Heavy-flavor production in proton-proton interactions

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We present a model based on quantum chromodynamics for calculating the production of particles containing heavy quarks. The production of heavy quarks is due to two-gluon annihilation. Soft-gluon emission is assumed to create a random-phase situation. For D, B, and K mesons we calculate transverse-momentum distributions.

I. INTRODUCTION

The production of particles made of c and b quarks has been estimated using quantum chromodynamics (QCD) and quarkonium considerations by several authors.[1,10] The usual treatment for production of pairs of mesons made of one heavy and one light quark is to calculate the cross section for production of a heavy-quark–antiquark pair by constituent annihilation. The contributions above $D\bar{D}$ or $B\bar{B}$ threshold are then identified with the cross section for production of a D or B meson as the case may be.[2]

When considering the production of heavy mesons made of quark-antiquark pairs of the same flavor (quarkonium) one either assumes annihilation of constituents directly into the produced state (e.g., gluon + gluon $\rightarrow \eta_c$) (Refs. 3, 4) or production of a state which decays into the final quarkonium state of interest. The production of ψ mesons, for example, is often considered as the result of production of P-wave c-$\bar{c}$ mesons by gluon-gluon annihilation followed by radiative decay into ψ. We have found that this mechanism proposed by Carlson and Suaya[5] yields a cross section which is a few times smaller than experiment. Furthermore, it appears that only a fraction of the produced ψ mesons are accompanied by a photon.[6,7] Again ψ' production cannot be explained in this way since the branching ratio from P states above the ψ' is very small due to their being above threshold for decay into $D\bar{D}$. Finally this mechanism leads to the wrong P_T distribution for the produced ψ mesons coming, as it were, as the recoil distribution of the accompanying photon. This mechanism leads to a distribution of the form $[(m_\chi - m_\psi)^2 - P_T^2]^{-1/2}$ whereas the experimental data[8] shows the distribution $e^{-1.67P_T}$.

We treat $D\bar{D}$ and $B\bar{B}$ production in this paper using QCD in lowest order, neglecting any Q^2 dependence of constituent distributions. The main difference with previous papers is the assumption that we consider, for example, $D\bar{D}$ production as the result of $c\bar{c}$ production followed by the capture of two light sea quarks. The usual procedure is to identify $D\bar{D}$ production with $c\bar{c}$ production above $D\bar{D}$ threshold. The procedure we use for $B\bar{B}$ production (and a bold extrapolation to $K\bar{K}$ production) is similar. We also calculate the transverse-momentum distributions of the produced D, K, and B mesons.

We give a new model for the production of quarkonium mesons and apply the results to $Q\bar{Q}$ systems where $Q = c, b$, and s quarks. The model assumes that a produced $Q\bar{Q}$ pair results in a bound state after emitting a gluon. The rate for this process is estimated using a dipole matrix element and nonrelativistic quarkonium wave functions. Most of the produced $Q\bar{Q}$ pairs lead to $D\bar{D}$, $K\bar{K}$, or $B\bar{B}$ production as the case may be. However, as noted above, there is a probability that a bound state results following gluon emission. There is expected to be a high probability for "soft"-gluon emission and absorption by the heavy quarks before the transition to a bound state by single-gluon emission (e.g., in ψ production) or light-quark capture (e.g., in $D\bar{D}$ production). This is assumed to create a random-phase situation so that the relative rate is determined by intensity ratios.

II. HEAVY-PSEUDOSCALAR-MESON PRODUCTION

We now calculate $D\bar{D}$, $B\bar{B}$, and $K\bar{K}$ production using lowest-order QCD. Lowest-order QCD yields the following result for $Q\bar{Q}$ production in proton-proton interaction[9,10]:

$$\frac{d^2\sigma(s,P_T,x_f)}{dP_T^2dx_f} = \int \frac{d\hat{\sigma}}{dt}(t,\hat{s}) \sum_{i=+,-} f(x_1^{(i)})f(x_2^{(i)}) \frac{\partial(x_1^{(i)},x_2^{(i)})}{\partial(P_T^2,x_f)}\,dt\,, \tag{1}$$

where, neglecting quark-antiquark fusion which could provide an appreciable contribution at low enough energy,

