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Large rapidity separation of baryonic number in hard processes

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A mechanism combining the parton scheme and dual topology with Y-shaped baryons is investigated in (a) the production of large-M^2 baryonic pairs in e^+e^- annihilation, (b) deep-inelastic scattering events in which a large momentum is transferred from the incident to the observed baryon, and (c) high-p_T baryon production in hadron + baryon collisions. One junction line is assumed to connect the observed final baryon to the antibaryon [in case (a)] or to the target baryon [cases (b) and (c)]. Accordingly, a Regge-type factor $M^{-2\beta}$ (or $|t|^{-\beta}$) is incorporated in the conventional parton formulas. Double (single) inclusive cross sections are worked out, and the limits of the model are discussed. The possibility of measuring this factor is discussed for cases (a) and (b). In case (c), the mechanism considered here may be larger than the conventional one.

I. INTRODUCTION

In ordinary (low-p_T) hadron-hadron collisions, baryon-antibaryon pairs are believed to be created via a multiperipheral mechanism (Fig. 1) leading to an $(M^2)^{-\beta}$ decrease of the double inclusive cross section at large $B\bar{B}$ invariant mass. This factor, which can also be written in terms of rapidity separation as $e^{-\beta\Delta y}$, represents the price one has to pay for the displacement of the baryonic junction in the dual diagram. In dual topological unitarization (DTU),[1,2] the exponent β is related to the Regge intercepts of M_0^J and M_2^J (Fig. 2) by the relation

$$\beta = \alpha_2^J - \alpha_M = \alpha_0^J - \alpha_{\text{Pomeron}}\,. \tag{1}$$

In this paper we will extend this topological approach to baryon production in hard processes such as e^+e^- annihilation (Fig. 3)

$$e^+e^- \to B\bar{B}' + X\,, \tag{1a}$$

deep-inelastic electron scattering (Fig. 4)

$$e + B \to e + B' + X\,, \tag{1b}$$

and high-p_T collisions (Fig. 5)

$$h + B \to B'(\text{high } p_T) + X\,. \tag{1c}$$

In the spirit of jet universality[3] we again associate an exponential factor to the migration of the junction in rapidity space,

$$e^{-\beta\Delta y} \sim \begin{cases} M^{-2\beta} & [\text{reaction (a)}] \\ |t|^{-\beta} & [\text{reaction (b) and (c)}] \end{cases} \sim (p\cdot p')^{-\beta}\,,$$

where $t = (p_B - p_{B'})^2$. However, jet universality itself is controversial, so the exponent β may not be the same as in low-p_T phenomena. For the moment we leave it as a free parameter, which might be fixed later by a counting rule analogous to those of the constituent-interchange model. This mechanism has been proposed independently by Aurenche and Bopp[4] to explain the observed p_T spectrum of protons in proton-proton collisions, with different parametrization and input, however.

In what follows we shall state the power laws in M^2 and t more precisely, work out formulas for cross sections, and discuss the domain in which our mechanism is relevant.

II. $e^+e^- \to B\bar{B}' + X$

According to the parton model, this reaction is a two-step process: (1) quark pair production $e^+e^- \to i\bar{i}$, given by QED and (2) jet fragmentation, $i\bar{i} \to B\bar{B}' + X$. Convenient kinematical variables are (in the center-of-mass frame): $s = q^2$ = invariant center-of-mass energy squared, $z = 2p\cdot q/q^2 = 2E/\sqrt{s}$, $p_{\parallel i}$ = longitudinal momentum with respect to the jet $(\bar{i}i)$ axis, $\vec{p}_{T/i}$ = momentum transverse to the jet axis (supposed to be limited, ~0.5 GeV/c), $z_\pm = E \pm p_{\parallel i}/\sqrt{s}$.

We will describe the jet fragmentation by the following formula [see the Appendix, formula (A6)]:

$$dN = C\mathfrak{D}_{B/i}(z_+)\mathfrak{D}_{\bar{B}'/\bar{i}}(z'_-)\frac{dz_+}{z_+}\frac{dz'_-}{z'_-} \times [(1-z_+)(1-z'_-)p\cdot p'/m^2]^{-\beta}\,, \tag{2}$$

where N is the number of pairs per jet, $(1/z)\mathfrak{D}_{B/i} \equiv D_{B/i}$ the usual single fragmentation function, and C a normalization constant. Integration over $\vec{p}_T$ is understood. This formula is valid only for large $(1-z_+)(1-z'_-)p\cdot p'$ and for $z_+ > z'_+$ (rapidity ordering implied by Fig. 3).

