Phase of ϵ_K' and the sign of sinδ

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New Kobayashi-Maskawa angular regions characterized by $\sin\delta < 0$ are explored. Such regions are found to satisfy all existing phenomenological constraints, and bear interesting implications for future phenomenology. Prior theoretical lower bounds on the magnitude of ϵ_K' are respected by the new angular regions, but a change in sign provides the opportunity to confirm or eliminate the new regions in the forthcoming $|\eta_{00}/\eta_{+-}|$ experiments.

I. INTRODUCTION

In the standard Kobayashi-Maskawa (KM) parametrization[1] of the six-quark model, the quark mixing matrix contains three real, Cabibbo-type mixing angles θ_i and one complex, *CP*-violating phase $e^{i\delta}$. θ_1, θ_2, and θ_3 may be conventionally chosen to lie in the first quadrant $(0<\theta_i<\pi/2)$, in which case the *CP*-violating phase δ must *a priori* be allowed to assume all values $(0<\delta<2\pi)$. It is often held that phenomenological constraints from *CP* violation in K^0 decays restrict sinδ to positive values, constraining the physical region to the first two quadrants $(0<\delta<\pi)$. This incomplete viewpoint leads, in turn, to specific constraints on the other unknown angles (θ_2, θ_3), upon which many phenomenological predictions have been based. It is, however, equally likely that sinδ is negative, a possibility which leads to very different and interesting phenomenological predictions.

II. K^0-$\bar{K}^0$ MIXING AMPLITUDE

CP violation in K^0 decay stems primarily from the kaon mass matrix.[2] The *CP*-impurity parameter ϵ_K arises from complex couplings associated with the exchange of heavy quarks in the $2W^{\pm}$ exchange processes (Fig. 1), causing a *CP*-violating phase in the effective $|\Delta S|=2$ interaction responsible for kaon mixing.[3] The evaluation of Fig. 1 has been simplified in part by ignoring momentum dependence in the *W* propagators. Although rapid convergence provided by Glashow-Iliopoulos-Maiani cancellation makes such an approximation possible, the present experimental situation regarding the *t*-quark mass raises doubts concerning this "lowest-order-in-m_t^2/M_W^2" approach, *especially* in the present context: the proposed new KM angular regions emerge as m_t increases. We instead compute Fig. 1 in a general R_ξ gauge,[4] with full attention to the *W* propagators, and obtain the following gauge-invariant result[5]:

$$
\begin{aligned}
A=\frac{-G^2}{8\pi^2}\{\bar{s}\gamma^\mu(1-\gamma_5)d\}^2\sum_{i,j=u}^{c,t}\xi_i\xi_j\Bigg\{&\frac{m_i^2(M^2+m_j^2/4)}{M^2-m_j^2}\left[\frac{-M^4}{(M^2-m_i^2)^2}\ln\frac{M^2}{m_i^2}+\frac{m_j^4}{(m_i^2-m_j^2)^2}\ln\frac{m_j^2}{m_i^2}+\frac{M^2}{M^2-m_i^2}+\frac{m_j^2}{m_i^2-m_j^2}\right]\\
&+\frac{m_j^2(M^2+m_i^2/4)}{M^2-m_i^2}\left[\frac{-M^4}{(M^2-m_j^2)^2}\ln\frac{M^2}{m_j^2}+\frac{m_i^4}{(m_j^2-m_i^2)^2}\ln\frac{m_i^2}{m_j^2}+\frac{M^2}{M^2-m_j^2}+\frac{m_i^2}{m_j^2-m_i^2}\right]\\
&+\frac{m_i^2m_j^2}{2}\left[\frac{m_i^2}{(M^2-m_i^2)(m_i^2-m_j^2)}\ln\frac{M^2}{m_i^2}+\frac{m_j^2}{(M^2-m_j^2)(m_j^2-m_i^2)}\ln\frac{M^2}{m_j^2}\right]\\
&+2m_i^2m_j^2M^2\left[\frac{m_i^2}{(M^2-m_i^2)^2(m_i^2-m_j^2)}\ln\frac{M^2}{m_i^2}+\frac{m_j^2}{(M^2-m_j^2)^2(m_j^2-m_i^2)}\ln\frac{M^2}{m_j^2}\right.\\
&\left.-\frac{1}{(M^2-m_i^2)(M^2-m_j^2)}\right]\Bigg\}
\end{aligned}
\tag{1a}
$$

