# Two-component duality and flavoring in the $P+f$ model

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(Received 22 October 1979; revised manuscript received 18 January 1980)

We show that modern Regge fits to rising $\pi N$ total cross sections $\sigma_{\pi N}$ using the Harari-Freund $P+f$ model of diffraction are not consistent with two-component duality. If a conventional Pomeron is chosen (dominant $j$-plane pole plus weak cuts), the resulting $f$ is "dual" to the resonances plus one-half the background. Conversely, constraining the $f$-pole amplitude by duality does not allow a reasonable fit to $\sigma_{\pi N}$. In contrast, the $P$-$f$ identity model of diffraction is shown to satisfy a modified form of two-component duality. We show that by incorporating flavoring renormalization, the $P+f$ picture can be made consistent with duality. The unflavored $\hat{P}$ intercept is 0.91 and the flavored $P$ intercept is 1.1. Significant absorptive $j$-plane cuts are also required, though these are small enough to be consistent with dominant short-range order. Thus flavoring, which is so essential in $P$-$f$ identity phenomenology, seems to play a positive role in diffraction scattering generally.

## I. INTRODUCTION

The idea of two-component duality has been instrumental in motivating the Harari-Freund (HF) version of the $P+f$ model of diffraction.[1] The duality prediction

$$\mathrm{Im}\,T_{f\,\mathrm{pole}} \approx \langle \mathrm{Im}(\mathrm{resonances})\rangle \tag{1}$$

along with

$$\mathrm{Im}\,T_{\mathrm{Pomeron\,(nonplanar)}} \approx \langle \mathrm{Im}(\mathrm{background})\rangle$$

is perhaps the most important phenomenological distinction of this scheme.[2] Theoretically, this is supposed to mean that the $f$ is dominantly planar while the Pomeron is a new effect [e.g., closed dual string, quantum-chromodynamics (QCD) glueball] which is unrelated to simple resonance production. However, one should note that the ($P \approx$ glueball) assignment is best motivated without quark loops at $N_c = \infty$ where infrared gluon dynamics is relevant. Diffraction at $N_c = 3$ is arguably quite different, depending on quark-loop confinement-related hadron mass scales.[3]

More recently, a different idea, called the Pomeron-$f$ identity,[4] has developed. Here there is only one high-lying trajectory, which is curved and goes through the $f$ meson. This scheme is, in fact, consistent with hadron data provided careful attention is paid to "flavoring," renormalization effects due to the quasithreshold production of strange quarks, diquarks, charmed quarks, etc.[4-6] Two-component duality here evidently does not hold in the above form.

Although the Pomeron $P$ is somewhat enigmatic, two important constraints exist. First, observed dominant short-range order (SRO) in rapidity dictates that the Pomeron amplitude is dominated by a $j$-plane pole, not cuts. Second, Veneziano's topological expansion[7] (TE) tells us that the bare Pomeron of the Reggeon field theory is the leading behavior of the cylinder. The nonleading behavior of the cylinder is an object of much discussion[3,6] (it can even "wipe out" the standard $f$ as in the $P$-$f$ identity[4]), but the HF scheme is based on the assumption that the $f$-pole amplitude is highly exchange degenerate and dominates the $I=0$ planar amplitude. Thus, the unambiguous TE prediction

$$\mathrm{Im}\,T_{\mathrm{planar}} \approx \langle \mathrm{Im}(\mathrm{resonances})\rangle \tag{2}$$

reduces to Eq. (1) in the HF model to a presumably good approximation.[8]

It is important to note that this relation is really an important consistency requirement for the HF model. It is also, in principle, easy to test if one adopts the usual supposition that asymptotic Regge-*pole* amplitudes are trivial to continue to low energies when expressed in the laboratory energy $\nu$, an $s$-$u$ crossing-symmetric variable.[9]

We show here[6,10] that actually two-component duality is badly violated by modern $P+f$ fits. If the $P$ at $t=0$ is principally a $j$-plane pole plus weak $j$-plane cuts (as implied by dominant short-range rapidity correlations in inelastic final states at present energies), then the amount of violation can be stated rather precisely. Instead of the HF prediction $\mathrm{Im}\,T_{f\,\mathrm{pole}} \approx \langle \mathrm{Im}(\mathrm{resonances})\rangle$, typical fits published after 1974 produce an $f$-pole amplitude satisfying

$$[\mathrm{Im}T_{f\,\mathrm{pole}}]_{\mathrm{fit}} \approx \langle \mathrm{Im(resonances)}\rangle + \tfrac{1}{2}\langle \mathrm{Im(background)}\rangle, \qquad (3)$$

provided that the established resonances are inserted on the right-hand side.[11] This is the case even under differing views regarding the extent to which the $f$ is exchange degenerate, its intercept, etc. The crucial point is the existence of rising total cross sections.

