Simple Lagrangian for a superspace affine theory

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Arguments are advanced for considering a particular Lagrangian in superspace. It is constructed solely from the connection associated with the graded group of real, general linear transformations, and is especially simple in form. The resulting equations of motion and the spontaneous-symmetry-breaking solutions, which maintain global supersymmetric invariance, are examined.

I. INTRODUCTION

In previous papers we developed and studied some aspects of the formalism associated with imposing Yang-Mills gauge invariance on theories in superspace.[1-3] In particular, we limited our considerations to the invariance induced by general coordinate transformations. The appropriate group was found to be the graded Lie group of real, general linear transformations. There has been considerable related activity using different approaches, also in superspace, and we list just a few of the papers that are most closely related to the work presented here.[4]

In the present paper a Lagrangian is proposed which yields slightly different equations of motion than previously studied,[1-3] but which compensates by being very simple. In Sec. II we exhibit the proposed Lagrangian. In Sec. III we derive the associated equations of motion, while Sec. IV deals with the spontaneous symmetry breaking of the vacuum. This breaking preserves global supersymmetric invariance. A summary and conclusion is given in Sec. V.

II. THE PROPOSED LAGRANGIAN

We use z^A to denote collectively commuting Bose (x^μ) as well as anticommuting Majorana ($\theta^{\alpha m}$) coordinates (α is a Dirac index, and m is an internal-symmetry index which will be suppressed whenever no confusion arises).

In Ref. 1 we found that we were led to introduce an affine connection Γ_C for the purpose of constructing convariant derivatives. We also showed that[5]

$$\Gamma_C = G_B{}^A \Gamma_{\hat{A}}{}^B{}_C \,, \tag{2.1}$$

where $G_B{}^A$ are the generators of the group of Sec. I, satisfying the algebra

$$G_A{}^B G_C{}^D - (-1)^{(a+b)(c+d)} G_C{}^D G_A{}^B = [\delta^B{}_C \delta^E{}_A \delta^D{}_F - (-1)^{(a+b)(c+d)} \delta^D{}_A \delta^B{}_F \delta^E{}_C] G_E{}^F \,. \tag{2.2}$$

The lower case latin indices, e.g., $a=0,1$ according to whether A is a Bose or Fermi index, respectively. The group parameters $\omega_{\hat{B}}{}^A(z)$ are related to the elements U of the group by

$$U = \exp[G_A{}^B \omega_{\hat{B}}{}^A(z)] \,. \tag{2.3}$$

If we consider a coordinate transformation $z'^A = z^A + \xi^A(z)$, then for infinitesimal ξ^A, $\omega_{\hat{B}}{}^A$ is given by

$$\omega_{\hat{B}}{}^A \simeq \frac{\partial_L \xi^A}{\partial z^B} \,, \tag{2.4}$$

where the subscript L means left derivative.[1] Furthermore, under this infinitesimal transformation, we found that[1] the $\Gamma_{\hat{B}}{}^A{}_C$ were required to satisfy

$$\Gamma'_{\hat{B}}{}^A{}_C(z') \simeq \Gamma_{\hat{B}}{}^A{}_C(z) - (-1)^{c(1+d)} \Gamma_{\hat{B}}{}^A{}_D \omega_{\hat{C}}{}^D - \omega_{\hat{B}}{}^D \Gamma_{\hat{D}}{}^A{}_C + (-1)^{c(a+d)} \Gamma_{\hat{B}}{}^D{}_C \omega_{\hat{D}}{}^A - \omega_{\hat{B}}{}^A{}_{,C} \,. \tag{2.5}$$

In the last term of Eq. (2.5), the comma denotes the right derivative.