We can obtain a rough estimate of the normalization factor C by noting that, when we integrate (2) over z', we get the single fragmentation function (the probability of finding an antibaryon, once a baryon is detected, is equal to one). For this purpose we make the following approximations

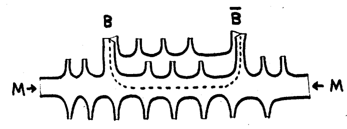

FIG. 1. Multiperipheral mechanism for high mass baryon-antibaryon pair production, in the Y-shaped baryon string model.

[for the threshold in $p\cdot p'$, see (A9)]:

$$z'_- \ll 1, \quad \int \frac{dz'_-}{z'_-}\cdots \simeq \int_{(1-z_+)}^{\infty} \frac{d(p\cdot p')}{p\cdot p'}\cdots, \tag{3}$$

$$\mathfrak{D}_{\bar B'/\bar i}(z'_-) \equiv \mathfrak{D}_{B'/i}(z'_-) \simeq \mathfrak{D}_{B'}(0), \tag{4}$$

where $\mathfrak{D}_B(0)$ is the rapidity density of baryons of type B in the central region. We are led to the normalization constraint

$$1 \sim C \sum_{B'} \mathfrak{D}_{B'}(0) \int_{m^2/(1-z_+)}^{\infty} \frac{d(p\cdot p')}{p\cdot p'} \times [(1-z_+)p\cdot p'/m^2]^{-\beta}. \tag{5}$$

Hence

$$C \sim \frac{\beta}{\sum_B \mathfrak{D}_B(0)}. \tag{6}$$

We stress that this value of C is only an order of magnitude, due to the fact we have extrapolated an asymptotic form of the double fragmentation function down to the threshold in $p\cdot p'$.

So far we have used kinematical quantities ($z^{\pm}$, $\vec{p}_{T/i}$, etc.) which depend on the definition of the jet axis. It is more practical to express the cross section in terms of new variables depending only on $\vec{p}$ and $\vec{p}\,'$. To fix the idea, let us suppose $|\vec{p}| \geq |\vec{p}\,'|$. Then, M^2 being large requires the proton to be relativistic ($|\vec{p}| \gg m$), and the angle between the jet axis and the proton momentum is small:

$$\theta_{p,i} \simeq \frac{p_{T/i}}{|\vec{p}|}.$$

Accordingly we will use the new variables $\vec{p}\,'_{T/p}$ and $p'_{\parallel p}$ relative to the proton axis (Fig. 6). They are related to the old ones by

$$\vec{p}\,'_{T/p} \simeq \vec{p}\,'_{T/i} - \frac{\vec{p}\cdot\vec{p}\,'}{\vec{p}^{\,2}}\vec{p}_{T/i}, \tag{7}$$

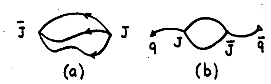

FIG. 2. (a) M_0^J baryonium. (b) M_2^J baryonium.

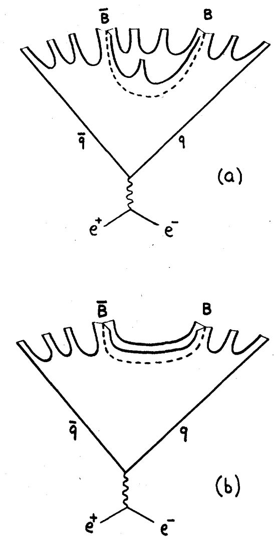

FIG. 3. Massive $B\bar B$ pair production in e^+e^- annihilation: (a) without rapidity gap between B and $\bar B$ (general case); (b) with a rapidity gap.

$$p\cdot p' \simeq E(E' - p'_{\parallel p}), \tag{8}$$

$$z_+ \simeq z, \quad z'_- \simeq \frac{E' - p'_{\parallel p}}{\sqrt{s}}, \quad \frac{dz'_-}{z'_-} \simeq -\frac{dp'_{\parallel p}}{E'}, \tag{9}$$

and we can write the double inclusive cross section (integrated over $p'_{T/p}$) as

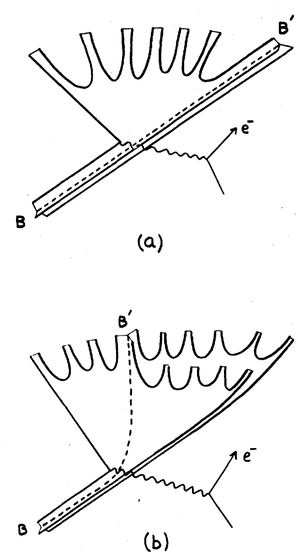

FIG. 4. Deep-inelastic event (valence contribution): (a) ordinary event, with a leading final baryon; (b) rare event, with a high momentum transfer to the baryon.