The nonplanar $\approx\langle$Im(background)$\rangle$ relation is more difficult to examine due to difficulties in extrapolating $j$-plane-cut amplitudes to low energies. Hence, our attitude is that only Eq. (1) is feasible to check phenomenologically, but that this is enough to provide a severe constraint.

It might be argued that cylindrical mixing causes a significant deviation of $T_{f\,\mathrm{pole}}$ from $T_{\mathrm{planar}}$. This is an *a priori* plausible idea. In the $P$-$f$ identity, for example, cylinder mixing produces a very significant effect, and Eq. (2) is expected to hold but not Eq. (1). In the $P+f$ HF picture, the breaking of exchange degeneracy (EXD) is not supposed to be much (this is, after all, supposed to be an attractive feature of the scheme), so here one does expect Eq. (1) to hold. In any event, we show in the Appendix that the signs are probably wrong. Given an $f$-pole amplitude with cylinder mixing that is too big to average the resonances, the $f$-pole amplitude with the cylinder subtracted out violates duality even more. This follows under reasonable hypotheses involving the generation of the $P+f$ model from underlying (e.g., QCD) dynamics.

If we relax the preferred condition that the Pomeron be dominantly a $j$-plane pole, then many results are possible. We shall present several examples. Of course, one could always *define* the $P$ and $f$ to satisfy two-component duality. Our point is that, however one may be tempted to do this, there is no longer any phenomenological support for the idea from canonical Regge fits to total cross sections.

Conversely, we show that if one attempts to constrain the $f$ pole by duality and the $P$ to be $j$-plane-pole dominated, a bad description of $\sigma_{\pi N}$ results.

In contrast to the failure of two-component duality in the usual $P+f$ model, we show that the form of two-component duality in Eq. (2) can be satisfied in the $P$-$f$ identity model framework. Here a model[12] is needed to separate the planar and cylinder amplitudes, as well as $P$-$f$ identity phenomenology in which flavoring has been included (we repeat that flavoring is essential in $P$-$f$ identity phenomenology[5,6]).

Finally, we show that by incorporating flavoring renormalization, the $P+f$ picture can be made consistent with duality. The unflavored $\hat{P}$ intercept is 0.91 and the flavored $P$ intercept is 1.1. Significant absorptive $j$-plane cuts are also required, though these are small enough to be consistent with dominant short-range order.

The rest of this paper is organized as follows. In Sec. II we compare conventional $P+f$ phenomenology with $f$-resonances duality [Eq. (1)]. In Sec. III we exhibit our attempt to make the usual $P+f$ model satisfy $f$-resonances duality and fit $\sigma_{\pi N}$ (which fails). Section IV contains the analysis of the $P$-$f$ identity and duality.[10] Section V analyzes flavoring, $\sigma_{\pi N}$, and duality in the $P+f$ context. Section VI contains some conclusions. The Appendix deals with $f$-cylinder mixing and duality in the $P+f$ model.

## II. TWO-COMPONENT DUALITY IN THE STANDARD $P+f$ MODEL

The primary experimental test of two-component duality in the $P+f$ framework was made in 1969 by Harari and Zarmi.[13] They showed that the low-energy extrapolations of the $f$ and $P$ amplitudes of the existing fits, respectively, did indeed average the resonance and background components of the absorptive $\pi N$ amplitude. However, we know that total cross sections $\sigma$ rise, and this imperils the idea. Roughly, rising $\sigma$ means that the $P$ part $\sigma^{(P)}$ rises, i.e., $\sigma^{(P)}$ decreases as we go to lower energies. But then the $f$ contribution $\sigma^{(f)}=\sigma-\sigma^{(P)}$ is bigger at low energies than it would be if $\sigma^{(P)}$ were constant. The bigger $\sigma^{(f)}$ now tends to be too high to average the resonances.

