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Some problems bearing on the concept of space-time quanta

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A critical assessment of a recent empirical space-time-quantization hypothesis based on the ρ -meson half-width is made with the object of clarifying the ambiguities and uncertainties of the proposal. We show it is necessary to await a precise resolution of the unstable-particle pole positions before the theory can be adequately tested. An epistemological critique of the application of the space-time-quantization concept to electromagnetic radii is also presented.

In the years immediately following the creation of quantum theory there existed considerable discussion concerning the possibility of extending the quantization program to space-time itself. Attempts to construct such a theory have lately been inspired by many motives, including the desire to purge quantum electrodynamics of divergent self-energy integrals, to implement the calculational tools of nonlocal field theories, to explain the origin of the internal symmetries, and to overcome the problems of the radiating classical electron.¹ In many of these approaches the Compton wavelength, $\lambda = \hbar/mc$, was imputed the role of a fundamental length. Its special significance resided in the fact that a single relativistic particle cannot be localized in space to a volume smaller than λ^3 due to the inevitable materialization of its confinement energy in the form of particle-antiparticle pairs.² The one-particle state under observation is thus lost. The overt mass dependence of λ , however, necessarily limits its general utility as a fundamental length parameter of hadronic dimensions, so the search for a universal fundamental length λ_0 evidently reduces to the search for a universal mass. The obvious candidate is the quark mass, but the absence of a theory-free determination of the free or bound-state quark mass as well as the variety of quarks, all fundamental, to choose from still poses a crucial limitation. Many other candidates for λ_0 , some of which are presented in Table I, have been considered. These include the electron and nucleon Compton wavelengths, empirical parameters like the nucleon radius, and universal constants like the Planck length and the classical electron radius.

The elementary length λ_0 is usually intended to characterize only the length scale of the strong interactions. The existence of mass splitting suggests that relevant scales associated with the other interactions may also exist. The proposed fundamental lengths presented in Table I are grouped according to this criterion. However, in the discussion that follows, λ_0 will be taken to represent processes arising only from the strong interaction, unless otherwise indicated.

In order to avoid semantic ambiguity it is important to distinguish between some of the different usages of the space-time-quantization concept. Excluding proposals involving exotic space-time topologies, these fall into five main types.

(1) In what we shall term the standard approach, space and time themselves are treated as continuous quantities but physical processes are conjectured to occur only over space and time intervals of magnitude λ_0 and τ_0 , respectively. This sense of space-time quantization is thus analogous to energy quantization, the quanta may occur in any size, but in a given process must occur in some specific size. We are consequently led to the idea of a process-dependent quantization and must distinguish between gravitational, weak, electromagnetic, and hadronic space-time quanta as outlined in Table I.

(2) Space-time quantization may also be meant to suggest that physical space-time itself exists as a (hypercubic) lattice of points rather than as a continuous manifold. In this radical approach distances or times smaller than the lattice spacing are necessarily devoid of meaning. In this form space-time quantization is analogous to the

TABLE I. Proposed space-time quanta.

Interaction	Quantity	λ_0 (fm)	τ_0 (10^{-24} sec)
Strong	Nucleon Compton wavelength	0.21	7.0×10^{-1}
	Proton charge radius ^a	0.81 ± 0.04	2.7 ± 0.1
	Quark Compton wavelength ^b	0.69 ± 0.04	2.31 ± 0.01
	$\hbar c/2\Gamma_\rho$ ^c	0.67 ± 0.03	2.2 ± 0.1
	Nuclear zero-point radius	0.3	1.0
Electromagnetic	Classical electron radius ^d	2.82	9.4
	Electron fundamental time ^e	1.88	6.3
	Electromagnetic zero-point radius ^d	2.5×10^{-2}	8.3×10^{-2}
	Classical nucleon radius	1.5×10^{-3}	5.1×10^{-3}
	QED lower bound ^f	$\leq 4.0 \times 10^{-2}$	$\leq 1.3 \times 10^{-1}$
	Mossbauer limit ^g	$< 1.0 \times 10^{-7}$	$< 1.0 \times 10^{-6}$
Weak	W-boson Compton wavelength ^h	2.7×10^{-3}	9.0×10^{-3}
	Z-boson Compton wavelength ^h	2.3×10^{-3}	8.0×10^{-3}
	Weak zero-point limit ^d	4.7×10^{-5}	1.6×10^{-4}
	Fermi length ($G_F^{1/2}/\hbar c$)	6.8×10^{-4}	2.3×10^{-3}
Gravitational	Planck length ($\hbar G/c^3$) ^{1/2}	1.6×10^{-20}	5.3×10^{-20}
	Gravitational zero-point limit ^d	2.3×10^{-20}	7.7×10^{-20}
	Electron Schwarzschild radius	1.0×10^{-42}	3.3×10^{-42}

^a From a least-squares fit to the low-energy data of Ref. 3.

