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Muon-number violation in some horizontal gauge theories

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Horizontal gauge models have been studied which have a mechanism for suppressing flavor-changing processes involving only two flavors, i.e., processes like $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and the K^0-$\bar{K}^0$ transition. It is found that the rates for some muon-number-violating processes, in particular for μe conversion and kaon decays, can be close to experimental upper limits. This is in contrast to horizontal gauge models in which the K^0-$\bar{K}^0$ transition is not suppressed.

Recently the subject of muon-number violation in horizontal gauge models has received some attention.[1-3] Horizontal gauge models, like other extensions of the phenomenologically successful Weinberg-Salam model,[4] have been considered because the latter model does not answer some intriguing questions. For example, the Weinberg-Salam model does not explain why nature repeats itself with the electron family, the muon family, and so on. It does not say how many fermion generations may exist. The unanswered questions regarding generations make horizontal gauge models interesting, in that these models may answer some of the questions. Horizontal gauge models have been proposed to calculate the e-μ mass ratio,[5] to generate spontaneous CP violation,[6] and to calculate Cabibbo-type mixing angles[7] between different generations. A horizontal gauge model is a model in which a gauge symmetry between the different generations of quarks and leptons is introduced. This "horizontal" symmetry is spontaneously broken, leading to mass differences between different generations as required by experiments. In such models there exist horizontal gauge bosons which mediate flavor-changing processes like the K^0-$\bar{K}^0$ transition, μe conversion, etc. The observed limits on the rates for these processes require that the horizontal gauge bosons must pick up very heavy masses. The aim of this paper is to consider flavor violation in some horizontal gauge models. In particular I study a mechanism by which flavor-changing processes involving only two types of flavors are suppressed (Lincoln Wolfenstein brought this suppression mechanism to my attention). It has been suggested that the K_L-K_S mass difference constrains the mass of possible horizontal gauge bosons so strongly that muon-number-violating processes mediated by these bosons would have insignificant rates.[1,2] The flavor-change-suppressing mechanism described in this paper suppresses the K_L-K_S mass difference. As a result, some horizontal gauge models predict muon-number-violating rates which may be comparable with present experimental upper limits.

The mechanism for suppressing $\Delta S = 2$ currents can be illustrated in the model of Maehara and Yanagida.[6] This model was proposed as a means of getting CP violation with only two generations of quarks. In addition to the usual $SU(2)W \times U(1)$ symmetry of Weinberg and Salam this model has a horizontal $SU(2)_H$ symmetry. The quarks and leptons have the usual $SU(2)_W \times U(1)$ representation. Under the $SU(2)_H$ group they transform as doublets with the multiplets being (d,s), (u,c), (e,μ), (ν_e,ν_μ) (for both right- and left-handed particles). The model has three kinds of Higgs particles: ω, ϕ, and χ. Table I summarizes the transformation properties of these Higgs particles under the gauge groups. The χ bosons cannot couple to the fermions and are used to give heavy masses to the horizontal gauge bosons. The ϕ are needed to break the mass degeneracy between different fermion generations. The vacuum expectation values of the Higgs particles are as follows:

$$\begin{aligned}
&\omega:\ \begin{pmatrix} 0 \\ <\omega> \end{pmatrix} \Big/ \sqrt{2}\,, \\
&\phi:\ \begin{pmatrix} 0 & 0 & 0 \\ <\phi_1> & <\phi_2> & <\phi_3> \end{pmatrix} \Big/ \sqrt{2}\,, \qquad (1) \\
&\chi:\ \begin{pmatrix} <\chi_1> & <\chi_2> \end{pmatrix} / \sqrt{2}\,.
\end{aligned}$$

TABLE I. Transformation properties of the Higgs particles in the $SU(2)_W \times U(1) \times SU(2)_H$ model of Maehara and Yanagida.