$$\frac{E'z\,d\sigma}{dp'_{\parallel p}dz\,d\Omega_p}=\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\Omega}\bigg|_{\Omega=\Omega_p}3C[(1-z)(1-z'_-)p\cdot p'/m^2]^{-\beta}\sum_{\text{flavor } i}e_i^2\mathfrak{D}_{B/i}(z)\mathfrak{D}_{B'/i}(z'_-)\,, \tag{10}$$

where e_i stands for the quark charge in electron units, and $p\cdot p'$ and z'_- are given by (8) and (9). The factor 3 is due to color summation. An order-of-magnitude estimate for C is given by (6).

Until now, we have integrated over the $\vec{p}_T$'s, assuming them to be small. Their observable effect consists of a noncollinearity between $\vec{p}$ and $\vec{p}'$, given by (7). Typically, $p'_{T/P}\lesssim 1$ GeV. Now if we assume there is no correlation between $\vec{p}_{T/i}$ and $\vec{p}'_{T/i}$, we have more precisely

$$\langle p'_{T/p}{}^2\rangle=\langle p'_{T/i}{}^2\rangle+\frac{\vec{p}'^2}{\vec{p}^2}\langle p_{T/i}{}^2\rangle\,. \tag{11}$$

In the case where $|\vec{p}|<|\vec{p}'|$, we just have to switch primed and unprimed quantities in formulas (7)–(10) (with $z'_-\longrightarrow z_+$).

III. $e+B\to e+B'+X$

There is a strong similarity between electroproduction and e^+e^- annihilation, due to crossing. In fact, Fig. 4(b) is the crossed version of Fig. 3(a) when $\bar{B}$ has been pushed to the left-hand edge (i.e., is a leading particle). We shall define, in the laboratory frame,

$q=(\nu,\vec{q})=$ virtual-photon momentum, $q^2<0$;
$p, p'=$ initial and final baryon momenta;
$k, k'=$ initial and final momenta of the interacting quark;
$x=-q^2/2p\cdot q$;
$z=p\cdot p'/p\cdot q=E_{B'}/\nu$;
$s=$ mass squared of the hadronic final state $\simeq 2p\cdot q(1-x)$.

Again, we have a two-step process: (1) Electron-quark scattering $e^-i\to e^-i$, given by QED. Each species of quark appears with a probability $G_{i/B}(x)\,dx$. (2) Jet fragmentation. The probability of finding the scattered baryon at z will be given, for large $p\cdot p'$, by [see the Appendix (A13)]

$$dN_{B'}=C\,\mathfrak{D}_{B'/i}(z)\frac{dz}{z}[(1-x)(1-z)p\cdot p'/m^2]^{-\beta}\,. \tag{12}$$

If one does not have jet universality ($\beta_{\text{hard}}\neq\beta_{\text{soft}}$), one must use this formula only for $x\gtrsim\frac{1}{2}$. If $x\lesssim\frac{1}{2}$ [see, e.g., Fig. 12(a)], one must distinguish a hadronic region in rapidity space of length $\ln(1/x)$ where $\beta=\beta_{\text{soft}}$ and a current region of length $\ln|q^2|$, where $\beta=\beta_{\text{hard}}$. The generalization of the exponential Regge-type factor is

$$\beta e^{-\beta(Y-Y_{\text{threshold}})}\Rightarrow\beta(Y)\exp\left[-\int_{Y_{\text{threshold}}}^{Y}\beta(Y')dY'\right]\,. \tag{13}$$

Integrating over $\vec{p}'_{T/q}$, the double inclusive cross section reads

$$\frac{E_{B'}d\sigma}{dE_{B'}dq^2\,dx}=\frac{d\sigma(e^-\mu\to e^-\mu)}{dq^2}\bigg|_{\hat{s}=2E_e x m_B}C\,[(1-x)(1-z)E_{B'}/m]^{-\beta}\sum_{\text{flavor } i}e_i^2G_{i/B}(x)\mathfrak{D}_{B'/i}(z)\,. \tag{14}$$

Equivalently, in terms of virtual-photon–nucleon cross section,

$$\frac{1}{\sigma_{\gamma^*B}}\frac{z\,d\sigma}{dz}=C\,\frac{\sum_i e_i^2G_{i/B}(x)\mathfrak{D}_{B'/i}(z)}{\sum_i e_i^2G_{i/B}(x)}[z(1-z)s/2m^2]^{-\beta}\,. \tag{15}$$

A formula analogous to Eq. (11) can be found in Ref. 5 for the transverse momentum spreading (with respect to the virtual photon).