In Fig. 1 we exhibit the $t$-channel isoscalar $\pi N$ resonance part of $\nu\,\mathrm{Im}A'^{+}(\nu,0)$ taken from the phase-shift analysis of Ref. 14. Here $\nu$ is the laboratory energy and $t=0$, so

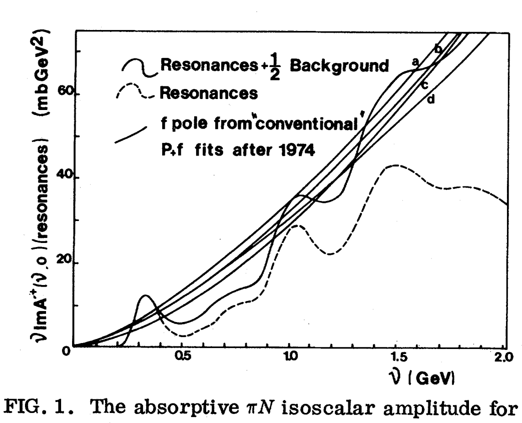


FIG. 1. The absorptive $\pi N$ isoscalar amplitude for the resonances (Ref. 14) (dashed line), the resonances plus one-half the background (solid line), and the absorptive $f$-pole amplitudes from four recent conventional $P+f$ fits (Ref. 15) (see Sec. II).

$$\mathrm{Im}A'^{+}(\nu,0)=\tfrac{1}{2}(\nu^2-m_\pi^{\,2})^{1/2}(\sigma_{\pi^+p}+\sigma_{\pi^-p})\,.$$

We have adopted the Harari-Zarmi prescription[13] for widths $\Gamma\alpha q^{2l+1}$ below resonance and $\Gamma=$ constant above resonance. There are only minor changes from 1969, mostly around $\nu=1.5$ to 2 GeV.

We also exhibit, for comparison, the resonances plus one-half the background as the solid line in Fig. 1.

Superimposed we plot the $t=0$ absorptive $f$-pole amplitudes a–d taken from several recent typical "conventional" $P+f$ published fits.[15] By conventional we mean that the $P$ is basically a $j$-plane pole plus small cuts at $t=0$, consistent with dominant short-range order. Although the authors of these fits differ markedly in their philosophy toward exchange degeneracy, and although the $f$ intercept ranges from 0.4 to 0.6, the results are quite uniform. *The $f$-pole amplitudes of these fits average the resonances plus one-half the background.*[11]

To test the generality of these results, we exhibit Fig. 2. Here the phase-shift analysis of Fig. 1 is seen to agree well with two other phase-shift analyses[16] below $\nu=1.5$ GeV, with some disagreement above this value. The $f$ amplitudes of the fits (a–d) of Fig. 1 are plotted along with the results from five others.[17] Fits e and f are older Regge fits that were performed before much of the newer cross-section data were available. They are included only for completeness. Fits g, h, and i are highly unconventional. Fits g and h parametrize the $P$ by unmotivated *ad hoc* functions which curve up at low energies. Fit i parametrizes the $P$ by a dipole and a large "shielding cut." The energy dependence of the shielding cut acts rather like a conventional $f$, which explains the resulting small $f$ amplitude in this fit.

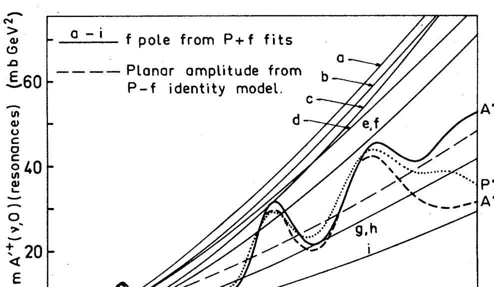


FIG. 2. The absorptive $\pi N$ isoscalar resonance amplitude is taken from three phase-shift analyses (Refs. 14 and 16) labeled P77, A74, A72. Fits a–d are as in Fig. 1. Fits e–i are either older fits that do not fit Fermilab data well, or else do not satisfy the constraint of $j$-plane-pole dominance of the $P$ implied by dominant short-range order and unitarity (see Sec II). Also shown is the comparison for the model-dependent (Ref. 12) $P$-$f$ identity planar amplitude (cf. Sec. IV).

None of the $f$ amplitudes in Figs. 1 or 2 average the resonances as well as any of the pre-1969 fits used to. The closest are the older fits e and f. Total cross sections measured in 1976 rise faster than fits e and f, thus marking the evolution in time from 1969 when two-component duality was satisfied.

We have also examined $KN$ scattering. Similar results are obtained.

Are there any loopholes within conventional $P+f$ phenomenology? Different $\Gamma(q)$ functions should not make qualitative changes, as Harari and Zarmi argued. Additional low-lying poles ($\epsilon,\sigma,P''$) with $\mathrm{Im}A'^{+}>0$ will not change $\sigma$ fits above 10 GeV but will increase the already too large low-$\nu$ Regge $\mathrm{Im}A'^{+}$. Extrapolation of all fits was done in $\nu$. Other variables like $\nu+\nu_0$ lack motivation [except for $\sigma^{(f)}=\beta s^{\alpha_f}/\lambda^{1/2}(s,m^2,m_\pi^{\,2})$, where $\nu_0=\frac{1}{2}$ GeV, which again makes the $f$ amplitude bigger than for our choice $\nu_0=0$]. Next, if the $P$ is $f$-dominated, perhaps a piece of the resonances should be reassigned from the $f$ to the $P$, but this also makes the situation worse.