Higgs particle	$SU(2)_W$ isospin I	$SU(2)_H$ isospin I_H	U(1) hyperchange Y
ω	$\frac{1}{2}$	0	1
ϕ	$\frac{1}{2}$	1	1
χ	0	$\frac{1}{2}$	0

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In addition to the usual two charged (W^+ and W^-) and two neutral (Z^0 and γ) gauge bosons the model has three neutral horizontal bosons S_1, S_2, and S_3. The mass matrix for the gauge bosons takes the form

$$L_{\text{mass}} = \tfrac{1}{2} \sum_{i,j=0}^{3} S_\mu^i (M^2)_{ij} S^{j\mu} + M_W{}^2 W_\mu^+ W^{-\mu} , \quad (2)$$

where S^0 is the notation for the Z boson, g, g', and g_S are the gauge coupling constants corresponding to $SU(2)_W$, $U(1)$, and $SU(2)_H$, respectively, and

$$\begin{aligned} M^2{}_{00} &= \frac{g^2+g'^2}{4} \; <\omega>^2 + <\phi_a><\phi_a>^* \;, \\ M^2{}_{a0} &= -\frac{g_s(g^2+g'^2)^{1/2}}{2} \epsilon_{abc} \mathrm{Im} <\phi_b><\phi_c>^* \;, \\ M^2{}_{ab} &= M^2{}_{ba} = \frac{g_S^2}{4} [(<\chi_d><\chi_d>^* + 4<\phi_c><\phi_c>^*)\delta_{ab} \\ &\qquad - 4\,\mathrm{Re}<\phi_a><\phi_b>^*] \;, \\ M_W{}^2 &= \frac{g^2}{4} (<\omega>^2 + <\phi_a><\phi_a>^*) \;. \end{aligned} \quad (3)$$

In the above expressions repeated indices are summed over and a, b, c, and d take appropriate values. From the above mass matrix we see that the χ Higgs bosons give a common heavy mass to the horizontal gauge bosons. The degeneracy of the gauge bosons is broken at most by terms of order $M_W{}^2/M_S{}^2$ if we assume that $g_S \approx g$. The mixing between Z^0 and the horizontal gauge bosons is also at most of this order. The near degeneracy of the S bosons is a consequence of putting the χ Higgs bosons in a $SU(2)_H$ doublet and of the fact that the two-dimensional representations of the SU(2) generators (i.e., the Pauli spin matrices) anticommute.

Flavor-changing processes due to the S bosons can be conveniently classified into three groups: (1) those which occur even if there is no mixing between fermions and the mass degeneracy of the S bosons is not broken, (2) those which occur after the fermions get mixed but the mass degeneracy of the S bosons remains, and (3) those which occur after this mass degeneracy is broken. Table II shows the level (from the levels described above) at which different flavor-changing proces-

TABLE II. Muon-number-violating processes in the model of Maehara and Yanagida when $\Delta M_S/M_S$ is of the order of $M_W^2/M_S{}^2$.

Process	Level at which event occurs	Definition for R	Order of magnitude for R_{theor}	Upper limit for R_{expt}	Reference for R_{expt}	Remarks
$\mu \to e\gamma$	Third	$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to \text{all})}$	2×10^{-16}	2×10^{-10}	10	a
$\mu \to ee\bar{e}$	Third	$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to \text{all})}$	3×10^{-16}	2×10^{-9}	11	a
$\bar{\mu}e \to \bar{e}\mu$	Third		3×10^{-16} $\times R_{\text{expt}}$		12	b,c
μe conversion	Second	$\frac{\Gamma(\mu^- + A \to e^- + A)}{\Gamma(\mu^- + A \to \nu + A')}$	5×10^{-6} for S	7×10^{-11} (S) 1.6×10^{-8} (Cu)	13,14	b
$K_L \to \mu e$	First	$\frac{\Gamma(K_L \to e\mu)}{\Gamma(K_L \to \text{all})}$	2×10^{-10}	2×10^{-9}	15	d,e,f
$K^+ \to \pi^+\bar{\mu}e$	First	$\frac{\Gamma(K^+ \to \pi^+\bar{\mu}e)}{\Gamma(K^+ \to \text{all})}$	3×10^{-8}	5×10^{-9}	15	b
$K^+ \to \pi^+\mu\bar{e}$	Second	$\frac{\Gamma(K^+ \to \pi^+\mu\bar{e})}{\Gamma(K^+ \to \text{all})}$	3×10^{-8}	7×10^{-9}	15	b
K^0-$\bar{K}^0$ transition	Third	K_L-K_S mass difference	Process determines M_S	$\Delta m_K =$ 3.5×10^{-15} GeV	1	b,g

