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Hyperon decays revisited

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We examine the standard quantum-chromodynamics-inspired description of nonleptonic hyperon decays. We point out that the discrepancy between s- and p-wave amplitudes which emerged from a PCAC (partial conservation of axial-vector current) analysis cannot be cured by the inclusion of factorizable diagrams. We include anomalous contributions to the axial-vector current divergence which appear in the presence of the effective local Fermi interaction. Like factorizable diagrams, these preserve the successful results of PCAC for s-wave amplitudes, but are insufficient to give a quantitatively satisfactory description. We estimate the decay rates for $\Omega^- \to \Xi^* \pi$ and $\Xi^0 l^- \nu_l$ in view of further tests of the standard model.

I. INTRODUCTION

The dynamics of nonleptonic decays has received renewed attention since the first applications[1] of operator-product and renormalization-group techniques to these processes in a quantum-chromodynamics (QCD) framework. The results of these analyses were encouraging in that a dynamical enhancement (suppression) was found for the effective Fermi operator responsible for $\Delta I = \frac{1}{2}$ ($\frac{3}{2}$) transitions. While the numerical values of the enhancement/suppression factors were insufficient to account for the observed amplitude ratios, there were a number of arguments,[2,3] based on the quark model and soft-pion analysis, which suggested a further enhancement/suppresion of the matrix elements of these operators. It was subsequently pointed out[4] that operators arising from the purely $\Delta I = \frac{1}{2}$ "penguin" diagrams, which vanish in both the SU(4) and free-quark limits, have chiral properties which give a large relative enhancement of their matrix elements. These techniques were used to predict[5] Ω^- decay amplitudes which turned out to agree with experiment[6,7] within the uncertainties on the theoretical parameters. A semiquantitative understanding of both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ amplitudes thus began to emerge.[8] More recently,[9] current ideas, including the MIT bag model for determining baryon-to-baryon current matrix elements, have been used to calculate parameter-free predictions which agree well with the s-wave data, and less well (except in sign) with p-wave data. The failure to describe adequately both s and p waves is an old problem related to the use of current algebra.[10]

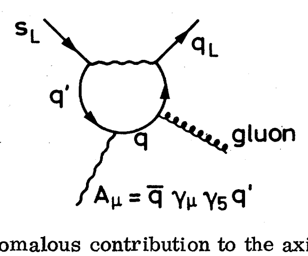

FIG. 1. Anomalous contribution to the axial-vector current divergence (hard-gluon renormalization of the Fermi coupling is implicit).

In this note we separate the amplitudes arising from different mechanisms so as to see more precisely just where the discrepancy lies. In particular we display a sum rule relating s- and p-wave amplitudes which follows directly from the standard current-algebra-cum-pole analysis,[10] and is insensitive to those factorizable diagrams which vanish in the soft-pion limit. We point out that the axial-vector current has an anomalous divergence[11] in the presence of the effective local weak interaction arising from the diagram of Fig. 1. The presence of this term introduces additional parameters into the analysis, but is insufficient to allow a satisfactory description of both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ amplitudes. We also present branching-ratio estimates for the decay modes $\Omega^- \to \Xi^* \pi$ and $\Omega^- \to \Xi^0 l^- \nu$, which should be measurable, and have in fact been observed, in the CERN hyperon-beam experiment currently under analysis, and which allow further tests of the nonleptonic decay mechanism. In particular, a measurement of the semileptonic decay matrix elements will sharpen the prediction for $\Omega^- \to \Xi\pi$.

II. ANALYSIS OF NONLEPTONIC HYPERON DECAYS

We first recall the procedure for the soft-pion analysis.[10] The amplitude for $A \to B\pi$ is evaluated in the limit of vanishing pion mass using chiral symmetry. The on-shell amplitude can be expressed in terms of the identity

$$\mathfrak{M}(p_\pi) \equiv \tilde{\mathfrak{M}}(p_\pi) + \mathfrak{M}(0), \tag{2.1}$$

where $\tilde{\mathfrak{M}}(p_\pi)$ contains the momentum dependence of the amplitude and $\mathfrak{M}(0)$ is its zero-energy value. Rapid variations with p_π are expected if there are nearby poles, and these are explicitly taken into account in evaluating $\tilde{\mathfrak{M}}$; other contributions are assumed to be negligible. The hyperon decay amplitudes are evaluated using the $V-A$ property of the weak current, approximate chiral SU(2) symmetry, approximate SU(3) symmetry for matrix elements, and retaining only the poles arising from $\frac{1}{2}^+$ intermediate states in $\tilde{\mathfrak{M}}$. We write the amplitude for the weak decay $a \to b + \pi^0$ as

$$i\mathfrak{M}(a \to b + \pi^Q) = G_F m_{\pi^+}{}^2 \bar{u}_b (A_{ba}^Q - B_{ba}^Q \gamma_5) u_b. \tag{2.2}$$

$[1+\gamma_5$ is a positive-helicity projection operator. With the definition (2.2) our sign conventions are the same as in the Particle Data Tables except for the sign of $\mathfrak{M}(\Sigma_0^+)$.$]$ Using the above prescription, one finds for s-wave amplitudes

$$G_F m_{\pi^+}{}^2 \bar{u}_b u_a A_{ba}^Q = \begin{cases} -\dfrac{1}{f_\pi} \langle b | [T^Q, \mathcal{H}] | a \rangle, \quad Q = \pm 1, & (2.3\text{a}) \\[2ex] \dfrac{1}{\sqrt{2} f_\pi} \langle b | \mathcal{H} | a \rangle \equiv \dfrac{1}{\sqrt{2} f_\pi} \mathcal{H}_{ba}, \quad Q = 0, & (2.3\text{b}) \end{cases}$$

where f_π is the pion decay constant ($f_\pi = 0.944 m_{\pi^+}$), $\mathcal{H}$ is the effective weak interaction Hamiltonian, and $T^\pm$ are the usual isospin raising and lowering operators; we have used the $\Delta I_3 = -\frac{1}{2}$ property of $\mathcal{H}$ in writing (2.3b). For p-wave amplitudes we obtain