## III. AN ATTEMPT TO RECONCILE DUALITY WITH THE CONVENTIONAL $P+f$ MODEL

We have tried to test the generality of the above observation by the following exercise. We write the $f$-pole contribution to $A'^{+}$ at laboratory energy $\nu$ as

$$\mathrm{Im}\,A_f'^{+}(\nu,0)=\nu\sigma_{\pi N}^{(f)}=\beta_f(\nu/\nu_0)^{1/2}\,, \qquad (4)$$

where $\nu_0=1$ GeV and $\beta_f=20$ mb GeV. This $f$-pole amplitude is consistent with the duality prediction Eq. (1). We parametrize the $P$-pole amplitude by taking $\sigma_{\pi N}^{(P)}=\beta_P(\nu/\nu_0)^{\alpha_P-1}$.

We then insist that the Fermilab data[18] for $\sigma_{\pi N}$ be accurately described. The results are shown in Fig. 3. The $P$ parameters are $\alpha_P=1.075$ and $\beta_P=15.3$ mb. As expected, forcing the duality constraint has led to values of $\sigma_{\pi N}$ much too low to agree with the data at moderate energies.

If one constrains the $f$ amplitude as above and tries to fit $\sigma_{\pi N}$ without insisting that the Fermilab points be accurately fit, a generally incorrect shape results.

We have modified this procedure by adding reasonably parametrized cuts and daughters, without noticeable changes in the conclusions. As it stands, our results employ a $P$ amplitude very close to that proposed by Quigg and Rabinovici.[15] Their predictions for Eq. (1) are shown in curve a in Figs. 1 and 2.

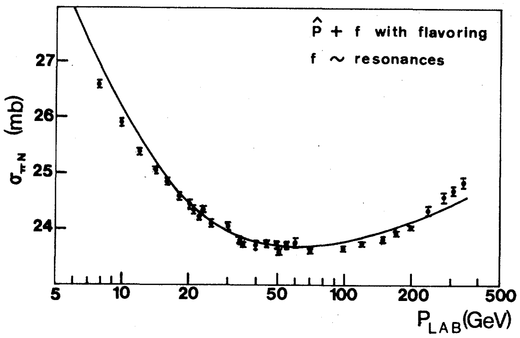


FIG. 3. The $I=0$ total $\pi N$ cross section $\sigma_{\pi N}$ obtained in a conventional $P+f$ parametrization with the $f$-pole amplitude constrained by duality (see Sec. III).

## IV. THE $P$-$f$ IDENTITY AND TWO-COMPONENT DUALITY

The $P$-$f$ identity is based on a specific realization of Veneziano's flavor quark topological expansion.[4,7] To the cylinder level we get a "two-component" picture $T=T_{\rm pl}+T_{\rm cyl}$, and we can ask whether the absorptive planar and cylinder amplitudes $\mathrm{Im}T_{\rm pl}$ and $\mathrm{Im}T_{\rm cyl}$ are dual to resonances and background, respectively. The difference here is that the $P$ and $f$ are the same object, hence the name $P$-$f$ identity. However, this $f$ is not the planar $f$, but rather is shifted by the cylinder amplitude according to the partial-wave equation for the $t=0$, $I=0$ amplitude,

$$A_j=A_j^{\rm pl}+gA_j^{\rm pl}C_jA_j\,. \tag{5}$$

All this is accomplished by taking the cylinder kernel $C_j$ as a nonsingular function of $j$ near $j=1$.

In order to specify the decomposition $T=T_{\rm pl}+T_{\rm cyl}$ we need a model. We choose the model of Ref. 12 which incorporates flavoring needed to obtain rising $\sigma$.[5] At low energies the unitarity sum is made up almost exclusively of $u$ and $d$ quarks and the relevant amplitude $\hat{A}_j$ is "unflavored."[5] $A_j^{\rm pl}$ has poles at $\alpha_{\rm pl}=0.55$ and $\alpha_d=\alpha_{\rm pl}-1$, namely the leading planar pole and a daughter.

The daughter is necessary to avoid the "$\omega$-problem" (without it $\alpha_\omega\approx 0$). In fact, the model produces $\alpha_\omega\approx 0.44$–$0.48$ and a mixing angle $\theta_\omega=-34°$. Even at low energies the (unflavored) $\hat{\omega}$ couples to $KN$ scattering through the cylinder kernel. This resolves the "$\omega$ problem".