$$
=\frac{-G^2}{8\pi^2}\{\bar{s}\gamma^\mu(1-\gamma_5)d\}^2\sum_{i,j=u}^{c,t}\xi_i\xi_j\left[\frac{m_i^2m_j^2}{m_i^2-m_j^2}\ln\frac{m_i^2}{m_j^2}+\frac{1}{M^2}\left(-\tfrac{3}{4}m_i^2m_j^2\right)+\frac{1}{M^4}\cdots\right],
\tag{1b}
$$

$$\xi_u=-c_1s_1c_3,\quad \xi_c=s_1c_2(c_1c_2c_3-s_2s_3e^{i\delta}),\quad \xi_t=s_1s_2(c_1s_2c_3+c_2s_3e^{i\delta}).$$

Expanding A in powers of $1/M_W^2$ recovers, in lowest order, the results of prior analyses [Eq. (1b)],[6] and substantiates the results of earlier work pertaining to $\sin\delta>0$ provided m_t is sufficiently small.

Other uncertainties in Eq. (1a) for the mixing amplitude are apparent. The absorptive contribution is altogether absent. In fact, we know there is an absorptive contribution from intermediate

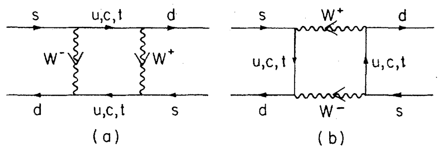

FIG. 1. The $2W^{\pm}$ exchange diagrams responsible for $K^0-\bar{K}^0$ mixing.

pion states. Furthermore, it is possible that additional dispersive contributions may occur in the vicinity of these absorptive resonances.[7] We also mention that the effects of asymptotic freedom upon A are non-negligible even for high-loop momenta. These radiative effects may be summed by means of the renormalization group,[8] and one can show that radiative corrections so obtained do not alter any of the results which follow.[9] We shall consider Eq. (1a) for A an adequate representation of the *dispersive* mixing amplitude.

III. CONSTRAINTS

The phase of A is constrained by experiment—A is proportional to M_{12}^K, for which experimental data on ΔM_K, $\Delta\Gamma_K$, and $\text{Re}\epsilon_K$ fix[10]

$$\frac{\text{Im}M_{12}^K}{\text{Re}M_{12}^K}\left(=\frac{\text{Im}A}{\text{Re}A}\right)=6.6\times10^{-3}\,(\text{experiment})\,. \qquad (2)$$

This imposes a constraint on the KM angles, which in addition to known solutions for $\sin\delta>0$ possesses solutions when $\sin\delta$ and $\cos\delta<0$. Equation (2) is solved implicitly for s_2 and plotted against $\sin\delta$ for quadrant III in Fig. 2. We observe the emergence of the new regions when s_3 exceeds 0.28 or 0.19 for $m_t=20$ and 40 GeV, respectively.[11] The new regions expand as s_3 is increased to its upper limit imposed by Cabibbo universality, $s_3\simeq0.5$.[12]

Yet there are additional constraints which the regions must satisfy. The K_L-K_S mass difference (ΔM_K) must agree with experiment, and the decay rate for $K_L\to\mu\bar{\mu}$ cannot be made too large. ΔM_K is related to the real part of A:

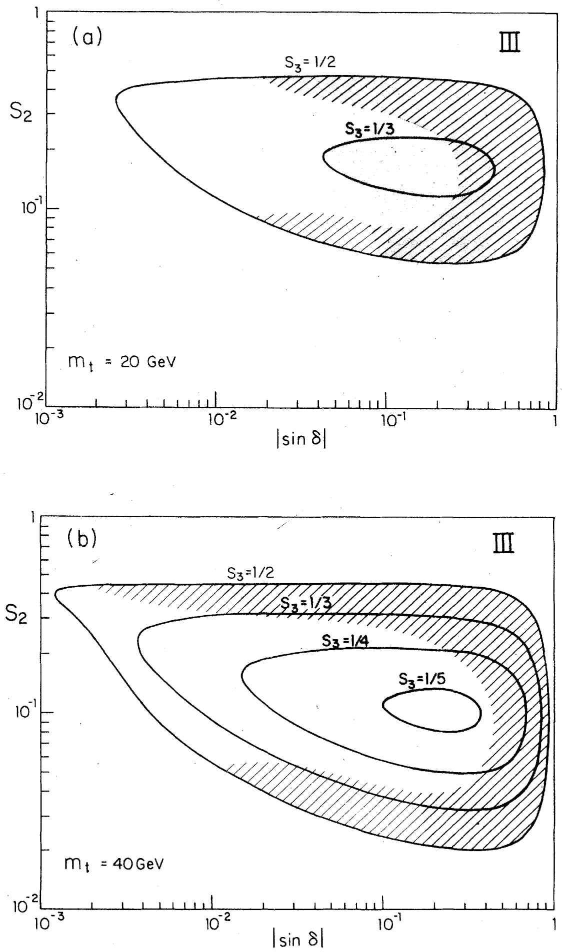

FIG. 2. Kobayashi-Maskawa angular regions for $\sin\delta<0$. Solid lines are solutions to the *CP* condition [Eq. (2)], which emerge as s_3 exceeds 0.28 and 0.19 for $m_t=20$ and 40 GeV, respectively. Shaded regions are eliminated by the experimental upper bound on $|\epsilon'/\epsilon|$. Slashed regions are ruled out by the K_L-K_S mass difference. The regions which remain, encircled by Cabbibo universality ($s_3\lesssim0.5$), satisfy all phenomenological constraints.

$$\Delta M_K=\frac{G^2}{16\pi^2 m_K}\langle\bar{K}|[\bar{\Psi}_s\gamma^\mu(1-\gamma_5)\Psi_d]^2|K\rangle\sum_{i,j=u}^{c,t}\text{Re}\xi_i\xi_j\left\{\frac{m_i^2(M^2+m_j^2/4)}{M^2-m_j^2}\left[\frac{-M^4}{(M^2-m_i^2)^2}\ln\frac{M^2}{m_i^2}\cdots\right]+\cdots\right\}, \qquad (3)$$

imposing a constraint which depends critically upon the evaluation of an unknown matrix element. We choose the method of evaluation based on the intermediate vacuum state ($\frac{8}{3}f_K^2m_K^2$) as an *upper bound* for the matrix element, in keeping with the bag analysis of Shrock *et al.*,[13] who assign a value half as large with a factor-of-2 uncertainty. Regions of Fig. 2 which are unable to account for ΔM_K on the basis of Eq. (3) must be eliminated (slashes), although the precise boundary must be viewed cautiously. The constraint from $K_L\to\mu\bar{\mu}$, as discussed by Shrock *et al.*[14] for the case of $\sin\delta>0$, is not found to be restrictive when applied to the quadrant III regions.

Finally, the experimental upper bound on $|\epsilon_K'|$ constrains the proposed new regions. Theoretical analyses of ϵ_K' in the six-quark model[15,16] place $|\epsilon_K'|$ proportional to $c_2s_2s_3\sin\delta$. Unlike quadrants I and II where this angular factor itself is tightly constrained by the *CP* condition [Eq. (2)], $c_2s_2s_3$

× sinδ varies dramatically over quadrant III. One must eliminate large regions (shaded) corresponding to values of $|\epsilon'_K|$ in excess of the experimental upper limit $|\epsilon'_K/\epsilon_K| \lesssim 0.017$.[10] (We remark that the nonvanishing of ϵ'_K would alter the *CP* condition [Eq. (2)], and effectively shift the KM curves. Such an effect is bound by the magnitude of ϵ'_K, and is justifiably ignored in the present context.[17]) The regions which remain satisfy all the phenomenological constraints, and exist on an equal footing with quadrants I and II. The final determination of the correct angular quadrant is purely a matter of experiment.