$$G_F m_{\pi^+}{}^2 B_{ba}^Q = -\frac{1}{f_\pi} (m_a + m_b) \sum_c \left(g_{bc}^Q \frac{\mathcal{H}_{ca}}{m_a - m_c} + \frac{\mathcal{H}_{bc}}{m_b - m_c} g_{ca}^Q \right), \tag{2.4}$$

where g_{ba}^0 is the axial-vector coupling constant

$$g_{ba}^Q \bar{u}_b \gamma_\mu \gamma_5 u_a = \langle b | A_\mu^Q | a \rangle \tag{2.5}$$

and is expressible in terms of the measured[12] parameters

$$F = 0.428, \quad D = 0.823. \tag{2.6}$$

Using (2.3b) in (2.4) we can write sum rules of the form

$$\frac{B_{ba}^Q}{m_a + m_b} = -\sqrt{2} \sum \left(g_{bc}^Q \frac{A_{ca}^0}{m_a - m_c} + \frac{A_{bc}^0}{m_b - m_c} g_{ca}^Q \right), \tag{2.7}$$

where the unmeasurable Σ^0 decay amplitude can be eliminated using Eqs. (2.3):

$$A(\Sigma_0^0) = \tfrac{1}{2} A(\Sigma_-^-) = 0.97 \pm 0.01 \tag{2.8}$$

in the conventional notation, $A(\Sigma_0^0) \equiv A_{n\Sigma^0}^0$, etc., and with the measured amplitudes[13] listed in Table I. Equations (2.3) also imply the relation (differing by the sign of Σ_+^+ from the $\Delta I = \frac{1}{2}$ relation)

$$A(\Sigma_-^-) + A(\Sigma_+^+) = -\sqrt{2} A(\Sigma_0^+), \tag{2.9}$$

which is in good agreement with the data of Table I:

$$2.00 \pm 0.02 = 2.09 \pm 0.07. \tag{2.10}$$

Then with *no further assumption* the p-wave amplitudes for Λ and Σ decays are completely determined in terms of the s waves. These predictions are also shown in Table I. If in addition we assume the $\Delta I = \frac{1}{2}$ rule and approximate SU(3) symmetry for the $\mathcal{H}_{ba}$ we get the relations

$$\begin{aligned} &2A(\Xi_0^0) + A(\Lambda_0^0) \simeq -(\tfrac{3}{2})^{1/2} A(\Sigma_0^+) \\ &(1.66 \pm 0.04 = 1.48 \pm 0.05), \\ &A(\Sigma_+^+) = 0 \\ &(0.07 \pm 0.02 = 0), \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} A(\Xi^0 \to \Sigma^0 \pi^0) &= \frac{1}{\sqrt{2}} (\Xi^- \to \Sigma^- \pi^0) \\ &\simeq \frac{1}{\sqrt{3}} [2A(\Lambda_0^0) + A(\Xi_0^0)] \\ &= -0.34 \pm 0.03. \end{aligned} \tag{2.12}$$

Equation (2.12) can be used to calculate p-wave amplitudes for Ξ decay, giving the values in parentheses in Table I. Except for Λ decay the soft-pion sum rules give a poor approximation to the data, and the discrepancy for Ξ decay is too large to be attributed to the small violations of the $\Delta I = \frac{1}{2}$ rule (generally 5%) and SU(3) symmetry (gene-

TABLE I. Experimental values (Ref. 13) of hyperon decay amplitudes and current-algebra predictions for p waves derived from s waves as described in the text. Bracketed entries include Coulomb corrections (Ref. 14). The predictions in parentheses require further assumptions (see text).

	s waves	p waves	
	A	B	Prediction
Λ_-^0	1.47 ± 0.01 [1.44]	9.98 ± 0.24 [9.80]	10.60 ± 0.56
Λ_0^0	-1.07	-7.14 ± 0.56	-8.12 ± 0.40
Σ_+^+	0.07 ± 0.02	19.04 ± 0.16	0.16 ± 0.33
Σ_0^+	-1.48 ± 0.05	11.99 ± 0.58	4.91 ± 0.17
Σ_-^-	1.93 ± 0.01	-0.65 ± 0.08	-5.32 ± 0.23
Ξ_0^0	1.55 ± 0.03	-5.96 ± 1.12	(-1.06 ± 0.56)
Ξ_-^-	2.04 ± 0.02	-6.70 ± 0.38	(-1.50 ± 0.78)

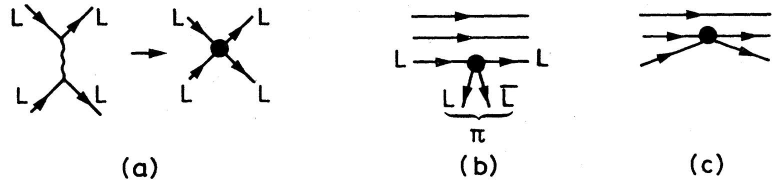

FIG. 2. (a) Diagram inducing effective local $V-A$ Fermi operators, (b) factorizable matrix element, and (c) baryonic matrix element.

rally 10%). The particularly large discrepancy in the prediction for the Σ_+^+ p-wave amplitude is easily understood if we recall[15] that a best fit to the s-wave amplitudes using Eq. (2.3) gives an f/d ratio for the (octet-dominated) matrix elements of $\mathcal{H}$ which is close to that for baryon mass splittings; equality of these ratios results in $B(\Sigma_+^+)\equiv 0$.

Since the conventional soft-pion procedure is not a good approximation to the data, we should examine the assumptions that went into it. We shall not question the $V-A$ nature of the weak current nor approximate chiral symmetry.

