Weak and electromagnetic form factors of baryons at large momentum transfer

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Perturbative quantum-chromodynamic predictions are given for the weak and electromagnetic elastic and transition form factors of baryons at large momentum transfer Q. The leading (helicity-conserving) octet and decuplet form factors can all be expressed as linear combinations of the proton and neutron magnetic form factors. The predictions for the spin structure and relative normalization of the baryon form factors reflect the assumed $SU(2)_L \times U(1)$ structure of the electromagnetic and weak currents, quark and gluon hard-scattering dynamics, and the helicity-flavor symmetry of baryon wave functions at short distances. The results hold to all orders in $\alpha_s(Q^2)$ and to leading order in m/Q. We also discuss the special features of the contribution of the end-point $x \sim 1$ region to baryon form factors.

I. INTRODUCTION

In this paper we show how the electroweak elastic and inelastic form factors of baryons at large momentum transfer can be used to systematically test the dynamics and symmetries of the quark currents and hadronic wave functions at short distances.

The predictions for the spin structure and relative magnitudes of the baryon form factors pertaining to the electromagnetic, neutral, and charged currents depend upon the assumed $SU(2)_L \times U(1)$ structure[1] of these currents. They also depend in detail upon the dynamics of hard-scattering processes involving quarks and gluons and the basic helicity-flavor symmetry of baryon wave functions at short distances.

As has been shown in Refs. 2–4, hadronic form factors and other exclusive processes in quantum chromodynamics are controlled at large momentum transfer $Q = (|q^2|)^{1/2}$ by two basic elements—the hard-scattering amplitude $T_H(x_i, Q)$ for the scattering of the valence quarks from the initial to final direction, and the hadronic distribution amplitudes $\phi(x_i, Q)$, the probability amplitudes for finding the valence quarks with longitudinal momentum fractions x_i at small transverse distance $\sim 1/Q$ in each hadron. The forms of T_H and ϕ reflect the dynamical and symmetry properties of hadrons at the quark level. Detailed perturbative quantum-chromodynamic (QCD) predictions for the power-law and anomalous logarithmic behavior of meson and baryon form factors to leading order in $\alpha_s(Q^2)$ and m/Q are given in Refs. 3 and 4.

The predictions for the electroweak baryon form factors $G(Q^2)_{AX^* \to B}$ which we discuss in this paper are actually general consequences of QCD helicity rules[2,4] and spin-flavor symmetry. In particular, form factors for any process in which the baryon helicity is changed $(h_A \neq h_B)$ or in which the initial or final baryon has nonminimal helicity $(|h_A| > \frac{1}{2})$ are suppressed by factors of m/Q. The results are all independent of the detailed form of the hard-scattering amplitude $T_H(x_i, Q)$ and thus hold to *all orders* in $\alpha_s(Q^2)$. Correction terms of order m/Q will not be considered. We discuss the special features of the contribution of the end-point $x \sim 1$ region to the baryon factors in the Appendix. The results apply for spacelike or timelike values of Q^2 sufficiently large such that the predicted leading power-law behavior $Q^4 G(Q^2) \sim$ const (modulo logarithms) is observed.[5]

An important feature of the perturbative QCD predictions—again true to all orders in $\alpha_s(Q^2)$— is that all of the helicity-conserving electroweak form factors involving only nucleons can be expressed as linear combinations of just two basic form factors—$G_{\parallel}(Q^2)$ and $G_{\overline{\parallel}}(Q^2)$—corresponding to amplitudes in which the current interacts with a valence quark with helicity parallel or antiparallel to the helicity of the nucleon, respectively. The coefficients are determined by the corresponding $SU(2)_L \times U(1)$ quark charges.[6] Thus the nucleon magnetic form factors $G_M^n(Q^2)$ and $G_M^p(Q^2)$ are sufficient to predict the weak nucleon form factors. The assumption of the standard helicity-flavor symmetry for the baryon wave functions at short distances then leads to the specification of all the leading electroweak octet

and decuplet form factors. The spatial wave functions are assumed to be symmetrical with respect to the quarks having the same helicity, a feature which is preserved under perturbative QCD evolution. As $Q^2 \to \infty$, the spatial wave function becomes totally symmetric,[4] $\phi_B(x_i, Q)$ $\sim x_1 x_2 x_3 (\ln Q^2/\Lambda^2)^{-\gamma_B}$, and thus the helicity-flavor structure of the baryon states satisfies exact SU(6) symmetry. The detailed results are given in the next sections.