IV. $h+B\to B'$ (high p_T) $+X$

We assume that the hard subprocess is quark-quark (or quark-antiquark) scattering. Then the experiment is not very different from deep-inelastic electron scattering, except that we have replaced a monoenergetic electron beam by a wide-band beam of quarks with a distribution $G_{a/h}(x)$. Also we take the standard parton formula,[12] but multiply it by our Regge-type factor:

$$\frac{E'\,d\sigma}{d^3p'}=\sum_{a,b,c}\int dx_aG_{a/h}(x_a)\int dx_bG_{b/B}(x_b)\frac{1}{\pi z}D_{B'/c}(z)\frac{d\hat{\sigma}}{d\hat{t}}(a+b\to c+d;\hat{s},\hat{t})C\,[(1-x_b)(1-z)p_B\cdot p_{B'}/m^2]^{-\beta}\,. \tag{16}$$

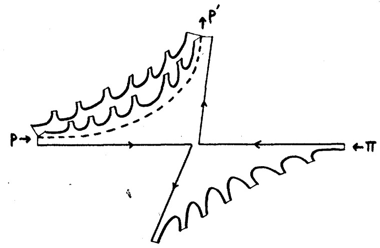

FIG. 5. Mechanism for production of high-p_T baryons in hadron-hadron collision, analogous to Fig. 4(b).

If one does not have jet universality, the integrand must be modified for $x_b \lesssim \frac{1}{2}$ according to (13). The constant C' is the same as in deep-inelastic lepton scattering, approximately given by (6). Assuming that the quark-quark cross section behaves at fixed angle like $\hat{s}^{-N}$, we get an inclusive cross section at fixed x_T and θ falling down like

p_T^{-2N} for mesons,

$p_T^{-2N-2\beta}$ for baryons,

i.e., a steeper slope for baryons, as it seems experimentally to be the case.

V. DISCUSSION

A. Limits of the model

In terms of rapidity, Eq. (2) can be rewritten in the central region as

$$dN = 2^\beta C\hat{D}(y' - y_{\min})dy' \, e^{-\beta\Delta y}\hat{D}(y_{\max} - y)\, dy \, , \quad (17)$$

with

$$y = y_{\max} + \ln z^+ = y_{\min} - \ln z^- \, ,$$

$$\Delta y = y - y' \, ,$$

and

$$C \sim \beta/\hat{D}(\infty) \, .$$

(Flavor indices are omitted and p_T is taken to be zero.) The exponential factor describes the $B\overline{B}$ correlation due to the junction line. $\hat{D}$ takes into account possible extra $B\overline{B}$ pairs on the right- and left-hand sides of the observed one. Thus, our model is not restricted to one single baryonic pair in the final state. It only specifies that no additional pair lies inside the interval $[y, y']$. However, at very large Δy, this is no longer true and there is a critical value Δ_1 (or a critical

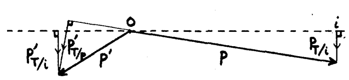

FIG. 6. Geometrical construction of transverse momenta in e^+e^- annihilation [Eq. (7)].

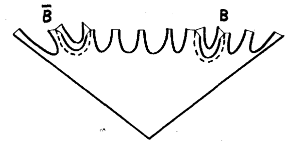

FIG. 7. Inclusive $B\overline{B}$ production coming from independent baryonic pairs.

mass M_1) above which the mechanism shown in Fig. 7 dominates. This new contribution is given by

$$dN^{(\text{independent pairs})} = \hat{D}(y' - y_{\min})\hat{D}(y_{\max} - y)\, dy\, dy' \, . \quad (18)$$

Thus

$$\frac{d\sigma^{(\text{same})}}{dy\, dy'} \Big/ \frac{d\sigma^{(\text{indep})}}{dy\, dy'} = 2^\beta C e^{-\beta\Delta y} \, , \quad (19)$$

and

$$2^{-\beta} e^{+\beta\Delta_1} = C \sim \frac{\beta}{\hat{D}(\infty)} \, , \quad (20a)$$

or, returning to our previous formulation, our mechanism is dominant for

$$\left.\begin{matrix}(1-z_+)(1-z_-')\\ \text{or}\\ (1-x)(1-z)\end{matrix}\right\} \times p \cdot p'/m^2 \lesssim \cosh\Delta_1 = C^{1/\beta} \sim \left(\frac{\beta}{\sum_B \mathfrak{D}_B(0)}\right)^{1/\beta} . \quad (20b)$$

On the other hand, there is a lower value of the left-hand side of (20b) below which the Regge form in (2) or (12) is not valid due to resonances and threshold effects.