The $\Gamma_{\hat{A}}{}^B{}_C$ may be used to construct the fourth-rank curvature tensor $R^C{}_{\hat{D}AB}$ and from this the contracted curvature $R_{\hat{A}B}$, which is given by

$$R_{\hat{A}B} = -(-1)^c \Gamma_{\hat{A}}{}^C{}_{C,B} + (-1)^{c(b+1)} \Gamma_{\hat{A}}{}^C{}_{B,C} + (-1)^{c+be} \Gamma_{\hat{A}}{}^E{}_B \Gamma_{\hat{E}}{}^C{}_C - (-1)^{ce} \Gamma_{\hat{A}}{}^E{}_C \Gamma_{\hat{E}}{}^C{}_B \,. \tag{2.6}$$

Since the $\Gamma_{\hat{A}}{}^B{}_C$ do not satisfy any particular symmetry constraint, there are three additional second-rank tensors that may be constructed with them. These are given in the Appendix.

It is possible to find a Lagrangian that will yield the equations of motion $R_{\hat{A}B;C}=0$ postulated in Ref. 1. (The semicolon denotes the right covariant derivative.) However, it is rather ugly and we look here for something much simpler. In particular, the simplest would be a scalar density which is just the square root of the determinant of a second-rank tensor. Since we wish to accommodate all fields (including gravitation) we choose this tensor to be $R_{\hat{A}B}$. *A priori* we could add other terms involving the other three tensors, but this introduces a set of arbitrary constants and, moreover, detracts from the simplicity of the model. The resulting Lagrangian is the superspace analog of Schrödinger's.[6] However, we will include all fields (Fermi as well as Bose) in the connection, so the difference will be enormous. Thus the action is given by

$$S=\int d^{4(N+1)}z\,\sqrt{-R}\ , \tag{2.7}$$

where $R=\det|R_{\hat{A}B}|$, and $4N$ is the number of Fermi coordinates. We note that S is dimensionless.[7]

There are two equivalent definitions of the determinant in superspace.[8,9] Using either one we find that an infinitesimal variation $\delta R_{\hat{A}B}$ induces a corresponding change in R given by

$$\delta R=(-1)^{b}RR^{BA}\delta R_{\hat{A}B}\ , \tag{2.8}$$

where R^{BA} is the inverse of $R_{\hat{A}B}$ (i.e., $R^{BA}R_{\hat{A}C}=\delta^{B}{}_{C}$).

Using Eq. (2.8) one may prove that the action of Eq. (2.7) is invariant under general coordinate transformations.

III. EQUATIONS OF MOTION

In order to obtain the equations of motion from the action, we need to know the covariant derivative of supervector densities for theories with torsion. Again using Eq. (2.8) one may show that if Ω^{A} is a vector density, given by $\Omega^{A}=\sqrt{-R}\,V^{A}$, where V^{A} is a vector, then

$$\Omega^{A}{}_{;B}=\Omega^{A}{}_{,B}+\Omega^{C}\Gamma_{\hat{C}}{}^{A}{}_{B}-(-1)^{c}\Omega^{A}\Gamma_{\hat{C}}{}^{C}{}_{B}\ . \tag{3.1}$$

Thus, the supercovariant divergence of Ω^{A} is given by

$$(-1)^{a}\Omega^{A}{}_{;A}=(-1)^{a}\Omega^{A}{}_{,A}+2\Omega^{A}\Lambda_{\hat{A}}\ , \tag{3.2}$$

where

$$\Lambda_{\hat{A}}=\tfrac{1}{2}(-1)^{b}[\Gamma_{\hat{A}}{}^{B}{}_{B}-(-1)^{a}\Gamma_{\hat{B}}{}^{B}{}_{A}]. \tag{3.3}$$

For theories in which $\Lambda_{\hat{A}}=0$, Eq. (3.2) reduces to the definition given in Ref. 8. However, in the present development, $\Lambda_{\hat{A}}\neq 0$ and the second term on the right-hand side of Eq. (3.2) is important.