II. GENERAL RESULTS IN QCD

In perturbative QCD, the dominant contribution to any weak or electromagnetic elastic or transition baryon form factor at large momentum transfer has the general structure[2-4]

$$G(Q^2)_{AX^* \to B} = \int_0^1 [dx] \int_0^1 [dy] \phi_B^*(y_i, \tilde{Q}_y) T_H(x_i, y_i, Q)$$

$$\times \phi_A(x_i, \tilde{Q}_x) \left[1 + O\left(\frac{m}{Q}\right) \right] \quad (1)$$

as $Q^2 = -q^2 \to \infty$, where $\tilde{Q}_x = \min(x_i Q)$, $[dx]$ $= \Pi_{i=1}^3 dx_i \, \delta(1 - \sum_j x_j)$ and similarly for $\tilde{Q}_y$ and $[dy]$. [The contribution to $G(Q^2)$ from the endpoint integration region where the struck quark has light-cone momentum fraction $x \sim 1$ is suppressed at large Q^2 due to the QCD Sudakov form factor.[7,8] This contribution is analyzed separately in the Appendix.] The quark distribution amplitude $\phi_A(x_i, Q)$ in Eq. (1) is the probability amplitude for finding three valence quarks in baryon A with fractions x_i of the baryon's longitudinal momentum, and collinear up to scale Q (i.e., $k_{\perp i} \lesssim Q$):

$$\phi_A(x_i, Q) \equiv d_F(Q)^{-3/2} \int^Q \prod_i \frac{d^2 k_{\perp i}}{16\pi^3} 16\pi^3 \delta^2\left(\sum_i k_{\perp i}\right)$$

$$\times \psi_A(x_i, k_{\perp i}). \quad (2)$$

[The factor $d_F^{-3/2}(Q)$ is due to wave-function renormalization of the quarks.] The hard-scattering amplitude $T_H(x_i, y_i, Q)$ is the amplitude for the collinear quarks to scatter from the initial to the final direction. This amplitude is defined to be collinear irreducible in that collinear mass singularities are removed by explicit subtractions. By definition, the collinear singularities are all absorbed into the distribution amplitudes ϕ. Consequently, all loop momenta are of order $k_\perp \sim Q$, and the quark and baryon masses are negligible (giving corrections suppressed by m/Q). This leads to three important consequences.[4]

(1) The hard-scattering amplitude T_H falls as $1/Q^4$ for Q large, up to logarithmic corrections due to the ultraviolet structure of QCD. This fol-

lows simply from dimensional arguments[9] since Q, and not the quark or baryon masses, must determine the scale of T_H. Since $\phi(x_i, Q)$ varies only logarithmically with Q, all the leading baryon form factors fall essentially as $1/Q^4$.

(2) Quark helicity is conserved along each quark line in T_H since the quark-gluon vertices and the electroweak vertices are all either vector or axial-vector couplings and such couplings conserve quark helicity for massless quarks. Only the components of the wave function having zero-orbital angular momentum along the direction of motion (i.e., $\vec{L} \cdot \vec{P} = 0$) contribute to (2) because of the angular integrations. Thus the baryon helicity equals the sum of quark helicities in T_H, and hadronic helicity is conserved in all leading form factors $G(Q^2)_{AX^* \to B}$ at large Q^2; i.e., $h_A = h_B$. In particular, form factors which change the hadronic helicity, such as the Pauli form factor $F_2(Q^2)$, are suppressed by factors of m/Q.