B. Numerical estimates

Little is known about β. If one makes a strong jet universality hypothesis (identity of soft and hard jets), then β is given by (1), according to DTU. For the intercepts we may use the empirical formula[1,2]

$$\alpha(0) = 1 - n_J/4 - n_{u,d}/4 - n_s/2 \, , \quad (21)$$

which gives (Ref. 6) $\beta = 0.5$. On the other hand, the value $\beta = 2$ could explain the observed p_T^{-12} proton spectrum at large p_T (Ref. 7) provided the mechanism of Fig. 5 is at work. (In fact, there is a large uncertainty in the exponent because neither at CERN ISR nor at Fermilab was the experiment done at fixed x_T.) In fact, there is no compelling reason to believe in the validity of DTU in hard processes. $\beta = 2$ is also a value obtained theoretically in a model where the junction is a parton.[11]

No precise data are available for the fragmentation function $D_B(z)$. $D_{\bar{p}}(z)$ has been measured in e^+e^- annihilation[8] only for $z \gtrsim 0.5$, $D_p(z)$ and $D_{\bar{p}}(z)$ in electroproduction[9] for $z \gtrsim 0.2$. These data are insufficient to give us an estimate of $\sum_B \mathfrak{D}_B(0)$.

To get an idea about this quantity, we may use its value in ordinary hadron-hadron collisions[10] as jet universality would require:

$$\mathfrak{D}_{\bar{p}}(0)\Big|_{e^+e^-} = \frac{1}{2}\frac{dN_{\bar{p}}}{dy}\Big|_{hh, y=0} \sim 0.05\,. \tag{22}$$

The factor $\frac{1}{2}$ comes from dual topology: In the hadronic case, there are two superposed jets (the upper one and lower one in Fig. 1). Assuming

$$\sum_B \mathfrak{D}_B(0) \sim 4\mathfrak{D}_{\bar{p}}(0)\,, \tag{23}$$

we get

$$\cosh\Delta_1 \sim \begin{cases} 6 & \text{for } \beta = 0.5\,, \\ 3 & \text{for } \beta = 2\,. \end{cases} \tag{24}$$

Let us see under what conditions our mechanism can show up in the three reactions that we have considered.

(a) e^+e^- *annihilation*. If β is large, the $B\bar{B}$ mass spectrum will be peaked near threshold [cf. Eq. (10)] and the independent pair mechanism [Eq. (10) without the Regge-type factor] will take over very soon. In the central region,

$$\beta = 2 \rightarrow M_{B\bar{B}} < M_1 \sim 3 \text{ GeV}$$

[assuming (22)]. Thus, it will be difficult to establish the power law behavior in M^2 (unless one can reject the four- or more-baryon events). Things are easier if β is small, although the mass interval may not be very large:

$$\beta = 0.5 \rightarrow M_{B\bar{B}} < M_1 \sim 4 \text{ GeV}\,.$$

One should try to test formula (10) with slow baryons of fixed z and z' (varying s).

(b) *Deep inelastic electron scattering*. Experimentally, the situation is easier because of the larger counting rate. Theoretically also, the power law behavior in t is expected to hold better than the one in M^2 because of the absence of resonant fluctuations. The final baryon energy, in the laboratory frame, at which the pair creation mechanism begins to dominate should be according to (24),

$$(1-x)E_{B'} \sim \begin{cases} 6 \text{ GeV} & \text{for } \beta = 0.5\,, \\ 3 \text{ GeV} & \text{for } \beta = 2\,. \end{cases}$$

In fact the crossover is probably at higher energy than the above ones, because the baryonic pair creation mechanism has a threshold effect; Eq. (18) is not valid at $\Delta y < 2$.

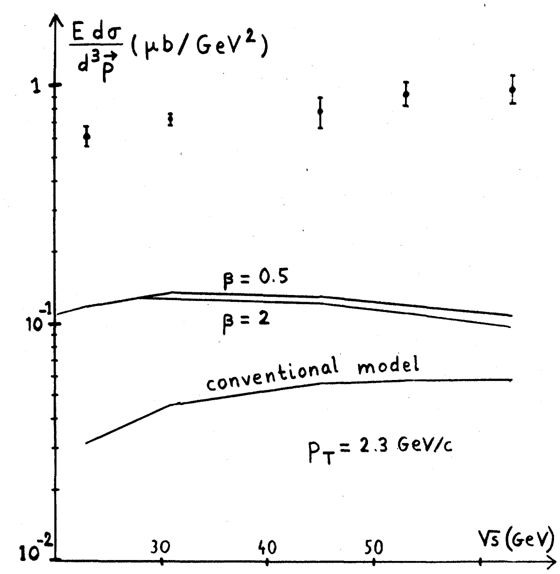

FIG. 8. Inclusive cross section for protons of $p_T = 2.3$ GeV/c at $\theta_{\text{c.m.}} = 0$ in the ISR energy range.

