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Decays of $L=1$ mesons to $\gamma\pi$, $\gamma\rho$, and $\gamma\gamma$

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The importance is stressed of measuring the rate and angular distributions in $f_0 \to \gamma\rho \to \gamma\pi^+\pi^-$. Predictions are made for this process on the basis of the single-quark-transition hypothesis in terms of the recently measured radiative widths $\Gamma(B\to\gamma\pi)$, $\Gamma(A_{1,2}^{\pm}\to\gamma\pi^{\pm})$. Additional predictions, based on vector dominance and measured rates for pion emission, permit the conclusions that $\Gamma(B\to\gamma\pi)=184\pm30$ keV, $\Gamma(A_1^{\pm}\to\gamma\pi^{\pm})=1$–$1.6$ MeV, $\Gamma(A_2^{\pm}\to\gamma\pi^{\pm})=375\pm50$ keV, and $\Gamma(f_0\to\gamma\rho)=1.35\pm0.2$ MeV. About 10% of this last partial width occurs in $M2$ and 90% in $E1$ radiation. The relative sign of the $M2$ and $E1$ amplitudes is predicted and one way is shown to measure their ratio. Another application of vector dominance leads to the prediction $\Gamma(f_0\to\gamma\gamma)=7.7\pm2$ keV, dominantly in the helicity-2 channel. An intrinsic limitation of vector dominance for heavy-quark systems is noted.

Recent experimental progress in the measurement of radiative decays of light mesons[1] and in the study of decays such as $f_0\to\gamma\gamma$ (Ref. 2) has led us to reexamine and bring up to date some predictions made several years ago[3] for these processes.

In this paper we wish to stress the importance of measurements of the decay $f_0\to\gamma\rho\to\gamma\pi^+\pi^-$. Definite predictions for the rate and angular distributions for this process can be made[4] on the basis of the single-quark-transition hypothesis[5]; additional relations stem from vector-meson dominance.[6]

Once more is known about the process $f_0\to\gamma\rho$ it is a simple matter (modulo kinematic uncertainties) to relate this process with the help of vector-meson dominance and SU(3) to $f_0\to\gamma\gamma$.

An extensive discussion of $f_0\to\gamma\rho$ and $f_0\to\gamma\gamma$ is contained in Ref. 3. We shall quote some results obtained there in the course of the present work. Our new results consist primarily of a dissection of the older predictions into their component hypotheses, in such a way that more recent measurements can be used to advantage. We find the following results.

(1) The process $f_0\to\gamma\rho$ should consist of electric-dipole ($E1$) and magnetic-quadrupole ($M2$) contributions:

$$\Gamma(f_0\to\gamma\rho)\equiv\Gamma_{E1}+\Gamma_{M2}\,. \qquad (1)$$

The electric-dipole amplitude is related to a linear combination of amplitudes for $B\to\gamma\pi$ and $A_1\to\gamma\pi$. The former involves no quark spin flip ($^1P_1\to\gamma+{}^1S_0$). The magnetic-quadrupole amplitude in $f_0\to\gamma\rho$ is uniquely determined by that in $A_2\to\gamma\pi$.

We find

$$\tilde{\Gamma}_{E1}=9\{[\tilde{\Gamma}_\gamma(B)]^{1/2}\pm[2\tilde{\Gamma}_\gamma(A_1)]^{1/2}/6\}^2\,, \qquad (2)$$

$$\tilde{\Gamma}_{M2}=3\tilde{\Gamma}_\gamma(A_2)/2\,, \qquad (3)$$

where $\tilde{\Gamma}_\gamma(X)\equiv\tilde{\Gamma}(X^+\to\gamma\pi^+)$, and $\tilde{\Gamma}$ denotes a partial width with kinematic factors divided out. In the approach of Refs. 3 and 4, $\tilde{\Gamma}\sim\Gamma/p_\gamma^{\,3}$, where p_γ is the photon momentum in the decaying particle's rest frame.

Equations (1)–(3) depend only on the single-quark-transition hypothesis.[5] If vector-meson dominance is assumed, the positive sign in Eq. (2) is favored.

(2) The relative $E1$ and $M2$ contributions to $f_0\to\gamma\rho$ may be isolated by measuring the dipion angular distribution in $f_0\to\gamma\rho\to\gamma\pi^+\pi^-$. (No $E3$ contribution is expected.[4]) In the dipion rest frame, the distribution in the angle θ between π^+ and γ is

$$W(\theta)d(\cos\theta)\sim(1+\beta\cos^2\theta)d(\cos\theta)\,. \qquad (4)$$

We shall express β in terms of $\Gamma_\gamma(B,A_1,A_2)$. Sign ambiguities are removed if vector dominance is assumed. We confirm an earlier result[3]: $\beta\simeq-0.6$, $\Gamma_{M2}/\Gamma_{E1}\simeq0.1$, very far from the pure $E1$ limit $\beta=-\frac{1}{7}$, $\Gamma_{M2}=0$.

