

Pseudoscalar-meson decay constants and weak form factors in a relativistic quark model

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Pseudoscalar-meson decay constants and weak form factors are studied on the basis of a relativistic quark model, where the quark-antiquark system with unequal masses is treated in the framework of ladder approximation. $f_K/f_\pi = 1.25$ and $f_K/f_\pi f_+^{K\pi}(0) = 1.27$ are obtained. Decay constants and form factors for charmed mesons are also predicted.

I. BASIC ASSUMPTIONS

Interest in the estimating¹ of the decay constants of pseudoscalar mesons and the weak form factors has been revived recently after the discovery of charmed mesons. In the present paper we study the decay constants and the weak form factors $f_\pm(q^2)$ on the basis of a relativistic quark model, but without using an explicit solution for the radial part of the Bethe-Salpeter (BS) amplitudes, since we confine our estimation to the ratios of the decay constants to f_π and the deviations of $f_\pm(0)$ from unity. A study of meson decays based on such a prescription was made in 1968 by Kitazoe and Teshima,² where the $q_a\bar{q}_b$ system was treated as one with equal masses $M_a = M_b$. The essential idea in our model lies in a new treatment of the $q_a\bar{q}_b$ system with $M_a \neq M_b$ in the framework of the ladder approximation and lowest-order calculation.

The wave function of the pseudoscalar meson P_a^b with momentum \vec{q} is given by²

$$(2\pi)^{3/2}(2E_q)^{1/2} \langle 0 | T(\psi_{ai\alpha}(x_1)\bar{\psi}_{bj\beta}(x_2)) | P_a^b(\vec{q}) \rangle = e^{-i q \cdot x} \frac{\delta_{ij}}{\sqrt{3}} P_a^b[\Phi_P(q)]_{\alpha\beta} \frac{1}{(2\pi)^4} \times \int d^4k \phi_P(k) e^{-i k \cdot (x_1 - x_2)}, \quad (1)$$

where

$$\Phi_P(q) = i(1 + \not{q}/\chi_P)\gamma_5, \quad (2)$$

$a, i,$ and α are flavor, color, and Lorentz indices, respectively (hereafter we omit the color and Lorentz indices), P_a^b denotes the unitary spin state, and P denotes $P = \pi, K, D,$ and so on. The four-dimensional center-of-mass coordinate X_μ is defined by

$$X_\mu = \alpha x_{1\mu} + \beta x_{2\mu}, \quad \alpha + \beta = 1, \quad (3)$$

where the coefficients α and β may deviate from $\frac{1}{2}$ in the case of $M_a \neq M_b$. The BS equation for $\Phi(q)\phi(k)$ is written by

$$(\not{k} + \alpha\not{q} - M_a)\Phi(q)(\not{k} - \beta\not{q} - M_b)\phi(k) = i g^2 \gamma_\mu \Phi(q) \gamma_\nu \frac{1}{(2\pi)^4} \int d^4k' -\frac{1}{2} D_F^{\mu\nu}(k - k') \phi(k') \quad (4)$$

in ladder approximation. (Although we assume a vector potential as the potential between q and \bar{q} , this assumption is not essential in the following discussions.)

Our basic assumption is as follows: We calculate only the lowest-order diagrams in the framework of ladder approximation. Then, in the normalization of the electromagnetic form factors $f_+^{(a)}(q^2)$ and $f_+^{(b)}(q^2)$ which are given by Figs. 1(a) and 1(b), respectively, disagreement which results in $M_a \neq M_b$ appears if $\alpha = \beta = \frac{1}{2}$. We assume that this disagreement can be compensated by shifting the coefficients α and β from $\frac{1}{2}$. We assume that the contributions from higher-order diagrams can be absorbed in this prescription for the normalization.