$$\frac{d\hat{\sigma}}{dt}(t,\hat{s})=\frac{g^4}{(t-m_Q^2)^2}\left(\frac{1}{12}\right)(-2m_Q^4-6tm_Q^2-2um_Q^2+2ut)+\frac{g^4}{(u-m_Q^2)^2}\left(\frac{1}{12}\right)(-2m_Q^4-2tm_Q^2-6um_Q^2+2ut)$$
$$+\frac{g^4}{\hat{s}^2}\left(\frac{3}{16}\right)(-28m_Q^4+20um_Q^2+20tm_Q^2-4(t+u)^2+4ut)+\frac{g^4}{(t-m_Q^2)(u-m_Q^2)}\left(-\frac{1}{96}\right)(-8m_Q^4-4tm_Q^2-4um_Q^2)$$
$$+\frac{g^4}{(t-m_Q^2)\hat{s}}\left(\frac{3}{32}\right)(-12m_Q^4+4um_Q^2+12tm_Q^2-4t^2)+\frac{g^4}{(u-m_Q^2)\hat{s}}\left(-\frac{3}{32}\right)(12m_Q^4-12um_Q^2-4tm_Q^2+4u^2)\quad\text{(1a)}$$

with $g^2/4\pi=\alpha_s$ and m_Q the mass of the quark Q; $f(x)$ the gluon distribution function is given by[11]

$$f(x)=3(1-x)^5/x. \qquad \text{(1b)}$$

We neglect any nonscaling behavior of the gluon distribution as the effect is not substantial.

The quantities $x_1^{(i)}$ and $x_2^{(i)}$ in (1) are given by

$$x_1^{(i)}=\frac{\frac{1}{2}x_f\sqrt{s}\pm(\frac{1}{4}x_f^2 s+4a^2+2a\tau\sqrt{s})^{1/2}}{2(a/\sqrt{\tau}+\frac{1}{2}\sqrt{\tau s})},$$
$$x_2^{(i)}=\frac{\tau}{x_1^{(i)}} \qquad \text{(1c)}$$

with $a=(1/\sqrt{s})(t-m_Q^2)$, $\tau=x_1^{(i)}x_2^{(i)}=-a^2/(P_T^2+t)$. The t-integration region is defined by

$$0\leq x_1^{(i)}x_2^{(i)}\leq 1,\quad 4m_Q^2+4P_T^2<\hat{s}<s\quad(\hat{s}=\tau s).$$

Integrating over $x_i^{(i)}x_2^{(i)}$ yields

$$\frac{d\sigma}{dP_T^2}=\int_0^1 F_{gg}(\tau)\frac{d\tau}{\tau}\frac{d\hat{\sigma}}{dt}(\hat{s},t)\frac{\hat{s}}{2(m_Q^2-t-\frac{1}{2}\hat{s})}\times\theta(\hat{s}-4m_Q^2-4P_T^2),$$

where

$$t=\tfrac{1}{2}(2m_Q^2-\hat{s})+\tfrac{1}{2}(\hat{s})^{1/2}(\hat{s}-4m_Q^2-4P_T^2)^{1/2}$$

and

$$F_{gg}(\tau)=\tau\int_\tau^1 f(x)f\left(\frac{\tau}{x}\right)\frac{dx}{x}.$$

Integrating over P_T^2 yields

$$\sigma(s)=\int_0^1 F(\tau)\sigma(\hat{s})\frac{d\tau}{\tau},$$

where[12]

$$\sigma(\hat{s})=\frac{\pi\alpha_s^2}{3\hat{s}^3}\left[(\hat{s}^2+4\hat{s}m_Q^2+m_Q^4)\ln\left(\frac{\hat{s}+\Lambda}{\hat{s}-\Lambda}\right)-(7\hat{s}+31m_Q^2)\frac{\Lambda}{4}\right] \qquad \text{(1d)}$$

with

$$\Lambda=(\hat{s}^2-4\hat{s}m_Q^2)^{1/2}.$$

The produced heavy $Q\overline{Q}$ pair, for the most part, captures each a light quark to produce $D\overline{D}$, $K\overline{K}$, and $B\overline{B}$ as the case may be. We therefore identify such production with heavy $Q\overline{Q}$ pair production. However, the threshold for production is taken as twice the mass of the produced quark Q and not twice the mass of the produced mesons. The rest of the mass comes from the light-quark energy. The capture rate of light quarks is unimportant here since the production of $D\overline{D}$, $B\overline{B}$, or $K\overline{K}$ is dominant. The production of $Q\overline{Q}$ bound states in competition with the dominant rates is of course dependent on the capture rate. We will then discuss the capture rate of light quarks when we discuss quarkonium production below.

Figures 1, 2, and 3 show the results for our calculation of $D\overline{D}$, $B\overline{B}$, and $K\overline{K}$ production using $\alpha_s=0.3$, 0.23, and 0.8, respectively, and quark masses as given in the figure captions, along with experimental data. The quark masses which best fit the data are $m_c=1.2$ GeV, $m_b\simeq 4$ GeV, and $m_s\simeq 0.4$ GeV. The masses for the c and b quarks are rather on the small side but not unreasonable. We comment further on this question below. The transverse-momentum distributions can be obtained from formula (1) and are shown in Figs. 5, 6, and 7.

