Four-momentum transfer between groups of secondary particles in proton-nucleus interactions at 200 GeV

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Data on interactions of 200-GeV protons in nuclear emulsion has been used to determine the lower limit of the four-momentum transfer between groups of secondary particles in jets. It is found that $\Delta_{\parallel}$ in a jet increases linearly with the multiplicity of the jet. All the jets with large multiplicity show clear pronounced maxima in $\Delta_{\parallel}$ plotted against log tan θ_L. The four-momentum transferred between groups of secondary particles produced in the above interactions has been studied to determine the nature of the particle exchanged between these groups. The results are in agreement with the prediction of the Pomeranchuk-pole-exchange model.

I. INTRODUCTION

The study of nucleon-nucleus interactions has been attracting wide attention for quite some time now. Earlier it was hoped that the study of multiparticle production in such interactions would provide information which normally was unavailable in hadron-hadron processes and thus would help in identifying the model of strong interactions of hadrons with hadrons at high energies. The problem is still unsolved and a study of some finer details of hadron-nucleus interactions has become essential.

An interesting parameter in the analysis of multiple production of mesons in hadron-nucleus interactions is the four-momentum transfer between groups of particles. This was first introduced by Niu.[1] The square of the four-momentum transfer (Δ^2) characterizes the dynamics of the interaction process and is invariant under Lorentz transformation. The knowledge of four-momentum transfer has added importance from the standpoint of the theory of peripheral interactions which is based upon the exchange of one or several virtual pions between the interacting objects.

It is observed by Niu[1] that at high energies the longitudinal component of the four-momentum transfer $\Delta_{\parallel}$ is $\sim 1-2\,\text{GeV}/c$. It is also seen that the frequency distribution of $\Delta_{\parallel}$ is rather a broad one. This observation has been discussed by various authors[2-4] from the points of view of theoretical models. However, one must note that the experimental data came from cosmic-ray events and it suffered from two drawbacks: firstly, imprecise knowledge of primary energy and, secondly, poor statistics.

The present paper is an extension of the application of the four-momentum transfer analysis to study the nature of the exchange particle between groups of secondary particles produced in proton-nucleus interactions together with broad interaction characteristics in high-multiplicity events. The distribution of $\Delta_{\parallel}$ against log tan θ_L for $N_h > 2$ with $20 \leq n_s \leq 46$ events show a clear maxima in most of the primary interactions. It is also observed that $\Delta_{\parallel}$ is proportional to the multiplicity of shower particles (n_s). The observations are in agreement with the prediction of the model of Pomeranchuk-pole exchange.

II. EXPERIMENTAL DETAILS

A stack of Ilford G-5 emulsion plates of dimensions $12 \times 10 \times 0.06\,\text{cm}^3$, used for this experiment, was exposed to 200-GeV protons at Fermilab. The plates were area scanned for nuclear interactions with $n_h \geq 2$. The experimental details and criteria for selecting p-nucleus interactions are discussed by Gurtu *et al.*[5,6]

A sample of 500 interactions, containing 104 events with $20 \leq n_s \leq 46$ and 396 events with $n_s < 20$, has been used for the present analysis. Angles of all secondary tracks with grain density, $g^* \leq 1.4 g_{\min}$ have been measured using Koristka R-4 microscope employing coordinate method, i.e., X, Y, Z coordinates for each track are measured. The least count of Z motion of the Koristika microscope and of X and Y graticules allowed the measurement of projected angles to the accuracy of $\sim 6'$ and of dip angles $\sim 30'$ over one field of view. However, for overlapping tracks the measurements were made after following the tracks to such lengths (usually $\sim 1000-5000\,\mu\text{m}$) that a clean separation between them was achieved. This considerably reduced the errors of measurement angles of tracks.

The data are presented in two subgroups characterized by the number of heavy prongs (N_h). The grouping is usually done to distinguish between the interactions with light (CNO) and heavy (Ag, Br) nuclei in emulsion.[5-11] The two subgroups taken are $2 \leq N_h \leq 6$ and $N_h > 6$. One should note that the sample with $N_h > 6$ represents exclusively the interactions with Ag Br nuclei of emulsion, but the sample with $2 \leqslant N_h \leqslant 6$ is a mixture of interaction with CNO and AgBr.

