

Second-order quantum-chromodynamic effect in J/ψ photoproduction

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Second-order quantum-chromodynamics (QCD) contributions are calculated for J/ψ photoproduction. We show that the measurement of the nonforward structure of the cross section $d\sigma/dt$ at large $-t$ (≥ 1 GeV²) could be an excellent test for QCD.

The cross section of J/ψ photoproduction has been calculated to order α_s in quantum chromodynamics (QCD) by the photon-gluon-fusion mechanism.^{1,2} The numerical result reproduces roughly the energy dependence of the cross section measured in the experiments.³ This mechanism has also been supported from the experimental data of J/ψ production in hadron-hadron collisions for which the corresponding gluon-fusion mechanism was shown to give very consistent explanations.⁴

As far as the lowest-order graphs are concerned, the cross section for J/ψ photoproduction is forward, that is, $-t \approx 0$, because by definition gluons must be nearly on-shell. This means that the higher-order QCD contributions give the cross section in the nonforward region.⁵ Thus in this paper we calculate the nonforward structure

of the cross section through order α_s^2 and show that the measurement of the slope of the cross section $d\sigma/dt$ at large $-t$ (≥ 1 GeV²) could be an excellent test for QCD.

All of the order- α_s^2 graphs are given in Fig. 1, where (A)–(D) and (a)–(h) give the contributions from light-quark and gluon components in a target nucleon, respectively. The gauge invariance is easily proved for each of the sets of graphs (A) and (B), (C) and (D), and (a)–(h). Figure 2 gives the notation for momenta of particles.

The on-shell condition of the initial and final light quark or gluon determines the momentum fraction x , which is defined as $p = xP$, to be $x = q^2/2P \cdot q$, where q ($= p_- + p_+ - k$) is the momentum transfer in the t channel. In the laboratory frame the cross section for the photoproduction of $c\bar{c}$ pairs is

$$\begin{aligned} \frac{d^2\sigma}{dt dW_c^2}(\gamma N \rightarrow c\bar{c} X) = & \frac{\alpha\alpha_s^2}{4\pi M^2 E^2 |t|} \int_{M^2} dM_X^2 \Sigma_0^2 (W_c^2 - 4m_c^2)^{1/2} \\ & \times \int d\Omega_{\vec{\Delta}} \frac{1}{(\Sigma_0^2 \sin^2\varphi + W_c^2 \cos^2\varphi)^{3/2}} \\ & \times \left\{ 2e_c^2 \sum_{i=u}^s f_{i/N}(x) |\mathfrak{M}_A + \mathfrak{M}_B|^2 + 2 \sum_{i=u}^s e_i^2 f_{i/N}(x) |\mathfrak{M}_C + \mathfrak{M}_D|^2 \right. \\ & + \frac{2}{3} e_c^2 G_N(x) \left[\sum_{j=A}^h |\mathfrak{M}_j|^2 + 2(\mathfrak{M}_A \mathfrak{M}_B^* + \mathfrak{M}_A \mathfrak{M}_C^* + \mathfrak{M}_A \mathfrak{M}_D^* \right. \\ & \left. \left. + 2(\mathfrak{M}_B \mathfrak{M}_C^* + \mathfrak{M}_B \mathfrak{M}_D^* + \mathfrak{M}_C \mathfrak{M}_D^*) \right) \right] \\ & \left. - \frac{1}{12} e_c^2 G_N(x) 2(\mathfrak{M}_A + \mathfrak{M}_B + \mathfrak{M}_C)(\mathfrak{M}_B^* + \mathfrak{M}_C^* + \mathfrak{M}_D^*) + \frac{3}{2} e_c^2 G_N(x) |\mathfrak{M}_E + \mathfrak{M}_H|^2 \right\}, \quad (1) \end{aligned}$$

where $e_i = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, $e_c = \frac{2}{3}$, M is the nucleon mass, E is the photon energy, $t = q^2$, $\Sigma = p_+ + p_-$, $\Delta = p_- - p_+$, φ is the angle between $\vec{\Sigma}$ and $\vec{\Delta}$, $f_{i/N}(x)$ ($i = u, d, s$), and $G_N(x)$ are the distribution functions of quarks and gluons, respectively, and \mathfrak{M}_i ($i = A, \dots, h$) are invariant amplitudes of the graphs given in Fig. 1. Here the numbers 2, 2, $\frac{2}{3}$, $-\frac{1}{12}$, and $\frac{3}{2}$ are the QCD color factors stemming from the color summation of the final partons and the color average of the initial partons. Σ_0 is given by the relation $\Sigma_0 = (s + t - M_X^2)/2M$. The upper bound of M_X^2 is determined by the condition $\cos\theta \leq 1$, where θ is the angle between $\vec{\Sigma}$ and \vec{k} ,

as

$$M_X^2 = s + t - M[(W_c^2 + |t|)^2 + 4E^2 W_c^2]/2E(W_c^2 + |t|).$$

We must remark here that the graph (D) in Fig. 1 causes a mass singularity of the type $\ln m^2$ in the integral (1) at the configuration \vec{k}/\vec{p}' , where $|a - m^2|$ takes the minimum value $m^2(1 + W_c^2/|t|)$. Here m is the mass of the light quark and $a = (p - \Sigma)^2$. It has not been proved whether this singularity remains or not when we introduce higher-order graphs than the ones included here. But fortunately the contributions coming from the graphs (C) and (D) do not seem to be large