[a] Vector and axial-vector currents.
[b] Vector currents.
[c] Process analogous to $\bar{K}^0$-K^0 transition.
[d] Axial-vector currents.
[e] Processes occur at one-loop level and not at tree level since the S bosons couple to vector currents only.
[f] $K^0 \to \bar{\mu}e$ occurs at first level and $K^0 \to \mu\bar{e}$ at second level. Both $\bar{\mu}e$ and $\mu\bar{e}$ have equal rates.
[g] Process is sensitive to matrix element of $\Delta S = 2$ current, not merely to square of matrix element.

ses occur. At the first level neutral currents will preserve to all orders the quantities $(N_{\nu_\mu}+N_{\nu_e})$, $(N_\mu+N_e)$, (N_c+N_u), (N_s+N_d), $(N_{\nu_\mu}+N_\mu+N_c+N_s)$, and $(N_{\nu_e}+N_e+N_u+N_d)$ where $N_\mu, N_e, \ldots$ represent the number of muons, electrons, ... minus the number of antimuons, positrons, ... involved in the process. The conservation of the first four quantities follows from the conservation of total lepton number, baryon number, and electric charge. The conservation of the remaining two quantities follows from the conservation of horizontal isospin. While the total lepton number and baryon number are conserved the horizontal gauge bosons violate the separate conservation of muon number, electron number, etc. At the first level flavor-changing processes which involve only two kinds of flavors, e.g., K^0-$\overline{K}^0$, $\mu\to ee\overline{e}$, $\mu\to e\gamma$, etc., are forbidden. This selection rule operates because the horizontal gauge bosons carry horizontal isospin and transfer it from one vertex to the other. As a result, if fermions carrying the same two flavors occur at both vertices of tree diagrams like the ones in Figs. 1 and 2, the flavor change occurring at one vertex is canceled by that occurring at the second vertex. Exotic processes mediated by the S bosons at the first level include $\overline{K}^0\to\mu\overline{e}$ and $K^0\to e\overline{\mu}$ (though not at tree level because the horizontal bosons couple only to vector currents), $K^-\to\pi^-\mu\overline{e}$, and $K^+\to\pi^+\overline{\mu}e$.

When the fermions mix among themselves (i.e., at the second level), in general the suppression of processes like K^0-$\overline{K}^0$ and $\mu\to ee\overline{e}$ is no longer complete and the horizontal gauge bosons may mediate such processes with the rates depending on the fermion mixing angles. However, the model of Maehara and Yanagida satisfies some special conditions [which are described later for a general class of $SU(n)_H$ models], and hence flavor-changing processes involving only two types of flavors are forbidden in this model even after fermion mixing. This is because the horizontal gauge bosons are assumed to be degenerate at the second level and one can mix these bosons among themselves and define new gauge-boson states so as to cancel the effect of fermion mixing. Even in the model of Maehara and Yanagida some flavor-changing processes are allowed at the second level which are forbidden at first level. These include $\overline{K}^0\to\overline{\mu}e$, $K^0\to\mu\overline{e}$, $K^+\to\pi^+\mu\overline{e}$, $K^-\to\pi^-\overline{\mu}e$, and μe conversion. The rates for these processes will depend on the fermion mixing angles. The quantities $(N_{\nu_\mu}+N_\mu+N_c+N_s)$ and $(N_{\nu_e}+N_e+N_u+N_d)$ are no longer conserved. Fermion mixing implies that $K^0\to e\overline{\mu}$ now occurs at tree level unless the left- and right-hand mixing angles in the d-quark sector are identical.

