|H|B^0,\ \vec{k}\rangle\left\{\frac{p}{q}[f_+(t_0)f_-(t)+(-1)^l f_-(t_0)f_+(t)]|B^0,\ -\vec{k}\rangle + [f_+(t_0)f_+(t)+(-1)^l f_-(t_0)f_-(t)]|\bar{B}^0,\ -\vec{k}\rangle\right\}. \tag{5.19}$$

Thus the amplitude for a meson with momentum $\vec{k}$ to decay to K^-X^+ at time t_0 and for the other meson to decay to $K_S + Y$ at time t is

$$A[(K^-X^+)_{\vec{k}},\ (K_S\,Y)_{-\vec{k}}] = \langle K^-X^+|H|B^0,\ \vec{k}\rangle\left\{\frac{p}{q}[f_+(t_0)f_-(t)+(-1)^l f_-(t_0)f_+(t)]\langle K_S Y|H|B^0, -\vec{k}\rangle + [f_+(t_0)f_+(t)+(-1)^l f_-(t_0)f_-(t)]\langle K_S Y|H|\bar{B}^0,\ -\vec{k}\rangle\right\}. \tag{5.20}$$

The CP-conjugate situation is described, in an obvious notation, by an amplitude

$$A[(K_S Y)_{\vec{k}},\ (K^+X^-)_{-\vec{k}}] = \langle K^+X^-|H|\bar{B}^0,-\vec{k}\rangle\left\{[f_+(t_0)f_+(t)+(-1)^l f_-(t_0)f_-(t)]\langle K_S Y|H|B^0,\ \vec{k}\rangle + \frac{q}{p}[f_-(t_0)f_+(t)+(-1)^l f_+(t_0)f_-(t)]\langle K_S Y|H|\bar{B}^0,\ \vec{k}\rangle\right\}. \tag{5.21}$$

Denote $M(\vec{k}) = \langle K_S Y|H|B^0,\ \vec{k}\rangle$, $\bar{M}(\vec{k}) = \langle K_S Y|H|\bar{B}^0,\ \vec{k}\rangle$, $F(\vec{k}) = \langle K^-X^+|H|B^0,\ \vec{k}\rangle$, and $\bar{F}(\vec{k}) = \langle K^+X^-|H|\bar{B}^0,\ \vec{k}\rangle$. If CP were a good symmetry, we would have

$$\begin{aligned} M(\vec{k}) &= -\bar{M}(-\vec{k})\,,\\ F(\vec{k}) &= -\bar{F}(-\vec{k})\,. \end{aligned} \tag{5.22}$$

In the KM model, instead,

$$\begin{aligned} M(\vec{k}) &= m(\vec{k})U_{bc}U_{sc}^*\,,\\ \bar{F}(\vec{k}) &= f(\vec{k})U_{bc}U_{sc}^*\,, \end{aligned} \tag{5.23}$$

where m and f are real functions, and

$$\begin{aligned} \bar{M}(\vec{k}) &= -M(-\vec{k})^*\,,\\ \bar{F}(\vec{k}) &= -F(-\vec{k})^*\,. \end{aligned} \tag{5.24}$$

For an apparatus with equal detection efficiency for $\vec{k}$ and $-\vec{k}$, we can drop the momentum dependence, obtaining

$$A(K^-K_S) = F\bar{M}\{\lambda[f_+(t_0)f_-(t)+(-1)^l f_-(t_0)f_+(t)] + [f_+(t_0)f_+(t)+(-1)^l f_-(t_0)f_-(t)]\}$$

and

$$A(K^+K_S) = \bar{F}\frac{q}{p}\bar{M}\{\lambda[f_+(t_0)f_+(t)+(-1)^l f_-(t_0)f_-(t)] + [f_-(t_0)f_+(t)+(-1)^l f_+(t_0)f_-(t)]\}. \tag{5.25}$$