(3) Vector dominance implies

$$\tilde{\Gamma}_\gamma(A_2)=54\{[\tilde{\Gamma}_\gamma(B)]^{1/2}\mp[2\tilde{\Gamma}_\gamma(A_1)]^{1/2}/6\}^2/5\,, \qquad (5)$$

where the minus sign in (5) goes with the plus sign in (2). With present experimental values,[1] (5) can be satisfied only with minus, implying plus in (2) as mentioned earlier. The partial widths $\Gamma_\gamma(B,A_1,A_2)$ can also be expressed in terms of corresponding pionic rates; we predict $\Gamma(B\to\gamma\pi)=184\pm30$ keV, $\Gamma(A_1^{\pm}\to\gamma\pi^{\pm})=1$–$1.6$ MeV, $\Gamma(A_2^{\pm}\to\gamma\pi^{\pm})=375\pm50$ keV.

(4) The value of β in (4) is directly related to the fraction of ρ's in $f_0\to\gamma\rho$ with transverse polarization:

$$r=\frac{\Gamma(f_0\to\gamma\rho_\perp)}{\Gamma(f_0\to\gamma\rho)}=\frac{2}{3+\beta}\,. \qquad (6)$$

This fraction is needed to learn about $f_0\to\gamma\gamma$ from $f_0\to\gamma\rho$ via a vector-dominance-model (VDM) expression

$$\Gamma(f_0 \to \gamma\gamma) \simeq (6.8\times 10^{-3})r\Gamma(f_0 \to \gamma\rho)\,. \tag{7}$$

(5) The ratio of helicity-0 and helicity-2 amplitudes in $f_0 \to \gamma\gamma$ may be expressed in terms of B, A_1, and A_2 radiative widths in many ways, one of which is

$$\frac{A_0^{(\gamma\gamma)}}{A_2^{(\gamma\gamma)}} = \frac{1}{\sqrt{6}}\left[\frac{1}{3}\left(\frac{2\tilde{\Gamma}_\gamma(A_1)}{\tilde{\Gamma}_\gamma(B)}\right)^{1/2} - 1\right]. \tag{8}$$

This tends to be a very small number; the value 0.11 was favored in Ref. 3, where many suggestions for measuring $A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)}$ in $e^+e^- \to e^+e^- f_0 \to e^+e^-\pi^+\pi^-$ were given.

(6) Vector dominance and the heavy-quark limit are incompatible with one another. Using the former (and the single-quark-transition hypothesis) we find

$$\tilde{\Gamma}(2^{++} \to \gamma\gamma)/\tilde{\Gamma}(0^{++} \to \gamma\gamma) \geq \tfrac{2}{5}\,, \tag{9}$$

whereas for heavy quarks, as in positronium,[7]

$$\tilde{\Gamma}(2^{++} \to \gamma\gamma)/\tilde{\Gamma}(0^{++} \to \gamma\gamma) = \tfrac{4}{15} < \tfrac{2}{5}\,. \tag{10}$$

This provides an intrinsic limitation to the accuracy with which $f_0 \to \gamma\rho$ can yield a prediction for $f_0 \to \gamma\gamma$.

We now present these results in more detail. Our notation is described in Ref. 3. We find it convenient to express the single-quark-transition hypothesis in terms of multipoles.[4,8] The amplitudes A_λ for decays of the form

$$X(\text{helicity }\lambda) \to \gamma(\text{helicity }+1,\ \text{along }+z\text{ axis}) + Y(\text{helicity }1-\lambda,\ \text{along }-z\text{ axis}) \tag{11}$$

may then be expressed in terms of these multipoles as shown in Table I.[9] The rates for these processes may be expressed in terms of the corresponding helicity amplitudes as[3,4]

$$\Gamma(X \to \gamma Y) = \frac{p_\gamma^3}{8\pi}\frac{2}{2J_X+1}\sum_{\substack{\lambda_\gamma=+1\\ \lambda\geq 0}} |A_\lambda|^2\,. \tag{12}$$

Specifically we find

$$\Gamma(B(1231) \to \gamma\pi) = \frac{p_\gamma^3}{8\pi}\frac{2}{3}|E1'|^2 \quad (p_\gamma = 608\text{ MeV}/c)\,, \tag{13}$$

$$\Gamma(A_1^+(1200) \to \gamma\pi^+) = \frac{p_\gamma^3}{8\pi}\frac{2}{3}|E1|^2 \quad (p_\gamma = 592\text{ MeV}/c)\,, \tag{14}$$

$$\Gamma(A_2^+(1317) \to \gamma\pi^+) = \frac{p_\gamma^3}{8\pi}\frac{2}{5}|M2|^2 \quad (p_\gamma = 651\text{ MeV}/c)\,, \tag{15}$$

and, for $f_0(1273) \to \gamma\rho$,

$$\left.\begin{aligned}\Gamma_{E1} &= \frac{p_\gamma^3}{8\pi}\frac{2}{5}15\left(E1' + \frac{\sqrt{2}}{6}E1\right)^2 \quad &(16\text{a})\\ \Gamma_{M2} &= \frac{p_\gamma^3}{8\pi}\frac{2}{5}\frac{3}{2}|M2|^2 \quad &(16\text{b})\end{aligned}\right\}\ (p_\gamma = 400\text{ MeV}/c)\,.$$

