Evidence for valence-quark clusters in nucleon structure functions

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By assuming the formation of quark clusters as valence quarks undergo Q^2 evolutions in quantum chromodynamics, we find evidences in the deep-inelastic neutrino scattering data that suggest their existence. From the moments of structure functions we extract direct information on the cluster distribution in a nucleon. Their physical significance is discussed.

I. INTRODUCTION

In the conventional parton model¹ the valence quarks do not play a prominent role since they are largely submerged under a sea of quarks and antiquarks. Yet the simple quark model² that has only three constituent quarks for nucleons and two for mesons without any mention of sea quarks has been quite successful in interpreting many aspects of hadron physics, such as the 2:3 ratio of the πp :pptotal cross sections. It is the purpose of this paper to unify these two views in the framework of quantum chromodynamics (QCD).

The key to the unification is in the identification of valence-quark clusters. If indeed we can associate with each valence quark its own cluster of sea quarks and gluons, then the three clusters in a nucleon can be identified with the constituents in the additive quark model, and a link between the two views can thereby be established. The idea of quark clusters if not new; it has been considered in the context of a broken-SU(6)_w×O(3) scheme,³ and in a specific phase-space model.⁴ It has also been found⁵ that hadron-nucleus scattering data reveal information on the number of guark clusters in nucleons and pions. But thus far the investigations have been rather model dependent. Fortunately, during the past year important advances have been made in the neutrino scattering experiments,^{6,7} and the predictions of QCD have been verified. We shall use the same data and the same theory to extract quantitative and direct information about the valence-quark clusters with basically no further assumptions beyond what defines the clusters.

II. VALONS

Suppose that a nucleon is a composite system of three constituent quarks. To determine their momentum distribution, one may perform a deepinelastic scattering experiment but would find that the data can be understood only if there are an infinite number of quarks and antiquarks. That is not contradictory to the picture of three constituent quarks if by the latter we mean three valence-quark clusters, each of which contain quarks, antiquarks, and gluons that can be resolved by high- Q^2 probes. At low Q^2 the resolution is so poor that only three clusters can be discerned in a nucleon, each having no recognizable internal structure. In bound-state problems they are called the constituent quarks. For brevity, we shall refer to the valence-quark clusters as valons.

The valons therefore serve as a bridge between hard and soft processes. As basic units in a bound-state $(low-Q^2)$ problem, they form the basis in terms of which the hadronic wave function can be described. It is here that the so-called uncalculable hadronic complication occurs. On the other hand, when the nucleon is probed at high Q^2 . it is really the valon structure that is probed, although on top of it there is also a smearing on account of the momentum distribution of the valons themselves. In the following we shall specify precisely what the valon structure is. Qualitatively, it has a cloud of quarks and gluons that evolve from a single quark in a way that is calculable in QCD. In that sense a valon is just a dressed valence quark.

Our aim is to learn about the wave function of a nucleon in the valon representation. From the study of the hadron spectroscopy it is reasonable to assume that a nucleon has three valons. However, it is not obvious whether the Hilbert space for the description of a hadron can be spanned by just the valon vectors only. Since the valons are not free, gluons are needed to bind them to form a hadron. In that case it appears that in addition to the valon coordinates one would need to include additional degrees of freedom for the gluons. Stated differently, the question is whether, in an (infinite-momentum) frame where a hadron moves fast, the valons exhaust the momentum of the hadron, or carry only a fraction γ of the hadron momentum with the balance $1 - \gamma$ being carried by gluons not included in the valons. The issue can-

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not be settled without a reliable theory for confinement. However, we can learn from experiments the relative importance of the valons and the gluons that bind them. As a working hypothesis, we assume that $\gamma = 1$. That is the basis of our valon model. If the assumption is wrong we should have difficulty in constructing a consistent description of the hadron that can accommodate the data.