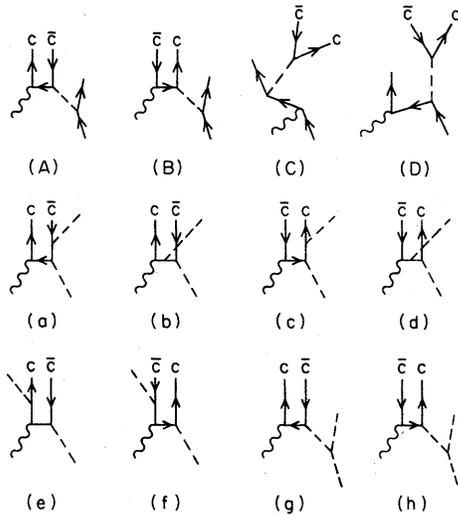


FIG. 1. Feynman graphs for $\gamma + N \rightarrow c\bar{c} + X$ up to order α_s^2 . The notation in this and subsequent figures is solid lines for quarks, dashed lines for gluons, and wavy lines for photons.

because of the large mass of J/ψ in the denominator of the gluon propagator. In this paper we assume the approximate relation $p = xP$ only for the purpose of making our calculation simple. Then $|\hat{u} - m^2|$ has the minimum $(W_c^2 + |t|)^3 / [4E^2 |t| - (W_c^2 + |t|)^2]$, which is different from the correct minimum $m^2(1 + W_c^2/|t|)$ but takes much smaller values than 1 GeV^2 in the region of $|t| \ll E^2$ and $|t| < W_c^2$. In this approximation, in fact, we found that the graphs (C) and (D) do not give important contributions in the region of $1 \text{ GeV}^2 \leq -t \leq 10 \text{ GeV}^2$ and $E \geq 50 \text{ GeV}$. Taking into account that the singularity is of a logarithmic type so that the contribution from the graph (D) does not depend so strongly on the minimum

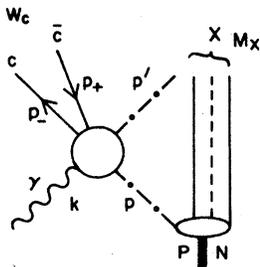


FIG. 2. The picture of the order- α_s^2 graphs to show the notation for momenta of particles. The nucleon labeled by N has the momentum P . The dash-dot lines denote quarks (or gluons) which have the momentum p and p' . Symbols p_- and p_+ denote momenta of charm quarks c and \bar{c} , and k is the one of photon. W_c is the invariant mass of the channel $c\bar{c}$ and M_X is the one of hadrons included in the anything X .

of $|\hat{u} - m^2|$, we can insist that our numerical results for $d\sigma/dt$ might not have a large contribution from the graph (D) and therefore are very reliable as far as $m \gtrsim 0.1-0.03 \text{ GeV}$ even when we treat this singularity correctly.

In the following step to connect Eq. (1) to J/ψ photoproduction, we use the same technique as in the photon-gluon-fusion model.^{1,2} The $c\bar{c}$ pair produced in the reaction given in Fig. 1 will undergo some final-state interaction (which must include a color-rearrangement process if the $c\bar{c}$ pair is a color-octet state) and will turn into some charmonium state such as J/ψ , χ , etc., or into a pair of charmed particles. Then the J/ψ production, in general, comes from two components: the direct J/ψ production from the $c\bar{c}$ pair and the decays of the higher-mass states into the J/ψ state.¹ Thus we find that the cross section of the J/ψ production is associated with the direct production of all charmonium states from the $c\bar{c}$ pair. Because the contributions from the decays are due to charmonium states with masses less than $2m_D$ [m_D is the mass of $D(1.86)$], the cross section for J/ψ production is supposed to be given approximately by some fraction of the value obtained by integrating Eq. (1) over W_c^2 from $4m_c^2$ (m_c is the mass of the charmed quark) to $4m_D^2$.

We introduce the number N_D to express the cross section for J/ψ production as the above integral divided by N_D . As discussed above this number will be determined roughly by the probabilities of the direct productions of charmonium states and the branching ratio for the decays of the higher-mass states into J/ψ . Although we do not have enough knowledge about these, we can fortunately use the approximate value of N_D obtained from comparing the cross section calculated to order α_s with the experimental data of the total cross section. N_D has been known to be around 8 if we use $m_c = 1.5 \text{ GeV}$ and the power $n = 5$ in the following form for the gluon distribution function which we shall use: $G_N(x) = 0.5(n+1) \times (1-x)^n/x$.⁶ As Jones and Wyld pointed out,¹ N_D depends strongly on m_c and n . Therefore we will mention later the dependences on m_c and n for our predictions calculated from Eq. (1).