It is worth noting that the cross section for electroproduction at small z is insensitive to the absolute values of the $\mathfrak{D}_B(0)$'s. Formula (15), together with (6), leads to

$$\frac{1}{\sigma^{\text{tot}}_{\gamma^* B}}\frac{z\,d\sigma}{dz} \sim \frac{\mathfrak{D}_{B'}(0)}{\sum_B D_B(0)}\beta\left(\frac{z\nu}{m'}\right)^{-\beta} \quad (z \text{ small})\,. \tag{25}$$

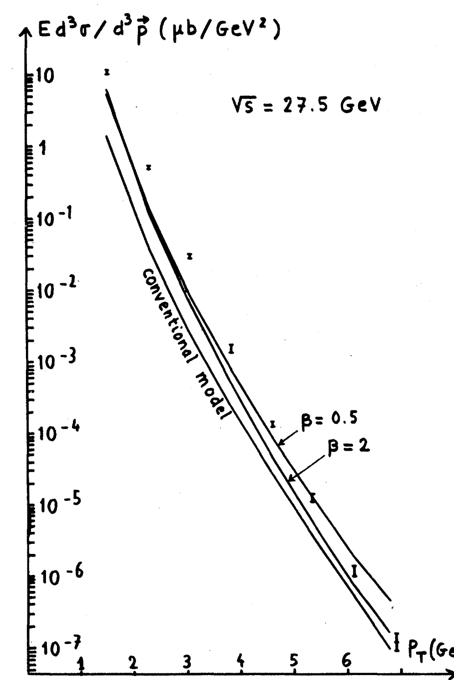

FIG. 9. High-p_T proton spectrum at $\theta_{\text{c.m.}} = 0$ and $P_{\text{lab}} = 400$ GeV/c ($\sqrt{s} = 27.5$ GeV).

Before extracting fragmentation functions from electroproduction data, one must be aware of the nonscaling contribution of our mechanism, which is the dominant one at $z \sim 0$, but also, according to (15), at $z \sim 1$.

(c) *Hadron + nucleon → high-p_T baryon + anything.* Here we can compare our formula (16) with existing data at c.m. angles near 90°.[7] Letting both C and β be free parameters would make it possible to fit any powerlike single-particle spectrum. But the normalization condition [Eqs. (5) and (6)] fixes the order of magnitude of our mechanism. Due to the trigger bias, which makes z close to one, the Regge-type factor does not damp very much the cross section. (We thank Dr. P. Aurenche for pointing out this effect to us.) In Figs. 8–10 we have compared our mechanism with the conventional one for typical ISR and Fermilab experiments. For the structure functions we take[13]

$$G(x) \equiv G_{u/p}^{\text{valence}} = [0.594(1-x^2)^3 + 0.461(1-x^2)^5 + 0.621(1-x^2)^7]x^{-1/2}; \tag{26}$$

for the fragmentation function[14]

$$\mathcal{D}(z) \equiv \mathcal{D}_{p/u}^{\text{valence}} = 0.16\sqrt{z}\,(1-z)^2\,, \tag{27}$$

$$\mathcal{D}_{p/u}(0) = 0.0276\,, \tag{28}$$

$$\sum_B \mathcal{D}_B(0) \simeq 2\mathcal{D}_{p/u}(0) \simeq 0.06\,, \tag{29}$$

we do not follow (23); we take a factor of 2 instead of 4 because the fragmentation function of Ref. 14 includes the contribution of hyperon decays. For $x_b < \frac{1}{2}$, the integrand of formula (16) was modified

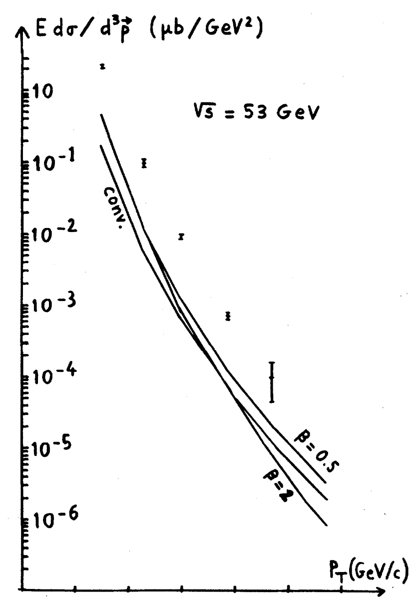

FIG. 10. High-p_T proton spectrum at $\theta_{\text{c.m.}} = 0$ and $\sqrt{s} = 53$ GeV.