(Masses have been taken from Ref. 10.) By combining Eqs. (13)–(16) and assuming that the reduced matrix elements $E1'$ and $E1$ are relatively real,[11] we arrive at Eqs. (1)–(3). These relations involve only the single-quark-transition hypothesis and thus are quite likely to be correct in view of all the other successes of this hypothesis.[12] If the momenta in Eqs. (13)–(16) are substituted into Eqs. (2) and (3), we find

$$\Gamma_{E1}(f_0 \to \gamma\rho) = 2.57\{[\Gamma_\gamma(B)]^{1/2} \pm 0.25[\Gamma_\gamma(A_1)]^{1/2}\}^2\,, \tag{17}$$

$$\Gamma_{M2}(f_0 \to \gamma\rho) = 0.35\Gamma_\gamma(A_2)\,. \tag{18}$$

The only ambiguity in Eqs. (3) and (18) comes from the assumption that $\Gamma \sim p_\gamma^3\tilde{\Gamma}$, which is appropriate for radiation by single quarks; in the single-quark-transition picture, details of the hadron wave function do not enter the calculation.[4] In the long-wavelength limit of a nonrelativistic model, one would expect 2^j-pole radiation to be associated with a kinematic factor p_γ^{2j+1} (p_γ^5 for $M2$ radiation). One would then expect

$$\Gamma_{M2}(f_0 \to \gamma\rho) = 0.13\Gamma_\gamma(A_2)\,, \tag{18$'$}$$

while Eq. (17) is unchanged. Equations (18) and (18′) give some idea of the range likely in dynamical models involving explicit wave functions. We shall use Eq. (18) in what follows, as it has been found quite satisfactory previously.[3,4,8]

Present preliminary values[1] for $\Gamma_\gamma(B) \approx$ (couple of hundred keV), $\Gamma_\gamma(A_1) \approx$ (several hundred keV), and[13]

$$\Gamma_\gamma(A_2) = 450 \pm 100\text{ keV} \tag{19}$$

imply $\Gamma(f_0 \to \gamma\rho) \sim 1$ MeV if the positive sign is taken in Eq. (17). The $M2$ contribution (18) is then

TABLE I. Multipole expressions for predictions of the single-quark-transition hypothesis for meson radiative decays.

Process	Helicity amplitude A_λ	Coefficients of $E1'$	$E1$	$M2$
$B \to \gamma\pi$	$\lambda=1$	1 (def)	0	0
$A_1^+ \to \gamma\pi^+$	$\lambda=1$	0	1 (def)	0
$A_2^+ \to \gamma\pi^+$	$\lambda=1$	0	0	1 (def)
$f \to \gamma\rho$[a]	$\lambda=0$	$\sqrt{6}/2$	$\sqrt{3}/6$	$-\sqrt{3}/2$
	$\lambda=1$	$3\sqrt{2}/2$	$\frac{1}{2}$	$-\frac{1}{2}$
	$\lambda=2$	3	$\sqrt{2}/2$	$\sqrt{2}/2$
$D \to \gamma\rho$[a]	$\lambda=0$	$3\sqrt{2}/2$	$-\frac{1}{2}$	$-\frac{1}{2}$
	$\lambda=1$	$3\sqrt{2}/2$	$-\frac{1}{2}$	$\frac{1}{2}$
$\epsilon \to \gamma\rho$[a]	$\lambda=0$	$\sqrt{3}$	$-\sqrt{6}/3$	0

[a] f, D, ϵ ideally mixed: $(u\bar{u}+d\bar{d})/\sqrt{2}$; $J^{PC} = 2^{++}, 1^{++}, 0^{++}$.

about 0.1 of the total width: It should be clearly discernible, as we shall show. If the negative sign (disfavored by vector dominance) were taken in Eq. (17), the $M2$ contribution to $f_0 \to \gamma\rho$ should be a much larger fraction of the total width.