Approximate SU(3) symmetry. Relaxing this assumption would allow for parity-violating baryon-to-baryon matrix elements of $\mathcal{H}$. This could introduce commutator terms in p waves, analogous to (2.3), and pole terms in s waves, analogous to (2.4). However, these would not really be poles since their residues vanish, so for such terms to give a significant variation to $\mathfrak{M}$ would require anomalously large matrix elements. If they did contribute significantly they would spoil the successful relations of Eq. (2.11) for the dominant $\Delta I=\frac{1}{2}$ amplitudes. Neglecting these "nonpole" contributions we would still have a p-wave commutator term, but it would not contribute to $B(\Sigma_+^+)$. Independently of SU(3), the effective local operator $\mathcal{H}$ has only parity-conserving matrix elements in the nonrelativistic quark model as well. Since baryon matrix elements generally satisfy SU(3) constraints fairly well, and since relaxing this assumption does not appear useful, we shall stick to it.

Other poles. There are also pole terms from higher resonances: $\frac{1}{2}^+$ for p waves and $\frac{1}{2}^-$ for s waves. In the nonrelativistic quark model these give vanishing contributions because the overlap integral vanishes in the matrix element of $\mathcal{H}$. Of the known states, the closest and thus potentially the most important is the SU(3) singlet $\Lambda(\frac{1}{2}^-, 1405)$. Its contribution can be directly measured from $A(\Sigma_+^+)$ which is indeed small; in our subsequent analysis we shall assume that this is the only contribution to $A(\Sigma_+^+)$. One might worry about the $\frac{3}{2}^+$ decuplet states $\Delta(1232)$ and $\Sigma(1385)$ which are close in mass to the Σ and Ξ, respectively. In a dispersion relation their contributions vanish both on-shell and in the soft-pion limit since either the spurion or the pion momentum vanishes. Again their effect would be expected to be most important in s waves and would induce violations of the Lee-Sugawara relation and $A(\Sigma_+^+)=0$, Eqs. (2.11).

Factorizable matrix elements. The factorizable matrix elements of the effective local $V-A$ current-current operator (Fig. 2) vanish in the soft-pion limit and are believed[3,4] to give an important contribution to the $\Delta I=\frac{3}{2}$ amplitudes. They also contribute to $\Delta I=\frac{1}{2}$ amplitudes and will be included in our analysis. However, they make no contribution to the sum rule for $B(\Sigma_+^+)$. Factorizable matrix elements of penguin operators [Fig. 3(c)] do not vanish in the soft-pion limit and are implicitly included in the sum rules (2.7) [although they also drop out for $B(\Sigma_+^+)$]. The baryon-to-baryon matrix elements of the dimension-6 penguin operator[4] [Fig. 3(a)] has contributions of the type of Fig. 3(b) which give no net contribution to the on-shell amplitudes. Their contribution in the soft-pion limit, proportional to $\langle q\bar{q}\rangle \simeq -m_\pi^2 f_\pi^2/2(m_u+m_d)$, can be shown to be independent of the quark mass renormalization prescription and to give a contribution

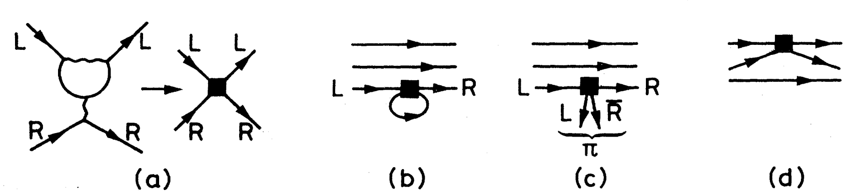

FIG. 3. (a) Penguin diagram inducing effective local $(V-A)$ $(V+A)$ Fermi operators with enhanced matrix elements which correspond to (b) in the soft-pion limit and the factorizable diagram (c) on mass shell. Baryonic matrix elements (d) are not enhanced.

identical to the on-shell factorizable penguin diagram of Fig. 3(c).

Anomalous current divergence. The diagram of Fig. 1 has a nonvanishing divergence in the limit of vanishing quark masses, giving an effective operator

$$q_u O^u \propto \tilde{F}^i_{\rho\sigma}\partial_\rho \bar{q}\lambda_i\gamma_\sigma(1-\gamma_5)q' , \tag{2.13}$$

where $\tilde{F}^i_{\rho\sigma}$ is the dual of the gluon field tensor and λ_i is a color SU(3) matrix. Since the operator (2.13) is a flavor octet it has a vanishing matrix element between Σ^+ and n, so it will not contribute to Σ^+_+ amplitudes. Its matrix elements also satisfy the Lee-Sugawara relation (2.11) to the extent that SU(3) breaking can be neglected. However, its matrix elements need not respect all the sum rules satisfied by factorizable contributions which are proportional to the divergences of quasiconserved currents. On the other hand, they can alter significantly the PCAC (partial conservation of axial-vector current) sum rule for $B(\Sigma^+_+)$ only through large contributions to s-wave amplitudes which are disfavored to some extent by successful calculations[9] of the latter. In addition these terms contribute to $|\Delta I| = \frac{3}{2}$ amplitudes which are small experimentally and approximately accounted for by factorizable terms.[4] A quantitative analysis, as described briefly below, shows that these contributions are insufficient to allow for a coherent description of all amplitudes.

In the nonrelativistic quark model, $\Delta I = \frac{3}{2}$ weak transition operators do not contribute to the matrix elements $\mathcal{H}_{ab}$ defined in Eqs. (2.3). Therefore in the standard approach the decay $\Sigma^+ \to n\pi^+$ receives no contribution. We define the remaining isospin amplitudes by

$$\begin{aligned} A^{\pi^0}_{ba} &\equiv A_{1/2}(a) + A_{3/2}(a), \\ B^{\pi^0}_{ba} &\equiv B_{1/2}(b) + B_{3/2}(b), \end{aligned} \tag{2.14}$$

and the $a \to b\pi^-$ amplitudes are then determined by isospin as linear combinations of $(A,B)_{1/2,3/2}$ and Σ^+_+. The factorizable diagrams of Fig. 2 give the contributions

$$\begin{aligned} A^F_{1/2}(a) &= \kappa(m_a - m_b) g^{ba}_V , \\ B^F_{1/2}(a) &= -\kappa(m_a + m_b) g^{ba}_A , \end{aligned} \tag{2.15}$$