^b Based on the dynamically generated quark mass, $m_q = 285 \pm 15$ MeV, computed in Ref. 4.

^c Value for the ρ width adopted from Ref. 16.

^d Zero-point radius denotes the minimum radius for localization resulting from quantum-mechanical field fluctuations $\sqrt{2f} \lambda_N$, where $f = g^2/4\pi\hbar c = 1$ for strong forces, $f = \alpha$ for electromagnetic forces, $f = (g_w^2/4\pi\hbar c)/\lambda_N^2$ for the weak decay of the muon, and $f = Gm_p^2/\hbar c$ for gravitation (Ref. 5).

^e Reference 6.

^f From QED tests of $e^+e^- \rightarrow$ leptons (Ref. 7).

^g From a study of the Mossbauer effect in ^{67}Zn modeled as radiation from a charged particle in an oscillator potential well (Ref. 8).

^h Reference 9. Mixing angle = 30° .

absolute quantization of charge or spin; the quanta occur as integral multiples of only one absolute size. The implications are utterly revolutionary: The notion of Archimedian continuity is abolished and with it the traditional geometry of points and infinitesimals. It is thus necessary to embrace discrete geometries, to accept the breakdown of Lorentz invariance at order λ_0 (and the consequent violation of translation and rotation invariance), and to eliminate the use of differential equations in favor of finite-difference methods.

(3) In the positivist sense, the uncertainty principle,

$$\begin{aligned} \Delta p_i \Delta x_i &\geq \hbar/2, \quad i = 1, 2, 3, \\ \Delta E \Delta t &\geq \hbar/2, \end{aligned} \quad (1)$$

is held to establish a natural limit to space-time resolution imposed by the creation of particle-antiparticle pairs in keeping with the Newton-Wigner localization criterion.² If Δp_{th} is the

lowest threshold for particle production, then the space-time concept itself is said to lose meaning at distances smaller than $\hbar/2\Delta p_{\text{th}}$. Indeed, the consistent exploitation of this feature has led to a virtual fiat against the use of space-time coordinates in the description of high-energy processes by the S-matrix theorists.¹⁰

(4) In the operationalist sense, the finite size of elementary particles is held to establish a natural limit to the meaning of spatial separation that requires the introduction of a fundamental length parameter. Since any space-time measurement is ultimately based on the existence of a scale and since the elementary particle is the smallest material scale in existence, the application of the concept of spatial separation to distances smaller than the particle diameter is meaningless. This principle does not necessarily alter our usual concepts of points and continua, but it does impose a lower limit on their domain of application.

(5) Finally, the least radical of the hypotheses,

which we term the conventionalist interpretation, asserts only the existence of a natural unit for space-time measurements. This concept treats space and time as continuous and permits subdivisions of λ_0 and τ_0 . It functions in a role similar to that of the Bohr radius in atomic physics. The repeated occurrence of strong-interaction scales on the order of one Fermi (see Table I) is suggestive for the truth of at least this interpretation.

These different ways of viewing space-time quantization are not meant to be mutually exclusive. Both the positivist and operationalist approaches are closely related, but the standard and radical approaches are necessarily in opposition. With the exception of the conventionalist approach, the literal application of any of these concepts obliges us to reconsider the meaning of all lengths associated with strong-interaction dynamics that are smaller than λ_0 .

Recently, a new point of view has been advanced by Ehrlich which adopts twice the full width of the ρ meson, Γ_ρ , as a universal energy. In a series of detailed papers¹¹⁻¹³ Ehrlich has discussed empirical evidence for the existence of an elementary length of magnitude

$$\lambda_0 = \hbar c / 2\Gamma_\rho = 0.62 \pm 0.03 \text{ fm} \quad (2)$$

and the associated elementary proper time,

$$\tau_0 = \lambda_0 / c = (2.2 \pm 0.1) \times 10^{-24} \text{ sec.} \quad (3)$$

Both λ_0 and τ_0 are presumably to be regarded as universal elementary constants associated primarily with the strong interaction and Ehrlich apparently formulates his variant of the space-time quantization hypothesis mainly in keeping with the standard approach, while drawing conceptual support from both the radical and positivist points of view.