Our model of a hadron in terms of the valons is not too far different from the usual picture of, say, a deuteron in terms of two nucleons. Even though pions are exchanged between the nucleons to effect binding, one usually regards a deuteron as a bound state of a proton and a neutron without including the pions as essential constituents. The wave function of the deuteron in the nucleon coordinates suffices to describe the effects of binding. In a similar way we treat a nucleon in terms of three valons and a meson in terms of two valons. The square of the wave function in momentum space, after integrated over all but one of the valon momenta, is the inclusive distribution of a valon v in a hadron h. We denote it by $G_{v/h}(y)$, where y is the momentum fraction carried by the valon. It is normalized by

$$\int_{0}^{1} G_{\nu/h}(y) dy = 1$$
 (2.1)

for each v. The momentum sum rule is

$$\sum_{\nu} \int_0^1 G_{\nu/h}(y) y \, dy = 1 , \qquad (2.2)$$

where the sum is over all valons in the hadron h.

Let $\mathfrak{F}^h(x, Q^2)$ denote the nucleon structure function, which can be either F_2 or xF_3 . Let $\mathfrak{F}^v(z, Q^2)$ be the corresponding structure functions of a valon; they are precisely known in QCD at least in the leading-order approximation. They are calculable because they have no bound-state complications which are entirely contained in $G_{v/h}(y)$. Starting from a quark at z = 1, the evolution equation of Altarelli and Parisi⁸ completely specifies $\mathfrak{F}^v(z, Q^2)$ at high Q^2 . On the basis of impulse approximation the interaction among the valons can be ignored when one of the valons is struck by a virtual photon at high Q^2 . We then have the convolution equation relating the hadron to valon structure functions,

$$\mathfrak{F}^{h}(x,Q^{2}) = \sum_{v} \int_{x}^{1} dy \ G_{v/h}(y) \mathfrak{F}^{v}(x/y,Q^{2}) .$$
 (2.3)

Defining the moments by

$$M^{h,v}(n,Q^2) = \int_0^1 dx \, x^{n-2} \mathcal{F}^{h,v}(x,Q^2) , \qquad (2.4)$$

$$M_{\nu/h}(n) = \int_0^1 dy \, y^{n-1} G_{\nu/h}(y) \tag{2.5}$$

we obtain from (2.3)

$$M^{h}(n, Q^{2}) = \sum_{\nu} M_{\nu/h}(n) M^{\nu}(n, Q^{2}) .$$
 (2.6)

Except for the sum over v, (2.6) is in the form of the solution of the renormalization-group equation,⁹ i.e., a product of two factors, one dependent on Q^2 and exactly calculable in QCD, and another dependent on the bound-state nature of the hadron and basically unknown. A relation similar to (2.6)was considered by Cabibbo and Petronzio.³ However, whereas the valons discussed here have no internal structure other than what is generated by evolution from $\delta(z-1)$,¹⁰ the constituent quark considered by them is assumed by ansatz to possess a nontrivial initial structure. Since in our approach $M^{\nu}(n, Q^2)$ is known, experimental data on $M^{h}(n, Q^{2})$ therefore make possible a phenomenological determination of $M_{\nu/h}(n)$ via (2.6). In so doing we gain direct insight on the hadronic wave function expressed in the valon coordinates.

III. THE NONSINGLET MOMENTS

In this paper we consider only the nonsinglet moments because we can extract from the nonsinglet data direct information about the valon distribution. For $F_2(x, Q^2)$ in e or μ scattering we have from (2.6)

$$M_2^{p} = 2M_{U/p}M_2^{U} + M_{D/p}M_2^{D}, \qquad (3.1)$$

$$M_2^n = M_{U/n} M_2^U + 2M_{D/n} M_2^D , \qquad (3.2)$$

where U and D denote u- and d-type valons, respectively. Owing to charge symmetry we thus have

$$M_{\rm NS}^{\mu N} \equiv M_2^{\rho} - M_2^{n} = (2M_{U/\rho} - M_{D/\rho})(M_2^{U} - M_2^{D}).$$
(3.3)