$$\frac{A^F_{3/2}(a)}{A^F_{1/2}(a)} = \frac{B^F_{3/2}(a)}{B^F_{1/2}(a)} = -\frac{8c_+}{3c_- + 2c_+} \equiv \delta_F , \tag{2.16}$$

where g_A, g_V are the usual axial-vector and vector-current matrix elements at zero momentum transfer,

$$\kappa = \sqrt{2}\sin\theta_C \cos\theta_C \frac{f_\pi}{m_\pi^2}\frac{(c_- + \frac{2}{3}c_+)}{6} ,$$

and $c_\pm$ are the coefficient functions in the operator-product expansion for the effective Fermi interaction

$$\begin{aligned} \mathcal{H}^{\text{eff}}(\Delta S = 1) &= \sqrt{2}\,(c_+ O_+ + c_- O_-) G_F + \text{penguin operators} \\ &\qquad + \text{higher dimension}, \\ O_\pm &= (\bar{u}_L\gamma_\mu s_L)(\bar{d}_L\gamma_\mu u_L) \pm (\bar{d}_L\gamma_\mu s_L)(\bar{u}_L\gamma_\mu u_L) . \end{aligned} \tag{2.17}$$

In the short-distance leading-logarithm approximation one gets

$$c_- \simeq 2.6, \quad c_+ \simeq 0.6, \quad \kappa \simeq 1\ \text{GeV}^{-1}, \quad \delta_F \simeq -\tfrac{1}{2}, \quad \delta_A = \tfrac{1}{3} , \tag{2.18}$$

where $\delta_A = 4c_+/(3c_- - c_+)$ is the $\frac{3}{2}/\frac{1}{2}$ ratio for the anomalous contribution and we have taken the QCD parameter $\Lambda \simeq 500$ MeV and a renormalization point $\mu \simeq 1$ GeV, but these numbers could well be renormalized further by soft-gluon exchange.

We simplify notation by defining "reduced" amplitudes:

$$a_{ba} = \frac{A_{ba}}{m_a - m_b} , \quad b_{ba} = \frac{B_{ba}}{m_b + m_a} . \tag{2.19}$$

Then assuming that the mechanisms of Figs. 1 and 2(b) are the only contributions to the $\Delta I = \frac{3}{2}$ amplitudes we may write

$$\begin{aligned} a_{3/2}(\Lambda) &= \frac{\sqrt{3}}{2}\,\delta_F\kappa + \delta_A a_A(\Lambda) , \\ a_{3/2}(\Sigma) &= \frac{1}{\sqrt{2}}\,\delta_F\kappa + \delta_A a_A(\Sigma) , \\ a_{3/2}(\Xi) &= -\frac{\sqrt{3}}{2}\,\delta_F\kappa + \delta_A a_A(\Xi) , \end{aligned} \tag{2.20}$$

$$\begin{aligned} b_{3/2}(\Lambda) &= -\frac{(3F+D)}{2\sqrt{3}}\,\delta_F\kappa + \delta_A b_A(\Lambda) , \\ b_{3/2}(\Sigma) &= \frac{(D-F)}{\sqrt{2}}\,\delta_F\kappa + \delta_A b_A(\Sigma) , \\ b_{3/2}(\Xi) &= \frac{(3F-D)}{2\sqrt{3}}\,\delta_F\kappa + \delta_A b_A(\Xi) , \end{aligned} \tag{2.21}$$

and if Figs. 1 and 2(b) are the only corrections to the PCAC treatment, the sum rules (2.7) are modified to

$$b_{1/2} \equiv \tilde{b}(a_{1/2}) + \Delta b , \tag{2.22}$$

$$\begin{aligned} \Delta b(\Lambda) = -\frac{\bar{\kappa}(3F+D)}{\sqrt{3}} + \frac{1}{\delta_A}\,[b_{3/2}(\Lambda) - (D+F)a_{3/2}(\Lambda) \\ + (\tfrac{2}{3})^{1/2} D a_{3/2}(\Sigma)] , \end{aligned} \tag{2.22a}$$

$$\Delta b(\Sigma) = \sqrt{2}\bar{\kappa}(D-F) + \frac{1}{\delta_A}\,[b_{3/2}(\Sigma) + (D-F)a_{3/2}(\Lambda)] , \tag{2.22b}$$

TABLE II. $\Delta I=\frac{1}{2}$ reduced amplitudes [see text; we assume the small amplitude for $a(\Sigma_+^+)$ arises from $Y_0^*(1405)$ exchange only and remove this contribution which contributes equally to all s-wave Σ decays], current-algebra predictions ($\tilde{b}$) for $b_{1/2}$, and contributions to $\Delta b = b_{1/2} - \tilde{b}$ arising from Figs. 1 and 2(b) using $\Delta I=\frac{3}{2}$ amplitudes (Table III) as input (see text).

	Experiment (Ref. 13) $a_{1/2}$	Experiment (Ref. 13) $b_{1/2}$	Calculation $\tilde{b}$	Expt. $\Delta b = b_{1/2} - \tilde{b}$	Calc. $\Delta b = b_{1/2} - \tilde{b}$
Λ	-5.93 ± 0.04 [-5.85]	-3.51 ± 0.11 [-3.49]	-3.80 ± 0.07 [-3.70]	0.29 ± 0.13 [0.21]	-3.73 ± 2.04 [-3.97]
Σ	-5.39 ± 0.08	6.23 ± 0.19	2.13 ± 0.03	4.10 ± 0.19	-0.89 ± 0.72
Ξ	7.38 ± 0.05	-2.11 ± 0.17	-0.24 ± 0.07	-1.87 ± 0.18	0.51 ± 1.02
Σ_+^+	0	8.93 ± 0.08	0.63 ± 0.10	8.30 ± 0.13	-1.17 ± 0.51 [-0.91]