III. ESTIMATION OF THE LOWER LIMIT OF FOUR-MOMENTUM TRANSFER AND THE RESULTS

Let us consider an interaction of a fast particle with a target nucleus in the rest system of the nucleus. We denote the energies and momenta of the produced particles belonging to the groups (i) and (j) as E_i, p_i and E_j, p_j, respectively. Then the four-momentum transfer between the groups of particles is given by

$$\Delta^2 = (E_i - E_j)^2 - (p_i - p_j)^2 . \qquad (1)$$

Let $\Delta_{\parallel}$ and $\Delta_{\perp}$ respectively denote the longitudinal and transverse components of the momentum transfer Δ. Neglecting the terms of higher order in the transverse momenta and the masses of the secondary particles in the relativistic approximation, the $\Delta_{\parallel}$ is given by[1,12]

$$\Delta_{\parallel} = 1.5\langle P_T\rangle \left(\sum \tan\theta_i \sum \cot\theta_j\right)^{1/2} , \qquad (2)$$

where $\langle P_T\rangle$ is the average transverse momentum and θ_i and θ_j are the space angles of the particles in the two groups. Factor 1.5 accounts for the neutral π mesons. Following Babecki *et al.*[13] we use $\langle P_T\rangle = 0.4\,\mathrm{GeV}/c$. Obviously, the above relation gives only the lower value of the four-momentum transfer because $\Delta_{\parallel}^2$ is less than Δ^2.

To estimate $\Delta_{\parallel}$, (Eq. 2), we divide the secondary particles in each jet into two groups, calling them groups (i) and (j). The group (i) contains shower particles which are emitted at an angle smaller than a certain value of θ_i in the laboratory system and the group (j) contains the remaining shower particles. Knowing the angles of each secondary particle, one obtains $\Delta_{\parallel}$. This is done for all the events and for all possible values of $i = 1$ to $(n_s - 1)$ and $j = (n_s - 1)$ to 1. The variation of $\Delta_{\parallel}$ with $\log_{10} \tan \theta_L$ for each shower track of an event having $N_h \geqslant 2$ and $20 \leqslant n_s \leqslant 46$ is studied for all 104 events. As already indicated the events are grouped on the basis of N_h as interactions with light $(2 \leqslant N_h \leqslant 6)$ and heavy $(N_h > 6)$ mass nuclei. The general behavior of these distributions is same. We have divided the events into 18 groups on the basis of N_h and n_s. The variation of $\Delta_{\parallel}$ for a typical event (chosen randomly) from each group is shown in Fig. 1. The upper part of each plot shows spectrum of the emission angles of particles of the jet, in the scale $x = \log_{10} \tan\theta_L$. It is observed from Fig. 1 that the majority of jets show clear pronounced maxima. For further analysis we consider three cases.

