Canonical neutral-current predictions from the weak-electromagnetic gauge group SU(3) × U(1)

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A straightforward SU(3) × U(1) model in which there is effectively one new neutral-current parameter (denoted by R) is shown to give the canonical neutrino neutral-current predictions for all values of R. For small R the "low-energy" theory is essentially SU(2) × U(1) while for R of the order of one it has a much richer "low-energy" gauge-boson mass spectrum. Even in the latter case, the predicted e-d asymmetry agrees with experiment. It is interesting that the atomic-physics parity violation depends sensitively on R.

I. INTRODUCTION

The remarkable experimental success of the $SU(2)_L \times U(1)$ weak-electromagnetic gauge theory[1] lies in its prediction of the varied pattern of neutral-current interactions.[2] Dividing the experiments into two classes, (a) neutrino neutral-current experiments and (b) electron neutral-current experiments, we presently know[2] that there are many experiments belonging to class (a) which agree with the theory while there are so far only two types of experiments belonging to (b). Of the latter, the polarized-electron–deuteron scattering asymmetry agrees with the theory while the polarized-photon–atom scattering asymmetry measurements are more controversial.

Since the gauge bosons and the Higgs boson predicted by $SU(2)_L \times U(1)$ have not yet been seen, it is certainly interesting to ask if there are other gauge theories which give the same neutral-current predictions. A number of authors[3] have shown that if the gauge group is of the form $SU(2)_L \times U(1) \times G$, and if certain additional restrictions on the Higgs structure and representation assignments are made, then the *neutrino* neutral currents will agree with those of $SU(2)_L \times U(1)$. In the present paper we extend the class of groups which can reproduce the canonical form of the neutrino neutral currents in a different direction: to a group of the form $G_L' \times U(1)'$. Of course, this would be trivial if $U(1)'$ is the same as the usual U(1) and if the usual $SU(2)_L$ is embedded in G_L' in such a way that the "low-energy" (i.e., (100 GeV) gauge bosons are just the $W^{\pm}$ and Z, effectively. However, we shall show that it is possible to have the same neutrino neutral currents even when the low-energy gauge-boson spectrum is very much richer. Furthermore, it will turn out that the predictions for e-d scattering agree with those of $SU(2)_L \times U(1)$ up to the precision of present experiments, while the predictions for the atomic-physics parity-violation experiments depend sensitively on a new parameter, R.

We shall utilize the gauge group $SU(3)_L \times U(1)$ to illustrate our result which holds for a class of $SU(n)_L \times U(1)$ gauge models (and probably others under appropriate conditions). Theories based on $SU(3)_L \times U(1)$ have been discussed by many authors[4] who have explored many possible fermion and Higgs-boson representation assignments. The present model is probably most similar to that of Georgi and Pais.[4] However, the two models are not identical and they also investigated questions different from the ones discussed here.

The notational conventions and representation assignments are given in Sec. II. Formulas for the gauge-boson mass spectrum are collected in Sec. III. In Sec. IV, the effective neutral-current Lagrangian which is due here to the exchange of two massive neutral vector bosons is given in a convenient form. Also, formulas for comparison with experiment and with $SU(2)_L \times U(1)$ are given. Our results on reproducing the canonical neutral-current predictions are stated and discussed in Sec. V. Finally, further discussion of the model is given in Sec. VI.

II. RAW MATERIALS OF THE MODEL

First the eight SU(3) gauge bosons are specified by a traceless tensor: $W^b_{a\mu}$, with $W^c_{c\mu}=0$. Ordinary β decay is mediated by $W^2_{1\mu}$. Conveniently normalized neutral fields are

$$F_\mu = -(\tfrac{3}{2})^{1/2} W^1_{1\mu}\,, \quad H_\mu = \frac{1}{\sqrt{2}}\,(W^2_{2\mu} - W^3_{3\mu})\,. \quad (2.1)$$

The U(1) gauge field is denoted D_μ. Linear combinations of D and F above are defined through

$$\begin{pmatrix} F \\ D \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix}, \quad (2.2)$$

wherein A_μ is the photon field. The diagonal fields Z_μ and H_μ can mix with each other to form the two physical heavy neutral gauge fields $Z^{(1)}_\mu$ and $Z^{(2)}_\mu$.