$$\Delta b(\Xi) = \bar{\kappa}\,\frac{(3F-D)}{\sqrt{3}} + \frac{1}{\delta_A}\left\{b_{3/2}(\Xi) + \tfrac{2}{3}D\left[2\left(\frac{m_\Lambda - m_N}{m_\Xi - m_\Sigma}\right)a_{1/2}(\Lambda) + \left(\frac{m_\Xi - m_\Lambda}{m_\Xi - m_\Sigma}\right)a_{3/2}(\Xi)\right] + (D-F)a_{3/2}(\Xi)\right\}, \tag{2.22c}$$

$$\Delta b(\Sigma_+^+) = -\frac{\sqrt{2}D}{\delta_A}\left[(\tfrac{2}{3})^{1/2}a_{3/2}(\Lambda) - a_{3/2}(\Sigma)\right], \tag{2.22d}$$

where

$$\bar{\kappa} = \kappa(1 - \delta_F/\delta_A).$$

We have used the Gell-Mann–Okubo mass relation

$$m_\Xi - m_\Sigma = 2(m_\Lambda - m_N) - (m_\Xi - m_\Lambda),$$

and the $\tilde{b}(a_{1/2})$ are the PCAC predictions (Table II) with no corrections. We have avoided making SU(3)-symmetry assumptions for the anomalous contribution as the operator (2.13) contains derivatives and is therefore sensitive to internal momenta. However, no assumption is necessary to see that the result is unsatisfactory. For example, Eq. (2.22d) gives (see Tables II and III)

$$8.30\pm0.13 = -\frac{1}{\delta_A}(0.39\pm0.17), \tag{2.23}$$

which not only requires a value $|\delta_A| \simeq 1/20$, much smaller than the calculated one [Eq. (2.18)], but has the wrong sign. On the other hand, the combination $2\Delta b(\Xi) + \Delta b(\Lambda) + (\frac{2}{3})^{1/2}\Delta(\Sigma)$ [Eqs. (2.22a)–(2.22c)] gives

$$1.58\pm0.45 = -\frac{1}{\delta_A}(1.25\pm1.00), \tag{2.24}$$

which again has the wrong sign but prefers $|\delta_A| \sim 1$. Taking the values of Eqs. (2.18) gives the last column of Table II, which generally worsens agreement with the data. It is not particularly reliable to use the $\Delta I=\frac{3}{2}$ amplitudes as input, since they have large errors and are sensitive to radiative corrections. However, the Coulomb corrections (bracketed entries in the tables), which are estimated[14] to be the most important radiative corrections, make little difference, and the large theoretical errors given should reflect sufficiently the experimental uncertainties. The point is that an adjustment of the coefficient functions in order to account in particular for the Σ decay amplitudes would not only do violence to their calculated values but would render fortuitous, as mentioned before, the agreement of s-wave predictions with data, as well as the fact that factorizable diagrams account reasonably well for the observed $\Delta I=\frac{3}{2}$ amplitudes as seen in Table III.

Similar remarks are probably applicable to any factorizable contributions (e.g., K poles and K^* poles) which contribute to $\Delta I=\frac{3}{2}$ amplitudes with strength dampened only by the coefficient function ratio c_+/c_- but have the virtue of preserving the Lee-Sugawara relation and the vanishing of $A(\Sigma_+^+)$ [Eqs. (2.11)]. Our analysis assumed only $\Delta I=\frac{1}{2}$ for the operator of Eq. (2.13), and the discrepancy between (2.23) and (2.24) is independent of the value of the $\frac{3}{2}/\frac{1}{2}$ amplitude suppression factor. There are other anomalous contributions coming from penguin operators in the expansion (2.17) which contribute only to $\Delta I=\frac{1}{2}$ amplitudes (as well

TABLE III. $\Delta I=\frac{3}{2}$ reduced amplitudes (see text) and factorizable contributions [Fig. 2(b)] using the values of Eq. (2.18).

	$a_{3/2}$ Experiment (Ref. 13)	$a_{3/2}$ Fact.	$b_{3/2}$ Experiment (Ref. 13)	$b_{3/2}$ Fact.
Λ	-0.12 ± 0.08 [-0.20]	-0.46	-0.05 ± 0.19 [-0.07]	0.32
Σ	-0.43 ± 0.14	-0.38	-0.61 ± 0.19	-0.28
Ξ	0.36 ± 0.11	0.46	-0.33 ± 0.19	-0.23

as anomalies with a gluon replaced by a photon which would mostly affect $\Delta I=\frac{3}{2}$ amplitudes) but they occur with considerably smaller coefficient functions.

It would seem that the PCAC approach (or the baryon-pole approximation which is essentially equivalent) is inadequate to describe the p waves, at least for Σ decay. Another indication that this is the case is the large value of the wave function at the origin,

$$|\psi(0)|^2 \simeq 1.97 m_\pi{}^3 ,$$

which is required to fit Σ_+^+ decay under the assumption of pole dominance. On the other hand the value

$$|\psi(0)|^2 \simeq 0.67 m_\pi{}^3$$

extracted from a pole dominance fit[5] to $\Omega^- \to \Lambda K^-$ is in much better accord with bag-model predictions and the value extracted from a fit to s waves.[16] The point is that in the standard picture the dominant (factorizable) contribution of the penguin operators drops out from both these decay amplitudes (as do anomalies and all other factorizable contributions). Since the nonfactorizable [Fig. 3(d)] baryon-to-baryon matrix element of penguin operators is expected to be small [we find