In the model of Maehara and Yanagida flavor-changing processes involving only two types of flavors, e.g., K^0-$\overline{K}^0$ and $\mu\to ee\overline{e}$, occur only at the third level, i.e., they occur only when the mass degeneracy of the S bosons is broken. These processes are suppressed relative to the other flavor-changing processes because the breaking of the S-boson mass degeneracy is small. The quantities $(N_{\nu_\mu}+N_{\nu_e})$, $(N_\mu+N_e)$, (N_c+N_u), and (N_s+N_d) are conserved to all orders and at all levels because they follow from conservation of total lepton number, baryon number, and electric charge.

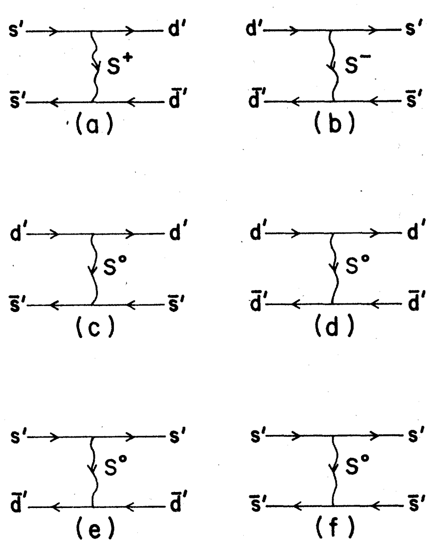

FIG. 1. Diagrams contributing to the K_L-K_S mass difference in the $SU(2)_W\times U(1)\times SU(2)_H$ model of Maehara and Yanagida with two generations of quarks and leptons.

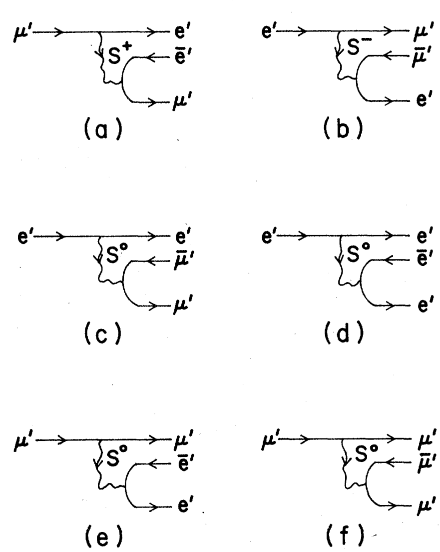

FIG. 2. Diagrams contributing to $\mu\to ee\overline{e}$ in the same model as for Fig. 1.

I will now illustrate the absence of flavor-changing processes involving only two kinds of flavors in the Maehara-Yanagida model at the second level by considering the K^0-$\bar{K}^0$ transition and $\mu \rightarrow ee\bar{e}$. In the following discussion I will use the horizontal-isospin raising and lowering gauge-boson states S^+, S^-, and S^0 as basis states. The plus and minus superscripts do not refer to electric charge. Figure 1 shows the various Feynman diagrams which contribute to K^0-$\bar{K}^0$ transitions. The quarks in the figure are left-handed horizontal gauge eigenstates s' and d'. The horizontal interaction currents are $S^+\bar{s}'d'$, $S^-\bar{d}'s'$, and $S^0(\bar{d}'d' - \bar{s}'s')/\sqrt{2}$. The gauge eigenstates can be represented in terms of mass eigenstates s and d as follows:

$$\begin{aligned} s' &= (\sin\theta)d + (\cos\theta)s , \\ d' &= (\cos\theta)d - (\sin\theta)s . \end{aligned} \tag{4}$$