Three triplets of Higgs fields $f^{(1)}$, $f^{(2)}$, and $f^{(3)}$ are assumed. Their electric charges are as follows:

$$f^{(1)} \to \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}; \quad f^{(2)}, f^{(3)} \to \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.3}$$

and it is assumed that the vacuum expectation values obey the simple pattern

$$\langle f_b^{(a)} \rangle = \delta_{ab} k_b . \tag{2.4}$$

This orthogonality of vacuum values can be enforced by having prominent terms like $|f^{\dagger(2)} f^{(3)}|^2$ in the Higgs potential. The SU(3) coupling constant g and the U(1) coupling constant g' may be specified by defining the gauge-covariant derivative of $f^{(1)}$:

$$(\mathfrak{D}_\mu f^{(1)})_a = \partial_\mu f_a^{(1)} - \frac{ig}{\sqrt{2}} W^b_{a\mu} f_b^{(1)} - \frac{2ig'}{3} D_\mu f_a^{(1)} . \tag{2.5}$$

The coupling constants are related to the proton charge e by

$$\tfrac{1}{2}\sqrt{3}\, gc = g's = -e . \tag{2.6}$$

A convenient parameter[5] which has exactly the same significance as the conventional $x = \sin^2\theta_W$ of the $SU(2) \times U(1)$ theory is

$$x = \tfrac{3}{4} c^2 . \tag{2.7}$$

Note that here one has the stronger bound $x \leq \frac{3}{4}$.

Finally, consider the fermions. All right-handed fields are taken to be SU(3) singlets. The left-handed leptons are assumed to belong to three antitriplets as follows:

$$\begin{pmatrix} e \\ \nu_e \\ \nu_e' \end{pmatrix}_L , \quad \begin{pmatrix} \mu \\ \nu_\mu \\ \nu_\mu' \end{pmatrix}_L , \quad \begin{pmatrix} \tau \\ \nu_\tau \\ \nu_\tau' \end{pmatrix}_L . \tag{2.8}$$

The first two entries in each antitriplet are the usual particles; the third will be considered to be (heavy) "neutrinos" and will also have right-handed singlets associated with them. The anomaly cancellation[6] in a model of the present type must be achieved by having an equal number of triplets and antitriplets and furthermore requiring the sum of all fermion charges to vanish.[7] A characteristic feature of the present class of models is the fact that anomaly cancellation does not occur generation by generation. Thus, we put the first two generations of left-handed quarks into six (counting color) triplets:

$$\begin{pmatrix} u \\ d \\ d' \end{pmatrix}_L , \quad \begin{pmatrix} c \\ s \\ s' \end{pmatrix}_L . \tag{2.9}$$

The fields in (2.8) and (2.9) are, of course, the gauge-group eigenstates which are related to physical fields by unitary transformations involving Cabibbo-type angles. d' and s' are new (heavy) quarks. The members of the third generation of left-handed quarks are put into three antitriplets:

$$\begin{pmatrix} b \\ t \\ t' \end{pmatrix}_L . \tag{2.10}$$

Here t and t' are presently unobserved heavy quarks with electric charge $\frac{2}{3}$. We shall assume negligible mixing between the primed and unprimed fermion fields and between the third generation of quarks and the first two generations. The latter assumption is needed to suppress some flavor-changing neutral currents for the first four flavors. These assumptions are perhaps an inelegant feature of this model but they are not unreasonable.