$$D^- = -F^- = 12F^P = 4D^P$$

for the reduced matrix elements of O_- and O_P, while the coefficient functions are in the ratio $c_P/c_- \simeq O(\alpha_s(\mu)/6\pi)$; see also Ref. 9] these decays should measure directly the matrix elements of O_- and therefore $|\psi|^2$ in a quark-model picture. However, it has been strongly argued[17] that penguin operators (which all have $\Delta I=\frac{1}{2}$) of dimension higher than six are so convergent that the higher dimensions are scaled by powers of light quark (constituent) masses rather than m_c (they cannot be scaled by m_W since they are suppressed by the Glashow-Iliopoulos-Maiani mechanism), and might therefore give a non-negligible contribution which could modify estimates of $\langle B|\mathcal{H}|B'\rangle$ as well as static SU(6) relations which follow from O_- dominance. However, unless they can be shown to give a rapidly varying contribution to decay matrix elements, they will not affect the above analysis which is independent of the value of $\langle B|\mathcal{H}|B'\rangle$ and to a large extent of assumptions on SU(3) symmetry. It has recently been conjectured[18] that gluon radiation may play a role in the apparently unexpectedly large decay width of the D^0, and may also give a significant contribution to exclusive two-body channels[19,20] via diagrams like that of Fig. 4(a). Since gluon exchange of this type allows quark diagrams which would otherwise be suppressed by helicity arguments, it could play a role in the dominant $\Delta I=\frac{1}{2}$ transition for $K\to 2\pi$. However, for baryon decay there is in any case no suppression of diagrams like that of Fig. 4(b) which are implicitly included in the PCAC-cum-pole analysis independently of the way in which gluons are exchanged; again they would have to be shown to give rise to a rapid variation in order to account for the discrepancy. The same statement applies to an alternative picture[19] which allows for a large gluon + color octet $(\bar{q}q)$ component of mesons. Extending this idea to baryons would not in principle invalidate the PCAC analysis although it could modify the estimation of matrix elements within the usual static SU(6) framework.

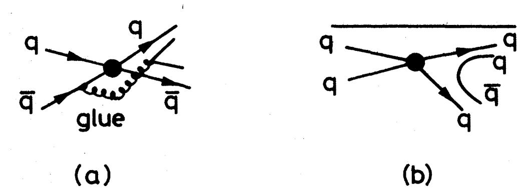

FIG. 4. (a) Two-body meson decay diagram which is helicity suppressed without gluon exchange between spectator and weakly interacting quarks. (b) Two-body baryon decay diagram which suffers no helicity suppression and is included in PCAC analysis with all possible gluon exchange.

III. Ω^- DECAYS

The initial treatment[5] of Ω^- decay within the standard QCD framework including penguin operators has been followed by a cleaner experimental situation[6,7] and another analysis[16] along the same lines where the bag model was used to calculate matrix elements. In this approach Ω^- decays are predominantly parity conserving, and predicted decay rates agree with experimental ones[21] within the uncertainties on the coefficient function $c_\pm, c_P$ and the other relevant parameters: The strong coupling constant $g_{\Omega^-\Xi^0 K}$ and the weak matrix element $\langle\Lambda|O_-|\Xi^0\rangle$ for $\Omega^-\to K\Lambda$ and the semileptonic decay matrix element $\langle\Xi|J_\mu|\Omega^-\rangle$ for $\Omega^-\to\Xi\pi$. Since the standard model appears successful in describing both two-body Ω^- decays and hyperon s-wave decay amplitudes it seems worth trying to test it further in spite of possible criticisms[17,19] of the assumptions used and the failure discussed above which is conceivably specific to the p-wave hyperon amplitudes. To this end we estimate the branching ratios for $\Omega^-\to\Xi^-\pi^+\pi^-$ and $\Omega^-\to\Xi^0 l^-\nu$.

A. Three-body nonleptonic decay

To estimate the decay rate for the observed decay

$$\Omega^- \to \Xi^-\pi^+\pi^- \tag{3.1}$$

we will use the naive SU(6) model as in Ref. 5. We assume a cascade decay

$$\begin{array}{l}\Omega^- \to \Xi^{*0}\pi^- \\ \quad\searrow \\ \quad\ \Xi^-\pi^+\end{array} \tag{3.2}$$

as suggested by the data, and first consider the two-body processes $\Omega^- \to \Xi^{*0}\pi^0$ and $\Omega^- \to \Xi^{*-}\pi^0$. According to the standard picture the factorizable diagram of Figs. 3(c) and 2(b) will dominate because of the matrix element enhancement[4,5,8] of the penguin operators (O_5 and O_6 in conventional notation[4]). Since O_- cannot contribute to the pole diagram of Fig. 4(b), and O_P does not have enhanced matrix elements in this case, we include only Figs. 3(c) and 2(b) for our estimate.

For the partial matrix element of the operator O_j one has

$$M_j(\Omega^- \to \Xi^{*0}\pi^-) = \sqrt{2}G_F \sin\theta_C \cos\theta_C c_j \langle \Xi^{*0}\pi^- | O_j | \Omega^-\rangle . \tag{3.3}$$

(Here we use conventional but more cumbersome notation: $O_1 = -O_-$, $O_+ = \frac{1}{5}O_2 + \frac{2}{15}O_3 + \frac{2}{3}O_4$, in order to use directly previous calculations.[4,5]) With the assumption of factorization we obtain

$$\begin{aligned}\langle \Xi^{*0}\pi^- | O_j | \Omega^-\rangle &= F_j \langle \pi^- | \bar{d}^i_L \gamma_\mu u^i_L | 0\rangle \langle \Xi^{*0} | \bar{u}^k_L \gamma^\mu s^k_L | \Omega^-\rangle \\ &= -\tfrac{1}{4} i f_\pi q^\pi_\mu F_j \langle \Xi^{*0} | u^k \gamma^\mu (1-\gamma_5) s^k | \Omega^-\rangle ,\end{aligned} \tag{3.4}$$

where the notation is that of Ref. 5. The Fierz factor F_j measures the interplay between the two contributing terms for each operator. As in the case[5] of $\Omega^- \to \Xi^0\pi^-$ we get

$$F_1 = -\tfrac{2}{3}, \quad F_2 = F_3 = F_4 = \tfrac{4}{3} , \tag{3.5}$$

while the penguin factors for the dominant vector contribution change sign:

$$F_5 = \tfrac{16}{3} F_6 \cong \pm \frac{16}{9} \frac{m_\pi^2}{m_u m_s} , \quad \text{for} \begin{cases} V \\ A \end{cases} . \tag{3.6}$$