The cross sections $\sigma^{\pm}$ for

$$e^+e^- \to (B_d^0 \bar{B}_d^0) + \cdots$$
$$\searrow K^{\pm}K_S + \cdots$$

are obtained at last by squaring these amplitudes and integrating over the reference times t_0 and t from 0 to ∞:

$$\begin{Bmatrix}\sigma^+\\ \sigma^-\end{Bmatrix} \sim \begin{Bmatrix}|p|^2|F|^2\\ |q|^2|\bar{F}|^2\end{Bmatrix} \times [\,|\lambda|^2(1-\alpha^2+y^2+x^2\alpha^2) + (1+\alpha^2+y^2-x^2\alpha^2) - 2\,\mathrm{Re}(y \mp ix\alpha^2)\lambda]\,. \quad (5.26)$$

Here we have restricted $B^0\bar{B}^0$ to be in an S wave in their rest frame. This will be the case if "Υ" is just above $B\bar{B}$ threshold but massive enough to produce additional particles. The CP-violating asymmetry between the conjugate channels is

$$\mathcal{A} \equiv \frac{\sigma^+-\sigma^-}{\sigma^++\sigma^-} = \frac{-2x\alpha^2\sin 2\phi}{1+y^2+y\cos 2\phi}\,, \quad (5.27)$$

where we have used $|p|^2 \cong |q|^2$, $|F|^2 \cong |\bar{F}|^2$, $|\lambda|^2 \cong 1$, as before.

If Υ_{4S} is above threshold for $\bar{B}B^*$ production, $\Upsilon_{4S} \to \bar{B}B^* \to \bar{B}B\gamma$ is a likely decay chain since $B^* \to B\gamma$ is probably the only kinematically allowed nonweak decay. The $\bar{B}B$ pair produced in this decay chain is in the relative S wave. The asymmetry presented in Eq. (5.27) will depend on the branching ratio of Υ_{4S} to this decay mode. Again we refrain from including such a reduction factor since this information on the branching ratio will become available before any asymmetry measurement is attempted. We thank Dr. B. Kayser for discussion on this point.

Using this kind of analysis, which is straightforward despite its tedium, one can expose other asymmetries in $B^0\bar{B}^0$ decays. For instance, asymmetries in the energy distributions of the charged kaons in $e^+e^- \to B\bar{B} \to K^+K^-K_S \cdots$ should be present. Note that all the asymmetries which involve B^0-$\bar{B}^0$ mixing are proportional to $\alpha = (1-y^2)/(1+x^2)$. Thus the CP-violating effects actually disappear if the mixing parameter x becomes too large. The value of $x\alpha$ is in fact maximal at $x \sim 1$. We have argued that the mixing, especially for the B_S^0-$\bar{B}_S^0$ system, might be very large indeed. Ironically, this might prevent observation of CP asymmetries of the type we discuss here.

VI. NUMERICAL ESTIMATES

In this section we shall briefly discuss the magnitude of a typical asymmetry of the type analyzed in the previous section, taking $\mathcal{A}$ of Eq. (5.27) as an example:

$$\mathcal{A} = \frac{\sigma^+-\sigma^-}{\sigma^++\sigma^-} = \frac{-2x\alpha^2\sin 2\phi}{1+y^2+2y\cos 2\phi}\,. \quad (6.1)$$

Present knowledge of the KM matrix[6,7] is crude, and we can expect no more than a rough estimate. For this purpose we shall use the free-quark estimates[2] of x and y:

$$x = \frac{m_t^{\,2}}{700\ \mathrm{GeV}^2}\,\frac{1}{(1+\xi\cos\delta)^2(1+\tau^2)}\,, \qquad y \cong \tfrac{1}{12}\,, \quad (6.2)$$

where $\xi \equiv s_3/s_2$ and $\tau \equiv \xi\sin\delta/(1+\xi\cos\delta)$. It is clear that $y \ll x$, except possibly in the region $\xi \gg 1$. Evaluating $\mathcal{A}$ from the KM matrix (2.1), it is easy to show that