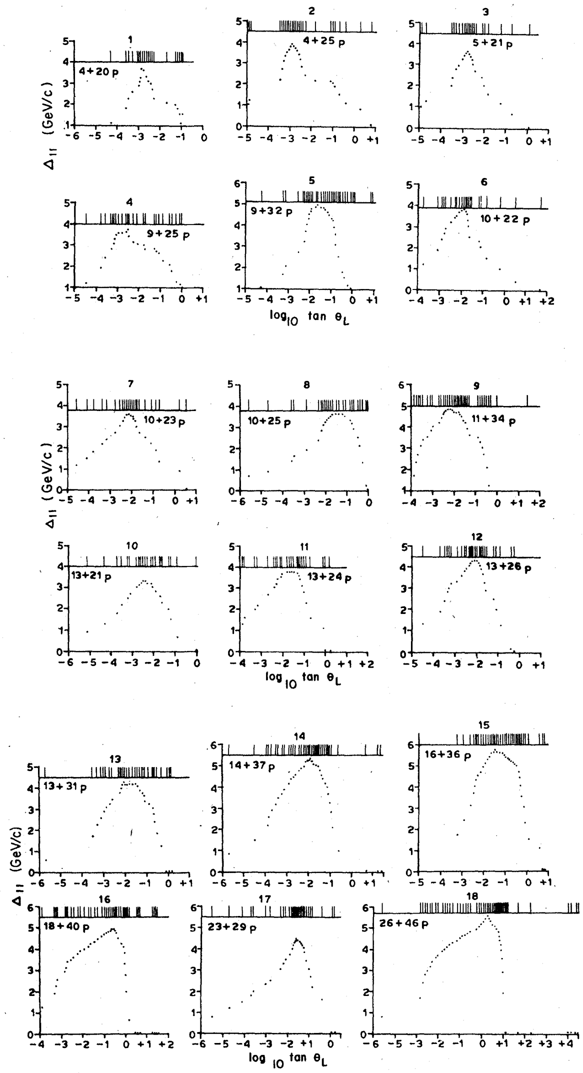

FIG. 1. Dependence of $\Delta_{\parallel}$ on $x = \log_{10}\tan\,\theta_L$ for jets (1) $4+20p$, (2) $4+25p$, (3) $5+21p$, (4) $9+25p$, (5) $9+32p$, (6) $10+22p$, (7) $10+23p$, (8) $10+25p$, (9) $11+34p$, (10) $13+21p$, (11) $13+24p$, (12) $13+26p$, (13) $13+31p$, (14) $14+37p$, (15) $16+36p$, (16) $18+40p$, (17) $23+29p$, (18) $26+46p$.

Case A. Symmetrical distribution of secondary particles (n_s) in the center-of-mass system (c.m.s.), i.e., $(n_s)_i = (n_s)_j$.

Case B. Asymmetrical distribution of particles in the c.m.s., i.e., $(n_s)_i \neq (n_s)_j$. The number of shower particles in the group (i) and group (j) are determined from the average value of $\log_{10} \tan\,\theta_L$ in the relation[14]

$$-\log_{10}\gamma_c = \langle \log_{10} \tan\theta_L \rangle .$$

The $\langle \log_{10} \tan\theta_L \rangle$ obtained for the event puts the demarcation between groups (i) and (j).

TABLE I. Longitudinal component of the four-momentum transfer between the groups of secondary particles at 200 GeV for high-multiplicity events with $N_h > 2$ and $20 \leq n_s \leq 46$.

	$(n_s)_i = (n_s)_j$ Case A				$(n_s)_i \neq (n_s)_j$ Case B				Case C			
Number of jet 1	$(n_s)_i$ 2	$(n_s)_j$ 3	$\Delta_{\parallel}^{\text{jet}}$ (GeV/c) 4	$\Delta_{\parallel}^{NN}$ (GeV/c) 5	$(n_s)_i$ 6	$(n_s)_j$ 7	$\Delta_{\parallel}^{\text{jet}}$ (GeV/c) 8	$\Delta_{\parallel}^{NN}$ (GeV/c) 9	$(n_s)_i$ 10	$(n_s)_j$ 11	$\Delta_{\parallel}^{\text{jet}}$ (GeV/c) 12	$\Delta_{\parallel}^{NN}$ (GeV/c) 13
1	10	10	3.303	1.775	14	6	2.267	1.218	10	10	3.303	1.775
2	12	13	3.794	2.039	13	12	3.674	1.975	10	15	3.870	2.080
3	10	11	3.590	1.930	11	10	3.546	1.906	10	11	3.590	1.930
4	13	12	3.323	0.965	12	13	3.431	0.997	10	15	3.488	1.013
5	16	16	4.764	1.384	18	14	4.683	1.361	10	22	4.546	1.321
6	11	11	3.637	1.057	16	6	2.498	0.726	10	12	3.627	1.054
7	11	12	3.479	1.011	12	11	3.441	1.000	10	13	3.483	1.012
8	12	13	3.433	0.997	11	14	3.433	0.997	10	15	3.341	0.971
9	17	17	4.665	1.356	21	13	4.412	1.282	10	24	4.387	1.275
10	10	11	3.183	0.925	11	10	3.221	0.936	10	11	3.183	0.925
11	12	12	3.638	1.057	11	13	3.606	1.048	10	14	3.513	1.021
12	13	13	4.164	1.210	11	15	3.857	1.121	10	16	3.727	1.083
13	16	15	4.194	1.219	15	16	4.182	1.215	10	21	3.996	1.161
14	18	19	5.174	1.504	17	20	5.109	1.485	10	27	4.163	1.210
15	18	18	5.549	1.613	20	16	5.411	1.572	10	26	5.095	1.481
16	20	20	4.718	1.371	18	22	4.542	1.320	10	30	3.718	1.080
17	14	15	4.291	1.247	11	18	3.699	1.075	10	19	3.448	1.002
18	23	23	5.162	1.500	21	25	4.887	1.420	10	36	3.860	1.122
Average value			1.342 ± 0.07				1.258 ± 0.07				1.251 ± 0.08	