III. HEAVY $Q\overline{Q}$ BOUND-STATE PRODUCTION

Our procedure for calculating $Q\overline{Q}$ bound-state production is to multiply the cross section for producing free $Q\overline{Q}$ as calculated using lowest-order QCD at a given $\hat{s}$ by the ratio R_1/R_2. Here R_1 is the rate for production of quarkonium ($Q\overline{Q}$ bound state) from a free $Q\overline{Q}$ pair by emission of a gluon. R_2 is the rate of capture of one light sea quark by the heavy Q or $\overline{Q}$ quark.

To calculate R_1, we use an effective Hamiltonian[13] for the perturbing interaction which induces gluon emission and a nonrelativistic wave function for the $Q\overline{Q}$ system. The initial state is taken as a plane wave and the final state as the bound state in a harmonic-oscillator potential with frequency chosen to yield the right spacing between lowest S-state levels. The rate

$$R_1=|\langle f|V_{\text{eff}}|i\rangle|^2 2\pi\delta(E_i-E_f-\omega)\frac{d^3k}{(2\pi)^3 2\omega}, \qquad (2)$$

where $|i\rangle$ is the plane wave of relative motion of the $Q\overline{Q}$ and $|f\rangle$ is the appropriate harmonic-oscillator wave function.

For V_{eff} we take[13] as effective dipole interaction

$$V_{\rm eff} = g\Lambda_a \vec{\nabla}\cdot\vec{A}_a(0)\,, \tag{3}$$

where g is the strong coupling constant, $\vec{A}_a(0)$ is the transverse color gauge field, and $\Lambda_a = \frac{1}{2}(\lambda_a + \lambda_a^*)$ are the color matrices for Q and $\overline{Q}$. $(\omega, \vec{k})$ is the four-momentum of the emitted gluon. We sum and average over color and spin of the quarks and gluons and we get (see Appendix)

$$R_1^{J/\psi} = \overline{\alpha}_s(\sqrt{\hat{s}} - m_\psi)^3 \frac{1}{V} e^{-P^2/\xi_c^2}(\hat{s} - 4m_c^2)\frac{8}{27}\pi^{3/2}\frac{1}{\xi_c^{\,7}}\,, \tag{4}$$

where $\overline{\alpha}_s$ is the analog of α_s for soft gluons and taken to be 1 in the following. V is from the box normalization of the plane wave of the produced quarks with three-momentum $P = \frac{1}{2}(\hat{s} - 4m_c^2)^{1/2}$ and $\xi = (\frac{1}{2}\omega_0 m_c)^{1/2}$ with ω_0 the spring frequency. The result is also valid for η_c if multiplied by the spin-statistical-weight ratio $\frac{1}{3}$.

We use the same forms for Υ and ϕ production with obvious changes in ω_0 and quark and vector-meson masses. We have for excited-state production using similar considerations

$$R_1^{\psi'} = \overline{\alpha}_s(\sqrt{\hat{s}} - m_{\psi'})^3 \frac{1}{V} e^{-P^2/\xi_c^2}(\hat{s} - 4m_c^2) \times\left(2\frac{P^2}{\xi_c^{\,2}} - 7\right)^2 \frac{4}{81}\,\frac{\pi}{\xi_c^{\,7}}^{3/2} \tag{5}$$

with appropriate parameter changes for Υ'. We have further

$$R_1^{\chi_J} = (2J+1)\frac{512}{729}\pi^{3/2}\overline{\alpha}_s\frac{1}{\xi_c^{\,5}}\left(3\frac{\xi_c^{\,2}}{P^2} + \frac{P^2}{\xi_c^{\,2}} - 2\right) \times(\sqrt{\hat{s}} - m_{\chi_J})^3 \tag{6}$$

and

$$R_1^{\Upsilon''} = \frac{64}{405}\overline{\alpha}_s(\sqrt{\hat{s}} - m_{\Upsilon''})^3\frac{1}{V}e^{-P^2/\xi_b^2}(\hat{s} - 4m_b^2) \times\left(-\frac{55}{8} + \frac{9}{2}\frac{P^2}{\xi_b^{\,2}} + \frac{1}{2}\frac{P^4}{\xi_b^{\,4}}\right)^2 \frac{\pi}{\xi_b^{\,7}}^{3/2}. \tag{7}$$