Consider $M_2^U = \sum_i e_i^{2} M_{q_i/U}$ where $M_{q_i/U}$ denotes the moments of the distribution of q_i quark in a U valon. If q_i is a u quark, we call it M_{fav} (for favored distribution); for all other quarks and antiquarks, they are the same M_{unf} (for unfavored distribution).¹⁰ Then assuming that the relevant number of flavors f for the data to be analyzed is three, we have

$$M_2^U = \frac{4}{9} M_{\text{fav}} + \frac{8}{9} M_{\text{unf}} , \qquad (3.4)$$

$$M_2^D = \frac{1}{9}M_{\text{fav}} + \frac{11}{9}M_{\text{unf}} , \qquad (3.5)$$

whereupon we obtain

$$M_2^U - M_2^D = \frac{1}{3} (M_{\text{fav}} - M_{\text{unf}}) = \frac{1}{3} M_{\text{NS}}^v, \qquad (3.6)$$

where M_{NS}^{ν} is the moment of the nonsinglet quark distribution in a valon. Equation (3.6) is actually

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independent of f. We thus have, in general, from (3.3)

$$M_{\rm NS}^{\mu N}(n,Q^2) = \frac{1}{3} [2M_{U/p}(n) - M_{D/p}(n)] M_{\rm NS}^{\nu}(n,Q^2).$$
(3.7)

A similar expression can be derived for M_3 of neutrino scattering. If $\sin^2\theta_c$ is neglected, the usual quark-model relations for νp and $\overline{\nu} p$ structure functions are

$$F_{3}^{\nu\nu}(x) = 2[d(x) - \overline{u}(x)], \qquad (3.8)$$

$$F_{3}^{\overline{u}p}(x) = 2[u(x) - \overline{d}(x)], \qquad (3.9)$$

where the overall sign of the F_3 functions are chosen for phenomenological convenience. The valon structure functions are similar:

$$F_{3}^{\nu U}(z) = 2[d_{U}(z) - \overline{u}_{U}(z)], \qquad (3.10)$$

$$F_{3}^{\overline{\nu} U}(z) = 2[u_{U}(z) - \overline{d}_{U}(z)], \qquad (3.11)$$

$$F_{3}^{\nu D}(z) = 2[d_{D}(z) - \overline{u}_{D}(z)] = 2[u_{U}(z) - d_{U}(z)], \quad (3.12)$$

$$F_{3}^{\overline{\nu}D}(z) = 2[u_{D}(z) - \overline{d}_{D}(z)] = 2[d_{U}(z) - \overline{u}_{U}(z)], \quad (3.13)$$

where the subscripts on the right-hand side refer to the valons for which the quark distributions are defined. If we use the distribution functions for the U valon only, and denote their moments by $M_a(n, Q^2)$, it then follows from (2.6) that

$$M_{3}^{\nu\rho} = 4M_{U/\rho}(M_{d} - M_{\overline{u}}) + 2M_{D/\rho}(M_{u} - M_{\overline{d}}) , \qquad (3.14)$$

$$M_{3}^{\overline{\nu}p} = 4M_{U/p}(M_{u} - M_{\overline{d}}) + 2M_{D/p}(M_{d} - M_{\overline{u}}), \qquad (3.15)$$

$$M_{3}^{\nu n} = 2M_{U/n}(M_{d} - M_{\overline{u}}) + 4M_{D/n}(M_{u} - M_{\overline{d}}), \qquad (3.16)$$

$$M_{3}^{\nu n} = 2M_{U/n}(M_{u} - M_{\overline{d}}) = 4M_{D/n}(M_{d} - M_{\overline{u}}) , \qquad (3.17)$$

where

$$M_3(n) = \int_0^1 x^{n-1} F_3(x) dx.$$

Using charge symmetry again for the valon distributions in nucleons, we obtain for isoscalar target

$$M_{3} = \frac{1}{4} \left(M_{3}^{\nu\rho} + M_{3}^{\nu n} + M_{3}^{\overline{\nu}\rho} + M_{3}^{\overline{\nu}n} \right)$$

= $(2M_{U/\rho} + M_{D/\rho}) (M_{u} + M_{d} - M_{\overline{u}} - M_{\overline{d}}) .$ (3.18)