Case C. Asymmetrical distribution of secondary particles in the c.m.s., i.e., $(n_s)_i \neq (n_s)_j$, where the number of shower particles in the group (i) and (j) are determined using the value of γ_c for the incoming energy $E = 200$ GeV and considering that the interaction is pp collision.

The results for 18 typical jets (Fig. 1) are listed in Table I (columns 4, 8, and 12). The variation of $\Delta_{\parallel}$ with n_s for two subgroups of events, with $2 \leq N_h \leq 6$ and $N_h > 6$ (in each case $4 \leq n_s \leq 20$) is shown in Fig. 2 for the 396 events. It is observed from the figure (also refer to Table I) that the four-momentum transferred in a jet during the interaction is linearly proportional to the multiplicity, n_s, of the given jet. The linear relation is obtained by the least-square fit to the experimental data. This is of the form

$$\langle \Delta_{\parallel} \rangle = a + b(n_s)\,, \tag{3}$$

where a is the intercept cut off by the straight line on the $\Delta_{\parallel}$ axis and b is the slope of the fit. The values of a and b for the two subgroups of events are tabulated in Table II. The linear relationship between $\Delta_{\parallel}$ and n_s may be visualized in terms of the impact parameter; the larger the impact parameter (as in peripheral collisions) the smaller is the multiplicity n_s as well as $\Delta_{\parallel}$ and vice versa Figures 1 and 2 indicate that $\Delta_{\parallel}$ does not depend on N_h, i.e., on the size of the target. The same

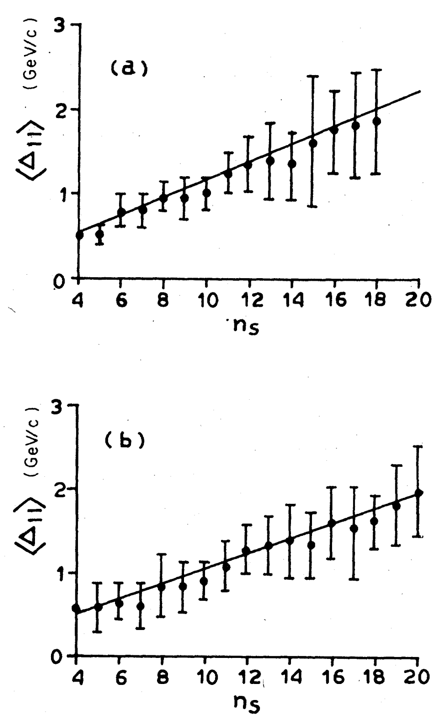

FIG. 2. Variation of $\langle \Delta_{\parallel} \rangle$ with n_s (a) for events with $2 \leq N_h \leq 6$ and (b) for events with $N_h > 6$. The solid curve is the best-fit line to the data points.

TABLE II. Values of constants a and b in GeV/c as determined from Eq. (3).

Constants	$2 \leq N_h \leq 6$	$N_h > 6$
a	0.180 ± 0.005	0.198 ± 0.024
b	0.094 ± 0.060	0.10 ± 0.002

result is also suggested by the near-constant values of a and b for the two subgroups of events as seen from Table II.