(3) Initial and final baryon helicities in $G(Q^2)_{AX^* \to B}$ must be minimal (i.e., $|h_A| = |h_B|$ $= \frac{1}{2}$) since the photon and weak bosons are vector particles. This is obvious in the Breit frame $(\vec{p}_A = -\vec{p}_B)$ where the change in hadronic angular momentum along the direction of motion (i.e., $\Delta \vec{J} \cdot \vec{P}_A = h_A + h_B = 2h_A$) must equal zero or one.

Nontrivial relations can be derived among the various form factors from the helicity-flavor structure of the wave functions and of the electroweak currents, without reference to the explicit form of T_H. The general distribution amplitude for $h = \frac{1}{2}$ baryons having isospin (I, I_3) has the structure

$$\sqrt{3}\, \phi_\uparrow(x_i, Q) = |\uparrow\downarrow\uparrow\rangle \left[|II_3\rangle_S \phi^S(x_i, Q) + |II_3\rangle_A \phi^A(x_i, Q) \right]$$

$$+ (1 \longleftrightarrow 2) + (3 \longleftrightarrow 2), \quad (3)$$

where the superscript S (A) implies symmetry (antisymmetry) under the exchange of particles 1 and 3. There is no antisymmetric state $|II_3\rangle_A$ for $I = \frac{3}{2}$ baryons. Such a state can exist for $I = \frac{1}{2}$ baryons. However, since we expect little asymmetry in ground-state wave functions (without heavy quarks) we shall ignore ϕ^A relative to ϕ^S for the $I = \frac{1}{2}$ baryons. (Note that the standard quark-model assumption that these baryon wave functions are S-wave states also implies ϕ^A $= 0$.) This approximation becomes exact in the limit $Q^2 \to \infty$ since[4] $(\beta = 11 - \frac{2}{3} n_f)$

$$\frac{\phi^A(x_i, Q)}{\phi^S(x_i, Q)} \to (x_1 - x_3) K \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-20/9\beta}$$

in QCD. Thus we neglect ϕ^A for all Q^2 in both $I = \frac{1}{2}$ and $I = \frac{3}{2}$ distribution amplitudes. The isospin wave functions $|II_3\rangle_S$ for the baryons of interest

here are given in Table I.

Each of the permutations in (3) contributes incoherently and equally since helicity is conserved along each quark line in T_H. Furthermore, it is important to realize that there are then only two dynamical amplitudes which determine each of the leading form factors—corresponding to whether the struck quark has the same helicity as the baryon or opposite helicity. Consequently, a form factor for $AX^* \rightarrow B$ with $h_A = \pm\frac{1}{2}$ $(=h_B)$ and with $X = \gamma$, W, or Z has the form

$$G^{(\pm)}(Q^2)_{AX^* \rightarrow B} = e_\parallel^{(\pm)}(AX^* \rightarrow B)G_\parallel^{AB}(Q^2)$$
$$+ e_\pi^{(\pm)}(AX^* \rightarrow B)G_\pi^{AB}(Q^2). \quad (4)$$

Here the constants $e_\parallel$ and e_π are the sums of the electroweak charges[6] carried by valence quarks in the baryon with helicities parallel and antiparallel, respectively, to the baryon's helicity. They are determined solely by the flavor wave functions $|II_3\rangle_S$ of the baryons (A and B), as given in Table I, and by the flavor-spin structure of the electroweak currents:

$$e_\parallel = {}_S\langle B(II_3)|Q(1) + Q(3)|A(II_3)\rangle_S ,$$
$$e_\pi = {}_S\langle B(II_3)|Q(2)|A(II_3)\rangle_S , \quad (5)$$

where $Q(1)$ is the electroweak charge operator for quark 1, etc. The QCD dynamics is contained in the form factors $G_\parallel^{AB}$ and G_π^{AB} where, for example,

$$G_\parallel^{AB}(Q^2) = \left(\frac{16\pi}{3}\frac{\alpha_s(Q^2)}{Q^2}\right)^2 \int_0^1 [dx][dy]\phi_B^*(y_i, Q)$$
$$\times T_\parallel(x_i, y_i, \alpha_s(Q^2))$$
$$\times \phi_A(x_i, Q) \quad (6)$$

is independent of the current (i.e., of X^*).