The last factor being $M_{\text{fav}} - M_{\text{unf}}$, we have

$$M_{3}(n, Q^{2}) = \left[2M_{U/p}(n) + M_{D/p}(n)\right] M_{\rm NS}^{v}(n, Q^{2}). \quad (3.19)$$

We can examine the implications of (3.7) and (3.19) for n = 1 and 2. Owing to the absence of renormalization of the electromagnetic and weak currents, we have $M_{NS}^{\nu}(1, Q^2) = 1$ for any Q^2 . It then follows from (2.1) and (2.5) that

$$M_{\rm NS}^{\mu N}(1,Q^2) = \frac{1}{3}, \qquad (3.20)$$

$$M_3(1, Q^2) = 3. (3.21)$$

The latter is just the Gross-Llewellyn Smith sum rule¹¹ and is consistent with the latest data.⁶ The former is also not new¹² and has the same physical content. For n=2, (2.2) serves as a constraint:

$$2M_{U/p}(2,Q^2) + M_{D/p}(2,Q^2) = 1.$$
(3.22)

Using this in conjunction with (3.7) and (3.19) we obtain

$$M_{\rm NS}^{\mu N}(2,Q^2) = \frac{1}{3} [4M_{U/p}(2) - 1] M_{\rm NS}^{\nu}(2,Q^2), \quad (3.23)$$

$$M_3(2,Q^2) = M_{\rm NS}^{\nu}(2,Q^2) \,. \tag{3.24}$$

In Sec. IV, $M_{NS}^{\nu}(n, Q^2)$ will be given for large Q^2 . If the n = 2 moments of $M_{NS}^{\mu N}$ and M_3 are accurately known at high Q^2 , then (3.23) and (3.24) can not only be used to check the validity of the valon model, but also to determine the average momentum fractions of the valons. Unfortunately, the present data are inadequate for those purposes.

For general n, $M_{U/p}$ and $M_{D/p}$ can be extracted from $M_{NS}^{\mu N}$ and M_3 using (3.7) and (3.19) provided that accurate data are available at high Q^2 . Currently, neutrino data^{6,7} have reached sufficiently high Q^2 to give successful test of QCD. To make use of those data while awaiting similar outcome of the muon data, we proceed on the basis of an approximation which can easily be lifted when the muon data become available. We assume that the valon distribution in a hadron is independent of flavor, i.e.,

$$M_{U/p} = M_{D/p} \equiv M_{v/N} \,. \tag{3.25}$$

We know that this is not exact, since the u- and d-quark distributions are somewhat different. However, with one less unknown we can proceed with just the neutrino data. What we shall obtain then will be an average valon distribution, which is worth knowing at this stage. Solution of the bound-state problem can at present do no better than that.

On the basis of (3.25) we obtain

$$M_{\rm NS}^{\mu N}(n,Q^2) = \frac{1}{9}M_3(n,Q^2) , \qquad (3.26)$$

$$M_3(n, Q^2) = 3M_{\nu/N}(n)M_{\rm NS}^{\nu}(n, Q^2).$$
(3.27)

Equation (3.26) indicates that $M_{\rm NS}^{\mu N}$ is about an order of magnitude less than M_3 , which is roughly true.^{6,12} Equation (3.27) will be used in the following to determine $M_{\nu/N}$. The implication of (3.25) is that each valon in a nucleon carries $\frac{1}{3}$ of the nucleon momentum, i.e.,

$$M_{\nu/N}(2) = \frac{1}{3}.$$
 (3.28)

It then follows that

$$9M_{\rm NS}^{\mu N}(2,Q^2) = M_3(2,Q^2) = M_{\rm NS}^{\nu}(2,Q^2) \,. \tag{3.29}$$

This provides a direct relationship between measurable quantities $(M_{NS}^{\mu N} \text{ and } M_3)$ and calculable function (M_{NS}^{ν}) .