When the gauge eigenstates are expressed in terms of mass eigenstates the contribution of each of the diagrams in Fig. 1 is as follows: diagram (a) is proportional to $-\cos^2\theta\sin^2\theta$; diagram (b) is proportional to the same combination; and diagrams (c)–(f) are proportional to $(\cos^2\theta\sin^2\theta)/2$. The sum of the diagrams is equal to zero as mentioned previously. While the cancellation has been illustrated only for diagrams involving left-handed quarks it is easy to see that one can group the remaining diagrams in such a way that the cancellation occurs in an exactly analogous manner, if the mixing angles in the right- and left-handed fermion sectors are equal. Figure 2 shows the suppression of $\mu \rightarrow ee\bar{e}$. The figure is in terms of gauge eigenstates μ' and e'. The fermion gauge eigenstates can be written in terms of mass eigenstates μ and e as

$$\begin{aligned} e' &= (\cos\theta)e - (\sin\theta)\mu , \\ \mu' &= (\sin\theta)e + (\cos\theta)\mu . \end{aligned} \tag{5}$$

The contribution of each of the diagrams in Fig. 2 to $\mu \rightarrow ee\bar{e}$ is as follows: diagram (a) is proportional to $\cos^3\theta\sin\theta$; diagram (b) is proportional to $-\sin^3\theta\cos\theta$; diagrams (c) and (f) are proportional to $(\sin^3\theta\cos\theta)/2$; and diagrams (d) and (e) are proportional to $-\cos^3\theta\sin\theta/2$. The sum of the diagrams is equal to zero.

As mentioned before, in the Maehara-Yanagida model the K_L-K_S mass difference does not constrain the muon-number-violating rates to be far below experimental limits, since $\Delta S = 2$ currents are suppressed. This is illustrated in Table II where orders of magnitude for different muon-number-violating rates are presented. In calculating these orders of magnitude it was assumed that the S-boson mass degeneracy breaking was of the order of $M_W{}^2/M_S{}^2$ times M_S, where M_S is the average mass of the S bosons. The degeneracy breaking would be of this order if the horizontal gauge coupling constant g_S is comparable to the weak coupling constant g. The mixing of the S and Z bosons is important only for third-level processes. This mixing modifies slightly the relative ratios of third-level entries in the tables and has no important consequences for the phenomenology. The left and right mixing angles in the fermion sector are assumed to be equal. All mixing angles are taken to be of order 1. The horizontal gauge coupling constant g_S is assumed to be equal to the $SU(2)_W$ coupling constant g. First- and second-level processes are assumed to have coupling constants of strength $g_S{}^2/M_S{}^2$. Third-level processes are assumed to have couplings of strength $g_S{}^2/M_S{}^2(\Delta M_s/M_s)$. Quantitative details of the assumptions regarding the coupling constants are given in Ref. 8. The mass of M_S is fixed by the K_L-K_S mass difference to be 200 M_W. (In calculating this, one needs the matrix elements of four-quark operators between the K^0 and $\bar{K}^0$ states. These matrix elements were taken from Ref. 9, and the vacuum insertion values of Ref. 9 were used with standard values for the current-algebra quark masses.) The experimental numbers in the table are from Refs. 10–15. From the table we see that some muon-number-violating rates are in fact higher than experimental limits. Thus, horizontal gauge models do not necessarily predict negligible muon-number-violating rates. Both $\Delta M_S/M_S$ and M_S can be made free parameters in the Maehara-Yanagida model. Table III shows the orders of magnitude for muon-number-violating rates in this case. In calculating the values M_S was fixed by the μe conversion rate to be 3000 M_W and $\Delta M_S/M_S$ was fixed by the K_L-K_S mass difference to be 0.004. μe conversion may not provide the most stringent constraint on the S-boson mass if the mixing between generations is small or if a cancellation between mixing angles occurs for sulphur. (A new experiment at TRIUMF for setting a limit on the μe conversion rate for argon will probably make the latter case unimportant.) If the mixing angles suppress μe conversion rates, the constraints on the S-boson masses come from muon-number-violating K^+ and K_L decays.

The cancellation mechanism illustrated by Figs. 1 and 2 can be generalized to $SU(n)_H$ models. A model incorporating a $SU(n)_H$ group will suppress flavor-changing processes involving only two types of flavors if the following four conditions are satisfied:

(1) All fermions must be put into fundamental

TABLE III. Muon-number-violating processes in the model of Maehara and Yanagida when $\Delta M_S/M_S$ and M_S are both free parameters.