III. GAUGE-BOSON MASS FORMULAS

From the terms in the Lagrangian

$$-\sum_{i=1}^{3} (\mathfrak{D}_\mu f^{(i)})_a^* (\mathfrak{D}_\mu f^{(i)})_a \tag{3.1}$$

together with (2.4) one finds the masses of the "off-diagonal" gauge fields

$$\begin{aligned} m^2(W_1^2) &= \tfrac{1}{2} g^2 (k_1^2 + k_2^2) , \\ m^2(W_1^3) &= \tfrac{1}{2} g^2 (k_1^2 + k_3^2) , \\ m^2(W_2^3) &= \tfrac{1}{2} g^2 (k_2^2 + k_3^2) , \end{aligned} \tag{3.2}$$

where k_i^2 denotes $|k_i|^2$. The masses of the two neutral gauge fields $Z^{(1)}$ and $Z^{(2)}$ are obtained by diagonalizing a 2×2 matrix; the result is

$$\begin{aligned} m^2(Z^{1,2}) &= \tfrac{1}{2}\{ m_Z^2 + m_H^2 \pm [(m_Z^2 - m_H^2)^2 + 4p^2]^{1/2} \} , \\ m_Z^2 &= \frac{2e^2}{9s^2c^2} (4k_1^2 + k_2^2 + k_3^2) , \\ m_H^2 &= \frac{2e^2}{3c^2} (k_2^2 + k_3^2) , \\ p &= \frac{2e^2}{3\sqrt{3}\, s c^2} (k_3^2 - k_2^2) . \end{aligned} \tag{3.3}$$

Since the Fermi constant $G_F = \sqrt{2} g^2 / 8 m^2(W_1^2)$ we have the following constraint on the vacuum values

$$k_1^2 + k_2^2 = \frac{\sqrt{2}}{4G_F} . \tag{3.4}$$

IV. NEUTRAL CURRENTS

First for comparison we give the effective neutral-current Lagrangian due to Z exchange[8] for the $SU(2) \times U(1)$ theory:

$$\mathcal{L}_{\rm eff} = 2\sqrt{2}G_F(\tfrac{1}{2}J_\mu^{(0)} - xJ_\mu^{\rm EM})^2 . \quad (4.1)$$

Here x and G_F have their usual meanings. $J_\mu^{\rm EM}$ is the electromagnetic current

$$J_\mu^{\rm EM} = -i\bar{e}\gamma_\mu e - i\bar{\mu}\gamma_\mu \mu + \tfrac{2}{3}i\bar{u}\gamma_\mu u - \frac{i}{3}\bar{d}\gamma_\mu d + \cdots \quad (4.2)$$

and $J_\mu^{(0)}$ is the "weak-isospin" current

$$J_\mu^{(0)} = i\bar{u}_L\gamma_\mu u_L - i\bar{d}_L\gamma_\mu d_L + i\bar{\nu}_e\gamma_\mu\nu_e - i\bar{e}_L\gamma_\mu e_L + i\bar{t}_L\gamma_\mu t_L - i\bar{b}_L\gamma_\mu b_L + \cdots . \quad (4.3)$$

For convenience we isolate the neutral-current interactions of the electron neutrino, for example, to get

$$\mathcal{L}_{\rm eff}(\nu_e) = 2\sqrt{2}iG_F\bar{\nu}_e\gamma_\mu\nu_e(\tfrac{1}{2}J_\mu^{(0)} - xJ_\mu^{\rm EM}) \quad (4.4)$$

[the current $J_\mu^{(0)}$ on the right-hand side of (4.4) is given by (4.3) but with an extra factor of $\frac{1}{2}$ for the $i\bar{\nu}_e\gamma_\mu\nu_e$ term]. Similarly, isolating the electron term one has

$$\mathcal{L}_{\rm eff}(e) = -2\sqrt{2}iG_F(\bar{e}_L\gamma_\mu e_L - 2x\bar{e}\gamma_\mu e)(\tfrac{1}{2}J_\mu^{(0)} - xJ_\mu^{\rm EM}) , \quad (4.5)$$

wherein again an extra factor $\frac{1}{2}$ should be supplied for the ee scattering term.