Under this assumption, for example, the matrix element of $\bar{K}^0 \rightarrow \pi^+$ current (Fig. 2) is given by

$$\mathfrak{M}_\mu^{K\pi} \equiv f_+^{K\pi}(q^2)(p+k)_\mu + f_-^{K\pi}(q^2)(p-k)_\mu = \frac{-2}{(2\pi)^4} \int d^4l \phi_K(l + \beta_K p) \phi_\pi(l + \beta_\pi k) \times \text{Tr}[\bar{\Phi}_\pi(k)\gamma_\mu \Phi_K(p)(\not{l} - M_q)], \quad (5)$$

where

$$\bar{\Phi}_P(q) \equiv -\gamma_5 C \Phi_P^T(q) C^{-1} \gamma_5 = -i\gamma_5(1 + \not{q}/\chi_P). \quad (6)$$

(See Appendix.) Generally the \not{l} term in Eq. (5) is

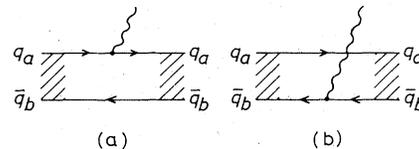


FIG. 1. Two diagrams for meson-photon interaction. We write the form factors which correspond to (a) and (b) as $f_+^{(a)}(q^2)$ and $f_+^{(b)}(q^2)$.

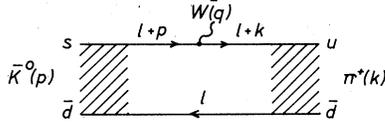


FIG. 2. Diagram for $\bar{K}^0(p) \rightarrow \pi^+(k)$ current. The diagram stands for the sum of the diagrams (a) and (b) in Fig. 4.

represented only in terms of \not{p} and \not{k} after the integration over l . Therefore, if we regard the mass parameter χ_P in Eq. (2) as the physical mass m_P of the pseudoscalar meson P_a^b , we obtain

$$\mathfrak{M}_\mu^{K\pi} = C \text{Tr}[\bar{\Phi}_\pi(k)\gamma_\mu\Phi_K(p)(A\not{p} + B\not{k} - M_d)] \\ = -4C(Am_K + Bm_\pi + M_d)(p_\mu/m_K + k_\mu/m_\pi),$$

$$\mathfrak{M}_\mu(q_a - q_a + \gamma) \equiv f_+^{(a)}(q^2)(p+k)_\mu.$$

$$= \frac{-2}{(2\pi)^4} \int d^4l \phi_P(l + \frac{1}{2}\beta_P q) \phi_P(l - \frac{1}{2}\beta_P q) \text{Tr}\{\bar{\Phi}_P(k)\gamma_\mu\Phi_P(p)[\not{l} - \frac{1}{2}\beta_P(\not{p} + \not{k}) - M_b]\}. \quad (10)$$

Since the function $\phi(l + \frac{1}{2}\beta_P q)\phi(l - \frac{1}{2}\beta_P q)$ is an even function of l_μ , we can drop the \not{l} term in the l -space integral (10) and we get

$$f_+^{(a)}(q^2) = \frac{8}{(2\pi)^4} \int d^4l \phi_P(l + \frac{1}{2}\beta_P q) \phi_P(l - \frac{1}{2}\beta_P q) \left[\frac{M_b}{\chi_P} - \frac{1}{2}\beta_P \left(1 + \frac{m_P^2}{\chi_P^2} \right) \right]. \quad (11)$$

In a similar way, we get the form factor $f_+^{(b)}(q^2)$ for $\bar{q}_b - \bar{q}_b + \gamma$ [Fig. 1(b)],

$$f_+^{(b)}(q^2) = \frac{8}{(2\pi)^4} \int d^4l \phi_P(l + \frac{1}{2}\alpha_P q) \phi_P(l - \frac{1}{2}\alpha_P q) \left[\frac{M_a}{\chi_P} - \frac{1}{2}\alpha_P \left(1 + \frac{m_P^2}{\chi_P^2} \right) \right]. \quad (12)$$

From the condition $f_+^{(a)}(0) = f_+^{(b)}(0) = 1$, we obtain

$$\frac{M_a}{\chi_P} - \frac{1}{2}\alpha_P \left(1 + \frac{m_P^2}{\chi_P^2} \right) = \frac{M_b}{\chi_P} - \frac{1}{2}\beta_P \left(1 + \frac{m_P^2}{\chi_P^2} \right) \\ = \frac{M_a + M_b}{2\chi_P} - \frac{1}{4} \left(1 + \frac{m_P^2}{\chi_P^2} \right) \\ \equiv \frac{B_P}{2\chi_P} \quad (13)$$

or

$$\alpha_P = \frac{1}{2} + \frac{M_a - M_b}{\chi_P(1 + m_P^2/\chi_P^2)} = \frac{M_a - \frac{1}{2}B_P}{M_a + M_b - B_P}. \quad (14)$$