Replacing the effect of all $\Delta I = \frac{1}{2}$ operators in the matrix element by an "effective" O_1,

$$-\tfrac{2}{3} c_1^{\text{eff}} = \sum_{i=1}^{3} c_i F_i + \sum_{i=5}^{6} c_i F_i , \tag{3.7}$$

$$\begin{aligned} M(\Omega^- \to \Xi^{*0}\pi^-) = &-\frac{\sqrt{2}}{6} i G_F \sin\theta_C \cos\theta_C (-c_1^{\text{eff}} + 2c_+) \\ &\times f_\pi q^\pi_\mu \langle \Xi^{*0} | \bar{u}\gamma^\mu (1-\gamma_5) s | \Omega^-\rangle , \end{aligned} \tag{3.8}$$

one finds, for example,

$$c_1^{\text{eff}(V)} \cong +12, \quad c_1^{\text{eff}(A)} = -18 .$$

(The authors of Ref. 4 prefer[22] the values $c_5 + \frac{3}{16} c_6 \simeq -0.25$, $c_1^{\text{eff}} = -18$ from their fit to hyperon decays, but more recent fits[9] do not require such a large penguin contribution.) In naive SU(6) the baryonic matrix element is given by

$$\begin{aligned} q^\pi_\mu \langle \Xi^{*0} | \bar{u}\gamma^\mu (1-\gamma_5) s | \Omega^-\rangle = &\sqrt{3}(m_\Omega - m_{\Xi^*}) \left(\frac{E_{\Xi^*} + m_{\Xi^*}}{2m_{\Xi^*}}\right)^{1/2} \delta_{mm'} \\ &+ 2Z\left(\frac{5\pi}{3}\right)^{1/2} (m_\Omega + m_{\Xi^*}) \left(\frac{E_{\Xi^*} - m_{\Xi^*}}{2m_{\Xi^*}}\right)^{1/2} \langle \tfrac{3}{2} 1 m' l | \tfrac{3}{2} m\rangle Y_1^l(\hat{q}) , \end{aligned} \tag{3.9}$$

where m and m' are the third components of spin for Ω^- and Ξ^*, respectively.

Taking now the unpolarized rate relative to the $\underline{10} \to \underline{8}$ one of Ref. 5, one obtains

$$\begin{aligned} \frac{\Gamma(\Omega^- \to \Xi^{*0}\pi^-)}{\Gamma(\Omega^- \to \Xi^0\pi^-)} = 2\frac{|\vec{k}_{\Xi^*}|}{|\vec{k}_\Xi|} \Bigg\{ &\left[\left(\frac{5}{8}\right)^{1/2} \left|\frac{\vec{k}_{\Xi^*}}{\vec{k}_\Xi}\right| \frac{(m_\Omega + m_{\Xi^*})}{(m_\Omega + m_\Xi)} \left(\frac{E_\Xi + m_\Xi}{E_{\Xi^*} + m_{\Xi^*}}\right)^{1/2}\right]^2 \\ &+ \left[\frac{3(m_\Omega - m_{\Xi^*})[(E_{\Xi^*} + m_{\Xi^*})(E_\Xi + m_\Xi)]^{1/2}}{2\sqrt{2}Z(m_\Omega + m_\Xi)|\vec{k}_\Xi|}\right]^2 \left(\frac{-c_1^{\text{eff}(V)} + 2c_4}{-c_1^{\text{eff}(A)} + 2c_4}\right)^2 \Bigg\} , \end{aligned} \tag{3.10}$$

where the axial renormalization constants for $\underline{10} \to \underline{8}$ and $\underline{10} \to \underline{10}$ are assumed to be comparable; consistency[23] with PCAC, which apparently works well[24] in Δ production, requires $Z \simeq 1$ and not $\frac{3}{4}$ as was used in Ref. 5.

Assuming now on-shell values for $|\vec{k}_{\Xi^*}|$ and E_{Ξ^*} in the intermediate $\Xi^*\pi$ state and taking[7]

$$B(\Omega^- \to \Xi^0\pi^-) = 0.234 \pm 0.013 \tag{3.11}$$

and

$$B(\Xi^{*0} \to \Xi^-\pi^+) \simeq \tfrac{2}{3} , \tag{3.12}$$

one finds

$$B(\Omega^- \to \Xi^-\pi^+\pi^-) \simeq \tfrac{1}{685} . \tag{3.13}$$

The decay is predominently parity violating; the contribution from the axial-vector current turns out to be ~1% because of the small final-state momentum $|\vec{k}_{\Xi^*}|$. Thus we expect the π^- to be nearly isotropic in the Ω^- rest frame, while the π^+ is in a

p wave in the $\Xi^-\pi^+$ center of mass.

Correcting this result for the Ξ^* finite width, we adjust for phase-space differences but assume off-shell effects in the matrix element to be negligible. The corrections thus obtained turn out to be small; the branching ratio (3.13) is multiplied by a factor of about 1.3. The peak in the $\Xi^-\pi^+$ invariant mass occurs about 2 MeV below the Ξ^{0*} mass, and the bulk of the $\Xi^-\pi^+\pi^-$ events lie in an invariant-mass range of one or two Γ_{Ξ^*} below $\sqrt{s}_{max}=m_{\Xi^{0*}}+0.8$ MeV. So the final prediction is

$$B(\Omega^-\to\Xi^-\pi^-\pi^+)\simeq\tfrac{1}{535}. \tag{3.14}$$

In the experiment with the highest statistics,[7] this final state has about the same detection efficiency as the $\Xi^0\pi^-$ state. With the branching ratio (3.14) about two events are expected and one was observed.[7]

Since the relative sign of the O_- and penguin contributions changes in going from $\Omega^-\to\Xi^-\pi$ to $\Omega^-\to\Xi^*\pi$, comparison of their matrix elements could provide a measure of the importance of the penguin operator provided Ξ^* dominance of the three-body decay and the general assumptions of the model are reliable.

B. Semileptonic Ω^- decay

The semileptonic decay mode of the Ω^- was given a very thorough treatment in an unpublished paper.[25] Here we will present the results of a simpler calculation, using more recent information on the N-Δ semileptonic coupling which should provide a fairly reliable estimate.