Clearly, there can be nothing latent in the properties of the ρ meson itself that warrants its isolation and special significance in the scheme of things. In view of the close numerical agreement between the quark Compton wavelength and Eq. (2), it is perhaps more useful to treat λ_0 as a basic parameter of the quark model.

One intriguing consequence of Ehrlich's hypothesis is the prediction that the hadronic resonance widths Γ_n are related to the natural numbers in a remarkably simple manner:

$$\begin{aligned} \Gamma_n &= 2\Gamma_\rho/n, \quad n = 1, 2, 3, \dots \\ &= 310.6, 155.3, 103.5, 77.6 \text{ MeV}, \dots \end{aligned} \quad (4)$$

Ehrlich presented "possible evidence" for this behavior by comparing the predictions of Eq. (4) with the resonance widths reported by the Particle Data Group.¹⁴ It is also natural to compare the

known particle masses to a similar prediction, $m_n c^2 = 2\Gamma_\rho/n$, resulting from the requirement that $\lambda = n\lambda_0$ if the smallest step in three-space is λ_0 . However, no hadron known to exist satisfies such a relation.

In the case of Eq. (4), the connection between the mean life τ of an unstable system and the width of the state, $\Gamma = \hbar/\tau$, on which Eq. (4) is based is open to question. Even though there is abundant experimental evidence that long-lived decaying systems obey an exponential decay law of the form $\exp(-t/\tau)$, a critical analysis of the actual experimental situation for short-lived systems shows that the lifetime of an unstable state may vary according to the experimental apparatus used, and as a result the relation between Γ and τ may be more complicated.¹⁵

In any case, a much more serious doubt occurs. Though Ehrlich addressed the difficulties associated with the validity of his conjecture arising from the model dependence of the resonance widths, he failed to stress the basic distinction that should be made between resonance parameters and resonance poles. Typically, the resonance mass m_R and width Γ_R are defined in terms of the behavior of the resonant phase shift δ_R and its energy derivative in the neighborhood of the peak in the formation cross section (as abstracted from the Breit-Wigner formula):

$$\begin{aligned} \delta_R(m_R) &= \pi/2, \\ \Gamma_R/2 &= [d\delta_R(W)/dW]_{W=m_R}^{-1}. \end{aligned} \quad (5)$$

For a resonance created in a production experiment some other criterion may be implemented—the peak energy in the distribution of the invariant mass of the decay particles, for example, or the maximum speed point on the Argand circle. It has been shown, with particular clarity for the ρ and Δ resonances,^{16,17} that Eqs. (5) are very sensitive to the mathematical forms used to parametrize δ_R . This model dependence of the resonance parameters may be further exaggerated by the presence of a strong background phase, leading to a further shift in the values of m_R and Γ_R .

From the standpoint of general principles,¹⁰ however, only the position of the second-sheet T -matrix pole,

$$E = mc^2 - i(\Gamma/2), \quad (6)$$

is of primary significance, not the energy at which δ_R assumes some particular value. Indeed, different resonance parametrizations analytically continued into the second sheet yield poles at virtually identical positions.^{16,17} This accounts for the major part of the discrepancy between mass and width values for the same particle re-

ported by different experimental groups, for example. Thus, to possess a fundamental model-independent value, Γ_p and Γ_n in Eq. (4) should actually be reinterpreted to denote twice the imaginary parts of the corresponding second-sheet T -matrix poles.¹⁶

With this change, Eq. (4) is susceptible to testing, using the results of a recent energy-dependent pion-nucleon partial-wave analysis¹⁹ over the region of the first Δ resonance. The Δ width, calculated from Eq. (4), using $\Gamma_p = 155.3 \pm 0.5$ MeV from the best determination of the ρ pole position made to date¹⁶ [and on which the values in Eqs. (2) and (3) are based], is $\Gamma_\Delta = 103.5 \pm 0.4$ MeV.