II. PSEUDOSCALAR-MESON DECAY CONSTANTS

In order to estimate the decay constants of the pseudoscalar mesons, we put further assumption on the radial wave function $\phi_P(l)$: We assume that the symmetry-breaking effect on $\phi_P(l)$ appears on-

where A , B , and C are functions of p^2 , k^2 , and q^2 , so that we are uniquely led to the ratio

$$\xi^{K\pi}(0) \equiv \frac{f_-^{K\pi}(0)}{f_+^{K\pi}(0)} = -\frac{m_K - m_\pi}{m_K + m_\pi} = -0.56. \quad (8)$$

The value (8) is in very poor agreement with the experimental value³

$$\xi^{K\pi}(0)_{\text{exp}} = -(\lambda_* - \lambda_0)(m_K^2 - m_\pi^2)/m_\pi^2 \\ = -(0.1-0.3), \quad (9)$$

where λ_* and λ_0 are the slope parameters of the vector and scalar form factors of K_{13} decays, respectively. Therefore, in this paper, we do not identify χ_P as the physical mass m_P .

Now, let us return to the problem of the normalization of $f_+^{(a)}(0)$ and $f_+^{(b)}(0)$. The matrix element of the current where a photon interacts on the quark q_a [Fig. 1(a)] is given by

ly through the form

$$\phi_P(l) = \rho_P \tilde{\phi}(\lambda_P l^2), \quad (15)$$

where $\tilde{\phi}(\xi^2)$ is a dimensionless function of ξ^2 , and ρ_P and λ_P are symmetry-breaking parameters which have dimension (mass)⁻². Then the decay constant f_P of the pseudoscalar meson P_a^b is given by

$$f_P = \frac{4\sqrt{3}}{\chi_P} \frac{-1}{(2\pi)^4} \int d^4l \phi_P(l) \\ = -\frac{\rho_P}{\chi_P \lambda_P^2} \frac{4\sqrt{3}}{(2\pi)^4} \int d^4\xi \tilde{\phi}(\xi^2), \quad (16)$$

where $\xi_\mu = (\lambda_P)^{1/2} l_\mu$, so that the relation

$$\frac{f_P}{f_\pi} = \frac{\rho_P \chi_\pi \lambda_\pi^2}{\rho_\pi \chi_P \lambda_P^2} \quad (17)$$

is derived. On the other hand, since Eq. (11) becomes

$$f_+^{(a)}(q^2) = \frac{\rho_P^2}{\lambda_P^2} \frac{B_P}{2\chi_P} F(\frac{1}{4}\lambda_P\beta_P^2q^2), \quad (18)$$

where

$$F(\xi^2) = \frac{8}{(2\pi)^4} \int d^4\xi' \bar{\phi}((\xi' + \xi)^2) \bar{\phi}((\xi' - \xi)^2), \quad (19)$$

we can obtain the relation

$$\frac{\lambda_P^2\chi_P}{\lambda_\pi^2\chi_\pi} = \frac{\rho_P^2B_P}{\rho_\pi^2B_\pi} \quad (20)$$

from the condition $f_+(0) = 1$. Therefore, from Eqs. (17) and (20), we get

$$\frac{f_P}{f_\pi} = \frac{\rho_\pi B_\pi}{\rho_P B_P} = \frac{\lambda_\pi}{\lambda_P} \left(\frac{\chi_\pi B_\pi}{\chi_P B_P} \right)^{1/2} \quad (21)$$

As seen in Eq. (18), in our model, the slope of the form factor $f_+^{(a)}(q^2)$ is given by $\lambda_P\beta_P^2/4$. We accept *a priori* that the electromagnetic form factor $f_+(q^2)$ is successfully described by the simple vector-meson-pole-dominance model: for example, in the K^+ meson, the form factors $f_+^{(u)}(q^2)$ and $f_+^{(s)}(q^2)$ are described by ρ - and ϕ -meson poles, respectively. Therefore, from the relations $\lambda_K\beta_K^2/4 = m_\rho^{-2}$ and $\lambda_K\alpha_K^2/4 = m_\phi^{-2}$, we get the relations

$$\alpha_K = m_\rho/(m_\rho + m_\phi), \quad \beta_K = m_\phi/(m_\rho + m_\phi) \quad (22)$$

and

$$\frac{\lambda_K}{4} = \left(\frac{m_\rho + m_\phi}{m_\rho m_\phi} \right)^2, \quad \frac{\lambda_\pi}{4} = \left(\frac{2}{m_\rho} \right)^2 \quad (23)$$