to account for the migration of the junction through the soft region [see formula (13)]. The quark-quark cross section was[12]

$$\frac{d\hat{\sigma}}{d\hat{t}} = 2.3 \times 10^6 \frac{1}{\hat{s}\hat{t}^3}\ \mu b\ \text{GeV}^6\,.$$

The three curves, $\beta = 0.5$, $\beta = 2$, and the conventional mechanism, are not very far apart. They lie below the experimental points (except at large x_T). Including the contribution of down quarks and of the sea, and adding the two mechanisms could reduce the discrepancy. On the other hand, our choice of D function is probably too optimistic at large z; $D(z) \sim (1-z)^2$ is not consistent with $G(x) \sim (1-x)^3$. Thus, the difficulty to get the right normalization still remains. (Here we must mention another model with junction which does not have this problem, i.e., the "countable junction model," in which the junction itself suffers the hard elastic scattering.[15])

VI. CONCLUSION

We have considered a mechanism for baryon production in hard processes which involve a large rapidity shift of baryonic quantum number (of the junction, in the string model). We applied it to three related processes

(a) $e^+e^- \to B\bar{B} + X$,

(b) $e^- + B \to B' + X$,

(c) $h + B \to B'$ (high p_T) $+ X$.

By analogy with soft processes, the migration of the junction was taken into account by a Regge-type factor in front of the usual parton formula. The normalization was roughly given by baryonic-number conservation. Our formulation is valid in a finite rapidity interval above which the mechanism with independent baryonic-pair creation will take over, restoring the usual parton formulas. The measurement of β in reaction (a) or, more easily, in reaction (b) could settle the question of jet universality between hard and soft processes. In reaction (c), our mechanism is of the same order of magnitude as the standard parton mechanism at present energies. In any case, it has to be considered seriously.

ACKNOWLEDGMENTS

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APPENDIX: REGGE-TYPE FACTOR FOR THE MIGRATION OF THE JUNCTION IN RAPIDITY SPACE

Let us calculate the ratio $R=\sigma_{\text{mig}}/\sigma_{\text{indep}}$ between the migration mechanism and the conventional one for the two processes

(a) $e^+e^- \to B\bar{B}' + X$,

(b) $eB \to e'B' + X$

in the framework of DTU and jet universality.

In reaction (a), according to jet universality, one can replace the virtual photon by a "resonant" meson-meson system of the same mass. We shall consider two extreme cases

(1) B and $\bar{B}'$ in the central region [Figs. 11(a) and 11(b)]. Applying the Regge-Müller formalism, we get

$$R=C\left(\frac{p\cdot p}{m^2}\right)^{\alpha^J_2-\alpha_M} \qquad \text{(A1)}$$

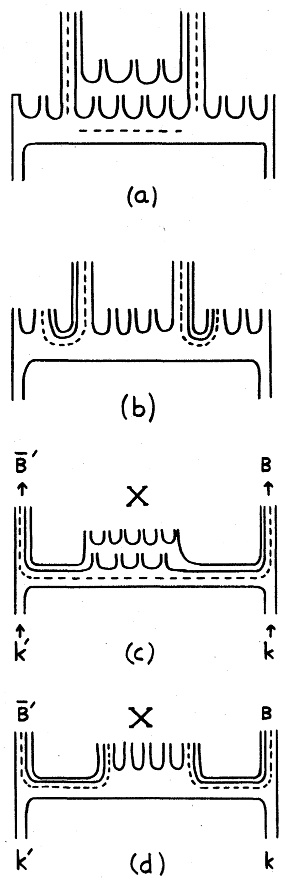

FIG. 11. $B\bar{B}$ production in a planar meson+meson jet: (a), (b) in the central region; (c), (d) at the ends of the spectrum; (a), (c) junction migration; (b), (d) independent baryonic-pair creation.