To measure the $M2$ intensity in $f_0 \to \gamma\rho \to \gamma\pi^+\pi^-$ one may measure the angular distribution $W(\theta)$ mentioned earlier in Eq. (4). It depends on the helicity amplitudes A_λ in the form[3]

$$W(\theta) \propto \tfrac{1}{2}(|A_0|^2 + |A_2|^2)\sin^2\theta + |A_1|^2\cos^2\theta . \quad (20)$$

Thus

$$\beta = \frac{2|A_1|^2}{|A_0|^2 + |A_2|^2} - 1 . \quad (21)$$

If we now substitute the multipole decompositions in Table I and normalize in such a fashion that

$$\mathcal{E}1 \equiv \left(\frac{p_\gamma^f}{8\pi}\right)^{3/2} \sqrt{15}\left(E1' + \frac{\sqrt{2}}{6}E1\right), \quad (22)$$

$$\mathfrak{M}2 \equiv \left(\frac{p_\gamma^f}{8\pi}\right)^{3/2} \sqrt{6}\,M2/2 , \quad (23)$$

$$p_\gamma^f \equiv (m_f^2 - m_\rho^2)/2m_f = 400\ \text{MeV}/c , \quad (24)$$

$$x \equiv \mathfrak{M}2/\mathcal{E}1 , \quad (25)$$

we find

$$\Gamma_{E1} = |\mathcal{E}1|^2 = (1+x^2)^{-1}\Gamma(f_0 \to \gamma\rho) , \quad (26)$$

$$\Gamma_{M2} = |\mathfrak{M}2|^2 = (1+x^{-2})^{-1}\Gamma(f_0 \to \gamma\rho) , \quad (27)$$

and

$$\beta = \frac{-1 - 6\sqrt{5}\,x - 5x^2}{7 + 2\sqrt{5}\,x + 25x^2/3} . \quad (28)$$

In terms of partial widths, we predict

$$\mathcal{E}1 = 1.60\{[\Gamma_\gamma(B)]^{1/2} \pm 0.25[\Gamma_\gamma(A_1)]^{1/2}\} , \quad (29)$$

$$\mathfrak{M}2 = \pm 0.59[\Gamma_\gamma(A_2)]^{1/2} . \quad (30)$$

Equations (17) and (18) are just the squares of these relations, according to our normalizations. An arbitrary choice of overall sign has been made in (29).

The quantities β and $\Gamma(f_0 \to \gamma\rho)/\Gamma_{M2}$ are plotted in Fig. 1, as functions of x. In Ref. 3 the range $0.18 \le x \le 0.45$ was considered, with $x \simeq 0.3$ favored. This favored value corresponds to $\Gamma/\Gamma_{M2} \simeq 12$ and $\beta \simeq -0.6$. As one can see from Eq. (28), a pure $E1$ decay distribution ($x=0$) would correspond to $\beta = -\frac{1}{7}$.

The range $-0.44 \le \beta \le -0.76$ corresponding to $0.18 \le x \le 0.45$ also could occur in principle for very large $|x|$, and, indeed, $|x| \to \infty$ would correspond to $\beta \to -0.6$. This unlikely possibility is easily checked since the predicted value for $\Gamma(f_0 \to \gamma\rho) = (1 + x^{-2})\Gamma_{M2}$ then would be very close to Γ_{M2} and would not exceed a couple of hundred keV according to Eqs. (18) and (19).

We now specify the vector-dominance constraint. The $\lambda = 0$ amplitude in Table I for $D \to \gamma\rho$ must vanish if the photon is dominated by a ρ. The D then would be decaying to two identical transversely polarized particles, which is impossible for a spin-1 particle.[14] The vanishing of this amplitude also is a consequence of demanding that the single-quark-transition descriptions of $\gamma\pi$ decays be the same whether treated directly or via the description for $\rho\pi$ using vector dominance.[3]

As a result of the above assumption, we have

$$3\sqrt{2}E1' - (E1 + M2) = 0 . \quad (31)$$

This may be expressed in terms of widths (with kinematic factors taken out) in Eq. (5). With p_γ^3 kinematic factors, Eq. (5) reads

$$\Gamma_\gamma(A_2) = 13.3\{[\Gamma_\gamma(B)]^{1/2} \mp 0.25[\Gamma_\gamma(A_1)]^{1/2}\}^2 , \quad (32)$$

where the upper sign goes with those in Eqs. (2), (17), and (29). With the preliminary values quoted in Ref. 1, Eq. (32) can be satisfied only with the minus sign, implying a plus sign in Eqs. (2), (17), and (29).