In the present model both $Z^{(1)}$ and $Z^{(2)}$ exchange contribute to the effective neutral-current Lagrangian.[9] The result can be written in the compact form

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2}(J_\alpha^Z \mu^{-2}{}_{ZZ}J_\alpha^Z + J_\alpha^H\mu^{-2}{}_{HH}J_\alpha^H + 2J_\alpha^H\mu^{-2}{}_{HZ}J_\alpha^Z) , \quad (4.6)$$

where the components of the (undiagonalized) inverse squared mass matrix are given by

$$\begin{aligned} \mu^{-2}{}_{ZZ} &= \frac{9s^2c^2}{8e^2}\,\frac{(k_2^2+k_3^2)}{(k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2)} , \\ \mu^{-2}{}_{HH} &= \frac{3c^2}{8e^2}\,\frac{(4k_1^2+k_2^2+k_3^2)}{(k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2)} , \\ \mu^{-2}{}_{HZ} &= \frac{3\sqrt{3}sc^2}{8e^2}\,\frac{(k_2^2-k_3^2)}{(k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2)} , \end{aligned} \quad (4.7)$$

and the currents are

$$\begin{aligned} J_\alpha^H &= \frac{g}{4}(J_\alpha^{(1)} - J_\alpha^{(0)}) , \\ J_\alpha^Z &= \frac{g}{4\sqrt{3}s}(3J_\alpha^{(0)} + J_\alpha^{(1)} - 6c^2J_\alpha^{\rm EM}) , \end{aligned} \quad (4.8)$$

where $J_\alpha^{(1)}$ is an isoscalar current, present in the $SU(3)\times U(1)$ theory, but not in the $SU(2)\times U(1)$ theory:

$$J_\alpha^{(1)} = i\bar{u}_L\gamma_\alpha u_L + i\bar{d}_L\gamma_\alpha d_L - i\bar{\nu}_e\gamma_\alpha\nu_e - i\bar{e}_L\gamma_\alpha e_L - i\bar{t}_L\gamma_\alpha t_L - i\bar{b}_L\gamma_\alpha b_L + \cdots .$$

We note that the third quark generation contributes to $J_\alpha^{(1)}$ in a different way from the first two generations. With our assumption of negligible mixing between the third quark generation and the others, this does not affect any of our results. Also note that we are not including any of the terms coming from charged intermediate boson exchange which can be Fierz transformed to neutral-current form, in our $\mathcal{L}_{\rm eff}$.

From (4.6) we extract the neutrino interactions as in (4.4) for the $SU(2)\times U(1)$ theory, to find

$$\mathcal{L}_{\rm eff}(\nu_e) = 2\sqrt{2}i\Lambda G_F\bar{\nu}_e\gamma_\alpha\nu_e \times\left\{\frac{1}{4}\left(2+\frac{k_1^2}{k_3^2}\right)J_\alpha^{(0)} - \frac{k_1^2}{4k_3^2}J_\alpha^{(1)} - xJ_\alpha^{\rm EM}\right\} , \quad (4.9)$$

where

$$\Lambda = \frac{(k_1^2+k_2^2)k_3^2}{k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2} . \quad (4.10)$$

The quantity x is defined by (2.7) and (2.2). The electron interactions which follow from (4.6) are

$$\mathcal{L}_{\rm eff}(e) = -2\sqrt{2}i\Lambda G_F(\bar{e}_L\gamma_\alpha e_L - 2x\bar{e}\gamma_\alpha e) \times\left\{\frac{1}{4}\left(2+\frac{k_2^2}{k_3^2}\right)J_\alpha^{(0)} + \frac{k_2^2}{4k_3^2}J_\alpha^{(1)} - x\left(1+\frac{k_2^2}{k_3^2}\right)J_\alpha^{\rm EM}\right\} . \quad (4.11)$$

In what follows we will need two formulas derived from (4.11). The first is for the asymmetry parameter in the scattering of polarized electrons off deuterium. Following Cahn and Gilman,[10] who work in the parton model, we get

$$\frac{A}{q^2} = \frac{-3\Lambda G_F}{20\sqrt{2}\pi\alpha}\left\{(2-\tfrac{20}{3}x)\left(1+\frac{k_2^2}{k_3^2}\right) + 1 + (1-4x)\left(3+\frac{2k_2^2}{k_3^2}\right)\left[\frac{1-(1-y)^2}{1+(1-y)^2}\right]\right\} . \quad (4.12)$$