The large-multiplicity events (with $N_h > 2$, $20 \leq n_s \leq 46$, Fig. 1) in p-nucleus interactions can be considered to be the result of several independent elementary NN interactions between the primary nucleon and the nucleons of the target nucleus. The number of such elementary NN interactions ($N_{\rm eff}$) inside the target nucleus has been worked out by Gurtu *et al.*[5,6] and Kaul *et al.*[15,16] using Glauber multiple-scattering theory.[17] Following these authors[18] we have used 1.86 for the (CNO) group and 3.44 for the (Ag Br) group as average number of collisions inside the target nucleus. The values of ($\Delta_\parallel^{NN}$) per elementary NN interaction thus estimated are listed in Table I (columns 5, 9, and 13). The mean values of $\Delta_\parallel^{NN} = (\Delta_\parallel^{\rm jet}/N_{\rm eff})$ together with the mean-square errors, obtained from our experimental data, along with those of Alekseeva[20] are listed in Table III. The two results appear to be in good agreement within experimental errors. This indicated that $\langle \Delta_\parallel \rangle$ is independent of the incident energy.

The average values of $\Delta_\parallel$, calculated for the entire sample, are listed in table IV. Here we have quoted the results for three subgroups of events, viz., $2 \leq N_h \leq 6$, $N_h > 6$ and $N_h \geq 2$. The distributions of $\sqrt{\Delta_\parallel{}^2}$ for these events are shown in Figs. 3a-3c respectively. The distributions show peaks around $\sqrt{\Delta_\parallel{}^2} \simeq 0.9$ GeV/c for $2 \leq N_h \leq 6$ and $\simeq 1.5$ GeV/c for both $N_h > 6$ and $N_h \geq 2$ events. The solid curve is the prediction of the Pomeranchuk-pole-exchange model which can be expressed as[21]

$$f(\Delta_\parallel) d\Delta_\parallel \propto \Delta_\parallel \exp(-a\Delta_\parallel{}^2) d\Delta_\parallel \,, \qquad (4)$$

where $a = 2\alpha(0)\ln(s/2Mn^2)$ with α as the derivative of the Pomeranchuk trajectory with respect to $\Delta_\parallel{}^2$,

TABLE III. Four-momentum transfer per elementary NN collision, $\langle \Delta_\parallel^{NN} \rangle$ (in GeV/c).

Case A	Case B	Case C	Energy (GeV)	References
1.34 ± 0.08	1.26 ± 0.07	1.25 ± 0.08	200	Present work
1.48 ± 0.06	1.39 ± 0.07	1.30 ± 0.09	12 to 2.5×10^4 per nucleon	Alekseeva[20]