Inserting Eq. (2.8) into Eq. (2.7) we have

$$\delta S=\tfrac{1}{2}\int d^{4(N+1)}z\,(-1)^{b}\mathcal{R}^{BA}\delta R_{\hat{A}B}\ , \tag{3.4}$$

where

$$\mathcal{R}^{BA}\equiv\sqrt{-R}\,R^{BA}\ . \tag{3.5}$$

$\delta R_{\hat{A}B}$ may be calculated using Eq. (2.6) and noting that $\delta\Gamma_{\hat{D}}{}^{C}{}_{A}$ is a tensor whose right covariant derivative is given by[1]

$$\begin{aligned}\delta\Gamma_{\hat{D}}{}^{C}{}_{A;B}=\{&\delta\Gamma_{\hat{D}}{}^{C}{}_{A,B}+(-1)^{a(c+e)}\delta\Gamma_{\hat{D}}{}^{E}{}_{A}\Gamma_{\hat{E}}{}^{C}{}_{B}\\ &-(-1)^{b(a+c+e)}\Gamma_{\hat{D}}{}^{E}{}_{B}\delta\Gamma_{\hat{E}}{}^{C}{}_{A}\\ &-(-1)^{a+ae}\delta\Gamma_{\hat{D}}{}^{C}{}_{E}\Gamma_{\hat{A}}{}^{E}{}_{B}\}.\end{aligned} \tag{3.6}$$

We then obtain

$$\begin{aligned}\delta R_{\hat{D}B}=\{&-(-1)^{c}\delta\Gamma_{\hat{D}}{}^{C}{}_{C;B}+(-1)^{c+cb}\delta\Gamma_{\hat{D}}{}^{C}{}_{B;C}\\ &-(-1)^{ce}\delta\Gamma_{\hat{D}}{}^{C}{}_{E}T_{\hat{C}}{}^{E}{}_{B}\},\end{aligned} \tag{3.7}$$

where $T_{\hat{C}}{}^{E}{}_{B}$ is twice the antisymmetric part of $\Gamma_{\hat{C}}{}^{E}{}_{B}$:

$$T_{\hat{C}}{}^{E}{}_{B}\equiv[\Gamma_{\hat{C}}{}^{E}{}_{B}-(-1)^{b+c+be+ce+bc}\Gamma_{\hat{B}}{}^{E}{}_{C}]. \tag{3.8}$$

Inserting Eq. (3.7) into Eq. (3.4) and using Eq. (3.2) gives (upon discarding surface terms)

$$\begin{aligned}\delta S=-\tfrac{1}{2}\int d^{4(N+1)}z\,(-1)^{b}\{&(-1)^{c}\delta^{B}{}_{D}[-(-1)^{ac}\mathcal{R}^{CA}{}_{;C}\\ &+2\Lambda_{\hat{C}}\mathcal{R}^{CA}]\\ &+(-1)^{ad}\mathcal{R}^{BA}{}_{;D}-2(-1)^{d+bd}\Lambda_{\hat{D}}\mathcal{R}^{BA}\\ &+(-1)^{d+bd}T_{\hat{D}}{}^{B}{}_{E}\mathcal{R}^{EA}\}\delta\Gamma_{\hat{A}}{}^{D}{}_{B}\ .\end{aligned} \tag{3.9}$$

In order that δS vanish, the coefficient of $(-1)^{b}\delta\Gamma_{\hat{A}}{}^{D}{}_{B}$ must do so, since $\Gamma_{\hat{A}}{}^{D}{}_{B}$ has no symmetry. Hence, the equation of motion is obtained by setting the contents of the curly brackets in Eq. (3.9) equal to zero.

Taking $(-1)^{d}\delta^{D}{}_{B}$ of that equation and using $(-1)^{d}\delta^{D}{}_{B}\delta^{B}{}_{D}=-4(N-1)$, one may show that

$$(-1)^{b+ab}\mathcal{R}^{BA}{}_{;B}=\frac{2(4N-2)}{(4N-3)}(-1)^{b}\Lambda_{\hat{B}}\mathcal{R}^{BA}\ . \tag{3.10}$$

[Note: $(-1)^{b}T_{\hat{B}}{}^{B}{}_{E}=-2(-1)^{e}\Lambda_{\hat{E}}$.]

