CP violation in charged kaons

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A calculation is performed of the asymmetry $A = [\Gamma(K^+\to\pi^+\pi^+\pi^-) - \Gamma(K^-\to\pi^-\pi^-\pi^+)]/[\Gamma(K^+\to\pi^+\pi^+\pi^-) + \Gamma(K^-\pi^-\pi^-\pi^+)]$ in the Kobayashi-Maskawa model using soft-pion results of current algebra. The prediction obtained is $\langle A\rangle = 0.74|\epsilon'|$, where ϵ' is the CP-violating parameter of neutral kaons involving violations of the $\Delta I = 1/2$ rule.

The Kobayashi-Maskawa model[1] is the simplest generalization of the Weinberg-Salam[2] model to include more than four quarks, and one of its nicest features is that it allows for CP violation in the gauge coupling to the quarks as a consequence of the mixing quarks matrix. Recently,[3] the standard current-algebra technique was used to relate the CP violations in the neutral-kaon sector $K\to 3\pi$ decays to the better known CP-violating decay $K^0\to 2\pi$ in the context of the Weinberg-Salam model, generalized to three left-handed doublets of quarks in the manner of Kobayashi and Maskawa. In this paper, we proceed in a similar way as in Ref. 3, carrying out an analysis of CP violation in the charged-kaon sector.

In the $K\to 3\pi$ amplitudes, with the kinematics given by

$$K^i(p)\to \pi_1^a(K_1)+\pi_2^b(K_2)+\pi_3^c(K_3)\ ,$$

we can define the Lorentz-invariant energy variables S_j as

$$S_j=(P-K_j)^2\ , \tag{1}$$

with $S_1+S_2+S_3=3S_0=3(M_\pi^{\ 2}+\frac{1}{3}M_K^{\ 2})$. The indices i label the isospin of the kaon, a, b, c label the isospin of the pions, and j labels pions.

In the usual approximation that the dependence of the Dalitz-plot on the energies of the pions is at most linear, the CP-conserving part of the $K\to 3\pi$ amplitudes can be parametrized as follows:[4]

$$\langle\pi^+\pi^+\pi^-|H_w^{\ +}|K^+\rangle = i[\,(2\alpha_1-\alpha_3)+(\beta_1-\tfrac{1}{2}\beta_3+\sqrt{3}\,\gamma_3)y]\ , \tag{2a}$$

where α_1, β_1 (α_3,β_3) come from the transition into the $I=1$ state of the three pions caused by the $\Delta I=\frac{1}{2}$ ($\Delta I=\frac{3}{2}$) part of the CP-conserving weak Hamiltonian H_w, and γ_3 comes from the $I=2$ state of the three pions, where $y=(S_3-S_0)/M_\pi^{\ 2}$. We proceed in a similar way with the CP-violating part:

$$\langle\pi^+\pi^+\pi^-|H_w^-|K^-\rangle = i^2[(2\alpha_1'-\alpha_3')+(\beta_1'-\tfrac{1}{2}\beta_3'+\sqrt{3}\,\gamma_3')y]\ . \tag{2b}$$

All α_1, α_1', α_3, α_3', β_1, β_1', β_3, β_3', γ_3, and γ_3' are real parameters.

Let us define

$$\alpha_{jc}\equiv\alpha_j+i\alpha_j',\quad j=1,3$$
$$\beta_{jc}\equiv\beta_j+i\beta_1',\quad j=1,3$$
$$\gamma_{3c}\equiv\gamma_3+i\gamma_3'\ .$$

We write

$$\langle\pi^+\pi^+\pi^-|H_w|K^+\rangle = i[(2\alpha_{1c}-\alpha_{3c})+(\beta_{1c}-\tfrac{1}{2}\beta_{3c}+\sqrt{3}\,\gamma_{3c})y]\ ,$$

where $H_w=H_w^++H_w^-$. (2c)