The rate for light-quark capture to form D, B, or K we take as

$$R_2 = \frac{N}{2V}\,\frac{\int e^{-P_q/T}\sigma_{qQ}^{\rm tot}(\overline{s})(\overline{s} - m_Q^{\,2})d^3P_q/E_qE_Q}{\int e^{-P_q/T}d^3P_q} \tag{8}$$

with $\overline{s} = (p_q^\mu + p_Q^\mu)^2$, p_q^μ and p_Q^μ being the four-momenta of the light and heavy quarks, respectively; p_q and p_Q are the magnitudes of the three-momenta. The cross section $\sigma_{qQ}^{\rm tot}(\overline{s})$ is the total cross section for $\overline{q}$ incident on Q as determined from photoproduction on complex nuclei[14,15] and associated phenomena. We take this cross section to be zero below threshold for production of the appropriate pseudoscalar and constant above threshold. We have used the additive quark model to deduce formula (8). For b quarks we assume[15] $\sigma_{qb}/\sigma_{qc} = m_c^{\,2}/m_b^{\,2}$. The quantity N is the average number of light sea quarks or antiquarks in the proton-proton system assumed thermodynamically distributed at temperature T in the center of mass of the gluon-gluon system. From the short-range correlations of $(d^2\sigma/dy_1dy_2)(p + p \to \pi + X)$ (y_1 and y_2 are rapidities) we take $\Delta y = |y_1 - y_2| = 0.5$ as the average difference in rapidity where we still have interaction between the quarks. The average number of π's per unit of rapidity is about 0.2.[40] Therefore we get about 0.2 quarks per 0.5 unit of rapidity.

The motivation for assuming such a distribution is as follows. Most light quarks end up being emitted in combination with other light quarks as pions and ρ mesons. Their transverse-momentum distributions are well described on the whole by the form $e^{-p_T/T}$. The longitudinal distributions are more complicated of course, involving the distribution of gluons in this picture.

We have then that the cross section for production of the heavy $Q\overline{Q}$ bound state M_i is given by

$$\sigma(p + p \to M_i + X) = \int d\tau F_{gg}(\tau)\sigma(\hat{s})R_1^{M_i}(\hat{s})/R_2^{M_i}(\hat{s})\,, \tag{9}$$

where $\hat{s} = \tau s$ and $F_{gg}(\tau)$, $\sigma(\hat{s})$ as defined in Eqs. (1).

IV. COMPARISON WITH EXPERIMENT

The results for $\psi(3.1)$ and $\psi'(3.685)$ production in proton-proton interaction calculated as described using (4), (5), and (9) agree well with experiment as can be seen in Fig. 1. We have taken $\alpha_s = 0.3$. The value $\xi = 0.42$ GeV follows from the ψ', ψ spacing. We take $T = 150$ MeV, which is more or less conventional. The results are in any case rather insensitive to T. [A value T of 2 GeV would make for a change of only a factor of 2 (smaller) in the ψ production cross section.] The mass of the charmed quark is the essential parameter and comes to be $m_c = 1.2$ GeV. A more realistic potential might produce a different value for m_c. It is interesting to note that the ratio $\sigma(pp \to \psi X)/\sigma(pp \to \psi' X) \simeq 8$ in our model compared with the experimental ratio[16] 10 ± 3 at $s = 750$ GeV.[2] Figure 1 also shows our calculation of the cross section for $D\overline{D}$ production evaluated using formula (1) integrated on the computer. Our calculations do not reproduce the relatively large beam-dump results[17] but otherwise the agreement with experiment is good.

We have also calculated the cross sections for the production of $\chi_0(3.415)$, $\chi_1(3.510)$, and $\chi_2(3.555)$ using formulas (6) and (9) with the same parameters as for ψ production. They are also plotted