Inserting Eq. (3.10) back into the equation of motion, one obtains the alternate form

$$\begin{aligned}\mathcal{R}^{BA}{}_{;D}-\frac{2}{(4N-3)}(-1)^{c+ad}\delta^{B}{}_{D}\Lambda_{\hat{C}}\mathcal{R}^{CA}-2(-1)^{d+ad+bd}\Lambda_{\hat{D}}\mathcal{R}^{BA}&\\ +(-1)^{d+ad+bd}T_{\hat{D}}{}^{B}{}_{E}\mathcal{R}^{EA}=0\ .&\end{aligned} \tag{3.11}$$

After some manipulation Eq. (3.11) may be cast in the form

$$\begin{aligned}R_{\hat{A}B;C}+\frac{2}{(4N-3)}(-1)^{c}R_{\hat{A}B}\Lambda_{\hat{C}}+\frac{2}{(4N-3)}(-1)^{b+bc}R_{\hat{A}C}\Lambda_{\hat{B}}&\\ +(-1)^{b+bd}R_{\hat{A}D}T_{\hat{B}}{}^{D}{}_{C}=0\ .&\end{aligned} \tag{3.12}$$

We note that if $T_{\hat{B}}{}^{D}{}_{C}=0$ (and hence also $\Lambda_{\hat{C}}=0$) Eq. (3.12) reduces to

$$R_{\hat{A}B;C}=0\ . \tag{3.13}$$

Equation (3.13) was the equation postulated in Ref. 1. Thus, it follows from the action of Eq. (2.7) when the connection is constrained to be "symmetric" (i.e., no torsion). In the general case Eq. (3.12) is a set of equations of motion for the particle fields, which are contained in $\Gamma_{\hat{A}}{}^{B}{}_{C}(z)$, with no *a priori* constraints placed on the torsion. The particle content is extracted by expanding the connection in powers of $\theta^{\alpha m}$ (we will suppress the internal-symmetry index m when no confusion arises). Thus

$$\Gamma_{\hat{A}}{}^{B}{}_{C}(z)=\Gamma_{\hat{A}}{}^{B}{}_{C}(x)_{[0]}+\sum_{n=1}^{4N}\Gamma_{\hat{A}}{}^{B}{}_{C}(x)_{[\alpha_1\cdots\alpha_n]}\theta^{\alpha_1}\cdots\theta^{\alpha_n}. \qquad (3.14)$$

The coefficients of the $\theta^{\alpha_1}\cdots\theta^{\alpha_n}$ are the dynamical particle fields of the theory. As an example, $\Gamma_{\nu}{}^{\mu}{}_{\lambda}(x)_{[0]}$ is the connection of the gravitational field and its determination (for the free field case) is the same as in Ref. 3. Similarly, $\Gamma_{\alpha}{}^{\beta}{}_{\mu}(x)_{[0]}=A_{a\mu}(x)(T_a)^{\beta}{}_{\alpha}+\cdots$ contains the vector potential. The spin-$\frac{3}{2}$ field[10] appears in

$$\Gamma^{\lambda}_{\mu\alpha[0]}=c_1(\bar{\psi}_{\rho}\sigma^{ab}\gamma^{c})_{\alpha}e_a{}^{\rho}(x)e_b{}^{\lambda}(x)e_{c\mu}(x)+c_2(\bar{\psi}_{\rho}(x)\gamma_c\gamma_5)_{\alpha}\epsilon^{abcd}e_a{}^{\rho}(x)e_b{}^{\lambda}(x)e_{d\mu}(x)+\cdots, \qquad (3.15)$$

where c_1 and c_2 are numbers and $e_b{}^{\lambda}(x)$ is the vierbein field. Each field is additionally contained in other terms as well. By taking the coefficients of $\theta^{\alpha_1}\cdots\theta^{\alpha_n}$ in Eq. (3.12) one obtains the equations of motion for the particle fields. This aspect of the problem is still under investigation.

IV. SPONTANEOUS SYMMETRY BREAKING

In this section we shall exhibit two classes of interesting solutions to Eq. (3.12) that break local gauge invariance under the graded group of real, general linear transformations, but preserve global supersymmetric invariance.