Process	Level at which event occurs	Definition for R	Order of magnitude for R_{theor}	Upper limit for R_{expt}	Reference for R_{expt}	Remarks
$\mu \to e\gamma$	Third	$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to \text{all})}$	2×10^{-16}	2×10^{-10}	10	a
$\mu \to ee\bar{e}$	Third	$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to \text{all})}$	3×10^{-16}	2×10^{-9}	11	a
$\bar{\mu}e \to \bar{e}\mu$	Third		$3\times10^{-16} \times R_{\text{expt}}$		12	b,c
μe conversion	Second	$\frac{\Gamma(\mu^- + A \to e^- + A)}{\Gamma(\mu^- + A \to \nu + A')}$	Process determines M_S	7×10^{-11} (S) 1.6×10^{-8} (Cu)	13,14	b
$K_L \to \mu e$	First	$\frac{\Gamma(K_L \to e\mu)}{\Gamma(K_L \to \text{all})}$	4×10^{-5}	2×10^{-9}	15	d,e,f
$K^+ \to \pi^+\bar{\mu}e$	First	$\frac{\Gamma(K^+ \to \pi^+\bar{\mu}e)}{\Gamma(K^+ \to \text{all})}$	6×10^{-13}	5×10^{-9}	15	b
$K^+ \to \pi^+\mu\bar{e}$	Second	$\frac{\Gamma(K^+ \to \pi^+\mu\bar{e})}{\Gamma(K^+ \to \text{all})}$	6×10^{-13}	7×10^{-9}	15	b
K^0-$\bar{K}^0$ transition	Third	K_L-K_S mass difference	Process determines $\Delta M_S/M_S$	$\Delta m_K = 3.5\times10^{-15}$ GeV	1	b,g

[a] Vector and axial-vector currents.
[b] Vector currents.
[c] Process analogous to K^0-$\bar{K}^0$ transition.
[d] Axial-vector currents.
[e] Processes occur at one loop level and not at tree level since the S bosons couple to vector currents only.
[f] $K^0 \to \bar{\mu}e$ occurs at first level and $K^0 \to \mu\bar{e}$ at second level. Both $\bar{\mu}e$ and $\mu\bar{e}$ have equal rates.
[g] Process is sensitive to matrix element of $\Delta S = 2$ current, not merely to square of matrix element.

(n-dimensional) representations of $SU(n)_H$.

(2) The fermion mixing angles in the left- and right-hand sectors must be identical.

(3) Fermions belonging to different n-dimensional representations must not mix.

(4) The horizontal gauge bosons must be degenerate and must not mix with the Z boson of $SU(2)_W \times U(1)$.

When these conditions are satisfied one can mix the horizontal gauge bosons and make them carry appropriate flavors such that tree diagrams leading to flavor-changing processes involving only two types of flavors are absent. I will now prove that the horizontal gauge bosons can indeed be mixed in such a way as to suppress these types of flavor-changing processes. Let τ^α, $\alpha = 1, \ldots, (n^2-1)$, represent (n^2-1) linearly independent traceless Hermitian $n\times n$ matrices normalized such that trace $(\tau^\alpha\tau^\beta) = \delta^{\alpha\beta}$. These matrices form a representation of the generators of $SU(n)$.[16] Let ψ'_L be an n-dimensional vector of left-handed horizontal-gauge-eigenstate fermion fields transforming as an n-multiplet under $SU(n)$ [for example, ψ'_L could be (d', s') in the Maehara-Yanagida model]. The interactions of the S bosons with the fermions are $\bar{\psi}'_L\gamma^\mu\tau^\alpha\psi'_L S_\alpha$. These interactions are such that if ψ'_L were mass eigenstates then flavor-changing processes which involve flavors of two kinds only would be forbidden. The fermions in the ψ'_L multiplet mix among themselves to form mass eigenstates. Let U be the $n\times n$ unitary matrix which expresses the gauge eigenstates ψ'_L in terms of the mass eigenstates ψ_L, i.e.,