The output vacuum $\hat{A}_j$ poles are the unflavored Pomeron $\hat{P}$ of Ref. 5 with the intercept $\hat{\alpha}=0.85$ and the shifted daughter at $\hat{\alpha}_d\approx 0$ which does not contribute to $\mathrm{Im}A'^{+}$.

We take the scale $\nu_0=m_\pi$ in the Mellin transform so that $\mathrm{Im}A'^{+}=0$ at threshold. This also suppresses the daughter in $(\mathrm{Im}A'^{+})_{\rm pl}$. We normalize the $\hat{P}$-model amplitude to the fit of Ref. 5 with a normalization constant $N=8.8$ mb GeV$^2$, and as in Ref. 12 set $\lambda_d=3$. The absorptive planar amplitude $\mathrm{Im}A'^{+}(\nu,0)_{\rm pl}$ should be dual to the resonances. It is given by

$$\mathrm{Im}A'^{+}(\nu,0)_{\rm planar}=\frac{N}{2m}\left[\left(\frac{\nu}{\nu_0}\right)^{\alpha_{\rm pl}}+\lambda_d\left(\frac{\nu}{\nu_0}\right)^{\alpha_d}\right]\theta\left[\ln\frac{\nu}{\nu_0}\right]. \tag{6}$$

The results are plotted as the heavy dashed line in Fig. 2. The biggest component is the first term. The daughter bends the curve up at low $\nu$ and is 30% of the total at $\nu=1$ GeV. The model planar amplitude is reasonably well dual to the resonances. The model cylinder amplitude is somewhat low to average the background but is qualitatively in agreement.

Hence, the $P$-$f$ identity, implemented via Ref. 12, does satisfy duality in the topological form Eq. (2).

## V. FLAVORING AND DUALITY

We now show that by making the unconventional choice of explicitly including flavoring renormalization in the context of the HF $P+f$ model, an acceptable $\sigma_{\pi N}$ fit can be obtained, consistent with the duality constraint Eq. (1). $J$-plane cuts are also needed, but these do not dominate the amplitudes.

Actually, flavoring is a concept that should be independent of the details of diffraction scattering (e.g., whether the $f$ exists as a separate entity from the $P$). Given a dominant SRO framework, flavoring renormalization must occur due to the sequence of effective thresholdlike excitations of $K\bar{K}$, $B\bar{B}$, $D\bar{D}$, ... pairs. This has been described in great detail in Ref. 5. The idea is a little like the successive renormalizations of the scaling law due to the crossing of thresholds and exciting new quantum numbers, familiar from $e^+e^-$ annihilation, with important dynamical differences of course.[6] The Regge scaling law reads $\nu^{\hat{\alpha}}$ below the above effective thresholds ($\nu<30$ GeV) where $\hat{\alpha}$ is the unflavored $\hat{P}$ intercept. The $\hat{P}$ is the leading pole of the unflavored partial-wave amplitude $\hat{A}_j$, whose Mellin transform accurately describes $\sigma(s)$ below the effective $K\bar{K}$, $B\bar{B}$, . . . thresholds. The flavoring-renormalized trajectory $\alpha$ gives the scaling law $\nu^{\alpha}$, valid at energies much higher than these effective thresholds. At energies comparable to the effective thresholds, things are complicated, and neither $\nu^{\hat{\alpha}}$ nor $\nu^{\alpha}$ accurately represents the physics. The number $\alpha$ is the intercept of the flavored $P$, the leading pole of the Froissart-Gribov partial-wave amplitude $A_j$ before $j$-plane cuts are included ($P$ is the bare Pomeron of the Reggeon field theory). We stress that there are no "energy-dependent trajectories" (cf. Ref. 5).

Flavoring effects on $\sigma$ (at least in $NN$ scattering) can be read off the inelastic data for $K\bar{K}$, $B\bar{B}$, . . . production[19] given the reasonable assumption that at most one heavy pair per event is produced at current energies. The effect is very large ($\Delta\sigma > 20$ mb between $s=200$ and 4000 GeV$^2$), and this statement is model independent.

Flavoring is translated into the above $j$-plane language by implementing SRO with generic strong-coupling multiperipheral models.[5] The dominance of SRO, implying weak absorption, also means absorption effects cannot cancel the large flavoring effects (though they can dampen them somewhat, as we will see). The estimate of the renormalization $\alpha - \hat{\alpha}$ due to this approach was found in Ref. 5 as

$$\Delta\alpha = \alpha - \hat{\alpha} \approx 0.2\,. \tag{7}$$

References 5 and 12 investigated the phenomenology of flavoring within the context of the $P$-$f$ identity model. It is now known that explicit inclusion of flavoring is needed for a viable $P$-$f$ identity phenomenology.[4–6,20,21] Flavoring saves the $P$-$f$ identity model from a variety of potential problems including those raised by Romao and Freund,[22] Duke,[23] Quigg and Rabinovici,[15] and Pennington, Schrempp, and Schrempp.[24]

Since flavoring effects are so large, it was argued in Refs. 5, 6, and 10 that consistency with flavoring presented a severe challenge to conventional $P+f$ physics.