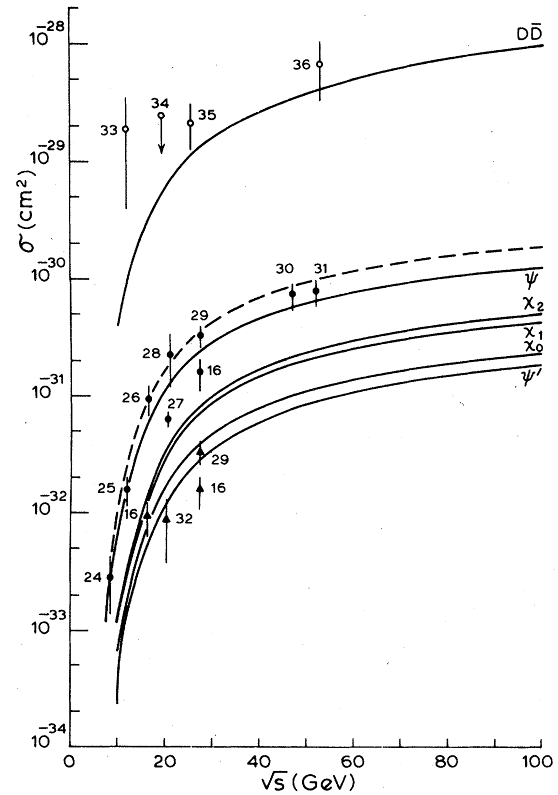

FIG. 1. The curves labeled ψ and ψ' represent our predictions for $\psi(3.1)$ and $\psi'(3.685)$ production cross sections in proton-proton interaction as a function of $\sqrt{s}$. The curves labeled χ_2, χ_1, and χ_0 represent the cross sections for $\chi_2(3.555)$, $\chi_1(3.510)$, and $\chi_0(3.415)$ production in proton-proton interaction, respectively. These results are obtained using (see text) $\xi = 0.42$ GeV, $m_c = 1.2$ GeV, $T = 0.15$ GeV, $N = 0.2$, and $\alpha_s = 0.3$. The dashed curve represents the ψ production when the χ_J states contribution is added to the curve labeled ψ. The curve labeled $D\bar{D}$ corresponds to the $D\bar{D}$ production evaluated in QCD with the same value of α_s and m_c; ●, ▲, O are the experimental data points for ψ, ψ', and $D\bar{D}$, respectively. The number attached to each point indicates the experimental reference.

in Fig. 1. The experimental branching ratios[18] for the radiative decay of these states into ψ are $B(\chi_0 \to \psi\gamma) = 0.033 \pm 0.010$, $B(\chi_1 \to \psi\gamma) = 0.234 \pm 0.008$, and $B(\chi_2 \to \psi\gamma) = 0.16 \pm 0.03$. We find using these results that there is an additional contribution of 20% to the ψ production cross section. Moreover, the cross section for χ_J production from gluon-gluon annihilation directly to χ_J (Carlson-Suaya mechanism) gives another 30% of the ψ-production cross section found using (5) and (9). Altogether then χ_J production accounts for about one third of ψ production. Some experiments seem to confirm substantial $\psi\gamma$ production. Kirk *et al.*[7] find that $(70 \pm 28)\%$ of the ψ particles are produced by radiative decay of χ particles and Kourkoumelis *et al.*[6] report a contribution of $(47 \pm 8)\%$.

Figure 2 shows our calculation of the cross section for production of the $\Upsilon(9.46)$ in proton-proton interaction. We adjust ξ to the Υ', Υ spacing obtaining $\xi = 0.8$ GeV. The coupling constant $\alpha_s = 0.23$. The b-quark mass $m_b = 4$ GeV to fit the experimental data [19,20] which are also shown in Fig. 2. Again different potentials might require a different value for the quark mass. Using the same parameters we calculate the cross section for $\Upsilon'(10.09)$ inclusive production in proton-proton interaction. We find $\sigma(pp \to \Upsilon'(10.09) + X) = 5.6$ pb at $s = 750$ GeV^2. Comparing with the experimental data available at this energy[19] $B\sigma = 0.09 \pm 0.02$ pb yields the branching ratio $B(\Upsilon' \to l\bar{l}) \simeq 1.6\%$ which is not unreasonable. Using the same parameters for $\Upsilon''(10.45)$ we find $\sigma(pp \to \Upsilon'' X) = 76.5$ pb at $s = 750$ GeV.[2] Comparing with the experimental data for $B\sigma(pp \to \Upsilon'' X)$ we find $B(\Upsilon'' \to l\bar{l}) \simeq 0.03\%$. Our cal-