Electromagnetic processes lead to a mass splitting in the Δ decuplet and thus to results which differ from this prediction. To first order in α it may be shown²⁰ that the expected charge splitting between Δ^{++} and Δ^0 is

$$\Gamma(\Delta^0) - \Gamma(\Delta^{++}) \approx \frac{4}{3} \frac{\eta}{\rho} \Gamma_\Delta = 1.44 \pm 0.01 \text{ MeV}. \quad (7)$$

This may be compared to the results from the πN analysis,²⁰

$$\Gamma(\Delta^0) - \Gamma(\Delta^{++}) = 8.79 \pm 0.57 \text{ MeV}, \quad (8)$$

which are seen to differ by several standard deviations. Admittedly, Eq. (7) is a model-dependent result, but not to an extent that eliminates the serious disagreement between Eqs. (7) and (8). Electromagnetic splitting thus adds another uncertainty which limits the predictability of Eq. (4).

Recently it has been possible to determine the pole positions of some of the higher resonances of the pion-nucleon system. For the $N^*(1470)$, Lee and Shaw²¹ quote $\Gamma(N^*)/2 = 108 \pm 5$ MeV so that $2\Gamma_p/\Gamma(N^*) = 1.44 \pm 0.07$. Though the errors the authors quote for the N^* width are only estimated, the indication again is that Eq. (4) fails to represent accurately the data.²² Thus, until the appropriate unstable particle poles are established with sufficient accuracy to test Eq. (4) it is not possible to support Ehrlich's fundamental-time hypothesis by an analysis of the widths of the resonant states.

In another paper¹³ Ehrlich has presented auxiliary evidence for his space-time-quantization hypothesis citing, for example, the observed or calculated electromagnetic radii of the pion, kaon, and proton as evidence for the validity of Eq. (2). These radii are defined by the usual prescription

$$r^2 = -6\hbar^2 [dG(q^2)/dq^2]_{q^2=0}, \quad (9)$$

where $G(q^2)$ is the appropriately normalized electric or magnetic form factor written as a function of q^2 , the invariant four-momentum transfer squared. It is important to point out that the con-

cept of a spatially extended electromagnetic structure is not rigorously applicable to relativistic hadrons, however. The link between relativistic form factors and the "static" charge (or current) densities on which Eq. (9) is based can only be forged in the Breit frame, i.e., the Lorentz system in which the virtual photon transfers no energy. Only in this frame is it possible to define the charge (or current) density of a hadron as a three-dimensional Fourier transform of the electric (or magnetic) form factor. However, as is well known, such a description is unsatisfactory because in the Breit frame the nucleon itself is moving, thus obscuring the concept of a static density.²³ Furthermore, there is no guarantee that the radius defined by Eq. (9) is positive definite, nor is there any fundamental rationale for the assumption of spherical symmetry on which it is based.

Even if these problems could in some fashion be overcome,²⁴ there is still a philosophical difficulty undermining present approaches to particle substructure. The "cloud" of virtual particles surrounding a hadron does not impart a real structure since virtual particles cannot be identified with real entities but appear only as mathematical symbols in the theory.²⁵ Briefly then, the concept of a hadronic electromagnetic radius, motivated primarily by analogy with the classical electrodynamics of rigid bodies, is purely symbolic, and should not be treated as indicative of a real spatial extension. In this context, the consideration of the electromagnetic radii can offer nothing in support of Eq. (2).

It may be objected that even though electromagnetic radii do not possess a literal significance they nonetheless are observables and Eq. (2) is only intended as a restriction on the magnitude of observable lengths. Yet even if this argument is accepted it is not correct to conclude, as Ehrlich does in Ref. 13, that present measurements of the electromagnetic radii of the pseudoscalar mesons offer support for Eq. (2). First of all, there is as yet no uncontested value for the pion radius. The only two direct measurements, based on low-energy electron-pion scattering, give conflicting results^{26, 27}: $r_\pi = 0.78 \pm 0.1$ and 0.56 ± 0.04 fm, respectively. Conventional theoretical treatments tend to favor the latter value whereas a careful analysis²⁸ based on the use of unstable particle propagators supports the former value.²⁹ Data on the radius of the charged kaon have been taken at Fermilab from an electron-kaon experiment (Fermilab Experiment No. E456) but results are not yet available. The value $r_{K^+} = 0.51 \pm 0.07$ fm was reported by Syganov³⁰ from the results of a similar experiment and is not consis-

tent with Eq. (2).