Equation (32) is a concise and stringent test of (a) the single-quark-transition hypothesis, (b) vector dominance, and (c) the specific kinematic prescription whereby all widths are proportional to p_γ^3. It can be combined with Eqs. (17) and (18) to eliminate the interference term (whatever its sign), yielding the prediction

$$\Gamma(f_0 \to \gamma\rho) = 5.1\Gamma_\gamma(B) + 0.31\Gamma_\gamma(A_1) + 0.15\Gamma_\gamma(A_2) . \quad (33)$$

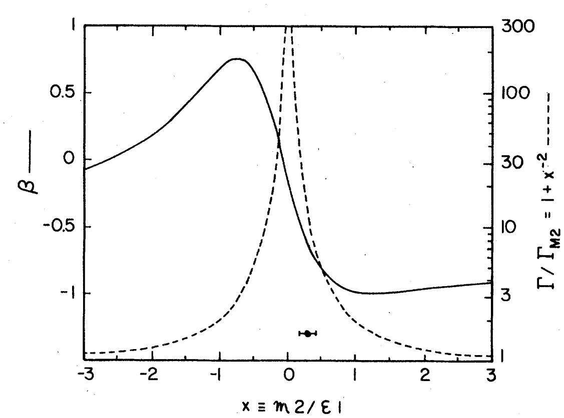

FIG. 1. Angular-distribution coefficient β [Eq. (4)] for $f_0 \to \gamma\rho \to \gamma\pi^+\pi^-$ (left-hand scale, solid line) and ratio $\Gamma(f_0 \to \gamma\rho)/\Gamma_{M2}(f_0 \to \gamma\rho)$ (right-hand scale, dashed line) as functions of $x = \mathfrak{M}2/\mathcal{E}1$ (see text for normalization). The point ⊷ corresponds to the range of x examined in Ref. 3.

Even with the very crudely known values[1] for the B, A_1, and A_2 radiative decays, Eq. (33) predicts values of at least an MeV for $\Gamma(f_0 \to \gamma\rho)$. Clearly this is a decay mode worth looking for.

From (31) and (32) one sees that unless $\Gamma_\gamma(A_1) \gtrsim 16\Gamma_\gamma(B)$, which is unlikely, the amplitudes $M2$ and $E1'$ will have the same sign. Since we have argued that $E1$ and $E1'$ should have the same sign in the VDM framework, this specifies the relative signs of all the amplitudes in $f_0 \to \gamma\rho$. In particular, positive signs are chosen in *both* Eqs. (29) and (30). Then x [related to β of Eq. (4) by Eq. (28)] is now specified in terms of observed partial widths by

$$x = \mathfrak{M}2/\mathcal{E}1$$
$$= 0.37[\Gamma_\gamma(A_2)]^{1/2}/\{[\Gamma_\gamma(B)]^{1/2} + 0.25[\Gamma_\gamma(A_1)]^{1/2}\}. \tag{34}$$

The constraint (32) may be used if desired to eliminate the least well known of the partial widths in (34).

Vector-meson dominance also implies quantitative relations between processes $X \to V_\perp^0 Y$ and $X \to \gamma Y$, where $V_\perp^0$ denotes a transversely polarized vector meson. Specifically, we have

$$\Gamma_\gamma(B) = \frac{1}{9}\left(\frac{p_\gamma}{p_\pi}\right)^3 \frac{\alpha}{(g_\rho^{\,2}/4\pi)}\Gamma(B \to \omega_\perp\pi), \tag{35}$$

$$\Gamma_\gamma(A_{1,2}) = \left(\frac{p_\gamma}{p_\pi}\right)^3 \frac{\alpha}{(g_\rho^{\,2}/4\pi)}\Gamma(A_{1,2}^+ \to \rho_\perp^0\pi^+). \tag{36}$$

The ω couples $\frac{1}{3}$ as strongly to the photon as the ρ, leading to the factor of $\frac{1}{9}$ in the rate (35). The momentum factors are specific to the processes at hand, and are noted in Table II. We may take[3] $g_\rho^2/4\pi = 2.7 \pm 0.3$. It remains to specify the rates $X \to V_\perp^0 Y$.

The helicity amplitudes for $1^+ \to 1^- 0^-$ may be parametrized as

$$A_1^{(\pi)} = S + D/\sqrt{2}, \tag{37}$$

$$A_0^{(\pi)} = S - D\sqrt{2}, \tag{38}$$

so that $\Gamma^{(\pi)} \propto 2|A_1^{(\pi)}|^2 + |A_0^{(\pi)}|^2 \propto |S|^2 + |D|^2$. For the B,[15]

$$D/S \simeq 0.3 \pm 0.1, \tag{39}$$

$\Gamma(B \to \omega_\perp\pi)/\Gamma(B \to \omega\pi) = 0.89 \pm 0.06$. With[10] $\Gamma_{tot}(B) = 129 \pm 10$ MeV $\simeq \Gamma(B \to \omega\pi)$, this implies $\Gamma(B \to \omega_\perp\pi) = 115 \pm 12$ MeV.