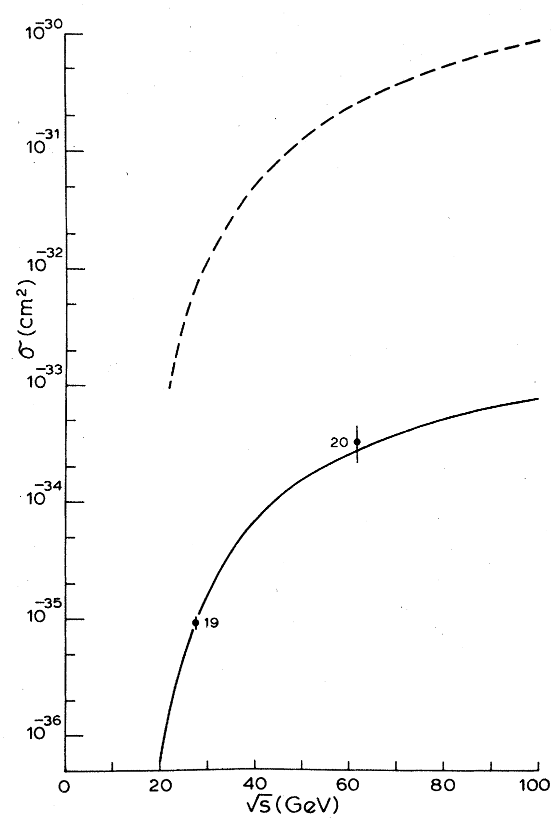

FIG. 2. The solid curve represents our predictions for $\Upsilon(9.46)$ production in proton-proton interaction as a function of $\sqrt{s}$. The following values have been used: $\xi = 0.8$ GeV, $m_b = 4.0$ GeV, $T = 0.15$ GeV, $N = 0.2$, $\sigma_{qb} \simeq m_c^2/m_b^2 \sigma_{qc}$, and $\alpha_s = 0.23$. The dashed curve corresponds to our QCD prediction for $B\bar{B}$ production with the same value α_s and m_b. ● are the data points for $\sigma(pp \to \Upsilon X)$. The number attached to each data point indicates the experimental reference.

culated results for $B\bar{B}$ production in proton-proton interaction are also shown in Fig. 2. There are no experimental data. We have also calculated $K\bar{K}$ and $\phi(1020)$ production in our model. The results are shown in Fig. 3. We have used $\alpha_s = 0.80$ and $\xi = 0.19$ GeV. The quark mass m_s is taken to be 0.4 GeV. The result for $K\bar{K}$ production is roughly a factor of 2 too small when compared with experimental data. The ϕ production is a factor of 3 too large at intermediate energy getting better at lower energies (a factor of 2 too large) and somewhat worse at higher energies (a factor of 6 too large). We of course are dealing here with a quark system which is far from nonrelativistic.

Figure 4 shows our predictions for η_c production according to

$$\sigma(p + p \to \eta_c + X) = \tfrac{1}{3}\sigma(p + p \to \psi + X)\,, \tag{10}$$

where we take the right-hand side from our Fig. 1. This is compared with a calculation[4] in which two gluons annihilate directly into the η_c. We see that as has been pointed out earlier,[4] for η_c production the direct annihilation dominates.

Figure 5 shows our calculations for the p_T distribution of D mesons produced in proton-proton interaction at various energies. For $\sqrt{s} = 53$ and 63 GeV our results in the range $0.8 < P_T < 2.4$ GeV may be approximated by ce^{-ap_T} with $a = 2.28$ GeV^{-1}. For the smaller energy $\sqrt{s} = 27.4$ GeV, $a = 2.75$ GeV^{-1}. This latter value agrees with the range of value $1.3 < a < 2.5$ GeV^{-1} reported in a recent experiment[21] at $\sqrt{s} = 27.4$ GeV.

We have also calculated the p_T distribution of K mesons produced in proton-proton interaction (see

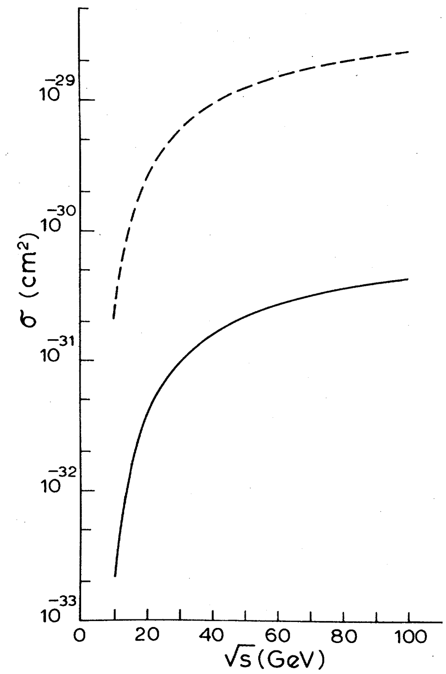

FIG. 4. The two curves represent predictions for $\sigma(pp \to \eta_c X)$. The lower curve corresponds to the result obtained from the model presented in the present paper (see text). The upper curve corresponds to the result obtained in Ref. 4.

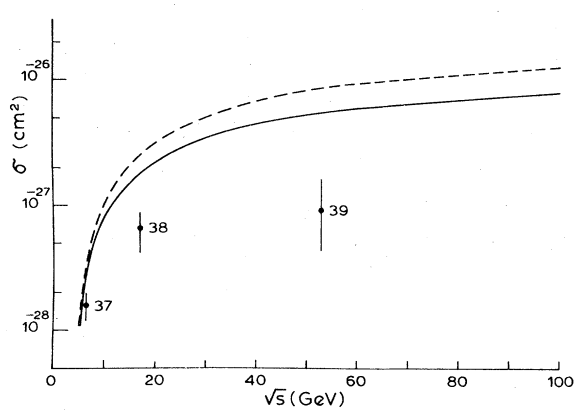

FIG. 3. The lower curve represents our prediction for $\phi(1020)$ production in proton-proton interaction as a function of $\sqrt{s}$. ● are the data points for $\sigma(pp \to \phi X)$. $\xi = 0.19$ GeV, $m_s = 0.4$ GeV, $T = 0.15$ GeV, $N = 0.2$, $\sigma_{qs} \simeq m_c^2/m_s^2\sigma_{qc}$ and $\alpha_s = 0.8$ have been taken. The upper curve represents our QCD prediction for $K\bar{K}$ production. As above, the number attached to each experimental data point indicates the experimental reference.