The parameters α_i, α_i', etc., are real if final-state interactions are neglected but in general have complex phases completely determined by CPT and unitarity from the final-state strong-interaction S matrix. It is well known[5,6] that the CPT theorem requires a particle and its antiparticle to have the same lifetime. However, for partial decay rates, which is our case, this may not be true[6,7] unless C or CP invariance holds, if we have strong interactions in the final state. To a good approximation,[8] the phase is determined by the isospin and the permutation symmetry of the 3π states; thus there is one phase for all the α's, the amplitudes of the completely symmetrical $I=1$ 3π state, one for all the β's, the amplitudes of the $I=1$ 3π state with mixed symmetry, and one for all the γ's, the amplitudes of the $I=2$ state. These three pions' phase shifts[8] depend on the energy of the pions and involve the pion-pion phase shifts which we take from the current-algebra calculation done by Weinberg.[9] We define

$$\begin{aligned}\tau^+&\equiv A(K^+\to\pi^+\pi^+\pi^-)\\ &=e^{i\delta_1}(2\alpha_{1c}-\alpha_{3c})\\ &\quad+e^{i\tilde\delta_1}(\beta_{1c}-\tfrac{1}{2}\beta_{3c})y+\sqrt{3}e^{i\tilde\delta_2}\gamma_{3c}y\end{aligned} \tag{3a}$$

$$\begin{aligned}\tau^-&\equiv A(K^-\to\pi^-\pi^-\pi^+)\\ &=e^{i\delta_1}(2\alpha_{1c}^*-\alpha_{3c}^*)\\ &\quad+e^{i\tilde\delta_1}(\beta_{1c}^*-\tfrac{1}{2}\beta_{3c}^*)y+\sqrt{3}e^{i\delta_2}\gamma_{3c}^*y\ .\end{aligned} \tag{3b}$$

By a straightforward calculation we obtain

$$|\tau^+|^2 = A + (B+C)y^2 + y(De^{i(\delta_1-\tilde\delta_1)} + D^*e^{-i(\delta_1-\tilde\delta_1)}) + \sqrt{3}\,y(Ee^{i(\delta_1-\delta_2)} + E^*e^{-i(\delta_1-\delta_2)}) + \sqrt{3}\,y^2(Fe^{i(\tilde\delta_1-\delta_2)} + F^*e^{-i(\tilde\delta_1-\delta_2)}) \qquad (4a)$$

and

$$|\tau^-|^2 = A + (B+C)y^2 + y(De^{-i(\delta_1-\tilde\delta_1)} + D^*e^{i(\delta_1-\tilde\delta_1)}) + \sqrt{3}\,y(Ee^{-i(\delta_1-\delta_2)} + E^*e^{i(\delta_1-\delta_2)}) + \sqrt{3}\,y^2(Fe^{-i(\tilde\delta_1-\delta_2)} + F^*e^{i(\tilde\delta_1-\delta_2)}) \,, \qquad (4b)$$

and for the experimental quantity of interest[5]

$$\frac{|\tau^+|^2-|\tau^-|^2}{|\tau^+|^2+|\tau^-|^2} = \frac{4y[\sin(\delta_1-\tilde\delta_1)\mathrm{Im}D + \sqrt{3}\sin(\delta_1-\delta_2)\mathrm{Im}E + y\sqrt{3}\sin(\tilde\delta_1-\delta_2)\mathrm{Im}F]}{2A+2(B+C)y^2+4y\cos(\delta_1-\tilde\delta_1)\mathrm{Re}D + 4\sqrt{3}\,y\cos(\delta_1-\delta_2)\mathrm{Re}E + 4\sqrt{3}\,y^2\cos(\tilde\delta_1-\delta_2)\mathrm{Re}F} \qquad (5)$$

where

$$A \equiv |2\alpha_{1c}-\alpha_{3c}|^2,\quad B \equiv |\beta_{1c}-\tfrac{1}{2}\beta_{3c}|^2,$$
$$C \equiv |\gamma_{3c}|^2,\quad D = (\beta_{1c}-\tfrac{1}{2}\beta_{3c})(2\alpha^*_{1c}-\alpha^*_{3c}), \qquad (6)$$
$$E = \gamma_{3c}(2\alpha^*_{1c}-2^*_{3c}),\quad F = \gamma_{3c}(\beta^*_{1c}-\tfrac{1}{2}\beta^*_{3c}) \,.$$