Note that Eqs. (4) and (5) are valid only because helicity is conserved along each quark line. This is a necessary consequence of a vector-gluon theory. By way of contrast, consider a scalar-gluon theory. In leading order, the helicity-conserving form factors have the general form

$$G^{(\pm)}(Q^2) = \tilde{e}_\parallel^{(\pm)}(AX^* \rightarrow B)G^{AB}(Q^2) \text{ (scalar gluon)},$$

where now the struck quark can only have parallel helicity. Since helicity is exchanged between the spectator quarks (in this order), $\tilde{e}_\parallel$ is not the same as $e_\parallel$ in Eqs. (4) and (5); additional permutation operators appear in the definition of $\tilde{e}_\parallel$ analogous to Eq. (5). Furthermore, there are helicity-nonconserving form factors, even in leading order in $1/Q$, for such a theory [e.g., $\gamma^* p(\frac{1}{2}) \rightarrow \Delta(-\frac{3}{2})$], again in marked contrast with QCD. Such differences reflect the gluon's spin. They allow us to experimentally distinguish a vector theory, which conserves quark helicity,

TABLE I. (1 $\longleftrightarrow$ 3)-symmetric flavor wave functions multiplying spin state $|\uparrow\downarrow\uparrow\rangle$ in Eq. (3).

$$|p\rangle = \frac{d(1)u(3) + u(1)d(3)}{\sqrt{6}}u(2) - (\tfrac{2}{3})^{1/2}u(1)d(2)u(3)$$

$$|n\rangle = (-|p\rangle \text{ with } u \leftrightarrow d)$$

$$|\Sigma^0\rangle = -\frac{u(1)d(3) + u(3)d(1)}{\sqrt{3}}s(2) + \frac{u(2)d(3) + u(3)d(2)}{2\sqrt{3}}s(1)$$
$$+ \frac{u(1)d(2) + u(2)d(1)}{2\sqrt{3}}s(3)$$

$$|\Sigma^-\rangle = (-|n\rangle \text{ with } u \leftrightarrow s)$$

$$|\Lambda^0\rangle = \frac{u(2)d(3) - u(3)d(2)}{2}s(1) + \frac{d(1)u(2) - d(2)u(1)}{2}s(3)$$

$$|\Delta^{++}\rangle = u(1)u(2)u(3)$$

$$|\Delta^+\rangle = \frac{1}{\sqrt{3}}[u(1)u(2)d(3) + \text{all permutations}]$$

$$|\Delta^0\rangle = (|\Delta^+\rangle \text{ with } u \leftrightarrow d)$$

$$|\Delta^-\rangle = (|\Delta^{++}\rangle \text{ with } u \rightarrow d)$$

$$|Y^0\rangle = \frac{1}{\sqrt{6}}[u(1)s(2)d(3) + \text{all permutations}]$$

$$|Y^-\rangle = \frac{1}{\sqrt{3}}[d(1)s(2)d(3) + \text{all permutations}]$$

from scalar- or tensor-gluon theories, which flip quark helicities.

III. SPECIFIC PREDICTIONS

The distribution amplitudes ϕ_S for protons and neutrons are essentially identical, by isospin symmetry. Consequently, all electromagnetic-, charged-, and neutral-current form factors involving nucleons alone are uniquely determined for large Q^2 given only the two functions $G_\parallel(Q^2)$ and $G_\pi(Q^2)$. The constants $e_\parallel$, e_π determining these various nucleonic form factors are given in Table II.

TABLE II. Algebraic coefficients determining the electroweak form factors of nucleons.