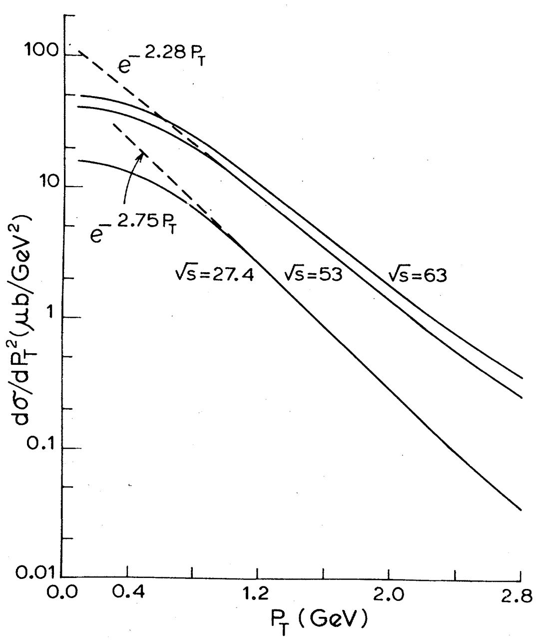

FIG. 5. The results of our calculations for the P_T distribution of D mesons produced in proton-proton interactions at various energies. The dashed lines correspond to a distribution e^{-aP_T} with $a = 2.28$ GeV^{-1} for $\sqrt{s} = 53$, and 63 GeV and $a = 2.75$ GeV^{-1} for $\sqrt{s} = 27.4$ GeV (see text). $m_c = 1.2$ GeV and $\alpha_s = 0.3$.

Fig. 6). The results may be approximated by ce^{-ap_T} with $a = 5.7$ GeV^{-1} at $\sqrt{s} = 53$ and 63 GeV and $a = 6.16$ GeV^{-1} at $\sqrt{s} = 27.4$ GeV for $0.2 < P_T < 0.9$ GeV. For this P_T range, the slope $a = 5.7$ GeV^{-1} we find at $\sqrt{s} = 53$ GeV compared with the experimental slope[22] $a \sim 4$ GeV^{-1}. A quark mass $m_s = 0.5$ GeV yields $a = 4.5$ GeV^{-1} in this model.

We also predict the p_T distribution of $B(5.3)$ produced in proton-proton collisions. The p_T distributions obtained in that case appears very flat, at the energies considered up to $p_T \sim 2$ GeV. For $2.2 < P_T < 4$ GeV a slope $a = 0.86$ GeV^{-1} is found at $\sqrt{s} = 53$ and 63 GeV and a slope $a = 1.5$ GeV^{-1} is found at $\sqrt{s} = 27.4$ GeV (see Fig. 7).

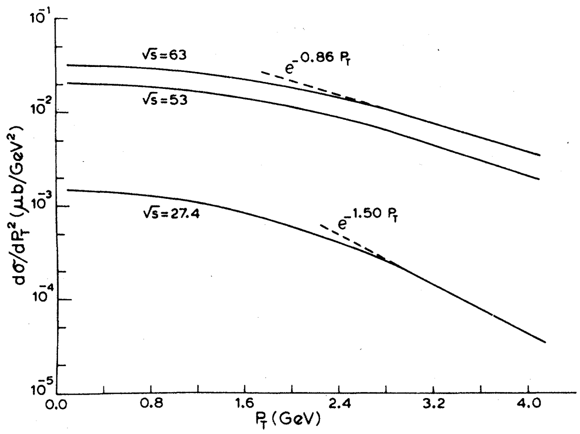

FIG. 7. The P_T distribution of $B(5.3)$ mesons produced in proton-proton interaction. The dashed lines correspond to a distribution e^{-aP_T} with $a = 0.86$ GeV^{-1} for $\sqrt{s} = 53$ and 63 GeV and $a = 1.5$ GeV^{-1} for $\sqrt{s} = 27.4$ GeV. $m_b = 4.0$ GeV and $\alpha_s = 0.23$.

V. SUMMARY AND CONCLUSIONS

We have presented a scheme based on QCD for calculating the production of particles containing heavy quarks. The results are rather good when compared with experiment and with previous estimates. Perhaps the use of more realistic quarkonium potentials will improve agreement with experiment further. The relatively low mass of the b quark and the c quark is imposed both by the data for quarkonium production and by the data for $D\bar{D}$ production. The transverse-momentum distributions of the produced D mesons are in good agreement with experiment.