IV. THE MOMENTS $M_{\rm NS}^{\nu}$ (n,Q^2)

We have stated that the structure of a valon is due to the gluon bremsstrahlung and quark-pair creation of an initial (bare) quark. At some small Q^2 , call it Q_v^2 , $\mathfrak{F}^v(z, Q_v^2) \propto \delta(z-1)$. As Q^2 increases, the δ function is smeared out.¹⁰ It is difficult to determine this smearing at low Q^2 , but at high Q^2 the leading-order result in QCD is a good approximation. Let the moments of the leading-order result be denoted by $\tilde{M}^v(n, Q^2)$; then our approximation at high Q^2 is

$$M_{\rm NS}^{v}(n,Q^2) \simeq \tilde{M}_{\rm NS}^{v}(n,Q^2) = [C \ln Q^2 / \Lambda^2]^{-d_n^{\rm NS}},$$
 (4.1)

where C is some constant and

$$d_n^{\rm NS} = \frac{4}{33 - 2f} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} \right].$$
(4.2)

Expressing (4.1) alternatively, we have

$$[M_{\rm NS}^{v}(n,Q^2)]^{-1/d_n^{\rm NS}} \simeq C \ln Q^2 / \Lambda^2.$$
 (4.3)

For n=2, we can use (3.29) to infer

$$[M_3(2,Q^2)]^{-1/d_2^{NS}} \simeq C \ln Q^2 / \Lambda^2 \,. \tag{4.4}$$

This linear dependence on $\ln Q^2$ has been verified by BEBC data,⁶ which give

$$\Lambda = 0.74 \text{ GeV}, \quad C = 4.8.$$
 (4.5)

CERN-Dortmund-Heidelberg-Saclay data⁷ unfortunately do not give the n=2 moment. It is important to stress that deviation from the straight line occurs at small Q^2 due to nonleading-order QCD corrections. If one were able to find an accurate formula to describe the behavior of $M_{NS}^{v}(2, Q^2)$ as $Q^2 \rightarrow Q_v^2$, then one would be able to meet the boundary condition $M_{NS}^{v}(2, Q_v^2) = 1$. However, that would be extremely difficult. Besides, we have no need for it here. So long as we limit Q^2 to large values where (4.4) is phenomenologically valid, we have (4.3) as an effective formula for all n. Note that in place of C we can define a Q_0 according to

$$C = (\ln Q_0^2 / \Lambda^2)^{-1}, \qquad (4.6)$$

so that by definition

$$\tilde{M}_{\rm NS}^{\nu}(n,Q_0^2)]^{-1/d_n^{\rm NS}} = 1.$$
(4.7)

Thus Q_0 is an effective value (different from Q_v) which specifies the starting point of Q^2 evolution in the linear approximation (i.e., $\tilde{M}_{\rm NS}^v$) in such a way that it mimics at high Q^2 the actual Q^2 evolution (i.e., $M_{\rm NS}^v$) commencing at Q_v with no approximation. Since we know neither Q_v nor the actual evolution, the effective formula

$$[\tilde{M}_{\rm NS}^{v}(n,Q^2)]^{-1/d_n^{\rm NS}} = \frac{\ln Q^2 / \Lambda^2}{\ln Q_0^2 / \Lambda^2}$$
(4.8)

is all we need at high Q^2 . It does *not* mean that we

are assuming the validity of the leading-order approximation for Q^2 near Q_0^2 , or that the usual leading logarithm formula for $\alpha_s(Q^2)$ is meaning-ful at Q_0^2 . Q_0 is just a parameter determined from the slope C in (4.4). From (4.5) and (4.6) we have

$$Q_0 = 0.82 \text{ GeV}$$
. (4.9)

The closeness of Q_0 to Λ implies, via (4.8), that the effective "age" of evolution even at $Q \approx 3$ GeV is quite advanced. This is a quantitative way of expressing precocious scaling, which means in the present context mature evolution.