$$\mathcal{A} \cong \frac{4x\tau}{(1+x^2)^2(1+\tau^2)}\,. \quad (6.3)$$

We obtain $\sin\delta$ for each m_t, s_2, and ξ by requiring

$$|\mathrm{Re}\,\epsilon_K| = \left|\frac{\mathrm{Im}M_{12}}{4M_{12}}\right| = 1.6\times 10^{-3}\,,$$

where the full expression for M_{12} is given in Ref. 10. (Some sample values of $\sin\delta$ for a given value of m_t, s_2, and ξ are given in Table I.) In contrast to the authors of Ref. 7, we have relaxed the constraint obtained from the K_L-K_S mass difference. We fell that this is justified since this constraint depends strongly upon the manner in which the matrix element of the effective four-quark operator is evaluated between K^0 and $\bar{K}^0$ states.

It should, however, be pointed out that the result of Ref. 7, which includes the constraint from M_{K_L}-M_{K_S} computed using the bag model, is $S_2 \gtrsim 0.2$. There is no such lower limit if M_{K_L}-M_{K_S} is computed using the free-quark model. There is an ambiguity in the determination of the sign of $\cos\delta$. In Figs. 5 and 6 we plot the asymmetry $\mathcal{A}$ for both $\cos\delta > 0$ and $\cos\delta < 0$.

From Figs. 5 and 6 we conclude that if nature chooses $s_2 \lesssim 0.1$ and $s_3/s_2 \lesssim 1$, there is a good pos-

TABLE I. Some sample values for given m_t (=20 GeV), s_2, and ξ used in Fig. 6.

ξ	$\sin\delta$
$s_2 = 0.2$	
0.012	0.97
0.04	0.30
0.10	0.12
0.20	0.06
0.4	0.03
1.0	0.013
$s_2 = 0.05$	
0.32	0.89
0.4	0.7
0.8	0.34
1.0	0.27

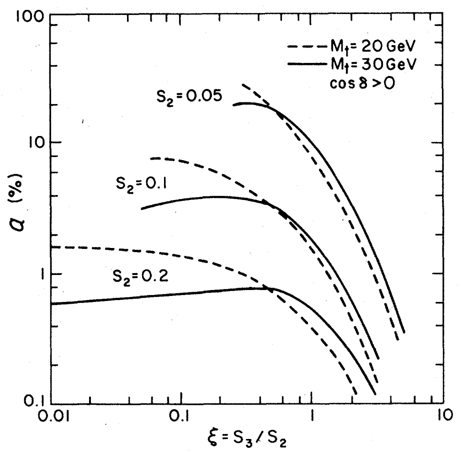

FIG. 5. The asymmetry $\mathcal{a}$ for $\cos\delta > 0$, $m_t = 20$ and 30 GeV. If $s_2 \lesssim 0.1$ and $\xi \lesssim 1$, $\mathcal{a}$ should be large enough to be measured in e^+e^- colliding-beam experiments.

sibility that $\mathcal{a}$ is large enough (2–20%) to be observed in e^+e^- colliding-beam experiments.

VII. CONCLUSIONS

The methods and examples we have described demonstrate the possibility of truly dramatic CP-violating effects in the bottom sector. Asymmetries of the type (5.4), (5.14), and (5.27) are confined to the range between +1 and −1 and must of course be proportional to $\sin\delta$. The best one can do theoretically is to search for an effect for which $\sin\delta$ is multiplied by a coefficient of order unity, as we have done. Asymmetries such as the ones we describe are thus the maximum obtainable within the KM model.

There is every reason to believe that if CP violation is present in bottom decay, then it will persist, and even grown more pervasive, as experimenters scale the flavor spectrum. This is true in almost any scheme which makes CP violation a natural and pervasive feature of the weak interactions. For the moment one can regard the KM model as a convenient parametrization of these effects and as a guide to experimental search.