This latter transformation is given by

$$\xi^{\mu}=i\bar{\epsilon}\Gamma^{\mu}\theta, \quad \xi^{\alpha}=\epsilon^{\alpha}, \qquad (4.1)$$

where ϵ^{α} is an infinitesimal constant Majorana spinor. Using Eq. (2.4) we determine the $\omega_{\hat{B}}{}^{A}$ to be

$$\omega_{\mu}{}^{\nu}=\omega_{\alpha}{}^{\beta}=\omega_{\mu}{}^{\alpha}=0$$

and

$$\omega_{\alpha}{}^{\mu}=i(\bar{\epsilon}\Gamma^{\mu})_{\alpha}, \qquad (4.2)$$

where

$$\Gamma^{\mu}=\gamma^{\mu}M_V+\gamma_5\gamma^{\mu}M_A. \qquad (4.3)$$

M_V and M_A are real internal-symmetry matrices (with dimensions of mass) satisfying the conditions $M_V=M_V{}^T$ and $M_A=-M_A{}^T$ and hence,[11] $(\eta\Gamma_{\mu})^T=(\eta\Gamma_{\mu})$.

In Ref. 2 we had given the most general vacuum $\Gamma_{\hat{A}}{}^{B}{}_{C}$ that are form invariant under Eqs. (2.5), (4.1), and (4.2). They are

$$\begin{aligned}&\Gamma_{\mu}{}^{\alpha}{}_{\nu}=0,\quad \Gamma_{\nu}{}^{\mu}{}_{\lambda}=0,\\ &\Gamma_{\beta}{}^{\alpha}{}_{\mu}=-(I_{\mu})^{\alpha}{}_{\beta},\quad \Gamma_{\mu}{}^{\alpha}{}_{\beta}=-(K_{\mu})^{\alpha}{}_{\beta},\\ &\Gamma_{\nu}{}^{\mu}{}_{\alpha}=i(\bar{\theta}\Gamma^{\mu}K_{\nu})_{\alpha},\quad \Gamma_{\alpha}{}^{\mu}{}_{\nu}=-i(\bar{\theta}\Gamma^{\mu}I_{\nu})_{\alpha},\\ &\Gamma_{\alpha}{}^{\delta}{}_{\beta}=i(\bar{\theta}\Gamma^{\lambda})_{\alpha}(K_{\lambda})^{\delta}{}_{\beta}-i(\bar{\theta}\Gamma^{\lambda})_{\beta}(I_{\lambda})^{\delta}{}_{\alpha},\\ &\Gamma_{\alpha}{}^{\mu}{}_{\beta}=(\bar{\theta}\Gamma^{\mu}I_{\nu})_{\alpha}(\bar{\theta}\Gamma^{\nu})_{\beta}-(\bar{\theta}\Gamma^{\mu}K_{\nu})_{\beta}(\bar{\theta}\Gamma^{\nu})_{\alpha}-i(\eta L^{\mu})_{\alpha\beta}.\end{aligned} \qquad (4.4)$$

In Eq. (4.4), I_{μ}, K_{μ}, and L_{μ} are constant matrices of the same form as Γ_{μ} in Eq. (4.3), e.g.,

$$I_{\mu}=\gamma_{\mu}I_V+\gamma_5\gamma_{\mu}I_A, \text{ etc.}$$

However, the internal-symmetry matrices I_V, I_A, etc., have no *a priori* symmetry constraints.

For the remainder of this section, $R_{\hat{A}B}$, etc., shall be the vacuum quantities calculated using Eqs. (4.4). We then insert these results into Eq. (3.12) to determine those I_{μ}, K_{μ}, and L_{μ} which result in the spontaneously broken solutions.