V. VALON DISTRIBUTION

We are now in a position to extract the valon distribution $G_{v/N}(y)$ from data. We start with the moment equation (3.27) for which we approximate $M_{\rm NS}^{v}(n,Q^2)$ at high Q^2 by $\tilde{M}_{\rm NS}^{v}(n,Q^2)$ given by (4.8); consequently, we have

$$[M_3(n,Q^2)]^{-1/d_n^{NS}} = S(n) \ln Q^2 / \Lambda^2, \qquad (5.1)$$

$$S(n) = [3M_{\nu/N}(n)]^{-1/d_n^{NS}} [\ln Q_0^2 / \Lambda^2]^{-1}.$$
 (5.2)

In Refs. 6 and 7 the left-hand side in (5.1) has been plotted against $\ln Q^2$, and straight-line fits for all moments analyzed have been obtained. Because it is only in Ref. 6 that the n=2 moment is given, from which we obtain Q_0 , we shall in the following concentrate only on the data in Ref. 6.¹³ The slopes of the straight lines S(n) are not predicted in QCD but they should not be ignored. They contain information about the hadronic wave function, which is precisely what we want to extract. In our picture they are related to the valon distribution, which is a particular representation of the low- Q^2 nucleon wave function. From the experimental plots that verify (5.1) we determine S(n) for n= 2,...,5, which are shown in Fig. 1.

To fit S(n) we assume that the valon distribution has the form



FIG. 1. Experimental and theoretical values for the slope S(n) as functions of the order of moments n. Data are derived from Ref. 6.

where B is the beta function. Equation (3.28) requires

$$b = 2a + 1$$
. (5.4)

Hence, the moments are

 $M_{\nu/N}(n) = B(n + \frac{1}{2}(b-1), b+1)/B(\frac{1}{2}(b+1), b+1).$ (5.5)

Substituting this into (5.2) and using f=3 (a choice made in Ref. 6), we calculate S(n) for various values of b. In Fig. 1 are shown the results for b = 1, 2, and 3. Obviously, b=2 gives an excellent fit. It implies then

$$G_{\nu/N}(y) = \frac{105}{16} y^{1/2} (1-y)^2 .$$
 (5.6)

This is the result that we have aimed to extract. Note that the large-y behavior does not correspond to the result by the spectator-counting rule,¹⁴ since the valons themselves do not experience large momentum transfers.

The form of the valon distribution in (5.6) is reasonable. At moderate Q^2 , the valon structure function $F_2^v(z, Q^2)$ is finite at z = 1, so the large-x behavior of $F_2^N(x, Q^2)$ for nucleon, according to (2.3) and (5.6), is $(1 - x)^3$, just as it is observed. At small v the valon distribution is suppressed, whereas the valence-quark distribution $q_n(x)$ diverges as $x^{-1/2}$ according to Regge behavior. That is because a valon carries not only the momentum of a valence quark but also those of the sea quarks and gluons in the cluster. Hence its average momentum is greater than that of valence quark, and the probability for it to carry zero momentum fraction is therefore suppressed by comparison. The particular behavior $y^{1/2}$ is, however, hard to understand. To have a singularity at y = 0 is incompatible with the solution of the bound-state problem in which the valons are confined to a finite region in spatial extension. That is, a wave function in coordinate space that vanishes at infinity should be regular at the origin in momentum space. Probably, the behavior $y^{1/2}$ is due to the fact that (5.6) describes the momentum distribution of an average valon. In reality the U and Dvalons need not behave the same way near y = 0, and they may separately have regular behaviors there.

VI. CONCLUSION

Our principal result is the valon distribution given in (5.6). It is obtained by using the high- Q^2 data of neutrino scattering. Indeed, deep-inelastic scattering data at high Q^2 must depend on the hadronic wave function at low Q^2 . By assuming the existence of valons and identifying them as dressed valence quarks whose structures are calculable in QCD, we have provided a representation for the hadronic wave function. We have achieved this without extrapolating the theoretical calculation to low Q^2 where the present method in QCD fails. Thus our result is reliable, subject to the approximation that flavor dependence is ignored. That will be remedied in our next more extensive investigation. We have found a number of sum rules, viz., (3.23) and (3.26), which should be checked when high- Q^2 muon data become available. We believe that the present neutrino data already suggest the meaningfulness of the valon picture. Firstly, (3.21) is derived in this picture and is consistent with data. Secondly, the data on S(n) are well fitted by our choice of valon distribution, which has the correct behavior at large momentum fraction. Thirdly, the value of Q_0 implies a reasonable effective size for the valons in nucleon. At the same time it provides a quantitative description of precocious scaling.