$$\psi'_L = U\psi_L \,. \tag{6}$$

U can be expressed in terms of the generators τ^α:

$$U = e^{i\epsilon_\beta\tau^\beta}\,, \tag{7}$$

where repeated indices are to be summed from 1 to (n^2-1). In terms of mass eigenstates the S-boson interactions can be written as $\bar{\psi}_L\gamma^\mu U^\dagger\tau^\alpha U\psi_L S_\alpha$. Consider the matrix $U^\dagger\tau^\alpha U$. Its trace is zero from the cyclic property of the trace. It is Hermitian because τ^α is Hermitian. It can be expressed as a linear combination of the

τ^α since they form a basis set for $n\times n$ traceless Hermitian matrices. Let

$$U^\dagger \tau^\alpha U = a_{\alpha\alpha'} \tau^{\alpha'} . \quad (8)$$

The $(n^2-1)\times(n^2-1)$ matrix $a_{\alpha\alpha'}$ is unitary as I now prove. Using the definition of $a_{\alpha\alpha'}$ twice we get

$$U^\dagger \tau^\alpha U U^\dagger \tau^\beta U = U^\dagger \tau^\alpha \tau^\beta U = a_{\alpha\alpha'} a_{\beta\beta'} \tau^{\alpha'} \tau^{\beta'} . \quad (9)$$

Taking the trace of both sides gives

$$\delta_{\alpha\beta} = a_{\alpha\alpha'} a_{\beta\beta'} \delta_{\alpha'\beta'} = a_{\alpha\alpha'} a_{\beta\alpha'} . \quad (10)$$

Thus, the matrix $a_{\alpha\alpha'}$ is unitary. The interactions of the S bosons with the fermions can be written as $\psi_L \gamma^\mu \tau^{\alpha'} \psi_L a_{\alpha\alpha'} S^\alpha$. Since one has the freedom to use redefined fields $S^{\alpha'} = a_{\alpha\alpha'} S^\alpha$ instead of the original S the result just proved shows that interactions involving only the horizontal gauge bosons and fermions are not affected by fermion mixing in ψ_L. Since processes like K^0-$\overline{K}^0$ transitions, $\mu \to ee\overline{e}$, etc., are forbidden before fermion mixing they remain forbidden even after fermion mixing. While the above proof was for left-handed fermions it also works for interactions involving both left- and right-handed fermions provided the mixing angles in the left- and right-hand sectors are equal.

In the above proof the fermions had to be put into fundamental representations of SU(n) because $U^\dagger \tau^\alpha U$ had to be expressible in terms of the τ^α. If the fermions are in an $(n+m)$-dimensional representation with $m \neq 0$, $(U^\dagger \tau^\alpha U)$ would still be Hermitian and traceless but it would be of dimensions greater than $n\times n$. In general it would not be expressible in terms of the generators of SU(n).

The model of Maehara and Yanagida incorporates only four quarks and four leptons. It has to be extended to take into account the discovery of the τ lepton and the Υ. One might consider adding another generation of quarks and leptons and using $SU(3)_H$ to suppress $\Delta S = 2$ currents. However, this leads to anomalies in the lepton sector.[7] One might adopt the viewpoint that the anomalies do not matter because the gauge group considered are part of a larger group (say a group which leads to grand unification) and the anomalies might vanish when the larger group is considered. The anomalies can also be removed by introducing right-handed neutrinos.[17] There is no obvious generalization to $SU(3)_H$ models of the Higgs multiplet assignment which gave nearly degenerate masses to the S bosons in the Maehara-Yanagida model. The anticommutation of the generators which accounted for the near degeneracy seems to be a particular property of SU(2). Another way to take the new particles into account is to consider a $SU(2)_H$ group with four generations of fermions placed in doublets of $SU(2)_H$ to suppress $\Delta S = 2$ currents. A way has to be found to prevent different horizontal doublets from mixing while allowing particles containing bottom quarks to decay (possible due to Higgs-particle interactions).