Before proceeding to numerical calculations, we must specify the distribution functions $f_{i/N}(x)$ for the light quarks in the target nucleon. As will soon be found, our main prediction does not almost depend on these functions. So we use here only the Field-Feynman parametrization.⁷

We first temporarily choose the values $m_c = 1.5 \text{ GeV}$ and $n = 5$.⁸ Figure 3 shows the numerical result of $d\sigma/dt$ for J/ψ photoproduction at the photon laboratory energy $E = 20 \text{ GeV}$, where we

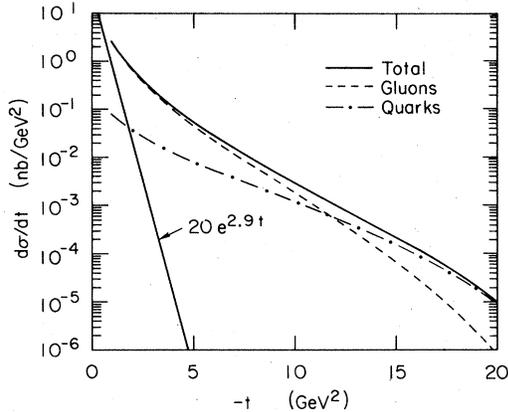


FIG. 3. The numerical result of $d\sigma/dt$ versus $-t$ for $\gamma + N \rightarrow \psi + X$ at $E = 20$ GeV. Here we set $m_c = 1.5$ GeV and $n = 5$ in Eq. (2). A dashed line shows that contribution from gluons, a dash-dot line the one from quarks and a solid line is the total of them. At the same time we show the curve of $20e^{2.9t}$ nb/GeV² as the typical diffractive curve suggested by the experiment.³

take $N_D = 8$ and $\alpha_s = (12\pi/25) \ln M_{J/\psi}^2 / \Lambda^2 (= 0.41)$ with $\Lambda = 0.5$ GeV. The main pattern of the contributions from quarks and gluons is that the contribution from gluons dominates the cross section in the lower $-t$ region ($-t < 15$ GeV²) and the contribution from quarks emerges prominently at higher $-t$. In this figure we show the typical diffractive curve $20e^{2.9t}$ nb/GeV² which was suggested by the SLAC experiments.³ Comparing the numerical result to this diffractive curve, we can predict that the slope of $d\sigma/dt$ should turn out to change very much apart from the diffractive curve around $-t = 1$ GeV².

We have studied to what extent the above prediction depends on m_c and n , i.e., the form of $G_N(x)$ in the range $1.25 \text{ GeV} \leq m_c \leq 1.75 \text{ GeV}$ and $3 \leq n \leq 7$. The numerical results show a striking feature that the slope of $d\sigma/dt$ does not almost depend on m_c in the region $-t \leq 15$ GeV² where the gluon contribution dominates. On the other hand, the slope has a slight n dependence, but is not sufficiently large to be discriminated in experiments. Therefore we can conclude that the slope of $d\sigma/dt$ will be expected to reflect the significant feature peculiar to the higher-order interactions of QCD and irrelevant to the values of m_c and the form of $G_N(x)$ in the region $-t \leq 15$

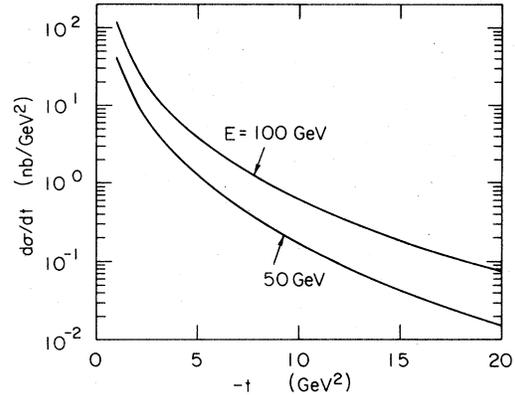


FIG. 4. The calculated curves of $d\sigma/dt$ versus $-t$ for $\gamma + N \rightarrow \psi + X$ at $E = 50$ and 100 GeV.

GeV².

We finally discuss the energy dependence of the cross section $d\sigma/dt$ calculated to order α_s^2 . Here we take the values $m_c = 1.5$ GeV and $n = 5$. In Fig. 4 we give the calculated curves of $d\sigma/dt$ versus $-t$ at $E = 50$ and 100 GeV. This figure shows that the curves become flatter as E increases but the change of the slope does not strongly depend on E . This is supported also from our numerical result that the energy dependence is almost due to the phase space included in Eq. (1) and has a tendency similar to the total cross section calculated to order α_s in the photon-gluon-fusion model.

After we finished this calculation, we found the paper of the experimental data for the virtual photoproduction of elastic J/ψ events which have been measured by the European Muon Collaboration.⁹ Although we cannot directly compare our results with this data, we can find the same tendency of the slope change at about $-t = 1$ GeV² in it as we predict in this paper.

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