The matrix element for $\Omega^-\to\Xi^0 l\bar\nu$ is related by SU(3) to that for $\nu+N\to l+\Delta$; in the notation of Ref. 23 we get

$$\begin{aligned}M(\Omega^-\to\Xi^0 l\bar\nu_l)=&-\left(\frac{3}{2}\right)^{1/2}G_F\tan\theta_C\,\bar u(p)\\&\times\left(\frac{C_3^V}{m'}\gamma_\nu\gamma_5F^{\mu\nu}+C_5^Aj^\mu\right)u_\mu(k),\\F^{\mu\nu}=&\,q^\nu j^\mu-q^\mu j^\nu,\end{aligned}\tag{3.15}$$

where u_μ is a Rarita-Schwinger spinor, j^μ is the leptonic current, and C_3^V, C_5^A are coupling constants which can be extracted from ν-induced Δ production; we use the values[23]

$$C_5^A(0)=-1.2\,,\quad C_3^V(0)=2.05\pm0.04\,. \tag{3.16}$$

The kinematics are defined in Fig. 5. In the SU(3) limit the parameter m' is just the common baryon mass. Symmetry breaking introduces an ambiguity; here we set $m'=m_{\text{nucleon}}$. Defining dimensionless invariants κ and τ by

$$\begin{aligned}\kappa^2&=(p_1+p_2)^2/M^2\,,\\\tau&=(k-p_1)^2/M^2\,,\\\mu&=m/M\,,\quad\mu'=m'/M\,,\end{aligned}\tag{3.17}$$

one obtains the unpolarized rate

$$\frac{d^2\Gamma}{d\kappa^2d\tau}=\frac{3G_F{}^2M^5\tan^2\theta_C}{32\pi^3}\frac{1}{M^4}j^{\alpha\beta}H_{\alpha\beta}\,, \tag{3.18}$$

where

$$H_{\alpha\beta}=H^V_{\alpha\beta}+H^A_{\alpha\beta}+H^{\text{int}}_{\alpha\beta}$$

is the hadronic tensor. The interference term drops out upon integration over the angular variable τ:

$$\begin{aligned}\frac{1}{M^4}\int_{\tau_{min}}^{\tau_{max}}d\tau\,j^{\alpha\beta}H^V_{\alpha\beta}=&\frac{1}{72}\left|\frac{C_3^V}{\mu'}\right|^2\kappa^2\lambda^{1/2}(\kappa^2,\mu^2,1)\\&\times\{2(3+\mu^2-\kappa^2)\lambda(\kappa^2,\mu^2,1)\\&\quad+12\kappa^2[(1-\mu^2)-\kappa^2]\}\,,\\\frac{1}{M^4}\int_{\tau_{min}}^{\tau_{max}}d\tau\,j^{\alpha\beta}H^A_{\alpha\beta}&\\=\tfrac{1}{72}|C_5^A|^2\{&[(1+\mu)^2-\kappa^2]\lambda^{1/2}(\kappa^2,\mu^2,1)+12\kappa^2\}\,,\end{aligned}\tag{3.19}$$

$$\lambda(\kappa^2,\mu^2,1)=[\kappa^2-(\mu+1)^2][\kappa^2-(\mu-1)^2]\,.$$

The resulting rate is sensitive to the q^2 dependence of the form factors:

$$\Gamma(\Omega^-\to\Xi^0\,e^-\,\bar\nu_e)=10^8\text{ sec}^{-1}\times\begin{cases}1.06\\1.29\end{cases} \tag{3.20}$$

for a constant form factor and including q^2 dependence, respectively. The larger number uses the vector form factor of Ref. 23 and the A_1 mass for a dipole axial form factor. One expects a higher mass scale for strangeness-changing matrix elements, so one expects the result to lie between the numbers in Eq. (3.20); with a measured lifetime of 0.82×10^{-10} sec^{-1} they give, respectively,

$$B(\Omega^-\to\Xi^0\,e^-\,\bar\nu_e)\simeq10^{-2}\times\begin{cases}0.87\\1.06\end{cases}. \tag{3.21}$$

Our partial lifetime is in approximate agreement with the results of Yeou-wei Yang. The axial-vector contribution is dominant; we found a vector contribution of 7% (4%) to the total rate for con-

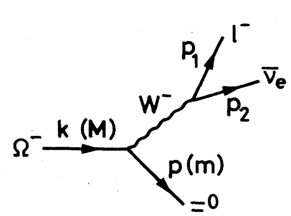

FIG. 5. Definition of kinematics for semileptonic Ω^- decay.

stant (varying) form factors. Therefore a measurement of the semileptonic branching ratio should determine the squared axial-vector matrix element at zero momentum transfer—which governs the partial rates for $\Omega^- \to \Xi\pi$ in the standard model—to within 20 or 30%. Our branching ratio of 1% is compatible with the observed number (~3.5 events) of semileptonic decays in the hyperon-beam experiment.[7]

IV. CONCLUSIONS

We have shown that the well-known failure of PCAC to describe adequately both s and p waves in hyperon decay persists when new developments within the framework of QCD are taken into account. A hitherto neglected anomalous contribution to the axial-vector current divergence fails to cure satisfactorily this discrepancy. On the other hand, calculations of s-wave hyperon and Ω^- decay amplitudes[5,9,16] fare well in confrontation with the data and this success is supported further by the (albeit scanty) observed rates[7] for three-body decays of the Ω^-. These results suggest that there is no difficulty in understanding either the dominance of $\Delta I = \frac{1}{2}$ amplitudes or the general nonleptonic enhancement within the framework of the standard model. They suggest further that there is not one simple explanation for these phenomena but rather that they result from the interplay of various dynamical effects. However, we cannot be completely satisfied until all amplitudes are calculable, and a new approach is apparently needed for the description of p-wave hyperon decay amplitudes. One possibility might be the extension of newly developed techniques[26] for calculating proton decay in a bag model ($B \to \pi$ + leptons) to the even more complicated case $B \to B' + \pi$.

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