IV. CONCLUSIONS

Having established the viability of sinδ < 0 in light of existing constraints, we look for means to distinguish these regions as new data become available.

The most salient feature of quadrant III is that ϵ'_K/ϵ_K *is negative*.[18] If ϵ'_K/ϵ_K is observed in the forthcoming $|\eta_{00}/\eta_{+-}|$ experiments, so will its sign be observed. Theoretical considerations equate the sign of ϵ'_K/ϵ_K with that of sinδ,[16] making $\epsilon'_K/\epsilon_K < 0$ a unique characteristic of the third-quadrant regions.

As in quadrants I and II, the magnitude of $|\epsilon'_K/\epsilon_K|$ is bound from below near 2.3×10^{-3} and 1.2×10^{-3} for $m_t = 20$ and 40 GeV, respectively.[16,17]

Although a complete phenomenological profile would be inappropriate, we would like to add a few remarks in brief.

(1) The new regions include (but are not restricted to) those where the $b\to c$ quark vertex is suppressed relative to $b\to u$.[19]

(2) In these same regions, the B_d-$\bar{B}_d$ would mix completely and should produce same-sign dilepton events copiously in e^+e^- at the Υ'''.[20]

(3) *CP* violation in B^0-$\bar{B}^0$ decay, which appears as a charge asymmetry in these same-sign dilepton events, necessarily changes sign (assumes the direction of positive charge), although total charge asymmetries remain at a level where detection appears unlikely.[21]

(4) If $b\to c$ were in fact suppressed, "rare" decay modes would become important, opening to observation numerous tests of *CP* violation, appearing as asymmetries in the partial decay widths of charge-conjugate b hadrons.[22]

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[2] This and other statements we shall make depend upon the absence of other *CP*-nonconserving interactions, which could arise, for example, if the Higgs sector were sufficiently complicated. See S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).

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[5] Terms in Eq. (1b) which would vanish upon summation over quark flavors have been dropped.

[6] R. E. Shrock, S. B. Treiman, and L. L. Wang, Phys. Rev. Lett. 42, 1589 (1979); V. Barger, W. F. Long, and S. Pakvasa, *ibid.* 42, 1585 (1979); J. S. Hagelin, Phys. Rev. D 20, 2893 (1979).

[7] L. Wolfenstein, Nucl. Phys. B160, 501 (1979).

[8] Strictly speaking, this treatment suffices only for the leading contribution to the mixing amplitude—terms proportional to $1/M_W^4$ (or G_F^2) in Eq. (1). Such an analysis has been carried out by F. Gilman and M. Wise, Phys. Lett. 93B, 129 (1980).

[9] After the submission of this article, this analysis was performed by B. Gaiser *et al.* which substantiates our finding. See B. Gaiser, T. Tsao, and M. Wise, SLAC Report No. SLAC-PUB-2523, 1980 (unpublished).

[10] K. Kleinknecht, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-23.

[11] We remark, at this point, that the effects of asymptotic freedom are to *hasten* and not retard the onset of the new regions—they exist for even smaller values of m_t and s_3.

[12] R. E. Shrock and L. L. Wang, Phys. Rev. Lett. 41, 1692 (1978).

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[17] For a discussion of KM angular constraints when $\epsilon' \neq 0$, see J. S. Hagelin, Harvard University Report No. HUTP-79/A081, 1979 (unpublished).

[18] Experimental data fix ϵ_K and ϵ'_K parallel (or antiparallel) in the complex plane.

[19] This same possibility may also exist for quadrant II. See J. Hagelin, Phys. Rev. D 20, 2893 (1979); V. Bar-

ger, W. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979).

[20]J. Hagelin, Phys. Rev. D 20, 2893 (1979); V. Barger, W. Long, and S. Pakvasa, *ibid.* 21, 174 (1980).

[21]See Ref. 20. In fact, charge asymmetries remain small even if nonspectator-type diagrams are incorporated into the analysis of B decay. See J. S. Hagelin and M. B. Wise, Harvard University Report No. HUTP-80/A070 (unpublished).

[22]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979).