For the decay $A_1 \to \rho\pi$, there does not appear to be any appreciable D-wave contribution. A small amount, interfering destructively in $A_1^{(\pi)}$ and constructively in $A_0^{(\pi)}$, is expected.[16] The expected range of $\Gamma(A_1 \to \rho_\perp\pi)/\Gamma(A_1 \to \rho\pi)$ is then from $\frac{2}{3}$ (if $D = 0$ so $A_1^{(\pi)} = A_0^{(\pi)}$) to about $\frac{1}{2}$ (if the scale of D is set from other processes according to the single-quark-transition hypothesis[16]). This last situation would correspond to only a 3% D-wave contribution to the $A_1 \to \rho\pi$ rate, which could easily have escaped detection up to now. With $\Gamma_{tot}(A_1) \cong 300$ MeV and $\Gamma(A_1^+ \to \rho^0\pi^+) = \Gamma(A_1^+ \to \rho^+\pi^0) = \frac{1}{2}\Gamma_{tot}(A_1)$, we find $\Gamma(A_1^+ \to \rho_\perp^0\pi^+) \cong$ 75–100 MeV.

The decay $A_2 \to \rho\pi$ always gives rise to transversely polarized ρ's, so only an isospin calculation is needed for the $A_2^+ \to \rho_\perp^0\pi^+$ rate. The expected $X \to V_\perp^0\pi$ and $X \to \gamma\pi$ rates are summarized in Table II.

Of the three rates in the last column of Table II, the least well known is $\Gamma_\gamma(A_1)$. We can use Eq. (32) to predict its value, obtaining $\Gamma_\gamma(A_1) = 1.13$ MeV from the predicted values of $\Gamma_\gamma(B)$ and $\Gamma_\gamma(A_2)$ in Table II. This is consistent with the range obtained in Table II from $A_1 \to \rho\pi$. If we substitute into expressions for $f_0 \to \gamma\rho$, we expect

$$\Gamma(f_0 \to \gamma\rho) = 1.35 \pm 0.2 \text{ MeV}, \tag{40}$$

$$x \equiv \mathfrak{M}2/\mathcal{E}1 = 0.33. \tag{41}$$

These are very close to the preferred values in Ref. 3.

Preliminary experimental results are consistent with the B and A_2 predictions of Table II, but the A_1 radiative width is probably somewhat less. This could pose a problem for vector dominance, if the relation (32) turns out to be violated. However, if (32) holds, and the explicit results of (35) and (36) are compatible with experiment for B and A_2, one would be tempted to use the measured

TABLE II. Vector-dominance predictions for $\Gamma(X \to \gamma\pi)$ in terms of $\Gamma(X \to V\pi)$, $V = \rho$ or ω.

Process $X \to V\pi$	Total rate[a] (MeV)	Rate[b] for $X \to V_\perp^0\pi$ (MeV)	p_π (MeV)	p_γ (MeV)	Predicted[c] $\Gamma(X \to \gamma\pi)$
$B(1231) \to \omega\pi$	129 ± 10	115 ± 12	348	608	$B \to \gamma\pi$: 184 ± 30 keV
$A_1(1200) \to \rho\pi$	~300	75–100	329	592	$A_1^+ \to \gamma\pi^+$: 1–1.6 MeV
$A_2(1317) \to \rho\pi$	71 ± 4	36 ± 2	414	651	$A_2^+ \to \gamma\pi^+$: 375 ± 50 keV

[a] Reference 10.
[b] See text for explanation.
[c] Equations (35) and (36).

properties of A_1 in Primakoff production[1] $\pi+Z\to A_1+Z$ to predict what should be observed in $A_1\to\rho\pi$, rather than *vice versa*. The properties of the A_1 in hadronic reactions still are not fully clear; in particular, the observed masses range over more than 200 MeV in different reactions.[10,17]

To learn about $f_0\to\gamma\gamma$ from $f_0\to\gamma\rho$, we need the fraction of ρ's with transverse polarization, defined in Eq. (6). Since

$$r=|A_0|^2+|A_2|^2/(|A_0|^2+|A_1|^2+|A_2|^2),$$

we obtain the second equality in (6) from Eq. (21). The fraction r is plotted as a function of $x\equiv\mathfrak{M}2/\mathcal{E}1$ in Fig. 2. It is $r=0.85$ for $x=0.33$. The predicted $f_0\to\gamma\gamma$ rate is then

$$\Gamma(f_0\to\gamma\gamma)=\left(\frac{p_{\gamma\gamma}}{p_\gamma}\right)^3\frac{1}{2}\left(\frac{10}{9}\right)^2\frac{\alpha}{g_\rho^{\,2}/4\pi}r\Gamma(f_0\to\gamma\rho)$$

$$=[(6.8\pm0.8)\times10^{-3}]r\Gamma(f_0\to\gamma\rho)\,, \qquad (42)$$

for $p_{\gamma\gamma}=m_f/2$, p_γ as given in Eq. (16), and $g_\rho^2/4\pi=2.7\pm0.3$. The factor of $\frac{1}{2}$ accounts for the identity of the two photons. The factor $(\frac{10}{9})^2$ expresses an elementary SU(3) relation. 90% of the $f_0\to\gamma\gamma$ amplitude comes from $f_0\to\gamma\rho\to\gamma\gamma$, while 10% comes from $f_0\to\gamma\omega\to\gamma\gamma$.