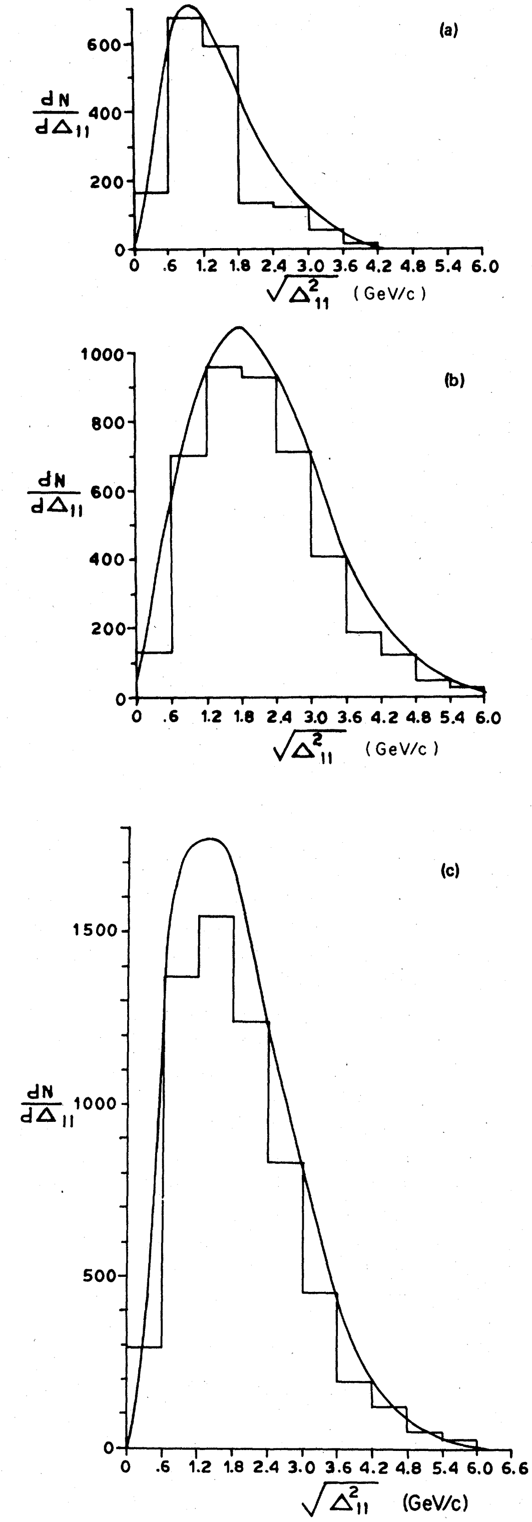

FIG. 3. $\Delta_\parallel$ distribution for interactions of different N_h. (a) $2 \leq N_h \leq 6$. (b) $N_h > 6$. (c) $N_h \geq 2$. The solid curve represents the fit of the function $f(\Delta_\parallel) d\Delta_\parallel \propto \Delta_\parallel \exp(-a \Delta_\parallel{}^2) d\Delta_\parallel$.

TABLE IV. Average four-momentum transfer between the groups of particles for different N_h events, $\Delta_{\parallel}^{\text{jet}}$ (in GeV/c).

$2 \leq N_h \leq 6$	$N_h > 6$	$N_h \geq 2$
1.53 ± 0.15	2.10 ± 0.16	1.89 ± 0.15

TABLE V. Values of constant a in $(\text{GeV}/c)^{-2}$ as determined from Eq. (4).

$2 \leq N_h \leq 6$	$N_h > 6$	$N_h \geq 2$	References
0.38 ± 0.04	0.16 ± 0.034	0.11 ± 0.017	Present work
00.87			Jain *et al.*[21]
0.65			Kobayakawa *et al.*[23]

s being the square of the center-of-mass energy and M_n is the mass of n pions. The best-fit values of a, besides the mean errors, calculated by the residual method,[22] for the three groups of N_h events are tabulated in Table V along with those of Jain *et al.*[21] and Kobayakawa *et al.*[23] The observed difference between our results and those of Jain *et al.*[21] and Kobayakawa *et al.*[23] may be attributed to the smaller statistics gathered by these authors at indiscrete cosmic-ray energies. One can also observe from Table V a clear dependence of a on the mass of the target.

The large value of the four-momentum transfer between groups of particles in a jet and the observed peak in $\Delta_{\parallel}$ distribution around $0.9 - 1.5\ (\text{GeV}/c)^2$ may be attributed to the formation of independent clusters in the multiperipheral chain.[24,25] The argument seems to be well supported by the agreement between the present data and the Pomeranchuk-pole-exchange model.

IV. CONCLUSIONS

The analysis of the data presented here leads to the following conclusions:

(*i*) The variation of $\Delta_{\parallel}$ with $\log_{10} \tan\theta_L$ shows clear maxima in the events studied. This may be regarded as due to the clustering of shower particles in jets,[11] as also found in the study of correlations among shower particles in p-nucleus interactions by Daftari *et al.*[11,26]

(*ii*) There is a good indication that four-momentum transfer between the groups of secondary particles increases linearly with the multiplicity (n_s) of the jet and the variation does not depend upon the target mass.

(*iii*) The four-momentum transfer ($\Delta_{\parallel}$) between the groups of secondary particles produced in proton-nucleus interactions at 200 GeV/c is in agreement with the prediction of Pomeranchuk-pole-exchange model.

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[18]$\nu_A = \Sigma \nu P_\nu / \Sigma P_\nu$ where P_ν is the probability of interactions inside the target nucleus. Following Glauber (Ref. 17) $\nu_A = A\delta_{in}/\delta_{in}(A)$ where $\delta_{in}(A) = 2\pi \int [1 - e^{-\delta_{in} T(b)}] b\, db$, $T(b) = A \int \rho(r)\, dZ$, and $r^2 = Z^2 + b^2$. Saxon-Woods distribution has been considered for $\rho(r)$. δ_{in} is (pp) inelastic cross section which have been evaluated from the empirical relation given by Morrison (Ref. 19).

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