We first note that the $\Gamma_{\hat{A}}{}^{B}{}_{C}$ of Eq. (4.4) satisfy

$$(-1)^c\Gamma_{\hat{A}}{}^{C}{}_{C}=(-1)^c\Gamma_{\hat{C}}{}^{C}{}_{A}=0, \qquad (4.5)$$

and thus $\Lambda_{\hat{A}}=0$. Hence, Eq. (3.12) becomes, upon using Eq. (3.8), and expressing $R_{\hat{A}B;C}$ in detail,

$$R_{\hat{A}B,C}-(-1)^{c(b+d)}\Gamma_{\hat{A}}{}^{D}{}_{C}R_{\hat{D}B}-(-1)^{c(b+c+d)}R_{\hat{A}D}\Gamma_{\hat{C}}{}^{D}{}_{B}=0. \qquad (4.6)$$

Furthermore, we find that the $R_{\hat{A}B}$ of Eq. (2.6) are given by

$$R_{\mu\nu}=C\eta_{\mu\nu}, \qquad (4.7a)$$

$$R_{\mu\alpha}=-R_{\alpha\mu}=iC(\bar{\theta}\Gamma_{\mu})_{\alpha}, \qquad (4.7b)$$

$$R_{\alpha\beta}=i[\eta(\Gamma^{\lambda}-L^{\lambda})K_{\lambda}]_{\alpha\beta}-i[\eta(\Gamma^{\lambda}-\tilde{L}^{\lambda})I_{\lambda}]_{\beta\alpha}+C(\bar{\theta}\Gamma^{\lambda})_{\alpha}(\bar{\theta}\Gamma_{\lambda})_{\beta}, \qquad (4.7c)$$

where

$$C=\tfrac{1}{4}\operatorname{tr}(I^{\lambda}K_{\lambda}) \qquad (4.8)$$

and

$$\tilde{L}^{\lambda}\equiv-\eta^{-1}(\eta L^{\lambda})^T. \qquad (4.9)$$

In Eq. (4.8) the trace is over the Dirac and the internal-symmetry indices.

Inserting Eqs. (4.4) and (4.7) into (4.6), then yields the following algebraic equations relating I^{μ}, K^{μ}, L^{μ}, and Γ^{μ}:

$$C(\Gamma_{\mu}+L_{\mu})+\tilde{K}_{\mu}S=0, \qquad (4.10a)$$

$$C(\Gamma_{\mu}-L_{\mu})-SI_{\mu}=0, \qquad (4.10b)$$

$$SK_{\mu}+\tilde{I}_{\mu}S=0, \qquad (4.10c)$$

where

$$S=(\Gamma^{\lambda}-L^{\lambda})K_{\lambda}-\tilde{I}_{\lambda}(\Gamma^{\lambda}+L^{\lambda})\,. \tag{4.10d}$$

We present two classes of solutions.

Case I. $K^{\mu}=I^{\mu}$ and $L^{\mu}=\tilde{L}^{\mu}$. If we require that $\Gamma^{\mu}\Gamma_{\mu}\neq 0$ and $C\neq 0$, then we may define a metric tensor $g_{\hat{A}B}$, as was done in Ref. 1, by $g_{\hat{A}B}\equiv 1/\lambda R_{\{\hat{A}B\}}$. $R_{\{\hat{A}B\}}$ is the "symmetric" part of $R_{\hat{A}B}$ (i.e., $R_{\{\hat{A}B\}}=\frac{1}{2}[R_{\hat{A}B}+(-1)^{a+b+ab}R_{\hat{B}A}]$). If we normalize the vacuum $g_{\hat{A}B}$ so that $g_{\mu\nu}=\eta_{\mu\nu}$, then $\lambda=C$. However, we find that the Fermi dimensionality of $4N$ is fixed with $N=2$. In this case $g_{\hat{A}B}$ does have an inverse. We also note that the vacuum $\Gamma_{\hat{A}}{}^{B}{}_{C}$ is "symmetric"; the torsion playing no role in the spontaneous symmetry breaking.