$AX^* \rightarrow B$	$e_\parallel^+$	e_π^+	$e_\parallel^-$	e_π^-
$p\gamma \rightarrow p$	1	0	1	0
$n\gamma \rightarrow n$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$
$pW^- \rightarrow n$ $nW^+ \rightarrow p$	0	$-\frac{c}{3}$	$\frac{4}{3}c$	0
$pZ^0 \rightarrow p$	$-w$	$-\frac{1}{6}$	$\frac{2}{3} - w$	0
$nZ^0 \rightarrow n$	$\frac{w}{3}$	$\frac{1}{6} - \frac{w}{3}$	$\frac{w}{3} - \frac{2}{3}$	$-\frac{w}{3}$

$$G^{(\pm)}(Q^2)_{AX^* \rightarrow B} = e_\parallel^{(\pm)}G_\parallel(Q^2) + e_\pi^{(\pm)}G_\pi(Q^2)$$

$$|c| = |\cos\theta_C| = 0.974 \pm 0.003$$

$$w = \sin^2\theta_W = 0.22 \pm 0.01$$

The form factors $G^{(\pm)}$ [Eq. (4)] are defined such that the expectation value of the electroweak current between nucleon states is

$$\langle p' | J_\mu | p \rangle = \bar{u}(p') \left[\gamma_\mu \frac{1+\gamma_5}{2} G^{(+)}(Q^2) \right.$$

$$\left. + \gamma_\mu \frac{1-\gamma_5}{2} G^{(-)}(Q^2) \right] u(p).$$

In general these form factors dominate as $Q^2 = -(p'-p)^2 \to \infty$ since only these currents conserve hadronic helicity in that limit. All other form factors are suppressed by powers of m/Q. Thus for electromagnetic interactions, $G^{(+)} = G^{(-)}$ is the usual magnetic form factor G_M, and from Table II we have

$$G_M^p = G_\shortparallel, \quad G_M^n = -\frac{G_\shortparallel}{3} + \frac{G_\bar\shortparallel}{3}.$$

The data[10] for magnetic form factors ($Q^2 \lesssim 1$ GeV^2) can be roughly parametrized as follows:

$$G_M^p(Q^2) \simeq -1.46 \, G_M^n(Q^2) \simeq \frac{2.79}{(1+Q^2/0.71 \text{ GeV}^2)^2}$$

(7)

and therefore

$$G_\shortparallel(Q^2) \simeq G_M^p(Q^2), \quad G_\bar\shortparallel(Q^2) \simeq -1.05 \, G_M^p(Q^2). \quad (8)$$

These functions together with the algebraic constants in Table II determine all the asymptotic nucleon form factors. For example, the isovector axial-vector charge form factor

$$F_A^{I=1}(Q^2) = \tfrac{1}{2} [G^+(Q^2) - G^-(Q^2)]^{I=1}$$

$$\xrightarrow[Q^2 \text{ large}]{} \tfrac{1}{2} \cos\theta_C \left(-\tfrac{4}{3} G_\shortparallel - \tfrac{1}{3} G_\bar\shortparallel\right)$$

is predicted to be $\simeq -0.48 \, G_M^p(Q^2)$ if we use Eqs. (7) and (8). The measured value is $\simeq -0.44 G_M^p(Q^2)$, which agrees surprisingly well with theory given the low Q^2 involved.

To the extent that flavor SU(3) is a good symmetry, the same functions $G_\shortparallel$ and $G_\bar\shortparallel$ [Eq. (8)] determine the strangeness-changing transition form factors from nucleons to other members of the nucleon octet. The algebraic constants $(e_\shortparallel, e_\bar\shortparallel)$ determining these form factors follow from the wave functions given in Table I. The constants are given in Table III.

Finally, the transition to particles in the $J = \tfrac{3}{2}$ decuplet ($\Delta, \ldots$) can also be analyzed. Since only helicity $\pm\tfrac{1}{2}$ baryons interact at large Q^2, the definition (6) of the form factor $G^{(\pm)}$ can be retained for these transitions even though the decuplet particles have spin $\tfrac{3}{2}$. Again, if SU(3) is a good symmetry, all transition form factors are specified by only two Q^2-dependent functions.

TABLE III. Algebraic coefficients determining the strangeness-changing weak form factors of nucleons.