We also accept that masses of vector mesons can be well described by linear mass formulas, $2m_{K^*} \simeq m_\rho + m_\phi$, $2m_{D^*} \simeq m_\rho + m_\phi$, and so on. Therefore, we assume that the quark mass difference is equal to the mass difference of the vector mesons concerned:

$$M_s - M_u \simeq m_{K^*} - m_\rho \simeq \frac{1}{2}(m_\phi - m_\rho), \quad (24)$$

and so on. Then we obtain

$$\frac{1}{2}\chi_K(1 + m_K^2/\chi_K^2) \simeq \frac{1}{2}(m_\rho + m_\phi) \simeq m_{K^*} \quad (25)$$

from Eqs. (14), (22), and (24). The relation (25) is generalized for any pseudoscalar mesons:

$$\chi_P = m_V + (m_V^2 - m_P^2)^{1/2}, \quad (26)$$

where the vector meson V belongs to the same unitary-spin state as the pseudoscalar meson P . Since, from Eqs. (14) and (26), we get the relations $m_\rho = M_u + M_u - B_\pi$, $m_{K^*} = M_u + M_s - B_K$, and so on, we must take $B_\pi = B_K = \dots \equiv B$. [Note that, in

our model, $(M_q - \frac{1}{2}B)$ corresponds to the so-called "constituent" quark mass m_q .^{4]}

Now we can evaluate the ratio (21). From Eqs. (23) and (26), we obtain

$$f_{K^*}/f_\pi = (\lambda_\pi/\lambda_K)(\chi_\pi/\chi_K)^{1/2} = 1.25, \quad (27)$$

which is in good agreement with experiment. Similarly we get

$$f_D/f_\pi = 1.94, \quad f_F/f_\pi = 2.90, \quad f_B/f_\pi = 1.75, \quad (28)$$

where we use $m_{D^*} = (m_\rho + m_\phi)/2$, $m_{F^*} = (m_\phi + m_\phi)/2$, and $m_{B^*} = (m_\rho + m_{T(9,4)})/2$ in places of the physical masses m_{D^*} , m_{F^*} , and m_{B^*} , respectively, and the relation $m_V^2 - m_P^2 = 0.55 \text{ GeV}^2$ which is extrapolated from the empirical relation $m_{F^*}^2 - m_F^2 \simeq m_{D^*}^2 - m_D^2 \simeq m_{K^*}^2 - m_K^2 \simeq 0.55 \text{ GeV}^2$ (cf. $m_\rho^2 - m_\pi^2 = 0.58 \text{ GeV}^2$). If we use the physical masses for m_{D^*} , m_D , m_{F^*} , and m_F , we get $f_D/f_\pi = 1.91$ and $f_F/f_\pi = 2.93$. Since the linear mass formula for vector mesons has an error of a few percent, the values in (28) also have the error of the same order.

III. WEAK FORM FACTORS

Finally we estimate the deviations of the weak form factors $f_\pm(q^2)$ at $q^2 = 0$ from unity. We assume that the function $\bar{\phi}(\xi_1^2)\bar{\phi}(\xi_2^2)$ can be approximately regarded as a function of $\xi_1^2 + \xi_2^2$, although it is generally a function of $\xi_1^2 + \xi_2^2$ and $|\xi_1^2 - \xi_2^2|$. Then we can evaluate the matrix element (5) by shifting the integration variable from l_μ to

$$l'_\mu = l_\mu + (\lambda_K\beta_K p_\mu + \lambda_\pi\beta_\pi k_\mu)/(\lambda_K + \lambda_\pi),$$

and we get

$$f_\pm^{K\pi}(q^2) = \frac{1}{2} \frac{4\rho_K\rho_\pi}{(\lambda_K + \lambda_\pi)^2} F(\xi_{K\pi}^2) \times \left[\left(\frac{M_d}{\chi_K} - \frac{\lambda_K\beta_K + \lambda_\pi\beta_\pi m_\pi^2/\chi_K\chi_\pi}{\lambda_K + \lambda_\pi} \right) \pm \left(\frac{M_d}{\chi_\pi} - \frac{\lambda_\pi\beta_\pi + \lambda_K\beta_K m_K^2/\chi_K\chi_\pi}{\lambda_K + \lambda_\pi} \right) \right], \quad (29)$$