Here A is the asymmetry $(d\sigma_R - d\sigma_L)/(d\sigma_R + d\sigma_L)$, q^2 is the four-momentum transfer squared, $\alpha = e^2/4\pi$, x is the quantity in (2.7), while y is the percentage energy loss of the incoming electron. The second formula is for a quantity related to the parity-violating asymmetry in the scattering of polarized light off an atom with proton number Z and neutron number N. This quantity is essentially the axial-vector part of the effective electron neutral current times the vector part of an appropriate hadron piece. Following the notation of Abbott and Barnett[11] we call this parameter "$g_A V_{\rm had}$" and find

$$g_A V_{\text{had}} = -\Lambda\left\{Z\left[\frac{1}{4} - x\left(1+\frac{k_2^2}{k_3^2}\right)+\frac{1}{2}\,\frac{k_2^2}{k_3^2}\right] - \frac{1}{4}\left(1-\frac{k_2^2}{k_3^2}\right)N\right\}. \quad (4.13)$$

V. INTERESTING SPECIAL CASES

The present model contains three Higgs vacuum values k_1, k_2, k_3 as opposed to only one in the usual SU(2) × U(1) model. Because of the constraint (3.4), this gives us two relevant free neutral-current parameters. Here we shall demonstrate the following:

(i) The limit where k_3 becomes very large is the usual SU(2) × U(1) theory. This result is known,[12] but is a helpful calibration.

(ii) The limit where k_1 becomes very small gives, *for any value* of the ratio k_2^2/k_3^2, the same neutrino neutral currents as the SU(2) × U(1) theory. Furthermore, even for fairly large values of k_2^2/k_3^2 the predicted polarized-electron–deuteron asymmetry parameter agrees with experiment as well as the SU(2) × U(1) prediction. There is the possibility of more disagreement with SU(2) × U(1) in the atomic-physics parity-violation prediction, but here the experimental statement is not so clear.

(iii) The limit where k_2 becomes very small gives, for any value of the ratio k_1^2/k_3^2, the same electron neutral currents as the SU(2) × U(1) theory. However, the neutrino neutral currents differ here so this case is not so reasonable experimentally.

First consider case (i) where k_3 becomes very large. Formally we shall take $k_3 \to \infty$. From the mass formulas (3.2) we see that the masses of the "K-meson-type" W bosons become infinite and hence the second-order processes mediated by their exchange vanish. From (3.3) it can be seen that $m^2(Z^{(1)}) \to \infty$ while

$$m^2(Z^{(2)}) \simeq \frac{m_Z^2 m_H^2 - p^2}{m_Z^2 + m_H^2} \simeq \frac{8e^2(k_1^2+k_2^2)}{3c^2(3s^2+1)} = \frac{\sqrt{2}e^2}{8G_F x(1-x)}, \quad (5.1)$$

where (3.4) and (2.7) were used in the last step. This formula is seen to agree with the one for $m^2(Z)$ in SU(2) × U(1). Thus W_1^2, W_2^1, and $Z^{(2)}$ as well as the photon are the only vector bosons with finite mass. Furthermore, the neutral-current effective Lagrangian (4.6) becomes a degenerate quadratic form showing that in this limit neutral-current processes are effectively mediated by a single massive neutral boson; explicitly,

$$\mathcal{L}_{\text{eff}} \to \frac{3s^2 G_F}{\sqrt{2}g^2}\left(J_\alpha^Z - \frac{1}{\sqrt{3}s}J_\alpha^H\right)^2, \quad (5.2)$$

which, using (4.8), is seen to agree with (4.1).

To demonstrate (iii) above merely note that as $k_2 \to 0$, $\Lambda \to 1$ [see (4.10)] and (4.11) → (4.5).