With $\Gamma(f_0\to\gamma\rho)=1.35\pm0.2$ MeV [Eq. (40)] and $r=0.85$, Eq. (42) predicts

$$\Gamma(f_0\to\gamma\gamma)=7.7\pm2.0\text{ keV}\,, \qquad (43)$$

a value very close to the favored one in Ref. 3. A value about three times smaller than this has been reported.[2] If vector dominance [including the kinematic prescription in (42)] is correct, this would imply instead $\Gamma(f_0\to\gamma\rho)\simeq\frac{1}{2}$ MeV. This still looks measurable. It is at the lower limit to be compatible (through the relations we have noted previously) with preliminary rates for the radiative decays of B, A_1, and A_2.

We have noted previously[3] the prediction that $f_0\to\gamma\gamma$ should be dominated by helicity 2. In our present notation, we expect

$$\frac{A_0^{(\gamma\gamma)}}{A_2^{(\gamma\gamma)}}=\frac{1}{\sqrt{6}}\left(\frac{1-\sqrt{5}x}{1+\sqrt{5}x/3}\right). \qquad (44)$$

This ratio is plotted as the dashed line in Fig. 2. It is a rapidly changing function of x. Nonetheless, the present relations allow us to express it in terms of B, A_1, and A_2 radiative widths in many ways, one of which is Eq. (8). With kinematic factors, this is

$$\frac{A_0^{(\gamma\gamma)}}{A_2^{(\gamma\gamma)}}=\frac{1}{\sqrt{6}}\left[0.49\left(\frac{\Gamma_\gamma(A_1)}{\Gamma_\gamma(B)}\right)^{1/2}-1\right]. \qquad (45)$$

As long as $1\le\Gamma(A_1^+\to\gamma\pi^+)/\Gamma(B\to\gamma\pi)\le10$ (a range satisfied both by the preliminary experimental values and by the parameters considered in Ref. 3),

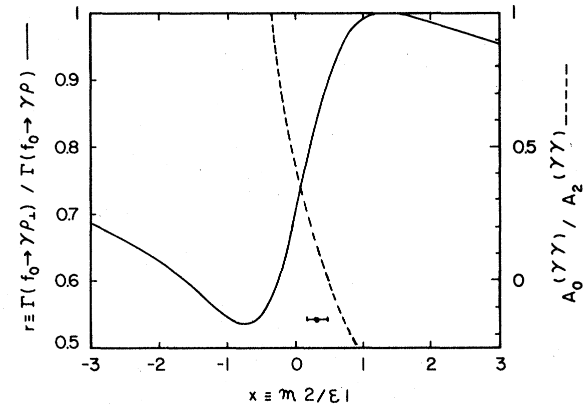

FIG. 2. Fraction of ρ's (solid line) predicted with transverse polarization in $f_0\to\gamma\rho$, and ratio $A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)}$ (dashed line) of helicity amplitudes in $f_0\to\gamma\gamma$, as functions of $x=\mathfrak{M}2/\mathcal{E}1$. The point ⊢●⊣ corresponds to the range of x examined in Ref. 3.

we will then have $|A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)}|\lesssim\frac{1}{4}$. The ratio $A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)}=0.11$ is expected from (44) if $x=0.3$. Many suggestions have been given[3] for measurement of this ratio in $e^+e^-\to e^+e^-f_0\to e^+e^-\pi^+\pi^-$. We recall the simplest: If θ' is the angle between one pion and the photon direction in the c.m. system of $\gamma\gamma\to f_0\to\pi^+\pi^-$, the angular distribution should be

$$W(\theta')\sim\tfrac{3}{4}|A_2^{(\gamma\gamma)}|^2\sin^4\theta'+\tfrac{1}{2}|A_0^{(\gamma\gamma)}|^2(3\cos^2\theta'-1)^2\,. \qquad (46)$$

Azimuthal correlations (if at least one final $e^\pm$ is detected) can resolve the sign of $A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)}$.[3]

A consistency check of the helicity structure associated with vector dominance would be to measure x from $f_0\to\gamma\gamma$ via Eq. (44) and then to compare it with that extracted from $f_0\to\gamma\rho$.

In the heavy-quark limit the dominant amplitude in Table I is $E1'$; the others are suppressed by a power of (photon energy/quark mass).[18] In this limit the heavy-quark analog of $D\to\gamma\rho$ [such as $\chi(3508, 1^+)\to\gamma\psi$] can give rise to transversely polarized vector mesons: The $\lambda=0$ amplitude in Table I does not vanish. Since the corresponding $\gamma\gamma$ decay [e.g., $\chi(3508)\to\gamma\gamma$] must vanish,[14] naive vector dominance must fail.