The radii of the neutral hadrons evidently pose a significant problem for the space-time-quantization hypothesis. Ehrlich was unable to reproduce the neutron charge radius using a string-lattice model and the usual pnn quark structure supplemented by the restriction of Eq. (2). Fishbane *et al.*³¹ have pointed out a similar difficulty in meshing the experimental value of r_n with standard models of quark/parton constituency without upsetting the observed spectrum of excited states. A neutral kaon regeneration experiment was performed recently³² that provided a value for the charge radius, $r_{K^0} = -0.054 \pm 0.026$ fm², a result again at variance with Eq. (2).

Since form factors result from the mixing of strong and electromagnetic interactions by virtue of the photon-vector meson coupling, it would be a curious feature of nature if the form factors and the parameters depending on them (e.g., the radii) were in fact characterized purely by λ_0 rather than by some electromagnetic fundamental length (see Table I). To really explore the validity of Eq. (2), then, it is necessary to probe the density of hadronic matter in strongly interacting particles.

The average intrinsic "size" of a particle induced purely by its virtual hadronic interactions has been defined and calculated for the pion and photon by Griffith.³³ He specifies only an upper bound for the pion radius of 0.82 fm and an effective radius of the photon (induced by the ρ meson) of 0.26 fm, too small to agree with Eq. (2), but with unknown errors.

It is theoretically conceivable that the charge and hadronic matter densities are intimately related, a suggestion first put in testable form by Chou and Yang.³⁴ This would immediately permit a determination of the particle radius from purely hadronic scattering processes. On the basis of such a consideration Ehrlich was led to conclude¹³ that the calculated radii of the charged pion and kaon, $r_\pi = r_K = 0.62$ fm, are consistent with Eq. (2). However, a detailed analysis³⁵ of the Chou-Yang model indicates that the proton opacity is very sensitive to the precise parametrization of the form factor, and this opacity enters into the determination of the pion and kaon radii from πp and πK scattering processes. A recent study³⁶ offers further support for this conclusion. Furthermore, the Chou-Yang results quoted by Ehrlich, supplemented by the usual assumption of the universality of the ρ coupling, suggests the neutral kaon radius must vanish within errors, a result, as we have seen, in disagreement with experiment.

In the limit of asymptotic freedom quantum

chromodynamics suggests that the four fundamental interactions merge into a single force at some critical distance b , signifying the breakdown of perturbation theory. From an examination of the transverse momenta in the outgoing state of inclusive processes, an impact-parameter argument gives $b \approx 10^{-3}$ fm. This tends to place a theoretical upper limit to λ_0 roughly in agreement with tests of quantum electrodynamics.

Our final remarks concern some relatively incidental aspects of the space-time-quantization hypothesis. It is worth pointing out that the "remarkable relation" between the proton mass and τ_0 ,

$$m_p c^2 = h/\tau_0, \quad (10)$$

first reported by Schwarz and Volk,³⁷ though numerically viable, is of no greater significance than any of the other numerical coincidences which occur throughout particle physics ($m_e = \alpha m_\tau$, e.g.). Furthermore, the derivation of Eq. (10) is erroneous for two reasons. It relies on the approximate form $\Delta E \Delta t \approx h$ for the energy-time uncertainty principle rather than the precise form Eq. (1), and makes the all-too-common error of equating the mean standard deviation of a measurement with possible values of the measurement, in this case $\Delta E = m_p c^2$ and $\Delta t = \tau_0$.³⁸ As such, Eq. (10) has no bearing on the existence of a quantized time.

A similar misapplication of the energy-time uncertainty principle is evident in the work of Messen,³⁹ who relates λ_0 to the total energy content of the universe, E_U , according to the formula $\lambda_0 = hc/2E_U$. In his work it is stated that the smallest wavelength λ_0 is reciprocally related to the largest energy E_U . This alleged relation of reciprocity, though widely used, is incompatible with the requirement that ΔE and Δt refer to the same, not different, quantum-mechanical states.⁴⁰

Finally, a disclaimer. The comments in this paper are intended only as criticisms of Eqs. (2)-(4) as specific implementations of the quantized-space-time concept and are not addressed to the more general use of the concept itself. There is little doubt that it may be necessary to revise our concepts of space and time at the level of microphysical structures. As Whitehead has succinctly stated: "The continuity of space apparently rests upon sheer assumption unsupported by any *a priori* or experimental grounds."⁴¹ The utility of the continuum apparently arises from its value in implementing the principles of determinism, causality, contiguity, and locality. Thus the continuum may be useful not because of its physical reality, but by virtue of its mathematical utility.

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