Case II. $K^{\mu}=-I^{\mu}$ and $L_{\mu}=\tilde{L}_{\mu}$. If we define $g_{\hat{A}B}\equiv(1/\lambda)R_{\{\hat{A}B\}}$, then $g_{\hat{A}B}$ will not have an inverse, since the vacuum $g_{\alpha\beta}$ is given by

$$g^{0}_{\alpha\beta}=(\bar{\theta}\Gamma^{\lambda})_{\alpha}(\bar{\theta}\Gamma_{\lambda})_{\beta} \tag{4.11}$$

with no constant θ-independent term. However, $R_{\hat{A}B}$ does have an inverse, since the "antisymmetric" part of $R_{\hat{A}B}$ does have a constant θ-independent term. This then permits us to expand about the broken-symmetry solution as the background, when quantizing the theory in the Feynman path-integral formalism. For this case the vacuum $\Gamma_{\hat{A}}{}^{B}{}_{C}$ has no particular symmetry. However, if $L_{\mu}=0$, then it is "antisymmetric," so that all of the spontaneous symmetry breaking comes from the torsion.

In case II as in case I we find that $N=2$. However, one expects that there will be quantum corrections which will relax this restriction.[13]

As a last remark, we note that one may rewrite the vacuum $R_{\alpha\beta}$ of Eq. (4.7c) in terms of S of Eq. (4.10d) as

$$R_{\alpha\beta}=i(\eta S)_{\alpha\beta}+C(\bar{\theta}\Gamma^{\lambda})_{\alpha}(\bar{\theta}\Gamma_{\lambda})_{\beta}\,. \tag{4.12}$$

Hence, the invertibility of $g_{\hat{A}B}$ depends upon (ηS) having a piece that is antisymmetric under transposition.

V. CONCLUDING REMARKS

In this paper, we have given arguments for considering a very simple Lagrangian constructed solely from the affine connection associated with the graded group of real, general linear transformations on superspace. No dimensioned constants had to be introduced to make the action dimensionless. We then obtained the resulting equations of motion and indicated how they would result in a set of equations for the particle fields. A set of algebraic equations which set conditions on spontaneously symmetry-breaking solutions were then derived. However, invariance under global supersymmetric transformations was maintained. Two examples of solutions to these equations were exhibited, both of which could be used as the background for quantization in the Feynman path-integral formalism.

The object of this work is to lay the foundation for the construction of a unified theory of all particles and their interactions. This goal is in common with most of the works cited in Ref. 4.

APPENDIX: SECOND-RANK TENSORS CONSTRUCTED WITH THE CONNECTION

In addition to $R_{\hat{A}B}$, there are three more independent second-rank tensors which may be constructed from the connection. We choose them to be $S_{\hat{A}B}$, $(-1)^{c+bc}T_{\hat{A}}{}^{C}{}_{B;C}$, and $U_{\hat{A}B}$. $T_{\hat{A}}{}^{C}{}_{B}$ is defined in Eq. (3.8), while the semicolon denotes the right covariant derivative. They are given by

$$S_{\hat{A}B}=(-1)^{a+c}[(-1)^{ab}\Gamma_{\hat{C}}{}^{C}{}_{B,A}-\Gamma_{\hat{C}}{}^{C}{}_{A,B}]\,, \tag{A1}$$

$$\begin{aligned}(-1)^{c+bc}T_{\hat{A}}{}^{C}{}_{B;C}=&(-1)^{c+bc}[\Gamma_{\hat{A}}{}^{C}{}_{B,C}-(-1)^{a+b+ab+ac+bc}\Gamma_{\hat{B}}{}^{C}{}_{A,C}]+(-1)^{c}[(-1)^{be}\Gamma_{\hat{A}}{}^{E}{}_{B}\Gamma_{\hat{E}}{}^{C}{}_{C}-(-1)^{a+b+ab+ae}\Gamma_{\hat{B}}{}^{E}{}_{A}\Gamma_{\hat{E}}{}^{C}{}_{C}]\\&-(-1)^{ce}[\Gamma_{\hat{A}}{}^{E}{}_{C}\Gamma_{\hat{E}}{}^{C}{}_{B}-(-1)^{a+b+ab}\Gamma_{\hat{B}}{}^{E}{}_{C}\Gamma_{\hat{E}}{}^{C}{}_{A}]\,,\end{aligned} \tag{A2}$$

and

$$U_{\hat{A}B}=\frac{(-1)^{a+c}}{2}[(-1)^{b+ab}\Gamma_{\hat{B}}{}^{C}{}_{C,A}-(-1)^{a}\Gamma_{\hat{A}}{}^{C}{}_{C,B}]\,. \tag{A3}$$