We now use the soft-pion limits of the $K \to 3\pi$ amplitude obtained from the usual current-algebra techniques. Using Eqs. (19a) through (19e) of Ref. 3, we obtain the following relevant relations:

$$\beta_1 = -3x\alpha_1 \,,\quad \beta_3 = \tfrac{15}{4}x\alpha_3,\quad \gamma_3 = \tfrac{-27}{8}\sqrt{3}\,\alpha_3\,,$$
$$\beta_1' = -3x\alpha_1' \,,\quad \alpha_3 = \sqrt{3}\frac{f_3}{f_1}\alpha_1,\quad \beta_3 = -\frac{f_3}{f_1}\frac{5}{2\sqrt{2}}\beta_1,$$

and

$$\gamma_3 = \frac{f_3}{f_1}\frac{9}{4}\left(\frac{3}{2}\right)^{1/2}\beta_1 \,, \qquad (7)$$

where $x = M_\pi^2/(M_K^2-M_\pi^2)$ and

$$(f_3/f_1) = -\langle I=2|H_w|K_S\rangle/\langle I=0|H_w|K_S\rangle$$

gives the violation of the $\Delta I=\frac{1}{2}$ rule in neutral-kaon decays, known to be of the order of $\frac{1}{20}$.

It is worth noting that the current-algebra relations relate only magnitudes of amplitudes. The parameters entering Eqs. (2a) and (2b) were assumed real numbers and the three pion phase shifts were introduced separately in Eqs. (3a) and (3b).

The phase convention chosen by Kobayashi and Maskawa[1] implies that the CP-violating Hamiltonian satisfies the $\Delta I=\frac{1}{2}$ rule, therefore,

$$\alpha_3' = \beta_3' = \gamma_3' = 0 \,, \qquad (8)$$

and only the CP-conserving piece H_w^+ violates the $\Delta I=\frac{1}{2}$ rule.

Using the relations (7) and the convention given by (8), we obtain

$$A = \beta_1^2\left[\frac{4}{9x^2} + \frac{2}{9x}\left(\frac{f_3}{f_1}\right)^2 - \frac{4\sqrt{3}}{9x}\frac{f_3}{f_1} + \frac{4}{9x}\left(\frac{\alpha_1'}{\alpha_1}\right)^2\right] \,, \qquad (9a)$$

$$B = \beta_1^2\left[1 + \frac{1}{4}\left(\frac{f_3}{f_1}\right)^2\frac{25}{8} + \frac{f_3}{f_1}\frac{5}{2\sqrt{2}} + \left(\frac{\alpha_1'}{\alpha_1}\right)\right] \,, \qquad (9b)$$

$$C = \beta_1^2\,\frac{9}{2}\left(\frac{f_3}{f_1}\right)^2\frac{81}{16} \,, \qquad (9c)$$

$$\mathrm{Im}D = -\alpha_1'\beta_1\frac{f_3}{f_1}\frac{9}{2\sqrt{2}} \,, \qquad (9d)$$

$$\mathrm{Im}E = -\alpha_1'\beta_1\frac{f_3}{f_1}\frac{9}{2}\left(\frac{3}{2}\right)^{1/2} \,, \qquad (9e)$$

$$\mathrm{Im}F = -\alpha_1'\beta_1\frac{f_3}{f_1}\,3x\,\frac{9}{4}\left(\frac{3}{2}\right)^{1/2} \,, \qquad (9f)$$

$$\mathrm{Re}D = \beta_1^2\left[-\frac{2}{3x} + \frac{2}{3}\frac{f_3}{f_1} + \frac{1}{3x}\left(\frac{1}{\sqrt{2}}\frac{f_3}{f_1}-1\right)\frac{f_3}{f_1}\frac{5}{2\sqrt{2}} - \frac{2}{3x}\left(\frac{\alpha_1'}{\alpha_1}\right)^2\right] \,, \qquad (9g)$$

$$\mathrm{Re}E = \beta_1^2\left(-\frac{1}{x} - \frac{f_3}{f_1}\frac{2}{3x}\right)\frac{f_3}{f_1}\frac{9}{4}\left(\frac{3}{2}\right)^{1/2} \,, \qquad (9h)$$

$$\mathrm{Re}F = \beta_1^2\left(1 + \frac{1}{2}\frac{f_3}{f_1}\frac{5}{2\sqrt{2}}\right)\frac{f_3}{f_1}\frac{9}{4}\left(\frac{3}{2}\right)^{1/2} \,. \qquad (9i)$$

We now turn to relate our result to the CP-violating parameters of the neutral-kaon sector. From Ref. 3

$$\sqrt{2}\epsilon' = -i\,(f_3/f_1)[g_3/f_3 - g_1/f_1] \qquad (10a)$$

where g_3 and g_1 are the $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ of the CP-violating Hamiltonian in the neutral kaon to two pions. The Kobayashi-Maskawa prescription thus implies $g_3=0$ and therefore

$$\sqrt{2}\epsilon' = i(f_3/f_1)(g_1/f_1) = i(f_3/f_1)(\alpha_1'/\alpha_1) \,, \qquad (10b)$$

where again the soft-pion results have been used. From the same Ref. 3, we have

$$\epsilon = \rho + i(\alpha_1'/\alpha_1) \,, \qquad (11)$$

from which we can estimate $\alpha_1'/\alpha_1 \sim 10^{-3}$, and thus $|\epsilon'| \sim 0.35\times10^{-4}$, which is slightly higher than the evaluation of Ref. 10.