$AX^* \to B$	$e_\shortparallel^+$	$e_\bar\shortparallel^+$	$e_\shortparallel^-$	$e_\bar\shortparallel^-$
$pW^- \to \Sigma^0$	0	$-\frac{\sqrt{2}}{3}s$	$-\frac{1}{3\sqrt{2}}s$	0
$pW^- \to \Lambda^0$	0	0	$(\tfrac{3}{2})^{1/2}s$	0
$nW^- \to \Sigma^-$	0	$-\frac{2}{3}s$	$-\frac{s}{3}$	0

$$|s| = |\sin\theta_C| = 0.220 \pm 0.003$$

Flavor-SU(6) symmetry implies that the distribution amplitudes for the decuplet are the same as for the octet. Consequently, the same functions $G_\shortparallel$ and $G_\bar\shortparallel$ from Eq. (8) will also approximately determine the octet-decuplet form factors. The algebraic constants relevant to these transitions are given in Table IV.

IV. CONCLUSIONS

It is increasingly apparent that higher-order corrections are very important in most QCD processes. Thus it is imperative that we examine those features of QCD which are valid to all orders in α_s. In this paper, we have shown that the large-Q^2 behavior of the electroweak form factors of baryons provides a variety of just such "all-orders" tests of QCD. The $1/Q^4$ falloff, helicity conservation, and the minimal-helicity selection rule for form factors are all valid in every order of QCD perturbation theory.

In Sec. III we presented a large number of non-

TABLE IV. Algebraic coefficients determining the electroweak transition form factors of nucleons.

$AX^* \to B$	$e_\shortparallel^+$	$e_\bar\shortparallel^+$	$e_\shortparallel^-$	$e_\bar\shortparallel^-$
$p\gamma \to \Delta^+$ $n\gamma \to \Delta^0$	$-\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$
$pW^+ \to \Delta^{++}$ $nW^- \to \Delta^{-a}$	0	$-(\tfrac{2}{3})^{1/2}c$	$(\tfrac{2}{3})^{1/2}c$	0
$pW^- \to \Delta^0$ $nW^+ \to \Delta^{+a}$	0	$\frac{\sqrt{2}}{3}c$	$-\frac{\sqrt{2}}{3}c$	0
$pZ^0 \to \Delta^+$ $nZ^0 \to \Delta^0$	$\frac{\sqrt{2}}{3}w$	$\frac{\sqrt{2}}{3}(1-w)$	$\frac{\sqrt{2}}{3}(w-1)$	$-w\frac{\sqrt{2}}{3}$
$pW^- \to Y^0$	0	$\frac{s}{3}$	$-\frac{s}{3}$	0
$nW^- \to Y^-$	0	$\frac{\sqrt{2}}{3}s$	$-\frac{\sqrt{2}}{3}s$	0

[a] The coefficients should be multiplied by (-1) for these processes.

trivial relations among the different electroweak form factors. Indeed all electroweak form factors involving baryons in the lowest mass octet and decuplet are essentially determined by only two independent form factors [Eq. (8)], using the purely algebraic constants tabulated in Tables II–IV. These relations are again valid to all orders in α_s, and critically test the spin of the gluon.

Finally, the properties described here not only test QCD, but provide a potentially useful tool for measuring the parameters determining the weak interactions of quarks and leptons.

APPENDIX

One can distinguish two distinct contributions to baryon form factors at large momentum transfer. The "hard-scattering" contribution is the dominant perturbative QCD contribution which arises from scattering of the three valence quarks quarks—each carrying a non-negligible fraction x_i of the baryon's momentum. This is the contribution analyzed in Secs. II and III of this paper. The other contribution arises from the end-point $x \sim 1$ integration region corresponding to the kinematic situation where the spectator quarks are essentially stopped and only the struck quark is forced to change direction.[7,11] Contributions from this region are related, by the Drell-Yan-West[7] connection, to the $x \sim 1$ behavior of inelastic structure functions. However, because the struck quark is close to the mass shell for $x \sim 1$, this end-point contribution to the baryon form factor is suppressed by the usual Sudakov quark form factor—corresponding to the probability ampli-

tude for a quark to scatter without gluon emission. (A detailed discussion is given in Refs. 4 and 12.) If one analyzes the QCD Sudakov form factor in the leading-logarithm approximation, the end-point contribution to hadron form factors is suppressed asymptotically by a power of m/Q relative to the hard-scattering contribution.[8]