If we were to substitute the VDM constraint (31) to eliminate the $M2$ amplitude in Table I, we would find

$$\frac{\tilde\Gamma(f_0\to\gamma\gamma)}{\tilde\Gamma(\epsilon\to\gamma\gamma)}=\frac{\tilde\Gamma(f_0\to\gamma\rho_\perp)}{\tilde\Gamma(\epsilon\to\gamma\rho_\perp)}$$

$$=\frac{2}{5}+\frac{12}{5}\left(1-\frac{\sqrt{2}}{3}\frac{E1}{E1'}\right)^{-2}>\frac{2}{5}\,. \qquad (47)$$

In the heavy-quark limit we have instead (10), in-

TABLE III. Summary of predictions for $f_0 \to \gamma\rho$ and $f_0 \to \gamma\gamma$ in terms of $\Gamma_\gamma(B) \equiv \Gamma(B \to \gamma\pi)$ and $\Gamma_\gamma(A_{1,2}) \equiv \Gamma(A^{\pm}_{1,2} \to \gamma\pi^{\pm})$. SQT = single-quark-transition hypothesis. VDM = vector-dominance model.

Relation	Equation number	Tests
$\Gamma_{E1}(f_0 \to \gamma\rho) = 2.57\{[\Gamma_\gamma(B)]^{1/2} \pm 0.25[\Gamma_\gamma(A_1)]^{1/2}\}^2$	(17)[a]	SQT
$\Gamma_{M2}(f_0 \to \gamma\rho) = 0.35\ \Gamma_\gamma(A_2)$	(18)	SQT
$\Gamma_\gamma(A_2) = 13.3\{[\Gamma_\gamma(B)]^{1/2} \mp 0.25[\Gamma_\gamma(A_1)]^{1/2}\}^2$	(32)[b]	SQT, VDM
$\Gamma(f_0 \to \gamma\rho) = 5.1\ \Gamma_\gamma(B) + 0.31\ \Gamma_\gamma(A_1) + 0.15\ \Gamma_\gamma(A_2)$	(33)[c]	SQT, VDM
$x \equiv (\mathfrak{M}2/\mathcal{E}1)_{f_0 \to \gamma\rho} = 0.37[\Gamma_\gamma(A_2)]^{1/2}/\{[\Gamma_\gamma(B)]^{1/2} + 0.25[\Gamma_\gamma(A_1)]^{1/2}\}$	(34)	SQT, VDM
$\Gamma(f_0 \to \gamma\rho) = 1.35 \pm 0.2$ MeV; $x = 0.33$	(40), (41)[d]	SQT, VDM
$\Gamma(f_0 \to \gamma\gamma) = (6.8 \pm 0.8) \times 10^{-3}\Gamma(f_0 \to \gamma\rho_\perp)$	(42)	VDM, SU(3)
$\Gamma(f_0 \to \gamma\rho_\perp) = 0.85\ \Gamma(f_0 \to \gamma\rho)$	Fig. 2[e]	SQT, VDM
$\Gamma(f_0 \to \gamma\gamma) = 7.7 \pm 2$ keV	(43)	SQT, VDM, SU(3)
$A_0^{(\gamma\gamma)}/A_2^{(\gamma\gamma)} = \{0.49[\Gamma_\gamma(A_1)/\Gamma_\gamma(B)]^{1/2} - 1\}/\sqrt{6}$	(45)	SQT, VDM

[a] Positive sign selected in Eq. (17) if vector dominance also assumed.

[b] Negative sign in (32) goes with positive sign in (18). These are the signs favored if (32) is to agree with present data.

[c] Equation (33) is not independent of (17), (18), and (32) but is presented for convenience.

[d] See text. Based on pionic decays of B and A_2, results of Table II, and corresponding prediction of Eq. (32): $\Gamma_\gamma(A_1) = 1.13$ MeV.

[e] Based on $x = 0.33$.

compatible with (47).

We suspect that the failure of naive vector dominance in the heavy-quark limit is due to the proximity of many 1^- poles, all contributing to the photon's coupling to heavy quarks. For light quarks, the ρ, ω, ϕ poles are so much closer to a mass-shell photon than their radial excitations that vector dominance should be better.

Our main results have been stated at the beginning of this paper. We have stressed the importance of the processes $f_0 \to \gamma\rho$ and $f_0 \to \gamma\gamma$ for testing quark-model selection rules and vector dominance. New measurements of B, A_1, $A_2 \to \gamma\pi$ rates are very helpful for these tests. We summarize some of the numerical predictions in Table III. It appears that vector dominance is mainly a tool for light-quark physics; it encounters contradictions in the heavy-quark limit.

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