The results justify our basic assumption that the three-valon representation of the nucleon without additional gluons is valid. If at Q_0 the three-valon states do not form a complete set to describe the nucleon, then (2.3) and (2.6) would be incorrect since gluons can have nontrivial structure functions. Equation (2.2) would certainly be wrong, thereby invalidating the entirely of our phenomenology. Evidently, the binding of the valons is adequately described by the distribution in (5.6) without attributing any momentum fraction to the binding agent (just as one would for triton). The fact that gluons carry nearly 50% of the nucleon momentum even at $Q^2 \simeq 2-3$ GeV² is to be understood in the same way that "precocious scaling" of the structure functions takes place also in the same Q^2 range. The point is that Q_0/Λ [which follows directly from data and the definition in (4.6)] is very nearly one; consequently, the parameter for evolution from Q_0 to Q is large even for $Q^2 \simeq 2-3$ GeV². It means that at such Q^2 evolution has mostly run its course and the valence quarks have already lost nearly half of their momenta to the gluons by bremsstrahlung. The Q_0/Λ parameter is therefore an important characterization of strong interaction. Its closeness to 1 implies that the strong interaction is very strong inside a valon at low Q^2 . There is no calculational scheme to elucidate the physics at such low Q^2 , but our present analysis at high Q^2 offers us the above insight on what effectively takes place there.

The significance of (5.6) is that it summarizes the so-called uncalculable hadronic matrix element. On the basis of it all hard processes can now be calculated without further unknowns. It closes the gap between quarks and hadrons; consequently, it should also play a key role in the problem of hadronization of quarks. It should also provide a clue to the study of the bound-state problem since $G_{v/N}(y)$ corresponds to the square of the constituent quark wave function in momentum space boosted to an infinite-momentum frame. Clearly, the development of the valon picture opens the way to the study of many hadronic processes in the framework of quarks and gluons.

- ¹R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).
- ²J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).
- ³G. Altarelli, N. Cabibbo, L. Maiani, and R. Petronzio, Nucl. Phys. <u>B69</u>, 531 (1974); N. Cabibbo and R. Petronzio, *ibid.* <u>B137</u>, 395 (1978).
- ⁴T. Kanki, Prog. Theor. Phys. <u>56</u>, 1885 (1976).
- ⁵A. Białas, W. Czyz, and W. Furmanski, Acta Phys. Pol. <u>B8</u>, 585 (1977); A. Białas, in *Proceedings of IX* Symposium on High Energy Multiparticle Dynamics, Tabor, 1978 (Czechoslovak Acad. Sci., Prague, 1978), p. C1. See also V. V. Anisovich, Yu. M. Shabelski, and V. M. Shekhter, Nucl. Phys. <u>B133</u>, 477 (1978), and references cited therein.
- ⁶P. C. Bosetti et al., Nucl. Phys. B142, 1 (1978).

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- ⁷J. G. H. de Groot *et al.*, Phys. Lett. <u>82B</u>, 292 (1979); <u>82B</u>, 456 (1979).
- ⁸G. Altarelli and G. Parisi, Nucl. Phys. <u>B126</u>, 298 (1977).
- ⁹See, for example, H. D. Politzer, Phys. Rep. <u>14C</u>, 130 (1974).
- ¹⁰T. A. DeGrand, Nucl. Phys. <u>B151</u>, 485 (1979).
- ¹¹D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14, 337 (1969).
- ¹²F. J. Gilman, Phys. Rep. <u>4C</u>, 95 (1972).
- ¹³An independent analysis of the data in Ref. 7 is given in R. C. Hwa, Univ. of Oregon Report No. OITS-122, 1979 (unpublished). The result agrees with the present one.
- ¹⁴R. Blankenbecler and S. J. Brodsky, Phys. Rev. D <u>10</u>, 2973 (1974).