where

$$\xi_{K\pi}^2 = \frac{\lambda_K\lambda_\pi}{\lambda_K + \lambda_\pi} \frac{1}{2} (\beta_K p - \beta_\pi k)^2, \quad (30)$$

and $F(\xi^2)$ is defined by

$$F(\xi^2) = \frac{8}{(2\pi)^4} \int d^4\xi' \bar{\phi}(\xi_1^2) \bar{\phi}(\xi_2^2) \Big|_{\xi_1^2 + \xi_2^2 = 2(\xi'^2 + \xi^2)} \quad (31)$$

in the place of the definition (19).

By putting $m_{K^*} = m_\rho(1 + \epsilon)$ and $m_\phi = m_\rho(1 + 2\epsilon)$, we can see that the slope of $f_\pm^{K\pi}(q^2)$ is $\simeq m_{K^*}^{-2}$ because of⁵

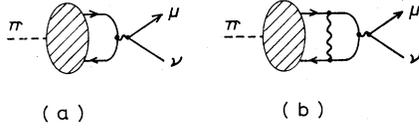


FIG. 3. Irreducible diagram (a) and reducible diagram (b) in $\pi \rightarrow \mu\nu$ decay.

$$\xi_{K\pi}^2 \simeq \frac{\lambda_K \lambda_\pi}{\lambda_K + \lambda_\pi} \frac{1}{2} \beta_K \beta_\pi q^2 \simeq \frac{1}{(1+2\epsilon)m_\rho^2} q^2 \simeq \frac{1}{m_{K^*}^2} q^2. \quad (32)$$

If we suppose that quark masses M_q are sufficiently large as compared to the usual hadron mass scale, we can approximate the factor within square brackets in Eq. (29) as $[B_K/2\chi_K \pm B_\pi/2\chi_\pi]$. Thus we derive the relation

$$f_{\pm}^{K\pi}(0) \simeq \frac{1}{2} \frac{f_K}{f_\pi} \left(\frac{2\lambda_K}{\lambda_K + \lambda_\pi} \right)^2 \left(1 \pm \frac{\chi_K}{\chi_\pi} \right), \quad (33)$$

where we use $F(\xi_{K\pi}^2)|_{q^2=0} \simeq F(0)$. Equation (33) leads to

$$\frac{f_K}{f_\pi} \frac{1}{f_{\pm}^{K\pi}(0)} = \left(\frac{\lambda_K + \lambda_\pi}{2\lambda_K} \right)^2 \frac{2\chi_\pi}{\chi_K + \chi_\pi} = 1.27, \quad (34)$$

which is in excellent agreement with the experimental value. On the other hand, the result

$$\xi^{K\pi}(0) = -(\chi_K - \chi_\pi)/(\chi_K + \chi_\pi) = -0.03 \quad (35)$$

is somewhat in disagreement with the experimental value (9). This means that our approximation is not so well justified in the evaluation of $f_{\pm}(q^2)$ as in that of $f_{\pm}(0)$ because there the minor terms are not negligible.

Our numerical results on $f_{\pm}(0)$ are summarized as follows:

$$\begin{aligned} f_{\pm}^{K\pi}(0) &= 0.984, & \xi^{K\pi}(0) &= -0.03, \\ f_{\pm}^{DK}(0) &= 0.92, & \xi^{DK}(0) &= -0.24, \\ f_{\pm}^{D\pi}(0) &= 0.84, & \xi^{D\pi}(0) &= -0.27, \\ f_{\pm}^{F\eta}(0) &= 0.81, & \xi^{F\eta}(0) &= -0.24, \\ f_{\pm}^{FK}(0) &= 0.77, & \xi^{FK}(0) &= -0.26. \end{aligned} \quad (36)$$

where, in the estimate of $f_{\pm}^{F\eta}(0)$, we use $\lambda_\eta/4 = [2(1/m_\rho + 2/m_\phi)]^2$ and $m_{\eta^*} = (m_\rho + 2m_\phi)/3$.