We show here that, although flavoring effects are indeed large, an acceptable $P+f$ description of $\sigma_{\pi N}$ can be obtained. Moreover, the resulting curvature of the $P$ amplitude allows a decreased magnitude of the $f$-pole amplitude, allowing $f$-pole $\sim$ resonances duality to be reinstated.

The way this happens is as follows. The $\hat{P}$ amplitude of the $P$-$f$ identity (before flavoring) winds up being similar to the $\hat{P}$ amplitude plus the $f$ amplitude in the $P+f$ model (before flavoring). This is made possible by increasing the unflavored intercept ($\hat{\alpha}=0.85$ for the $P$-$f$ identity,[5] $\hat{\alpha}=0.91$ here), and decreasing the $\hat{P}$ residue. The flavoring is then similar in the two cases. Absorptive effects must be added because the flavoring effects are in fact *too* strong. The absorptive cuts are a bit bigger in the $P+f$ case than in the $P$-$f$ identity case, but still small enough to be called compatible with $j$-plane pole dominance.

In no case do we get an acceptable description with either $\hat{\alpha}$ or $\alpha$ equal to one.

We now proceed to the details. We actually have to start with $NN$ scattering, because the only $ab \to c\bar{c}+X$ data ($c=K,B,\ldots$) that is compiled[19] is for $ab=NN$. We parametrize the $NN$ amplitude for the $P$ exactly as in Ref. 5. The absorptive triple-$\hat{P}$ cross section $\sigma_D$ (cf. Ref. 5) is bounded in magnitude by $\frac{1}{3}\sigma_{NN}$, which we take as a reasonable upper limit consistent with SRO. Most $P+f$ phenomenology is done with smaller absorption than that. To this we add an $f$-pole contribution of the form $\sigma_{NN}^{(f)} = \beta_f^{NN}(s/1\ \mathrm{GeV}^2)^{-1/2}$. We then obtain a fit to $\sigma_{NN}$, consistent with inelastic $K\bar{K}$, $B\bar{B}$ data. Details of this and other related matters will be presented as a separate communication.[25] The value of the flavored intercept $\alpha$ is $\alpha=1.1$, which is compatible with the above estimate $\Delta\alpha = \alpha - \hat{\alpha} \approx 0.2$.

Turning to $\pi N$ scattering, flavoring effects are determined via SRO from $NN$ scattering as described in Ref. 5. The numbers $\hat{\alpha}$ and $\alpha$ are also fixed from $NN$ scattering.

We fix the $f$-pole amplitude using Eq. (4) to satisfy duality Eq. (1). We are then in fact able to fit $\sigma_{\pi N}$ if we also include eikonal $\hat{P}\times\hat{P}$ cuts and the absorptive triple-$\hat{P}$ cross section $\sigma_D$. (We have tried including flavored cuts; they apparently do not work.[26]) The total absorptive cross section at 340 GeV is $\sigma^{abs}=-8$ mb, saturating the (rather arbitrary) $-\frac{1}{3}\sigma_{\pi N}$ bound for absorption taken to be necessary to ensure dominant SRO. All $\pi N$ parameters are as in Ref. 5, except $\hat{\alpha}=0.91$, $\alpha=1.1$, $\xi_{\pi N}=0.15$, $\beta=91.4$, and $b_0=0.5$.

The results for $\sigma_{\pi N}$ in the $\hat{P}+f+$flavoring model including the above absorptive cuts are presented in Fig. 4.

We emphasize that without including *both* flavoring and absorption we cannot reconcile the $P+f$ model with duality.

We conclude that an acceptable (and desirable) way out of the difficulties with two-component duality of the $P+f$ model is to include flavoring renormalization with some absorption. Of necessity, the description seems to be rather complicated, but it has the added virtue over conventional parametrizations of now being consistent with the successive excitation of quantum numbers expressed through $s$-channel unitarity.[5] It remains

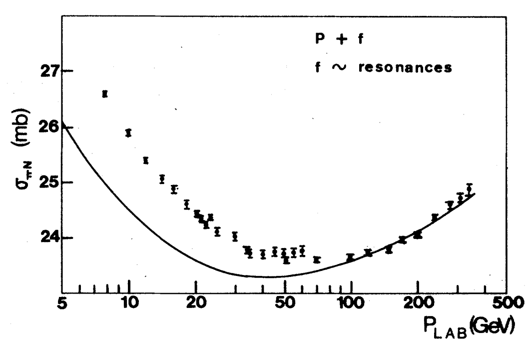


FIG. 4. Results for $\sigma_{\pi N}$ in the $P+f$ model including flavoring and absorptive effects, as described in Sec. V. The $f$-pole amplitude is constrained by duality, as in Fig. 3.

to be seen whether the rest of the legacy of the $P+f$ model is also compatible with flavoring.