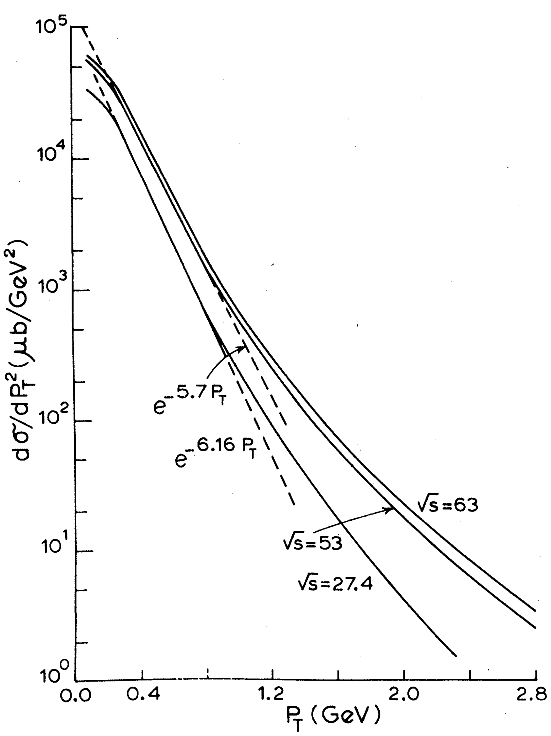

FIG. 6. The P_T distribution of K mesons produced in proton-proton interaction. The dashed lines represent a distribution e^{-aP_T} with $a = 5.7$ GeV^{-1} for $\sqrt{s} = 53$ and 63 GeV and $a = 6.16$ GeV^{-1} for $\sqrt{s} = 27.4$ GeV. $m_s = 0.4$ GeV and $\alpha_s = 0.8$.

APPENDIX

Using Eq. (2) and summing over gluon polarization, we obtain the rate[23]

$$\Gamma(i \to f + g) = \tfrac{4}{3}\,\tfrac{4}{27}\,\omega^3 \alpha_s |\langle f|\vec{r}|i\rangle|^2 , \quad \text{(A1)}$$

where ω is the emitted gluon energy and the color factor $\frac{4}{27}$ has already been extracted from the matrix element.

Using

$$|i\rangle = \frac{e^{ipr}}{\sqrt{V}} |s_a\rangle |s_b\rangle ,$$

$$s_a, s_b = \pm \tfrac{1}{2} ,$$

and

$$|f\rangle = R(r) F(s_a, s_b, \vec{r})$$

with the radial wave functions as

$$R_{1s}(r) = \left(\frac{4\xi^3}{\sqrt{\pi}}\right)^{1/2} e^{-\xi^2 r^2/2} , \quad \text{(A2)}$$

$$R_{2p}(r) = \left(\frac{8\xi^3}{3\sqrt{\pi}}\right)^{1/2} \xi r e^{-\xi^2 r^2/2} , \quad \text{(A3)}$$

$$R_{2s}(r) = \left(\frac{6\xi^3}{\sqrt{\pi}}\right)^{1/2} (1 - \tfrac{2}{3}\xi^2 r^2) e^{-\xi^2 r^2/2} , \quad \text{(A4)}$$

$$R_{3s}(r) = \left(\frac{32}{15\sqrt{\pi}}\right)^{1/2} (\tfrac{15}{8} - \tfrac{5}{2} r^2 \xi^2 + \tfrac{1}{2} r^4 \xi^4) e^{-\xi^2 r^2/2} , \quad \text{(A5)}$$

$$\left(\int R^2 r^2 dr = 1\right) ,$$

and the orbital spin factors:

$$F^{M}_{\psi,\psi'} = \sum_{s_1, s_2} |s_1 s_2\rangle\langle 1M|\tfrac{1}{2}s_1, \tfrac{1}{2}s_2\rangle \frac{1}{\sqrt{4\pi}} \quad \text{for } L=0 \text{ states,} \tag{A6}$$

$$F = \sum_{\substack{m_1 \\ m=0,\pm 1 \\ s_1, s_2 = \pm 1/2}} Y_1^m |s_1 s_2\rangle\langle 1m_1|\tfrac{1}{2}s_1, \tfrac{1}{2}s_2\rangle\langle JM|1m_1, 1m\rangle \quad \text{for } \chi(L=1, s=1) \text{ states,} \tag{A7}$$

averaging over $s_a, s_b = \pm\frac{1}{2}$ and summing over M, we get the result [Eq. (4)].

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