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APPENDIX: FEYNMAN-GRAPH RULES FOR COMPOSITE MESONS

(i) We evaluate only the matrix elements for the irreducible diagrams, not those for the reducible

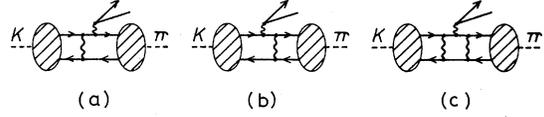


FIG. 4. Irreducible diagrams (a) and (b) and reducible diagram (c) in $K \rightarrow \pi\mu\nu$ decay.

diagrams. For example, Fig. 3(a) is the lowest irreducible diagram for $\pi \rightarrow \mu\nu$ decay, whereas Fig. 3(b) is the reducible diagram. Figures 4(a) and 4(b) are the lowest irreducible diagrams for $K \rightarrow \pi\mu\nu$ decay, whereas Fig. 4(c) is the reducible diagram.

(ii) We use factors

$$\frac{\delta_{ij}}{\sqrt{3}} \Phi_P(p) \phi_P(l) P_a^b \quad (A1a)$$

and

$$- \frac{\delta_{ij}}{\sqrt{3}} \bar{\Phi}_P(p) \phi_P(l) P_b^a \quad (A1b)$$

for initial and final pseudoscalar mesons, which are shown by Figs. 5(a) and 5(b), respectively, where $\Phi_P(p)$ and $\bar{\Phi}_P(p)$ are given by Eqs. (2) and (6), respectively, and $p = p_a - p_b$ and $l = \beta p_a + \alpha p_b$, or $p_a = l + \alpha p$ and $p_b = l - \beta p$. The coefficients α and β are defined by Eqs. (3) and (14).

(iii) We use factors

$$-i \frac{\delta_{ij}}{\sqrt{3}} K_+^{-1}(p_a) \Phi_P(p) K_+^{-1}(p_b) \phi_P(l) P_a^b \quad (A2a)$$

and

$$i \frac{\delta_{ij}}{\sqrt{3}} K_+^{-1}(p_b) \bar{\Phi}_P(p) K_+^{-1}(p_a) \phi_P(l) P_b^a \quad (A2b)$$

for initial and final mesons with one-gluon exchange, which are shown by Figs. 5(c) and 5(d), respectively, where $K_+^{-1}(p)$ is the inverse of quark propagator $K_+(p) = -\frac{1}{2} S_F(p) = i/(\not{p} - M_q + i\epsilon)$:

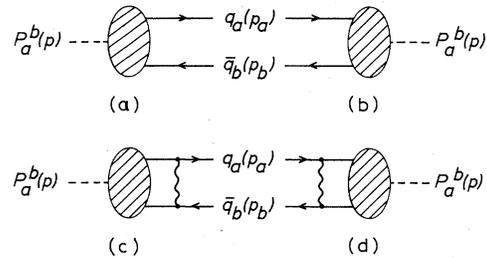


FIG. 5. Rules for Feynman diagrams: The diagrams (a), (b), (c), and (d) show initial meson, final meson, and those with one-gluon exchange, respectively.

$$K_*^{-1}(p) = -i(\not{p} - M_q). \quad (\text{A3})$$

Here we have used the relation (4).

(iv) In a similar way, we obtain the rules for vector meson $V_a^b(p)$ by substituting

$$\Phi_v(p)\phi_v(l)V_a^b = \left[\not{\epsilon}(p) + \frac{\not{p}\not{\epsilon}(p) - \not{\epsilon}(p)\not{p}}{2\chi_v} \right] \phi_v(l)V_a^b \quad (\text{A4})$$

for $\Phi_P(p)\phi_P(l)P_a^b$, where $\epsilon_\mu(p)$ is the polarization factor of the vector meson.

(v) We use a factor $(-1)^N$ where N is the number of closed fermion loops in the graph. Note that, for example, N is one for the diagrams shown in Figs. 3 and 4.

(vi) The other rules are identical with usual Feynman-graph rules for "elementary" particles.

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⁵Note that α_K and β_K for \bar{K}^0 are given by $\alpha_K = m_\phi / (m_\rho + m_\phi)$ and $\beta_K = m_\rho / (m_\rho + m_\phi)$, not by Eq. (22).