with

$$C=|g_{MBBM^J_2}/g_{MBBM}|^2 \,. \qquad \text{(A2)}$$

(2) B and $\bar{B}'$ in the opposite ends of the spectrum [Figs. 11(c) and 11(d)]. The right and left triple-Regge vertices $g_{BBM^J_2}$ depends, respectively, on

$$t\equiv(k-p)^2\simeq-\vec{p}_T^{\,2} \qquad \text{(A3a)}$$

and

$$t'\simeq-\vec{p}_T^{\,\prime 2} \;, \qquad \text{(A3b)}$$

and the missing mass is

$$M_X^{\,2}=q^2(1-z_+)(1-z'_-)\simeq 2p\cdot p'(1-z_+)(1-z'_-). \qquad \text{(A4)}$$

Thus, at fixed p_T and p'_T,

$$R=\frac{g_{BBM^J_2}(-p_T^{\,2})g_{BBM^J_2}(-p_T'^{\,2})}{g_{BBM}(-p_T^{\,2})g_{BBM}(-p_T'^{\,2})} \times[(1-z_+)(1-z'_-)p\cdot p'/m^2]^{\alpha^J_2-\alpha_M}\,. \qquad \text{(A5)}$$

Formulas (A1) and (A5) join smoothly in one single formula

$$R=C[(1-z_+)(1-z'_-)p\cdot p'/m^2]^{\alpha^J_2-\alpha_M} \;, \qquad \text{(A6)}$$

if one assumes

$$g_{BBM^J_2}{}^{(t)}=\sqrt{C}\;g_{BBM}{}^{(t)} \,. \qquad \text{(A7)}$$

At fixed z_+, we have a threshold in z'_-, due to the conditions

$$z'_->z_->\frac{m^2}{q^2z_+} \;, \qquad \text{(A8a)}$$

$$z'_->\frac{m^2}{q^2z'_+}>\frac{m^2}{q^2(1-z_+)} \,. \qquad \text{(A8b)}$$

These two conditions can be summarized in

$$z'_-\gtrsim\frac{m^2}{q^2z_+(1-z_+)} \;,$$

i.e.,

$$p\cdot p'\gtrsim\frac{m^2}{1-z_+} \,. \qquad \text{(A9)}$$

In reaction (b), we replace the virtual photon by an incoming meson k' plus an outgoing meson k with

$$q = k' - k \ ,$$

$$x = k^+/p^+ \ .$$

In the Regge-Müller analysis, there are many different cases to consider, according to the relative positions of B, B', k, and k' in rapidity space. We shall give only two examples

(1) $x \ll 1$, valence contribution, $m^2/s \ll z \ll m^2/|q^2|$ [Fig. 12(a)].

$$R = |g_{BBM_{\frac{J}{2}}}{}^{(m^2)} g_{M_{\frac{J}{2}}BBM}/g_{BBM}{}^{(m^2)} g_{MBBM}|(p\cdot p'/m^2)^{\alpha_{\frac{J}{2}}-\alpha_M}$$

$$= C(p\cdot p'/m^2)^{\alpha_{\frac{J}{2}}-\alpha_M} \ . \qquad \text{(A10)}$$

(2) x and z close to one [Fig. 12(b). Compare to Fig. 11(c)]. In a collinear $\vec{q}, \vec{p}$ frame,

$$t \equiv (k-p)^2 \simeq -k_T{}^2 \ , \qquad \text{(A11a)}$$

$$t' \equiv (p'-k')^2 \simeq -(k_T - p'_T)^2 \ , \qquad \text{(A11b)}$$

and the missing mass is

$$M_X{}^2 \simeq s(1-z) \simeq 2p\cdot p'(1-x)(1-z) \ . \qquad \text{(A12)}$$

This case is very similar to $e^+e^- \to B\bar{B}' + X$ at large z_+ and z'_-. Assuming (A7) again, we get

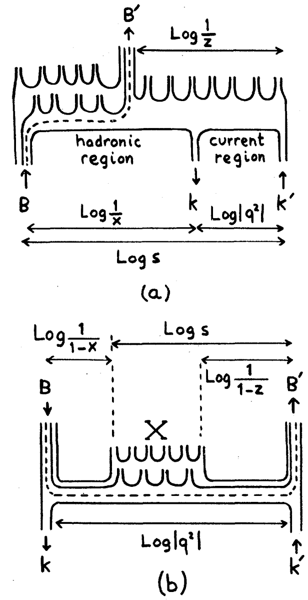

FIG. 12. Junction migration in "deep-inelastic *meson* scattering": (a) x and $z \ll 1$; (b) $x \sim 1$ and $z \sim 1$.

$$R = C[(1-x)(1-z)p\cdot p'/m^2]^{\alpha_{\frac{J}{2}}-\alpha_M} . \qquad \text{(A13)}$$

This formula generalizes (A10) to all (x, z) regions.

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