That these are tensors may be verified by the direct use of Eq. (2.5). However, the following relationships are informative, and may be used instead:

$$S_{\hat{A}B}=(-1)^{a}\delta^{D}{}_{C}R^{C}{}_{\hat{D}AB}\,, \tag{A4}$$

$$\begin{aligned}R_{[\hat{A}B]}&\equiv\tfrac{1}{2}[R_{\hat{A}B}-(-1)^{a+b+ab}R_{\hat{B}A}]\\&=U_{\hat{A}B}+\frac{(-1)^{c+bc}}{2}T_{\hat{A}}{}^{C}{}_{B;C}\,.\end{aligned} \tag{A5}$$

Since $T_{\hat{A}}{}^{C}{}_{B}$ is a tensor (being twice the antisymmetric part of $\Gamma_{\hat{A}}{}^{C}{}_{B}$), as is $R_{[\hat{A}B]}$, it follows from Eq. (A5) that $U_{\hat{A}B}$ is a tensor. The simplified form of Eq. (A2) results because of a cancellation occurring in $(-1)^{c+bc}T_{\hat{A}}{}^{C}{}_{B;C}$ when expressed completely in terms of $\Gamma_{\hat{A}}{}^{C}{}_{B}$.

We note that the three tensors of this section are all antisymmetric [e.g., $S_{\hat{A}B}=-(-1)^{a+b+ab}S_{\hat{B}A}$]. Furthermore, they vanish identically in a Riemannian theory.

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[5]$\hat{A}$ denotes an index that transforms as the left derivative of a scalar (for details see Ref. 1).

[6]E. Schrödinger, *Space-Time Structure* (Cambridge Univ. Press, London, 1954).

[7]We choose the dimension of the Majorana coordinates to be the same as those of the Bose, i.e., l. Thus $\Gamma_{\hat{A}}{}^{B}{}_{C}$ has the dimension l^{-1}. Hence $R_{\hat{A}B}$ has the dimension l^{-2}, while R^{AB} has dimension l^2. It has been shown (Ref. 8) that $R = \det|R_{\hat{A}B}| = (\det|R_{\mu\nu}|)(\det|R^{\alpha\beta}|)$ where these latter determinants are the customary Bose quantities defined in the (μ,ν) and (α,β) subspaces, respectively. Thus R has the dimension $l^{-8+8N} = l^{8(N-1)}$. The integral of the differential $\int d^{4(N+1)}z \cdots$ has dimension $l^{4-4N} = l^{-4(N-1)}$. This follows from the equality of integration and differentiation for Fermi coordinates (Ref. 12). Thus, the dimensions of $\sqrt{-R}$ and $(\int d^{4(N+1)}z \cdots)$ cancel leaving S dimensionless.

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[10]We use the Lorentz metric $\eta_{\mu\nu} = (-1,1,1,1)$ and a Majorana representation with $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, and $\gamma_5{}^2 = -1$. $\eta_{\alpha\beta}$ is related to the charge conjugation matrix C (in Dirac space) by $\eta_{\alpha\beta} = -(C^{-1})_{\alpha\beta}$. Thus, for the Majorana coordinates we may choose $\theta_\alpha = \theta^\beta \eta_{\beta\alpha}$, etc.

[11]The antisymmetric piece of $(\eta\Gamma_\mu)$ can be transformed away from the supersymmetric line element given by $ds^2 = (dx^\mu + i\bar{\theta}\Gamma^\mu d\theta)^2 + id\bar{\theta}Pd\theta$, where P is a real internal-symmetry matrix with $P = P^T$.

[12]For further discussion see F. A. Berezin, *Method of Second Quantization* (Academic, New York, 1966).

[13]P. Nath and R. Arnowitt, Phys. Rev. D 18, 2759 (1978).