We shall discuss (ii) above in more detail. It is most interesting, but of course not necessary, to consider a situation where the spectrum of vector mesons is very different from the SU(2) × U(1) case. Namely, we would like to consider all eight of them (excepting the photon) to have masses of roughly the same order. Thus we may consider

$$R \equiv \frac{k_2^2}{k_3^2} \quad (5.3)$$

to be around $\frac{1}{4}$ or so. Plots of vector-meson masses versus $1/R$, based on Sec. III, are displayed in Fig. 1. Although the formal limit $k_1 \to 0$ will be taken, this should be regarded as an approximation to k_1 small. A lower bound on k_1 can be obtained by holding the Yukawa term responsible for the mass of the bottom quark m_b and requiring the appropriate dimensionless coupling constant to be less than unity. This gives

$$\left(\frac{k_1}{k_2}\right)^2 > 2\sqrt{2}G_F m_b^2 \approx 10^{-3}. \quad (5.4)$$

To see that the neutrino neutral-current interactions are the same as in the SU(2) × U(1) theory note that as $k_1 \to 0$, $\Lambda \to 1$ and (4.9) → (4.4). This already means that the present model agrees with the majority of neutral-current experiments. Our prediction for the electron-deuteron asymmetry is given in (4.12). Note for comparison that $k_3 \to \infty$ gives the SU(2) × U(1) prediction. Because of the $(1-4x)$ factor the second term is negligible. Then

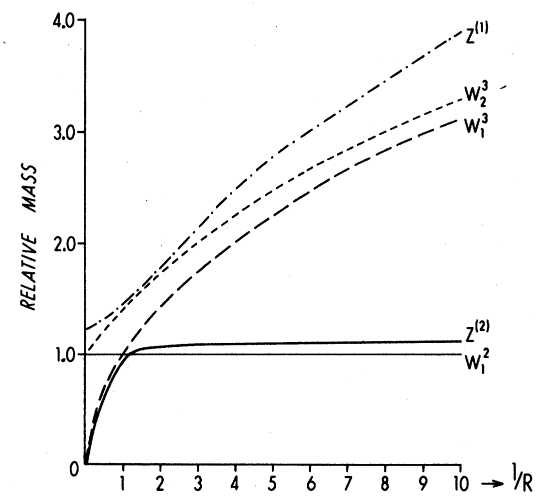

FIG. 1. Plots of gauge-boson masses relative to $m(W_1^2)$ [≅ 80 GeV] versus $1/R$ in the $k_1 \to 0$ limit.

we see (in the $k_1 \to 0$ limit)

$$\frac{(A/q^2)[\mathrm{SU}(3)\times\mathrm{U}(1)]}{(A/q^2)[\mathrm{SU}(2)\times\mathrm{U}(1)]}$$

$$\cong \frac{1+(2-\frac{20}{3}x)(1+R)}{3-\frac{20}{3}x} = 1+0.32R\,, \quad (5.5)$$

where we have taken[2] $x=0.23$. Thus, for R as large as $\frac{1}{3}$ the two theories give predictions which differ by 10%. Since the experimental accuracy[2] at present is about 20%, the two theories cannot be distinguished on this basis. It is interesting that the predictions for the atomic-physics parity-violation experiments distinguish more sharply between the two models. The $\mathrm{SU}(3)\times\mathrm{U}(1)$ prediction for small k_1 is given by (4.13) with $\Lambda\to 1$, while the $\mathrm{SU}(2)\times\mathrm{U}(1)$ prediction is given by the same formula wherein $\Lambda\to 1$ and $k_3\to\infty$. Thus we have

$$\frac{g_A V_{\mathrm{had}}[\mathrm{SU}(3)\times\mathrm{U}(1)]}{g_A V_{\mathrm{had}}[\mathrm{SU}(2)\times\mathrm{U}(1)]} = 1+R\,\frac{[(-x+\frac{1}{2})Z+\frac{1}{4}N]}{[(-x+\frac{1}{4})Z-\frac{1}{4}N]}\,. \quad (5.6)$$

For Bismuth, this becomes

$$1-1.81\,R\,, \quad (5.7)$$

which should be contrasted with (5.5). The effect of the present model is, since R is positive definite, to lower the amount of predicted parity violation. To fit the Seattle experiments[2] would require R to be about 0.55, while to agree with the Novosibirsk and Oxford experiments[2] would require R to be less than about 0.15. Of course, the $R=0$ limit is essentially just the $\mathrm{SU}(2)\times\mathrm{U}(1)$ theory at "low" energies.