Given that flavoring seems to enable both the $P$-$f$ identity and $P+f$ models to become consistent with important aspects of the data, it seems harder than ever to distinguish between them on phenomenological grounds alone. A decisive experimental distinction would evidently be welcome.

## VI. CONCLUSIONS

We have shown that existing $P+f$ phenomenology consistent with rising total cross sections and with the important constraint of $j$-plane pole dominance of the $P$ (i.e., dominant short-range order in rapidity) is *not* consistent with the classic duality equation $\mathrm{Im}\, T_{f\,\mathrm{pole}} \approx \langle \mathrm{Im}(\text{resonances})\rangle$ with the resonances being taken from phase-shift analyses. We also showed that the $P$-$f$ identity[4] with flavoring[5] properly incorporated[12] *is* consistent with the topological expansion version of duality $\mathrm{Im}\, T_{\mathrm{planar}} \approx \langle \mathrm{Im}(\text{resonances})\rangle$. We also showed that a modified $P+f$ phenomenology including flavoring and some absorption can be made consistent with rising $\sigma$ and with $\mathrm{Im}\, T_{f\,\mathrm{pole}} \approx \langle \mathrm{Im}(\text{resonances})\rangle$.

Further details will be published separately.[25]

## ACKNOWLEDGMENT

The work of S.T.J. was supported in part by NSF Grant No. PHY 77 05300.

## APPENDIX

We show here that, under reasonable hypotheses, if the $f$-pole amplitude is too large to satisfy duality [Eq. (1)], the planar amplitude is even bigger and therefore violates duality even more.

The demonstration is as follows. We utilize the topological expansion to the cylinder level, taking a solution which "naturally" produces the $P+f$ model in a good approximation. One can criticize this model on general phenomenological and theoretical grounds.[3,6] While such considerations may in fact be relevant for a choice as to the correct viewpoint of diffraction, the model below is a reasonable hypothesis in the current context insofar as it does naturally generate a spectrum with two leading poles with desired properties. It also incorporates a widely held attitude toward the Pomeron.[27]

Basically, one regards the Pomeron as generated by gluon dynamics at $N_c$ (number of colors) $=\infty$, as a pole at the cylinder level with intercept $\alpha^* \approx 1$.

The transition from $N_c=\infty$ to $N_c=3$ is assumed not to change this[28] (questions like this are at the heart of the controversy[3,6]). This pole is then allowed to mix in a limited way with the planar amplitude $A_j^{\mathrm{pl}}$, shifting the planar poles slightly but basically preserving their EXD. Since at the cylinder level one needs a $j$-plane-pole output ($j$-plane cuts presumably arise from higher-order topologies[4,7]), one writes a multiperipheral model for $A_j$. Ignoring inessential momentum-transfer integrals, we get

$$A_j = A_j^{\mathrm{pl}} + A_j^{\mathrm{pl}} C_j^* A_j \tag{A1}$$

with solution

$$A_j = (1 - A_j^{\mathrm{pl}} C_j^*)^{-1} A_j^{\mathrm{pl}} . \tag{A2}$$

Here as just mentioned the cylinder kernel $C_j^*$ is assumed generated by gluon dynamics as having a pole at $j=\alpha^*$,

$$C_j^* = \frac{g^*}{j-\alpha^*} . \tag{A3}$$

Nonsingular pieces of the cylinder kernel as well as flavoring are ignored (these are crucial in the $P$-$f$ identity[4]). $A_j^{\mathrm{pl}}$ is the planar amplitude with leading pole $A_j^{\mathrm{pl}} \approx \beta_0 (j-\alpha_0)^{-1}$.