While this work was being performed the author received a report which also deals with similar material. The report, by Cahn and Harari,[3] discusses a model with the same fermion representation as the $SU(2)_H$ model of Maehara and Yanagida. In contrast to the present work, Cahn and Harari concentrate more on the limits set by different processes on horizontal-gauge-boson masses than on the suppression mechanism studied here. As a result, they do not describe the details of the mechanism which suppresses flavor-changing processes involving only two types of flavors (such processes are called diagonal processes by them). They are also not interested in the conditions under which this suppression will occur in more general horizontal gauge models.

The paper of Cahn and Harari points out that different flavor-changing processes put constraints on essentially two mass parameters in the four-quark $SU(2)_H$ model, namely, the average mass of the horizontal gauge bosons and the order of the mass differences among the bosons. In agreement with their work, I find that the best limit on the boson mass differences comes from the K_L-K_S mass difference. Also, both papers agree that μe conversion provides the best limit on the average horizontal-gauge-boson mass if the mixing-angle suppression is not severe. The $K^+ \to \pi^+ \overline{\mu} e$ and $K_L \to \mu e$ processes are not suppressed by mixing angles. I find that with the conventional horizontal-multiplet assignments $K^+ \to \pi^+ \overline{\mu} e$ is more sensitive than $K_L \to \mu e$ to horizontal-gauge-boson masses, because the latter process requires axial-vector currents and hence would occur at the one-loop level. Cahn and Harari allow for unconventional horizontal-multiplet assignments which give axial-vector currents at the tree level, and find that $K_L \to \mu e$ is slightly more sensitive than $K^+ \to \pi^+ \overline{\mu} e$. Table IV shows my conclusions regarding flavor-changing processes for which improved measurements would be desirable, from the point of view of looking for the effects of the horizontal gauge models considered here. Table IV is based on my own work and the work of Cahn and Harari. It should be noted that the classification schemes of Ref. 3 and of this work, as well as the conclusions of Table IV, are valid only when the flavor-change-suppression mechanism of this paper is operative. In fact, if either this mechanism or some other mechanism does not

TABLE IV. Information given by different processes regarding horizontal gauge models.

Process	Gives information on	Limits (from Table III)	Remarks
K_L-K_S mass difference	horizontal-gauge-boson mass differences $\frac{\Delta M_S}{M_S}$	$\frac{\Delta M_S}{M_S} < 4\times10^{-10}\left(\frac{M_S}{M_W}\right)^2$	a
μe conversion	horizontal-gauge-boson mass M_S	$M_S > 3000\, M_W$	b
$K^+ \to \pi^+ \bar{\mu} e$	horizontal-gauge-boson mass M_S	$M_S > 300\, M_W$	c
$K_L \to \mu e$	horizontal-gauge-boson mass M_S	$M_S > 100\, M_W$	c, d

[a] Improved measurements will not be very useful because the uncertainties are mainly theoretical.

[b] Very sensitive to gauge-boson masses. Depends on fermion mixing angles.

[c] Occurs even in absence of fermion mixing. Hence useful if μe conversion is suppressed by small mixing angles.

[d] Useful process if nature prefers unusual fermion horizontal-multiplet assignments giving axial-vector currents at tree level.

suppress $\Delta S = 2$ currents due to horizontal bosons, the K_L-K_S mass difference will constrain all muon-number-violating rates to be uninterestingly low.

A mechanism which suppresses flavor-changing processes involving only two flavors has been studied. Models in which this mechanism operates can give muon-number-violating rates comparable with experimental limits. Table IV shows some interesting muon-number-violating processes, and the information given by improved measurements on these processes. The table is based on this work and the work of Cahn and Harari.

Note added in proof. A six-quark horizontal gauge model incorporating a suppression mechanism similar to the present one is considered in O. Shanker, TRIUMF Report No. TRI-PP-81-3, 1981 (unpublished). The flavor-violation possibilities in that model are more varied than those considered in this work.

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