Thus, the present model [$\mathrm{SU}(3)\times\mathrm{U}(1)$ with small k_1] agrees closely with all the well established neutral-current experiments even for reasonably large values of R. For the atomic-physics data, which may be considered controversial, this model provides, as a sensitive function of R, an interpolation between the $\mathrm{SU}(2)\times\mathrm{U}(1)$ prediction and the possibility of no atomic parity violation. From the present point of view the interest in the results of the atomic-physics experiments is very much enhanced. In any event it is always possible to imagine R small. Then one has the usual $\mathrm{SU}(2)\times\mathrm{U}(1)$ theory effectively at energies less than several hundred GeV while a more complicated $\mathrm{SU}(3)\times\mathrm{U}(1)$ interaction pattern emerges at higher (say thousands of GeV) energies. This might be a way of creating oases in Glashow's[13] "desert."

VI. ADDITIONAL DISCUSSION

The main purpose of the present paper has been to show that the neutral-current nonuniqueness theorems can be extended to gauge groups of the form $G_L\times\mathrm{U}(1)$. We have given an explicit illustration taking $G=\mathrm{SU}(3)$. The gauge theories based on $\mathrm{SU}(3)_L\times\mathrm{U}(1)$ are of course very much more complicated than the $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ theories and permit a large number of different variations by taking different Higgs structures, different assumed parameter ranges, different discrete symmetries, etc. Many of those possibilities have been discussed in detail in the literature.[4] Here we shall briefly mention some of the (other than neutral-current) characteristic features of the present model. These features could most likely be modified without materially changing the neutral-current predictions, by imposing further symmetries, but we shall not consider this here.

First consider the leptons, whose left-handed components are given in (2.8). All but ν_e, ν_μ, and ν_τ have right-handed singlets also. We assume separate mixing in the sets (ν_e,ν_μ,ν_τ) and $(\nu_e',\nu_\mu',\nu_\tau')$. Then the masslessness of (ν_e,ν_μ,ν_τ) implies that choosing the Kobayashi Maskawa[14] (KM) mixing matrix to be the unit matrix for W_1^2 interactions, there will be nontrivial and equal KM matrices for the W_1^3 and W_2^3 interactions. W_1^3 emission and reabsorption will mediate the exotic process[15] $\mu\to e\gamma$. This will vanish in the limits when either $m^2(W_1^3)\to\infty$ [$\mathrm{SU}(2)\times\mathrm{U}(1)$ limit] or when the KM matrix for W_1^3 interactions goes to the unit matrix. The latter mechanism is indicated as a suppression mechanism for the large-R limit of the theory. We have furthermore assumed $(\nu_e',\nu_\mu',\nu_\tau')$ to be very heavy. This will save the embarrassment of not yet having seen the lightest of these which should be stable.

Next consider the quarks, whose left-handed components are given in (2.9) and (2.10). We assume separate mixings in the sets (d,s,b) and (d',s'). Furthermore, for simplicity, we assume mixings between primed and unprimed quarks to be negligible. The characteristic feature here is that because there are two triplets and one antitriplet, there exists a "metric tensor"

$$\eta=\mathrm{diag}(1,1,-1)$$

in generation space. This results in some strangeness-changing neutral currents. To see this, note that if the physical d, s, and b quarks are assembled into a column vector D we will have

$$J_\alpha^{(1)} = i\bar{D}_L\gamma_\alpha\Omega^\dagger\eta\Omega D_L+\cdots\,,$$

where Ω is a 3×3 unitary matrix. The 12 matrix element of $(\Omega^\dagger \eta \Omega)$ controls the amplitude for $K_L \to \mu\bar{\mu}$, etc. Substituting into (4.11) gives, for example,

$$\mathcal{L}_{eff}(\Delta S=1)=\frac{-G_F\Lambda}{\sqrt{2}}\left(\frac{k_2}{k_3}\right)^2 \Omega_{32}^*\Omega_{31}\bar{\mu}\gamma_\alpha \times[(\tfrac{1}{2}-2x)+\tfrac{1}{2}\gamma_5]\mu\bar{s}(1+\gamma_5)d+\text{H.c.}$$

Thus we must suppress $K_L \to \mu\bar{\mu}$ by having $\mathrm{Re}(\Omega_{32}^*\Omega_{31})$ very small.

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