In particular, the methods we describe, emphasizing the on-shell transitions which make up cascade decays of heavy flavors, are immediately applicable to decay of the top (t) quark. Large symmetry-violating relative phases are accumulated as the heavy mesons make their way down to ordinary hadrons. However, effects which depend upon mixing might be small if, as discussed in Sec. III, the rate of T_0-$\overline{T}_0$ mixing is small compared to the decay rate.

ACKNOWLEDGMENT

This work was supported in part by the U.S. Department of Energy under Contract Grant No. DE-AC02-76ER02232B.

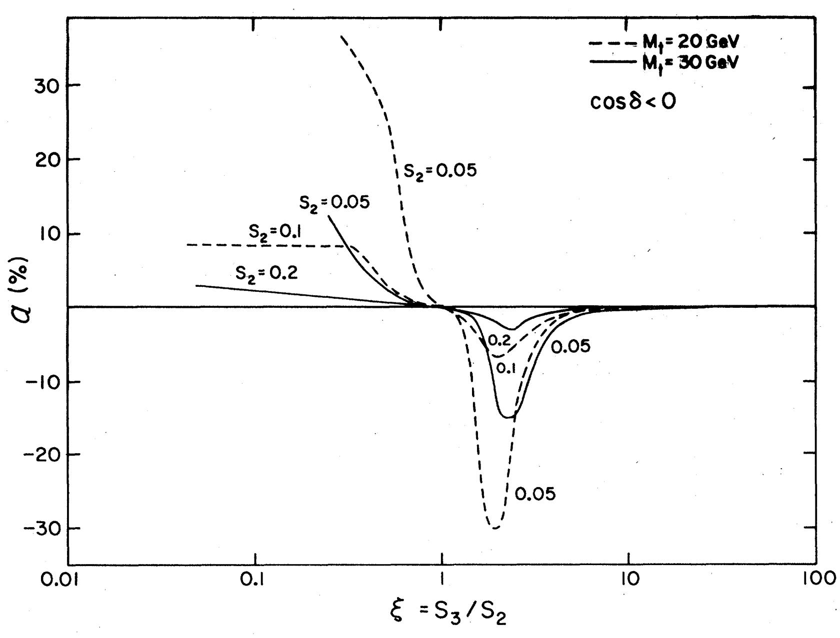

FIG. 6. The asymmetry $\mathcal{a}$ for $\cos\delta < 0$, $m_t = 20$ and 30 GeV.

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[14]A slightly more practical proposal (Ref. 3) involves producing a B^0-B^0 pair in e^+e^- annihilation. One then measures the number of events in which two positive leptons (N_{++}), two negative leptons (N_{--}), and a mixed pair (N_{+-}) are produced obtaining

$$\frac{N_{++}+N_{--}}{N_{++}+N_{--}+N_{+-}}=\frac{r+\bar{r}}{2+r+\bar{r}}\ .$$

CP violation is revealed in the charge asymmetry

$$\frac{N_{++}-N_{--}}{N_{++}+N_{--}}=\frac{r-\bar{r}}{r+\bar{r}}\ .$$

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[16]$|\epsilon_K|$ is actually an accurate reflection of the magnitude of *CP* impurity in the kaons, even though $|\epsilon_B|$ is not for the bottom mesons. Writing $|(1-\epsilon)/(1+\epsilon)|^2 \equiv (1-y)/(1+y)$, we have $y=2\,\mathrm{Re}\,\epsilon/(1+|\epsilon|^2)$. Within the phase convention established by the KM matrix, $|\mathrm{Re}\,\epsilon_K|\sim|\epsilon_K|\ll 1$, so $|\epsilon_K|$ gives an accurate measure of the physical quantity y_K. ϵ_B, however, is mostly imaginary, so that although $|\epsilon_B|$ is large, y_B is small. The issue of phase conventions is discussed briefly in Sec. IV.

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