It is conceivable that the end-point $x \sim 1$ contribution could play an important phenomenological role at moderate Q^2 in the baryon form factor. There are certain features of this contribution which distinguish it from the asymptotically dominant hard-scattering contributions. We first note that perturbative QCD predicts that the struck quark with $x \sim 1$ has the same helicity as the baryon.[13] If we assume that the baryon wave functions which describe structure functions for $x \sim 1$ are the SU(6) valence wave functions given in Table I (with no asymmetric ψ_A component[14] analogous to Eq. (3)], then perturbative QCD predicts[13]

$$G_{d/p}(x)/G_{u/p}(x) \xrightarrow[x \to 1]{} \tfrac{1}{5} . \tag{A1}$$

This prediction is in fact supported by recent deep-inelastic lepton-scattering data at large x.[15]

Applying this helicity rule to the $x \sim 1$ contribution to the baryon form factors leads to the prediction

$$G^{(\pm)}(Q^2)_{AX* \to B} \to e_\|^\pm (AX* \to B) \tilde{G}_\|^{AB}(Q^2) \quad \text{(end point)}$$
$$\tag{A2}$$

in contrast to Eq. (4). In particular, if Eq. (A2) is applicable, the proton to neutron ratio G_M^n/G_M^p is predicted to be $-\tfrac{1}{3}$ in contrast to the measured ratio (at low Q^2) which is closer to $-\tfrac{2}{3}$.

If both end-point and hard-scattering contributions are phenomenologically important, then the form of Eq. (4) holds with an extra contribution to $G^{AB}(Q^2)$ due to the end-point contribution. However, at large Q^2, only the hard-scattering contributions are predicted to survive.

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$$J_\mu^{\text{em}} = \tfrac{2}{3}\bar{u}\,\gamma_\mu\,u - \tfrac{1}{3}\bar{d}\gamma_\mu\,d, \quad J_\mu^{\text{CC}} = \bar{u}\gamma_\mu\,d_L\cos\theta_C + \bar{u}\gamma_\mu\,s_L\sin\theta_C,$$

$$J_\mu^{\text{NC}} = \tfrac{1}{2}\bar{u}\gamma_\mu\,u_L - \tfrac{1}{2}\bar{d}\gamma_\mu\,d_L - w J_\mu^{\text{em}}, \quad u_L = \frac{1-\gamma_5}{2}u \text{ and } w$$

$$= \sin^2\theta_W.$$

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[11]End-point contributions come from the region $1 - x \lesssim m/Q$, while "hard-scattering" contributions come only from $m/Q \ll 1 - x \lesssim 1$. In Ref. 4, it is shown that the intermediate region $m/Q \ll 1 - x \ll 1$ makes no contribution because of QCD evolution. Consequently, end-point and hard-scattering regions are readily distinguished in practical calculations.

[12]S. J. Brodsky and G. P. Lepage, in Quantum Chromodynamics, proceedings of the Summer Institute on Particle Physics, SLAC, 1979, edited by Anne Mosher (SLAC, Stanford, 1980).

[13]This was first shown in the tree-graph approximation by G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975); see also A. I. Vainshtein and V. I. Zakharov, Phys. Lett. 72B, 368 (1978). The analysis in perturbation QCD is given in Refs. 4 and 12.

[14]The absence of the asymmetric component ψ_A in the $x \sim 1$ wave function requires that spin-flip interactions between spectator quarks are negligible even in the region where the struck quark has all the momentum. This is not a priori obvious if quark-mass effects are important, but seems to be borne out by the structure-function data (see Ref. 15).

[15]BEBC collaboration, reported to the XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980 (unpublished).