We now neglect (f_3/f_1) and (α_1'/α_1) compared to 1 in Eqs. (9a) through (9c) and (9g) through (9i), which are the expressions appearing in the denominator of Eq. (5). Under this approximation,

$\mathrm{Re}E=0,\ \mathrm{Re}F=0,\ C=0,$

$A=4\beta_1^2/9x^2,\quad B=\beta_1^2\ ,$

and

$$\mathrm{Re}D=\beta_1^2\,(-2/3x)\ . \tag{12}$$

Thus, Eq. (8) becomes

$$\frac{|\tau^+|^2-|\tau^-|^2}{|\tau^+|^2+|\tau^-|^2}=\frac{27xy[\sin(\delta_1-\tilde{\delta}_1)+3\sin(\delta_1-\delta_2)+xy\frac{9}{2}\sin(\tilde{\delta}_1-\delta_2)]\,|\epsilon'|}{4+9x^2y^2-12xy\cos(\delta_1-\tilde{\delta}_1)} \tag{13}$$

Equation (13) shows that under the assumptions made in this work the asymmetry A of partial lifetimes of charged kaons decaying into three charged pions is proportional to $|\epsilon'|$. This first suggests a way to measure $|\epsilon'|$ independent of neutral kaons, and second a possibility to discriminate among theoretical models of CP violation.

Equation (13) depends on the energy of the three pions through y, δ_1, $\tilde{\delta}_1$, and δ_2. The explicit energy dependence of the phase shifts is given in Ref. 8. To study the values obtained from Eq. (13), we generate at random three pions constrained only by energy conservation. With this procedure the mean value of the asymmetry A given by Eq. (13) is $0.094\,|\epsilon'|$. A careful analysis of the energy distribution of the individual pions in a smaller sample suggested that a cut in the energy of the odd pion (whose electrical charge is different to the parent kaon) should enhance the mean value of Eq. (13). We generated a sample of pions in which we required that the energy of the odd pion was less than 7 MeV in the kaon center of mass, and the mean value obtained was $0.74\,|\epsilon'|$ with a standard deviation of 0.07. The sample used had 1000 events.

In order to measure the asymmetry given by Eq. (13), the experimental signature would be two even pions with center-of-mass with a mean value of 36 MeV each and a standard deviation of 6 MeV. The odd pion being cut at 7 MeV results in the energy being distributed with a mean value of 4 MeV, and changing the cut of the odd pion in ± 30% of its value changes the value of Eq. (13) by 1%. The number of events satisfying the cut are of the order of 10% of the total amount of generated events.

The existing experimental[11] limit on the value of Eq. (13) is that there is no CP violation of the one part in 10^3; our result suggests that one should find CP violation in charged kaons of one part in 10^5. We hope that our analysis may stimulate the experimental search of CP violation in the charged-kaon sector.

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[1]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics: Relativistic Groups and Analyticity* (*Nobel Symposium No. 8*), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[3]Ling-Fong Li and Lincoln Wolfenstein, Phys. Rev. D 21, 178 (1980).

[4]B. Holstein, Phys. Rev. 177, 2417 (1969).

[5]G. Lüders and B. Zumino, Phys. Rev. 106, 385 (1957); T. D. Lee, R. Oehme, and C. N. Yang, *ibid.* 106, 340 (1957).

[6]S. Okubo, Phys. Rev. 109, 984 (1958).

[7]Y. Ueda and S. Okubo, Phys. Rev. 139, B1591 (1965).

[8]B. Ya. Zeldovich, Yad. Fiz. 6, 840 (1967) [Sov. J. Nucl. Phys. 6, 611 (1968)].

[9]S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).

[10]F. Gilman and M. B. Wise, Phys. Lett. 83B, 83 (1979).

[11]W. T. Ford, P. A. Piroué, R. S. Remmel, A. J. S. Smith, and P. A. Souder, Phys. Rev. Lett. 19, 1370 (1970).