The poles of $A_j$ are at

$$2\alpha_\pm = \alpha_0 + \alpha^* \pm [(\alpha_0 - \alpha^*)^2 + 4\beta_0 g^*]^{1/2} . \tag{A4}$$

We write

$$A_j = \frac{\beta_+}{j-\alpha_+} + \frac{\beta_-}{j-\alpha_-} \tag{A5}$$

and we find

$$\beta_- = \beta_0 \frac{(\alpha^* - \alpha_-)}{(\alpha_+ - \alpha_-)} . \tag{A6}$$

Now $g^*$ and $\beta_0$ are positive (this is a general positivity requirement of all such equations). Hence,

$$\alpha_+ > \alpha^* \text{ and } \alpha_- < \alpha_0 , \tag{A7}$$

which just means that the mixing has repelled the original levels. Hence

$$\beta_- < \beta_0 . \tag{A8}$$

We write the $t=0$ Mellin transform as

$$\mathrm{Im}\, A'^{+}(\nu) = \int \frac{dj}{2\pi i} \left(\frac{\nu}{m_\pi}\right)^j A_j , \tag{A9}$$

where we have taken the scale $m_\pi$ so that the correct threshold behavior is ensured, i.e.,

$$\mathrm{Im}\, A'^{+}(\nu) = 0 \quad \text{for } \nu < m_\pi .$$

The connection to our parametrization in the text is clearly $\alpha_f = \alpha_-$ and $\beta_f = \beta_- (\nu_0/m_\pi)^{\alpha_-}$. Hence for $\nu > m_\pi$,

$$\beta_f(\nu/\nu_0)^{\alpha_f}=\beta_-(\nu/m_\pi)^{\alpha_-}<\beta_0(\nu/m_\pi)^{\alpha_0},$$

i.e.,

$$\mathrm{Im}A_f'^{+}(\nu)<\mathrm{Im}A_{\mathrm{planar}}'^{+}(\nu). \tag{A10}$$

Thus, if the $f$-pole amplitude is too big to average the resonances, the planar amplitude, being bigger, violates duality in a worse fashion. That is, the presence of cylinder mixing in the empirical $f$ is not an acceptable resolution of the duality difficulties of the $P+f$ model.

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[8]We are assuming that the planar amplitude is reasonably well described by, e.g., some narrow-width dual-resonance model whose planar unitarization does not introduce large nonresonant effects at least at low energies ($\nu<2$ GeV). This could be the case if there are effective thresholds for planar loops. Such thresholds exist in any strong-coupling multiperipheral model (see Ref. 5, Appendix). Explicit numerical calculations contain these thresholds [cf. H. M. Chan, J. E. Paton, S. T. Tsou, and S. W. Ng, Nucl. Phys. B92, 13 (1975), Fig. 12].

[9]Dynamical complications may exist in this continuation, however. We thank R. Warnock for a discussion on this point.

[10]J. W. Dash and Alex Martin, in *Hadron Physics at High Energies*, proceedings of Marseille Colloquium, edited by C. Bourrely, J. W. Dash, and J. Soffer (Centre de Physique Theorique, Marseille, 1978), p. III.89.

[11]By "resonances," we mean the $I_t=0$ $\pi N$ resonance contribution to $\mathrm{Im}\,A'^{+}(\nu,0)$ as established through phase-shift analyses (three examples are shown in Fig. 2). There is always the possibility of other broad resonances which have not been uncovered by these analyses. The inclusion of these effects, if they exist, would certainly ameliorate to some unknown extent the description of duality by these conventional $P+f$ fits, although given the serious $\sim50\%$ existing discrepancy, these effects would have to be quite large to resolve the difficulty completely (see Sec. II).

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[26]The absorptive amplitudes used in our $\hat{P}+f+$ flavoring + absorption model have the usual low-order perturbative $j$-plane-cut forms. The triple-$\hat{P}$ cross section $\sigma_D$ which contributes absorptively in the total cross section, describes $ab\to aX$ reactions near $x=1$ when the subenergies in the triple-$\hat{P}$ Regge graph are below the effective thresholds for $K\bar{K}$ and $B\bar{B}$ production (i.e., $s/M_X^2$ and $M_X^2/1$ GeV$^2\lesssim 60$). See J. Dash, Phys. Rev. D 9, 200 (1974). In fact, $ab\to aX$ reactions are consistent with the unflavored $\hat{P}$ of the $P$-$f$ identity with $\hat{\alpha}=0.85$; it remains to be seen whether the value $\hat{\alpha}=0.91$ found here is also compatible. Some (small) flavoring renormalization of $\sigma_D$ is present in the form-

ula we use (Ref. 5). The $\hat{P}\times\hat{P}$ eikonal cut is necessary in our $\sigma_{\pi N}$ fit, and serves mainly to decrease the model $\sigma_{\pi N}$ by about 2.5 mb at 10 GeV. The flavoring renormalization of the $\hat{P}\times\hat{P}$ cut has not been consistently examined theoretically. As mentioned in the text, the simple alternative of a flavored $P\times P$ cut has the wrong energy dependence to work empirically in detail.

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